# On the Design of Soft-Decision Fusion Rule for Coding Approach in Wireless Sensor Networks ⋆


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**Abstract.** In this work, two soft-decision fusion rules, which are respectively named the maximum *a priori* (MAP) and the suboptimal minimum Euclidean distance (MED) fusion rules, are designed based on a given employed sensor code and associated local classification. Their performance comparison with the distributed classification fusion using soft-decision decoding (DCSD) proposed in an earlier work is also performed. Simulations show that when the number of faulty sensors is small, the MAP fusion rule remains the best at either low sensor observation signal-to-noise ratios (OSNRs) or low communication channel signal-to-noise ratios (CSNRs), and yet, the DCSD fusion rule gives the best performance at middle to high OSNRs and high CSNRs. However, when the number of faulty sensor nodes grows large, the least complex MED fusion rule outperforms the MAP fusion rule at high OSNRs and high CSNRs.


## 1 Introduction

One of the general emerging visions for future applications is to deploy a large number of self-sustained wireless sensors to perform, e.g., environmental monitoring, battle field surveillance and health care maintenance. These wireless sensor nodes are typically battery-powered and made by economical techniques, and hence are vulnerable if they are employed in a harsh environment [1]. This makes energy efficiency and fault-tolerance capability becoming critical design factors in wireless sensor networks (WSNs). Another factor that distinguishes a WSN from other communication networks is that its end goal is to draw a


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discrete decision out of several possible events of interest, but not to convey information. For these reasons, research on collaborative signal processing, and in particular, collaborative detection and classification has been studied extensively in WSNs [2–5].

In order to achieve the desired robustness against sensor faults under limited energy support, a distributed classification fusion using error correcting codes (DCFECC) has been proposed to be used in WSNs [6]. In the DCFECC approach, the fusion center makes multi-hypotheses decision by receiving only one-bit information from each sensor to minimize the local energy consumption. As contrary to the hard decision decoding used in the DCFECC approach, a soft-decision DCSD approach was later proposed in [7]. It is suggested by the investigation on the DCSD approach in [7] that employing soft-decision can markedly enhance the fault-tolerance capability of the same code in WSNs. This motivates our further investigation on the design of soft-decision-based fusion rules in this work.

Three soft-decision fusion rules are investigated in this paper: The maximum *a priori* (MAP) fusion rule, the minimum Euclidean distance (MED) fusion rule, and the previously proposed DCSD fusion rule. It is obvious that the MAP fusion rule provides the best classification performance if no sensor nodes are faulty. However, when some faulty sensors do not follow the local classification rules that are mutually pre-agreed between the fusion center and the local sensors, the MAP fusion rule is expected to degrade considerably since among the three soft-decision fusion rules considered, it is the one that mostly trusts the local classification. Therefore, the DCSD and the MED, although suboptimal in performance at a fault-free situation, may be more robust, if several sensor faults are present. Our simulations do match our anticipation. Details will be given subsequently.

The paper is organized as follows. The distributed classification problem is described in the next section. The MAP and the MED fusion rules, as well as the DCSD fusion rule, are introduced in Section 3. Simulations on these soft-decision fusion rules are presented and remarked in Section 4. Conclusion is given in Section 5

## 2 System Model

Fig. 1 depicts a parallel fusion structure in which a number of sensors respectively make sensor measurements $\boldsymbol{z} = \{z_j\}_{j=1}^N$ given that one of the $M$ hypotheses is true, where $N$ is the number of sensors. The sensor measurement $\{z_j\}_{j=1}^N$ are conditionally independent given each hypothesis. Each sensor makes a preliminary decision $\boldsymbol{x} = \{x_j\}_{j=1}^N$, where $x_j \in \{-1, 1\}$, about the true hypothesis uncooperatively according to a pre-specified local classification rule, and sends the result to the fusion center. The received vector $\boldsymbol{y} = \{y_j\}_{j=1}^N$ may be subject to transmission errors due to the incorporation of link fading and interference. It is assumed that given $\{x_j\}_{j=1}^N$, $\{y_j\}_{j=1}^N$ are independent across sensors given each hypothesis. Also assume equal prior on the $M$ hypotheses. Denote by $h_{\ell|i}^{(j)}$ the probability of classifying measurement $z_j$ to $H_\ell$ given that the true hypothesis is $H_i$.

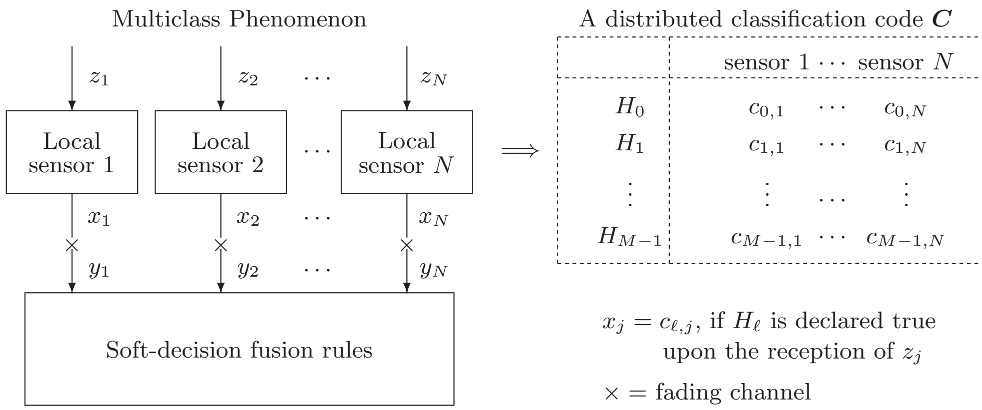


**Fig. 1.** System model for a WSN with distributed classification code.

In the coded distributed detection system considered, a $M \times N$ binary distributed classification code $\boldsymbol{C}$ is designed in advance. This code can be obtained based on the misclassification error criterion as used in [6] or by the efficient code search algorithm proposed in [8, 9]. In the code matrix, each row is associated with one hypothesis, and forms the codeword corresponding to this hypothesis. Specifically, the $\ell$th codeword in $\boldsymbol{C}$ is given by $\boldsymbol{c}_\ell \triangleq (c_{\ell,1}, c_{\ell,2}, \ldots, c_{\ell,N})$, where $c_{\ell,j} \in \{0, 1\}$. On the other hand, the column vector in $\boldsymbol{C}$ provides the local binary output according to the classified hypothesis at the respective sensor. Thus, if the $j$th sensor makes a local classification in favor of hypothesis $H_\ell$, it will transmit a binary decision whose value equals $(-1)^{c_{\ell,j}}$. As a result of the above setting,

$$\Pr\{x_j = -1 | H_i\} = \sum_{\ell=0}^{M-1} c_{\ell,j} h_{\ell|i}^{(j)}, \tag{1}$$

and

$$\Pr\{x_j = 1 | H_i\} = 1 - \Pr\{x_j = -1 | H_i\} = \sum_{\ell=0}^{M-1} (1 - c_{\ell,j}) h_{\ell|i}^{(j)}. \tag{2}$$

The communication channel between sensors and fusion center is assumed flat fading due to the assumption of very low bit rate. Perfect phase coherence is also assumed since the transmission range is usually small in most WSNs. Therefore, $y_j$ can be expressed as

$$y_j = \alpha_j x_j \sqrt{E_b} + n_j, \tag{3}$$

where $\alpha_j$ is the attenuation factor that models the fading channel, $E_b$ is the energy per channel bit, and $n_j$ is a noise sample from a white Gaussian process with single-sided power spectral density $N_0$. Our objective then becomes to investigate the robustness of the fusion rules given the local classification rules associated with the employed code $\boldsymbol{C}$.

# 3 Soft-decision fusion rules

## 3.1 MAP fusion rule

The MAP fusion rule makes the decision in favor of $H_i$ if $\Pr(H_i|\boldsymbol{y})$ is maximal for $0 \leq i \leq M-1$. It can be derived as

$$
\begin{aligned}
i &= \arg \max_{0\leq\ell\leq M-1} \Pr(H_\ell|\boldsymbol{y}) \\
&= \arg \max_{0\leq\ell\leq M-1} \Pr(\boldsymbol{y}|H_\ell) && (4)\\
&= \arg \max_{0\leq\ell\leq M-1} \sum_{\boldsymbol{x}\in\{-1,1\}^N} \Pr(\boldsymbol{x},\boldsymbol{y}|H_\ell) \\
&= \arg \max_{0\leq\ell\leq M-1} \sum_{\boldsymbol{x}\in\{-1,1\}^N} \Pr(\boldsymbol{x}|H_\ell)\Pr(\boldsymbol{y}|\boldsymbol{x},H_\ell) \\
&= \arg \max_{0\leq\ell\leq M-1} \sum_{\boldsymbol{x}\in\{-1,1\}^N} \Pr(\boldsymbol{x}|H_\ell)\Pr(\boldsymbol{y}|\boldsymbol{x}) \\
&= \arg \max_{0\leq\ell\leq M-1} \sum_{\boldsymbol{x}\in\{-1,1\}^N} \left(\prod_{j=1}^{N} \Pr(x_j|H_\ell)\right)\Pr(\boldsymbol{y}|\boldsymbol{x}) && (5)\\
&= \arg \max_{0\leq\ell\leq M-1} \sum_{\boldsymbol{x}\in\{-1,1\}^N} \left(\prod_{j=1}^{N} \Pr(x_j|H_\ell)\Pr(y_j|x_j)\right) && (6)\\
&= \arg \max_{0\leq\ell\leq M-1} \sum_{j=1}^{N} \log\left(\sum_{x_j\in\{-1,1\}} \Pr(x_j|H_\ell)\Pr(y_j|x_j)\right) && (7)
\end{aligned}
$$

where (4) follows from the assumption of equally likely hypotheses, (5) is valid since the local measurements are assumed spatially conditionally independent given each hypothesis and $x_j$ is determined uncooperatively across sensors, and (6) follows from the assumption of spatially independent communication channel statistics between local sensors and the fusion center. Notably, $\Pr(x_j|H_\ell)$ and $\Pr(y_j|x_j)$ in (7) are given by (1), (2) and (3).

## 3.2 DCSD fusion rule

For a given binary code $\boldsymbol{C}$, the DCSD fusion rule proposed in [7] chooses $H_i$ as the final decision, if

$$
\begin{aligned}
i &= \arg \max_{0 \le \ell \le M-1} \sum_{j=1}^{N} \log \left( \sum_{x_j \in \{-1,1\}} g(x_j|s = c_{\ell,j}) \Pr(y_j|x_j) \right) \\
&= \arg \min_{0 \le \ell \le M-1} \sum_{j=1}^{N} (\phi_j - (-1)^{c_{\ell,j}})^2,
\end{aligned}
$$

where $\phi_j$ is the bit log-likelihood ratio defined as

$$
\phi_j \triangleq \log \frac{\sum_{x_j \in \{-1,1\}} \Pr(y_j|x_j) \cdot g(x_j|s=0)}{\sum_{x_j \in \{-1,1\}} \Pr(y_j|x_j) \cdot g(x_j|s=1)},
$$

and

$$
g(x_j|s) \equiv \frac{\sum_{\ell=0}^{M-1} \mathbf{1}\{c_{\ell,j} = s\} \cdot \Pr(x_j|H_\ell)}{\sum_{k=0}^{M-1} \mathbf{1}\{c_{k,j} = s\}},
$$

and $\mathbf{1}\{\cdot\}$ is the indicator function. Since the DCSD fusion rule is not equivalent to the MAP fusion rule, it is suboptimal when there are no faulty sensors.

## 3.3 MED fusion rule

The DCSD fusion rule can be treated as averaging $\Pr(x_j|H_\ell)$ with respect to the adopted code. The MED fusion rule however is originated from an observation that the local classification is in general accurate. This observation can be mathematically termed as $\Pr\{x_j = (-1)^{c_{\ell,j}}|H_\ell\} \gg \Pr\{x_j \neq (-1)^{c_{\ell,j}}|H_\ell\}$, which immediately implies the approximation that $\Pr\{x_j = (-1)^{c_{\ell,j}}|H_\ell\} \approx 1$ and $\Pr\{x_j \neq (-1)^{c_{\ell,j}}|H_\ell\} \approx 0$. Taking this approximation to (7), we obtain:

$$
\begin{aligned}
i &= \arg \max_{0 \le \ell \le M-1} \sum_{j=1}^{N} \log \left( \sum_{x_j \in \{-1,1\}} \Pr(x_j|H_\ell) \Pr(y_j|x_j) \right) \\
&\approx \arg \max_{0 \le \ell \le M-1} \sum_{j=1}^{N} \log \left[ \Pr(y_j|x_j = (-1)^{c_{\ell,j}}) \right] \\
&= \arg \min_{0 \le \ell \le M-1} \sum_{j=1}^{N} (\varphi_j - (-1)^{c_{\ell,j}})^2,
\end{aligned}
$$

where

$$
\varphi_j = \log \frac{\Pr(y_j|x_j = 1)}{\Pr(y_j|x_j = -1)}.
$$

## 4 Simulation on robustness

In this section, we study the performance of the three aforementioned fusion rules through simulations. Both fault-free (without stuck-at faults) and faulty situations (sensors in the presence of stuck-at faults) are simulated. The hypothesis number $M$ and the sensor number $N$ are four and ten, respectively. We further assume that all sensor measurements have the same distribution given each hypothesis, and are randomly drawn from a unit-variance Gaussian distribution with means 0, $V$, $2V$ and $3V$ corresponding to hypotheses $H_0$, $H_1$, $H_2$ and $H_3$, respectively. Throughout this section, OSNR is defined as $20\log_{10}(V)$, while CSNR is given by $E_b/N_0 \times E[\alpha_j^2]$. Moreover, attenuation factor $\alpha_j$ is assumed to be Rayleigh distributed.

The code employed in this simulation is obtained by the *pruned exhaustive search algorithm* for the code with minimum decision error, which is listed in Table 1 [8]. It can be easily verified that the minimum pair-wise Hamming distance in this code is 5.

**Table 1.** The code obtained by the pruned exhaustive search algorithm.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $H_0$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $H_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $H_2$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| $H_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

In our simulations, $10^5$ Monte Carlo runs are performed for each OSNR and CSNR. The faulty sensors are uniformly drawn from the ten deployed sensor nodes, and always send one regardless of the local measurements.

Figures 2 and 3 summarize the performance of the three fusion rules at CSNR $=$ 5 dB. From Fig. 2, we observe that the MAP fusion rule has the best performance among all three rules at fault-free situation as anticipated. In addition, the DCSD fusion rule outperforms the MED fusion rule when no sensors are faulty. This can be justified by the fact that the MED fusion rule is simplified from the MAP fusion by making a "hard" assumption that the local classification is 100% accurate, while the DCSD "softly" approximates the MAP by replacing $\Pr(x_j|H_\ell)$ by its average counterpart $g(x_j|s = c_{\ell,j})$. From Fig. 3, we notice that when one faulty sensor is present, the DCSD fusion rule becomes the best at high OSNRs. The least complex MED fusion rule remains the worst among the three. Hence, we remark that at high OSNRs, the DCSD replacement $g(x_j|s = c_{\ell,j})$ is sufficient to compensate the impact due to the faulty sensor. When two faulty nodes are present, the robustness of the DCSD fusion rule extends to middle to high OSNRs.

Figure 4 presents the simulated performance when the number of faulty nodes further increases to 3. It can be seen that the least complex MED fusion rule becomes better than the MAP fusion rule at high OSNRs. This figure also shows that the DCSD fusion rule still possesses the best fault-tolerance capability at most simulated OSNRs.

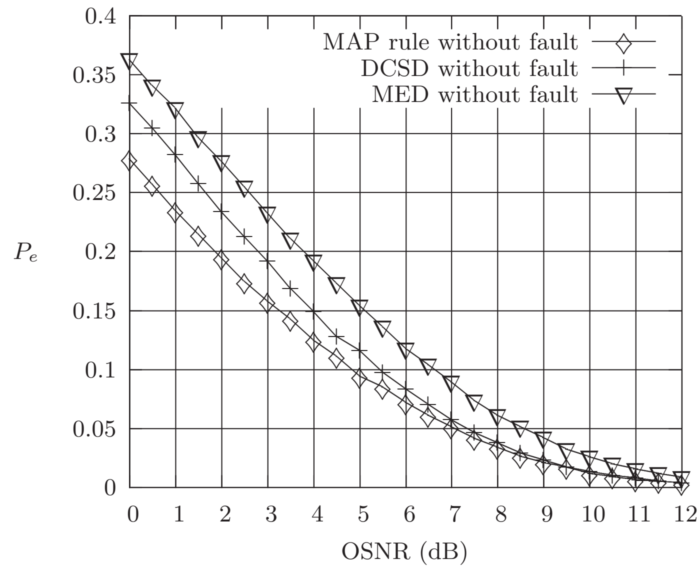


**Fig. 2.** Performance of the MAP rule, the DCSD rule, and the MED rule at CSNR=5 dB in fault-free situation.

Repeating the simulations with fault-free case, two faulty nodes, and three faulty nodes as in Figs. 2, 3 and 4 but fixing CSNR at 0 dB, we result Figs. 5 and 6. Figures 5 and 6 indicate that the DCSD fusion rule still provides the best performance at faulty situation at high OSNRs, but the OSNR range at which the DCSD performs the best decreases.

Figures 7 and 8 presents the simulation results corresponding to the situation when OSNR is fixed at 5 dB, and CSNR ranges from $-10$ dB to 10 dB. We observe from these two figures that the MAP fusion rule provides the best performance at low CSNRs either in the anticipated fault-free situation or in the sensor-faulty situation. The DCSD approach however performs the best at high CSNRs.

## 5 Conclusion

In this paper, we introduce two soft-decision fusion rules, and compare their robustness with the previously proposed DCSD fusion by simulations. We conclude our simulations that i) the MAP fusion rule gives the best performance in

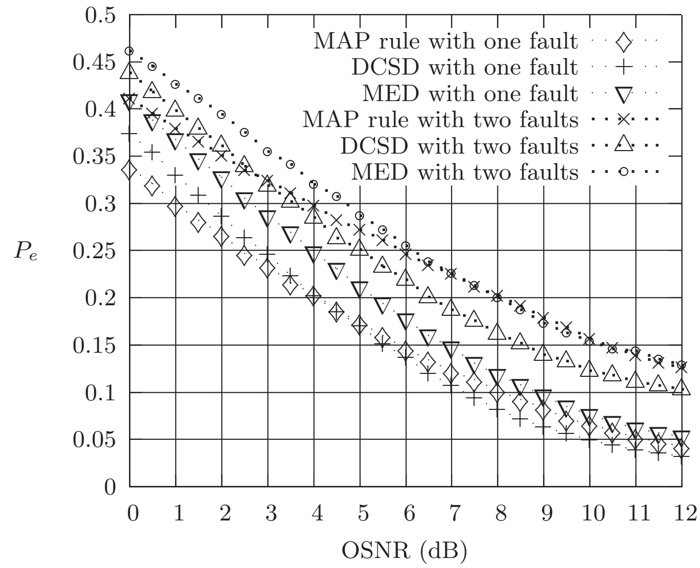


**Fig. 3.** Performance of the MAP rule, the DCSD rule, and the MED rule at CSNR=5 dB when one or two sensors suffer stuck-at fault.

fault-free situation as well as at low OSNRs or low CSNRs; ii) the DCSD fusion rule has a better fault-tolerance capability at middle to high OSNRs and at high CSNRs; iii) the MED fusion rule, although least complex, can perform better than the MAP fusion rule only when the number of faulty nodes is large, but is always worse than the DCSD fusion rule. These results can serve as a guide when the determination of suitable fusion rules for coding approach in wireless sensor networks is necessary.

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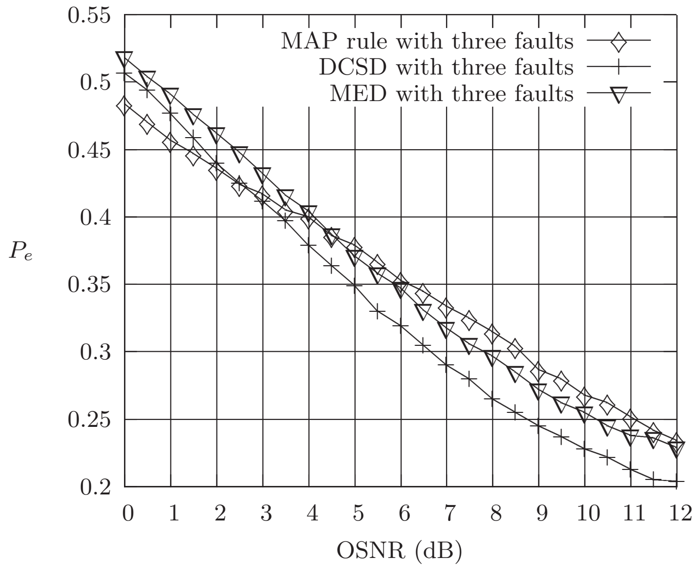


**Fig. 4.** Performance of the MAP rule, the DCSD rule, and the MED rule at CSNR=5 dB when three sensors are faulty.


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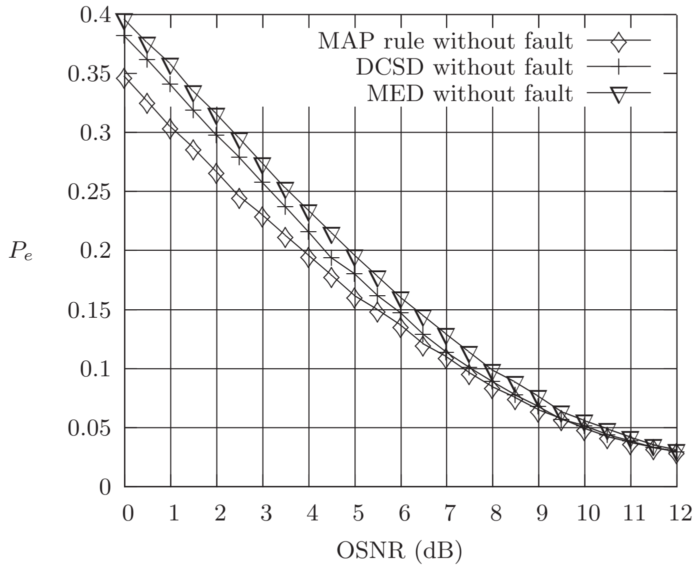


**Fig. 5.** Performance of the MAP rule, the DCSD rule, and the MED rule at CSNR=0 dB in fault-free situation.

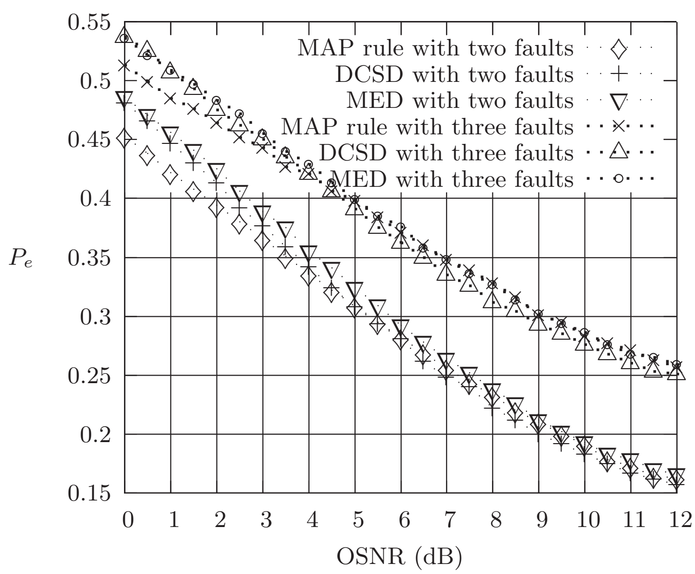


**Fig. 6.** Performance of the MAP rule, the DCSD rule, and the MED rule at CSNR=0 dB when two or three sensors are faulty.

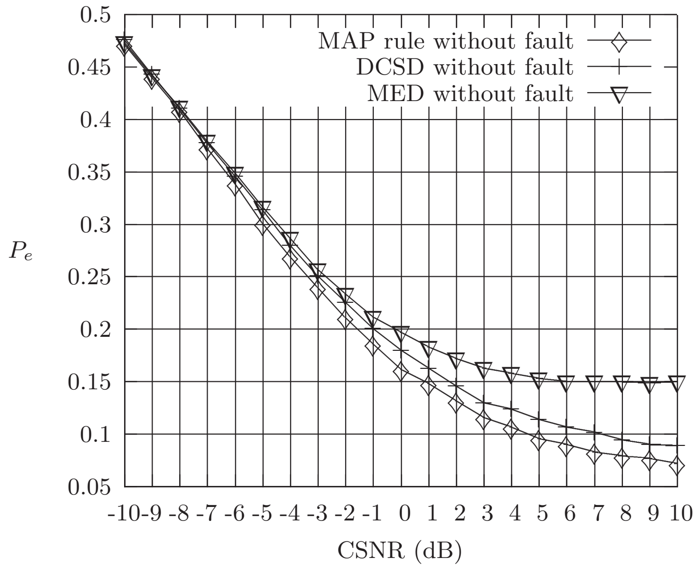


**Fig. 7.** Performance of the MAP rule, the DCSD rule, and the MED rule at OSNR=5 dB in fault-free situation.

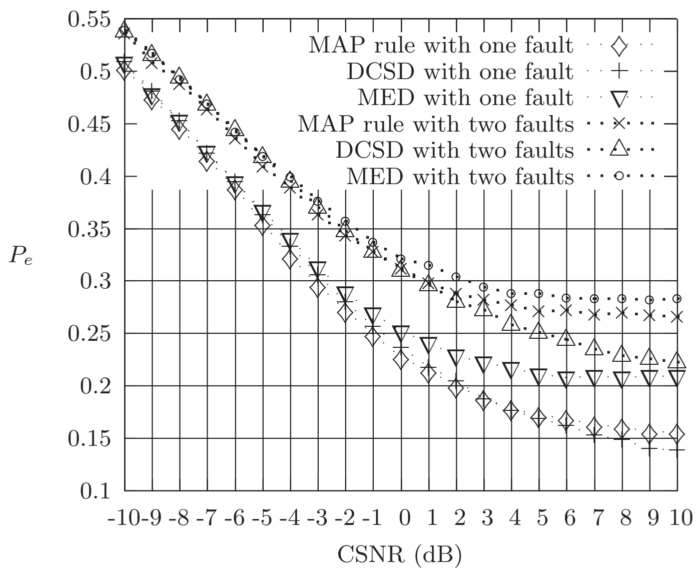


**Fig. 8.** Performance of the MAP rule, the DCSD rule, and the MED rule at OSNR=5 dB when one or two sensors are faulty.