# Analyzing Split Channel Medium Access Control Schemes with ALOHA Reservation ⋆

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**Abstract.** In order to improve the throughput performance of Medium Access Control (MAC) schemes in wireless communication networks, some researchers proposed to split the single shared channel into two subchannels: a control subchannel and a data subchannel. The control subchannel is used for access reservation to the data subchannel over which the data packets are transmitted, and such reservation can be done through the use of the dialogues such as RTS/CTS (Ready-To-Send/Clear-To-Send) dialogue. In this paper, we evaluate the maximum achievable throughput of split-channel MAC schemes that are based on RTS/CTS dialogues with pure ALOHA contention resolution mechanism. We derive and calculate numerically the probability density function (pdf) of the contention resolution periods on the control subchannel. We then apply these results to calculate the throughput of the split-channel MAC schemes, which we then compare with the performance of the corresponding single-channel MAC schemes. Our results show that, when radio propagation delays are negligible, the maximum achievable throughput of the split-channel MAC schemes is lower than that of the corresponding single-channel MAC schemes in the scenarios that we have studied. Consequently, our results suggest that splitting the single shared channel of the MAC scheme in a wireless network should be avoided. Simulation results are presented to support our analytical results.

## 1 Introduction

In wireless communication networks, Medium Access Control (MAC) schemes are used to control the access of active nodes to the shared channel [1]. As the

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throughput of the MAC schemes may significantly affect the overall performance of the wireless networks, some researchers proposed to split, either in time or in frequency, the single shared channel into two subchannels: a control subchannel and a data subchannel. The control subchannel is used for reservation of access to the data subchannel over which the data packets are transmitted, and such reservation can be done through the use of the RTS/CTS (Ready-To-Send/Clear-To-Send) dialogue. Examples of such split-channel MAC schemes can be found in [2], [3], [4], [5], [6], and [7].

In this paper, we analyze the performance of a generic split-channel MAC scheme, which is based on the RTS/CTS dialogue and with pure ALOHA [8] contention resolution on the control subchannel. A ready node sends an RTS packet on the control subchannel to reserve the use of the data subchannel. When the RTS packet is received, the intended receiver replies with a CTS packet to acknowledge the successful reservation of the data subchannel [9] [10].

Based on the previous work [11], we calculate the probability density function (pdf) of the contention resolution periods on the control subchannel. This pdf is then used to calculate the expected waiting time on the data subchannel and the throughput of the split-channel MAC schemes. We determine the maximum achievable throughput of the split-channel MAC scheme as a function of the ratio of the bandwidths of the control subchannel and the entire channel and compare the result to that of the corresponding single-channel MAC schemes. We show that, when pure ALOHA technique is used for contention resolution on the control subchannel and radio propagation delays are negligible, the throughput of the split-channel MAC schemes is inferior to that of the single-channel MAC schemes.

For notational convenience, we term single-channel MAC scheme as MAC-1 and split-channel MAC scheme as MAC-2. We further define MAC-2R as MAC-2 with parallel reservations; i.e., in the MAC-2R scheme, contention resolutions take place on the control subchannel in parallel with the transmission of data packets on the data subchannel.

The paper is organized as follows: Section 2 summarizes the related work. In Section 3, we present our main comparison results of comparing the MAC-1, the MAC-2, and the MAC-2R schemes. In Section 4, our numerical and simulation results are derived. We then conclude this work in Section 5.

## 2 Related Work

A dynamic reservation technique called split-channel reservation multiple access (SRMA) was introduced for packet switching radio channels in [2]. In SRMA, the available bandwidth was divided into three channels: two used to transmit control information and, one used for message transmission. Message delay of SRMA was studied in that paper and it was shown that SRMA out-performs other MAC schemes under some network settings.

Split channel MAC scheme was compared with single channel MAC scheme in [12]. The authors categorized "scheduling epochs," the periods of time needed to schedule the next data transmission, into two groups: bandwidth-dependent component (e.g., contention resolution of reservation packets) and bandwidth-independent component (e.g., radio propagation delay). It was found that, if a system has no bandwidth-independent component in its scheduling epochs, the split-channel schemes may achieve the same performance as the single-channel schemes do. However, the analysis in that paper considered the average contention resolution period only, rather than the random distribution of these periods.

Similarly, [5] compared the performance of the single-channel MAC schemes and that of the split-channel MAC schemes by considering only the expected value of the contention resolution periods. In [5] and [6], the authors further proposed to use partial pipelining technique to solve the problem of unbalanced separation of the control channel and the data channel. This approach is similar to the generalized MAC-2R scheme, even though busy signals but not RTS/CTS dialogues are transmitted on the control subchannel.

In [11], the authors studied the contention resolution period of the pure ALOHA channel and the CSMA channel. They derived the Laplace transform of the pdf of the contention resolution periods of the two channels. The expected value and the variance of the resolution periods were calculated. Our work differs from [11], in that we study the throughput of the split-channel MAC schemes and compare it to that of the single-channel MAC schemes. We analyze the contention resolution periods numerically and use these results to determine the maximum achievable throughput of the split-channel MAC schemes.

In [3], RTS/CTS dialogue packets are transmitted on a separate signaling (control) channel. The protocol conserves battery power at nodes that are not actively transmitting or receiving packets by intelligently powering them off. A Power Controlled Dual Channel (PCDC) scheme for wireless ad hoc networks was proposed in [7]. By transmitting RTS/CTS dialogues on the control channel with maximum power and data packets on the main channel with adjustable (lower) power, interference-limited simultaneous transmission can take place in the neighborhood of a receiving node. However, these studies used separate channels mainly to achieve energy efficiency and low interference between neighboring transmissions in multi-hop networks.

## 3 Throughput Comparisons

### 3.1 Assumptions and Notations

In order to compare the throughput of the MAC-1, the MAC-2, and the MAC-2R schemes, we make the following assumptions. The wireless communication network we study is assumed to be fully-connected, i.e., all nodes are in the transmission range of each other. We also assume that the packet processing delays and the radio propagation delays are negligible and that the traffic generated by active nodes (including retransmissions) is Poisson with rate $\lambda$.

We establish the following notation:

- $L_c$, $L_d$: the length of a control packet and that of a data packet, respectively
- $k$: the ratio of data packet length to the control packet length; i.e., $k = \frac{L_d}{L_c}$
- $R$, $R_c$, and $R_d$: the data rate of the entire shared channel, the control subchannel, and the data subchannel, respectively; i.e., $R = R_c + R_d$
- $r$: the ratio of the data rate of the control subchannel to the data rate of the entire channel in the MAC-2 and the MAC-2R schemes; i.e., $r = \frac{R_c}{R} = \frac{R_c}{R_c+R_d}$
- $\gamma_1$, $\delta_1$: the transmission time of a control packet and the transmission time of a data packet in the MAC-1 scheme, respectively; i.e., $\gamma_1 = \frac{L_c}{R}$ and $\delta_1 = \frac{L_d}{R} = k\gamma_1$
- $\gamma_2$, $\delta_2$: the transmission time of a control packet and the transmission time of a data packet, respectively, in the MAC-2 or the MAC-2R schemes; i.e., $\gamma_2 = \frac{L_c}{R_c} = \frac{\gamma_1}{r}$ and $\delta_2 = \frac{L_d}{R_d} = \frac{k\gamma_1}{1-r}$
- $\delta$: normalized data packet transmission time in the MAC-2 and the MAC-2R schemes; i.e., $\delta = \frac{\delta_2}{\gamma_2} = \frac{kr}{1-r}$.

### 3.2 Comparing the throughput of the MAC-1 and the MAC-2 schemes

Fig. 1 depicts an example of the operations of the MAC-1, the MAC-2, and the MAC-2R schemes. We treat the packet transmission on the channel as a renewal process. To send a data packet successfully, two control packets and a data packet need to be transmitted on the shared channel after the contention resolution period, which is the time between the end of the previous successful data transmission and the beginning of current successful RTS/CTS dialogue. According to [11], the expected value of the normalized contention resolution period in ALOHA channels ($\overline{w}$) is a constant, when normalized Poisson traffic arrival rate is fixed. In the MAC-1 scheme, the expected time of a data packet transmission cycle is:

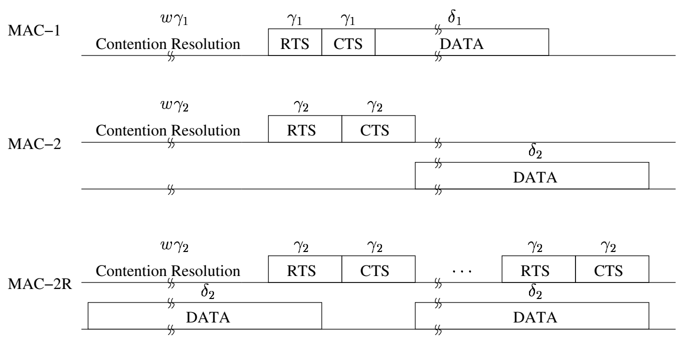


**Fig. 1.** Comparison of MAC-1, MAC-2, and MAC-2R

$$\overline{t_1} = \overline{w}\gamma_1 + 2\gamma_1 + \delta_1 = (\overline{w} + 2 + k) \cdot \gamma_1 \quad .$$

Thus, according to the property of renewal processes, the throughput of the MAC-1 scheme can be expressed as

$$S_1 = \frac{\delta_1}{\overline{t_1}} = \frac{k\gamma_1}{\overline{t_1}} = \frac{k}{\overline{w} + 2 + k} \quad . \tag{1}$$

In the MAC-2 scheme, the available bandwidth is split into two subchannels. Channel requests can only be transmitted after the current data transmission ends. The throughput of the MAC-2 scheme is a function of $r$. The expected time of a renewal cycle is

$$\overline{t_2} = \overline{w} \cdot \gamma_2 + 2 \cdot \gamma_2 + \delta_2 = \left( \frac{\overline{w} + 2}{r} + \frac{k}{1 - r} \right) \cdot \gamma_1 \quad .$$

Therefore, the throughput of the MAC-2 scheme is

$$S_2(r) = \frac{\delta_2}{\overline{t_2}} \cdot (1 - r) = \frac{k}{\frac{\overline{w}+2}{r} + \frac{k}{1-r}} \quad , \tag{2}$$

where the term $1 - r$ in the first equation represents the portion of the entire available bandwidth of the shared channel that is used as the data subchannel. Comparing (1) and (2), we conclude that $S_2(r) < S_1$ when $0 < r < 1$. Thus, the throughput of the MAC-2 scheme is always lower than that of the MAC-1 scheme.

### 3.3 Calculating the throughput of the MAC-2R scheme

For notational convenience, we normalize all variables with respect to $\gamma_2$ when calculating the throughput of the MAC-2R scheme, e.g., the data packet transmission time is now $\delta = \frac{\delta_2}{\gamma_2} = \frac{kr}{1-r}$.

As opposed to the operation of the MAC-2 scheme, in the MAC-2R scheme, contention resolutions take place on the control subchannel in parallel with the transmission of data packets on the data subchannel. A contention resolution period ($W$) begins on the control subchannel when the transmission of the data packet (for which the data subchannel was reserved in the previous reservation epoch) starts on the data subchannel. The contention period lasts until there is a successful RTS/CTS dialogue (see Fig. 2):

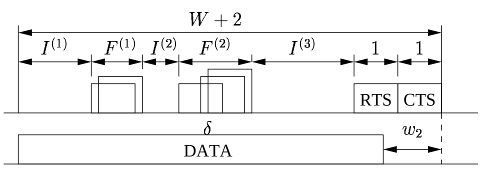


**Fig. 2.** An example of the contention resolution period in MAC-2R

$$W = \sum_{i=1}^{K-1} \left[ I^{(i)} + F^{(i)} \right] + I^{(K)} \quad , \tag{3}$$

where $K$ is the number of busy periods[4] during the $W+2$ contention resolution periods, of which the last one is successful, $I^{(i)}$ is the $i$-th idle period, and $F^{(i)}$ is the $i$-th failed busy period, in which RTS packet collisions occur. In (3), the summation term includes $K-1$ failed busy periods and the idle periods leading them. The second term, $I^{(K)}$, represents the idle period leading the successful busy period.

Let $p_s$ be the probability that a successful RTS/CTS dialogue starts after an idle period on the control subchannel. The Laplace transform of the pdf of the contention resolution period $W$ is [11]

$$W^*(s) = \frac{p_s}{\frac{1}{I^*(s)} - (1-p_s)F^*(s)} \ , \tag{4}$$

where $W^*(s)$ is the Laplace transform of $g(w)$, the pdf of $W$, $I^*(s)$ is the Laplace transform of $i(t)$, the pdf of the individual idle periods, and $F^*(s)$ is the Laplace transform of $f(t)$, the pdf of the individual failed periods.[5]

Since the inter-arrival times of packet reservations for the control subchannel (newly generated and those scheduled for retransmission) are identical, independent, and exponentially distributed with mean $1/G$ in time units of $\gamma_2$, where $G = \lambda\gamma_2$, the Laplace transform of the channel idle time ($I$) is

$$I^*(s) = \frac{G}{G+s} \ .$$

The probability of a successful transmission of a packet after an idle period is given by

$$p_s = e^{-G} \ .$$

The duration of an unsuccessful transmission period $F$ is given in [11] as

$$F^*(s) = \frac{Ge^{-(s+G)}\left[1-e^{-(s+G)}\right]}{\left(1-e^{-G}\right)\left[s+Ge^{-(s+G)}\right]} \ .$$

Thus,

$$W^*(s) = \frac{Ge^{-G}\left[s+Ge^{-(s+G)}\right]}{s^2 + sG\left[1+e^{-(s+G)}\right] + G^2e^{-2(s+G)}} \tag{5}$$

and consequently,

$$\overline{w} = E[W] = -\left.\frac{\partial W^*(s)}{\partial s}\right|_{s=0} = \frac{1}{G}e^{2G} - 1 \ .$$

In the MAC-2R scheme, when the value of $W$ (say, $w$) satisfies $w+2 \leq \delta$, the RTS/CTS dialogue succeeds before the end of the current data packet transmission on the data subchannel. Thus, the next data packet transmission

[4] We denote those contention periods with a packet transmission on the control subchannel as busy periods.

[5] We assume that all pdfs exist.

can start immediately after the current one ends. However, when $w + 2 > \delta$ (as shown in Fig. 2), the data subchannel will be left idle for a period of time, which we define as the waiting time on data subchannel ($w_2$). The expected value of this waiting time ($\overline{w_2}$) can be calculated as

$$\overline{w_2} = \int_{\delta-2}^{\infty} [w - (\delta - 2)] \cdot g(w)\, dw \ . \tag{6}$$

Note that the above equation holds even when $\delta - 2 < 0$.

Therefore, the throughput of the MAC-2R scheme can be expressed as

$$S_{2R}(r) = \frac{\delta}{\delta + \overline{w_2}} \cdot (1 - r) = \frac{1}{\frac{1}{1-r} + \frac{\overline{w_2}}{kr}} \ . \tag{7}$$

Note that the control subchannel access scheme is ALOHA for RTS packets. To maximize the throughput of the control subchannel, the RTS packet arrival rate in unit time on the control subchannel, $G = \lambda\gamma_2$, should be 0.5. In this case, the delay from when the control subchannel becomes available for reservation until a successful RTS/CTS dialogue takes place is minimized [11]. Thus, this value of $G$ minimizes $w$.

Before we proceed to calculate $\overline{w_2}$, it is worthwhile to evaluate the throughput if we only consider the average delay of contention resolution on the control subchannel. In this case, the average time of each reservation cycle on the control subchannel is $E[W] + 2 = \overline{w} + 2$ and the time of each transmission cycle on the data subchannel is $\delta$. The optimal throughput of the MAC-2R scheme occurs when $\delta = \overline{w} + 2$; i.e., the data packets are placed back-to-back and there is no waiting time needed on the data subchannel for conclusion of the contention resolution on the control subchannel. Thus,

$$\delta = \frac{kr^*}{1 - r^*} = \overline{w} + 2 \ ,$$

and the optimal $r$, which we label as $r^*$, based on the expected value of contention resolution delay is

$$r^* = \frac{\overline{w} + 2}{k + \overline{w} + 2} \ . \tag{8}$$

However, by substituting $r^*$ into (7), we obtain that

$$S_{2R}(r^*) = \frac{k}{\overline{w} + 2 + k} \left( \frac{1}{1 + \frac{\overline{w_2}}{\overline{w}+2}} \right) \ ,$$

which is lower than $S_1$ for $\overline{w_2} > 0$.

In order to calculate $\overline{w_2}$, we need to derive $g(w)$ explicitly. Instead of deriving a closed-form for $g(w)$, we use a numerical inversion of Laplace transforms presented in [13]. The value of $g(w)$ for a specified value of $w$ can be estimated as follows. First, $g(w)$ can be represented by a sequence of discrete values, $s_n(w)$,

$$g(w) = s_n(w) - e_d \text{ as } n \to \infty \ ,$$

where $e_d = \sum_{i=1}^{\infty} e^{-iA} g((2i+1)t)$ is the discretization error. Then, $g(w)$ can be approximated by the $s_n(w)$ sequence as:

$$g(w) \approx s_n(w) = \frac{e^{A/2}}{w} \left\{ \frac{1}{2} W^*(\frac{A}{2w}) + \sum_{i=1}^{n} (-1)^i Re(W^*) \left( \frac{A + 2i\pi j}{2w} \right) \right\} \quad , \quad (9)$$

where $A$ is a positive constant s.t. $W^*(s)$ has no singular points on or to the right of the vertical line $s = A/(2w)$ and $Re(W^*)(s)$ is the real part of $W^*(s)$ when $s$ is substituted by a complex number $x + yj$. In (9), $n$ represents the degree of discretization of $g(w)$, i.e., the larger the value of $n$ is, the more accurate the estimation of $g(w)$ by $s_n(w)$ is. In the numerical results shown later, we found that $n = 30$ provides accurate enough results when compared with our simulation results.

If $|g(w)| \leq 1$, the error is bounded by [13]

$$|e_d| \leq \frac{e^{-A}}{1 - e^{-A}} \quad .$$

When $A \geq 18.5$, the discretization error is $10^{-8}$. The constant $A$ can be further increased to improve the accuracy of the result.

## 4 Numerical and Simulation Results

We present our numerical and simulation results in this section. The available channel data rate is 1 Mbps and the control packet length is 48 bits.[6] Our simulation, written in C language, implements a network with 50 nodes, which are in the range of each other.

Fig. 3 depicts our numerical results of $g(w)$ for pure ALOHA-based MAC schemes and according to (5) and (9). We observe from this figure that, when normalized traffic load ($G$) is small, $g(w)$ decreases with the increase of $w$. As $G$ increases, there is a knee in $g(w)$ around $w = 2$, where the decline of $g(w)$ suddenly slows down.

These numerical results can be verified at $w = 0$. The pdf of the contention resolution period $w$ at $w = 0$ can be calculated as the pdf that exactly one RTS packet is sent out at $w = 0$ multiplied by the probability that no other RTS packets are transmitted on the control subchannel in the next unit time, i.e., $g(0) = Ge^{-Gw}\big|_{w=0} \cdot e^{-G \cdot 1} = Ge^{-G}$. For $G = 0.25$, 0.50, 0.75, 1.00, and 2.00, $g(0)$ is 0.1947, 0.3033, 0.3543, 0.3679, and 0.2707, respectively. These results match exactly those shown in Fig. 3.

Fig. 4 depicts our numerical results of expected waiting time on the data subchannel, $\overline{w_2}$, of the pure ALOHA-based MAC-2R scheme. These results are calculated according to (6) and the pdf obtained through numerical calculations for different network settings. To minimize $\overline{w_2}$ and maximize the throughput of the MAC-2R scheme, we choose normalized traffic load $G = 0.5$ in the calculation

[6] Of course, these system parameters may be changed. However, our results suggest that the conclusions for different parameters' values remain unchanged.

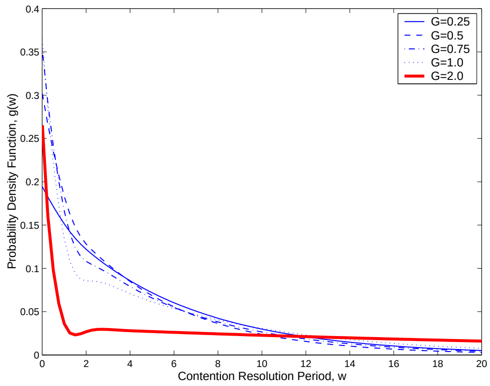


**Fig. 3.** Probability density function of $W$, $g(w)$, with different $G$ for MAC schemes

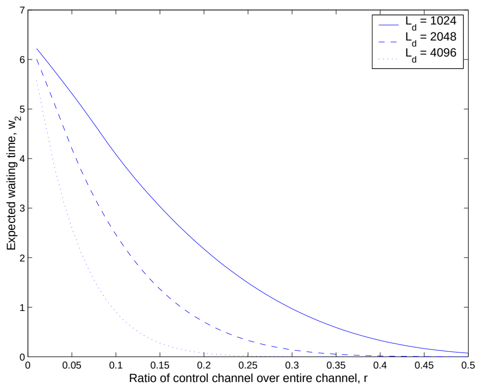


**Fig. 4.** Expected waiting time on the data subchannel, $\overline{w_2}$, for MAC-2R

of $g(w)$. In these results, the control packet length ($L_c$) is fixed at 48 bits, while the data packet length ($L_d$) takes on the values of: 1024, 2048, and 4096 bits to illustrate different operational overheads of the control packets.

As shown in Fig. 4, the expected waiting time on the data subchannel decreases exponentially as $r$ increases. Furthermore, this decrease is much faster when $k = \frac{L_d}{L_c}$ is larger. Thus, for the same value of $r$, the expected waiting time on the data subchannel is significantly shorter in networks with larger $k$. This is due to a much longer data packet transmission time, $\delta$. From this figure, we can also confirm the non-zero expected waiting time when $r$ is chosen as the optimal value of $r^* = \frac{\overline{w}+2}{\overline{w}+2+k}$, as shown in (8). The non-zero expected waiting time on the data subchannel leads to an inferior performance of the MAC-2R scheme, compared to the performance of the MAC-1 scheme.

In Fig. 5, we compare the throughput performance of pure ALOHA-based MAC-1 and MAC-2R schemes for different data packet lengths. The straight lines represent the throughput of the MAC-1 scheme. The throughput of the MAC-

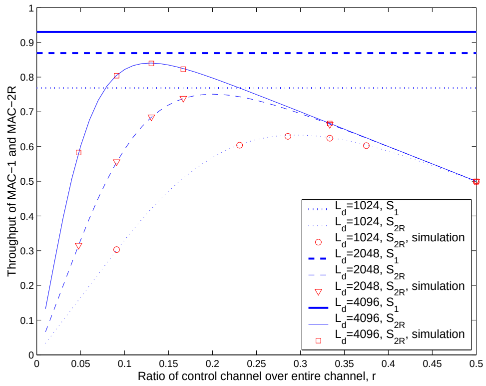


**Fig. 5.** Throughput comparisons between MAC-1 and MAC-2R

2R scheme increases as $r$ increases until the throughput reaches the maximum achievable value and then degrades. When $r$ is too small, the control subchannel needs much longer time to come up with a successful RTS/CTS dialogue. However, when $r$ is too large, the fraction of the entire available channel used to transmit data is too small, limiting the throughput of the MAC-2R scheme.

Comparing the throughput performance of the MAC-1 and the MAC-2R schemes, we observe that the MAC-1 scheme always out-performs the MAC-2R scheme, due to the non-zero waiting time on the data subchannel in the MAC-2R scheme. As expected, the throughput increases as $L_d$ (or $k$) becomes larger, approaching 1 as $L_d$ (or $k$) increases. In the same figure, Fig. 5, we also draw the simulation results of the MAC-2R scheme, demonstrating that our simulation results closely match those obtained by our analysis.

In Fig. 6, we show the ratio of the throughputs of the MAC-2R and the MAC-1 scheme, $S_{2R}/S_1$, as a function of $r$ for different data packet lengths $L_d$. It can be observed that the maximum achievable throughput of the MAC-2R scheme is closer to the throughput of the corresponding MAC-1 scheme as $L_d$ increases. Thus, the penalty for splitting the single channel is lower when data packet length is larger. As $L_d$ increases, the optimum $r$ that achieves the maximum throughput for the MAC-2R scheme becomes smaller.

In Fig. 6, we also draw symbols representing the performance of the MAC-2R scheme, when the single channel is split according to the expected value of the contention resolution periods. In these cases, $r$ is set to $r^* = \frac{\overline{w}+2}{\overline{w}+2+k}$, as shown in (8). As shown in the figure, the throughput of the MAC-2R schemes is offset from the optimum operation point of the MAC-2R scheme. Interestingly, we find that such a non-optimum scheme would operate at the same relative performance $S_{2R}/S_1$ for the different values of $L_d$, as the three symbols are all

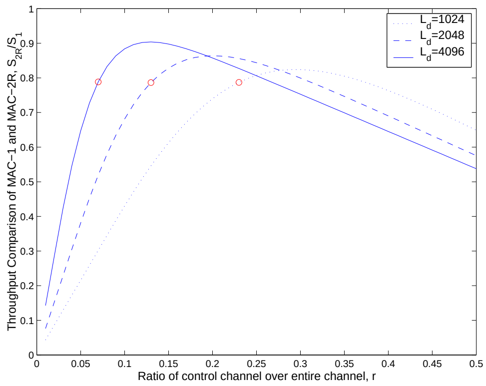


**Fig. 6.** Throughput comparisons between MAC-1 and MAC-2R

at 0.78.[7] When the MAC-2R scheme is optimized according to the expected value of the contention resolution periods, i.e., setting $r$ to $r^*$, we conclude that the throughput degradation of the MAC-2R scheme over the MAC-1 scheme can be as high as 22%.

## 5 Conclusions

In wireless communication networks, the Medium Access Control (MAC) scheme can significantly affect the performance of the network system. To improve the throughput performance of MAC schemes on random access channels, some researchers proposed to split the single shared channel into two subchannels: a control subchannel and a data subchannel. Control packets are sent on the control subchannel, while the data subchannel is used solely to transmit data packets. Therefore, separation of control packet transmission and data packet transmission is achieved.

Some previous publications in the literature claimed that the split-channel MAC scheme may achieve the same or better throughput as the corresponding single-channel MAC scheme does. However, as we show in this paper, these optimistic results were derived by considering only the expected value of the contention resolution periods, without taking into the account the random distribution of these periods. When the randomness of the contention resolution periods is considered, the split-channel schemes are inferior to the single-channel scheme in fully-connected networks and for the scenarios that we have studied here. According to our analysis, this result holds even if the split-channel schemes are optimized with respect to the ratio of the bandwidth of the control subchannel to the bandwidth of the entire channel.

[7] In fact, when $r = r^* = \frac{\overline{w}+2}{\overline{w}+2+k}$, $S_{2R}/S_1 = 1/\left[\left(\frac{1}{1-r} + \frac{\overline{w_2}}{kr}\right)\frac{k}{\overline{w}+2+k}\right]\Big|_{r=r^*} = 1/\left(1 + \frac{\overline{w_2}}{\overline{w}+2}\right)$. Since $\delta = \overline{w} + 2$ and it is not related to $k$, $\overline{w_2}$ is not related to $k$ according to (6). Therefore, the ratio $S_{2R}/S_1$ is not related to $k$.

The inferior throughput performance of split-channel schemes is due to the fact that the control subchannel cannot generate a successful channel reservation dialogue during the period of time when data packets are transmitted on the data subchannel. The randomness of these contention resolution periods requires a larger portion of the available bandwidth to be allocated to the control subchannel, so that long waiting time on the data subchannel would be unnecessary. However, as the overall throughput of split-channel schemes is limited by the capacity of the data subchannel, such allocation of a larger bandwidth to the control subchannel results in significant loss of performance of the data subchannel.

Even though our results are derived for MAC protocols that are based on the RTS/CTS dialogue, these results can be applied to other split-channel MAC schemes as well. In particular, these results can be useful for system engineering in evaluating the advantage and disadvantage of splitting a single shared channel.

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