Information Acquisition and the Equilibrium Incentive Problem

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Abstract

I study the optimal incentive provision in a principal-agent relationship with costly information acquisition by the agent. I emphasize that adverse selection or moral hazard is the principal's endogenous choice by inducing or deterring information acquisition. The principal designs the contract not only to address an existing incentive problem but also to implement its presence. Implementation of adverse selection relies on a steeper information rent to the agent than the standard menu, such that the agent is motivated to distinguish the efficient state of nature from the inefficient. Moral hazard is implemented by replacing the benchmark debt contract with a debt-with-equity-share contract, such that the agent does not attempt to acquire information to either avoid debt or to extract rent.

Keywords: Incentive contract, Information acquisition, Moral hazard, Adverse selection.
JEL Classification: D82, D83, D86

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1 Introduction

Incentive problems, i.e., adverse selection and moral hazard, have been the essence of standard agency theories, in which either or both are exogenously present. The exogeneity of incentive problems is attributed to the assumption that productive information structures are exogenous.[1] Information is, however, often realized as a result of endogenous and costly acquisition activities, as enunciated by Arrow.

> *A key characteristic of information costs is that...they typically represent an irreversible investment...I am thinking of the need for having made an adequate investment of time and effort to be able to distinguish one signal from another.* (Arrow, 1974: 39)

As the information structure is endogenous, so is the underlying incentive problem in the principal-agent relationship. Consider a principal contracting with an agent protected by limited liability, and both players are risk neutral. The principal's revenue is generated by the agent's hidden productive effort, and his productivity depends on the stochastic state of nature. The agent can acquire information on the realized state of nature but only at a sunk cost. The principal may thus implement adverse selection with a contract that induces the agent to acquire information, so that information is asymmetric and the agent manipulates the output with productive effort to communicate the state of nature to the principal. The principal may also implement moral hazard with a contract that deters the agent from acquiring information, in which scenario the principal and the agent are symmetrically informed, and the publicly observed output is an imperfect measure of the agent's private effort. Conventional incentive theories have previously discussed optimal contracts to cope with an existing incentive problem. I study how such a contract is designed to implement the presence of the incentive problem.

The emergence of the incentive problem and its interaction with the principal's information management has only received attention at one end: endogenous infor-

[1] Consider a principal contracting with an agent to execute a project that yields output to the principal, which depends on the agent's private productive effort and stochastic productive state of nature. Adverse selection arises when the agent has private information on the productive state of nature, while the principal observes only its stochastic distribution. The agent then manipulates output through private effort to communicate such information to the principal. Moral hazard arises when the productive effort is imperfectly measured by the output as the productive state of nature is imperfectly and symmetrically observed.

mation acquisition that generates adverse selection.[2] To the best of my knowledge, moral hazard as a consequence of deterrence of information acquisition has not yet been addressed in contract theory. I fill this gap by investigating how information management interacts with the equilibrium incentive problem and how the optimal contract is modified from the benchmark second-best in response to such interaction.

To implement adverse selection by inducing perfect information acquisition, the principal offers a menu contract that motivates the agent to distinguish efficient from inefficient states of nature and to reveal the truth. I discuss two possible ways for the principal to induce information acquisition. The first is to offer a contract with continuous transfer, in which a higher (lower) output than the second-best is specified for a sufficiently efficient (inefficient) state of nature to implement a steeper rent than its second-best counterpart. This method is a simple replication of Crémer, Khalil, and Rochet (1998a).

Alternatively, the principal can use the fact that the agent can only determine productive effort but not realized output when the agent has not acquire information. The principal can commit to pay a zero transfer over the set of outputs that an uninformed agent generates with probability 1; the proposed candidate in this paper is the set of rational numbers. The principal gains from this contract because it allows her to detect an uninformed agent with probability 1, which relaxes the constraint to induce information acquisition. The loss from this contract is that it relies on some pooling over some states of nature to have the output rational on the equilibrium path. Due to continuity of the model, there is an arbitrarily close rational output for any irrational output, and the loss from pooling is only of second-order and is outweighed by the first-order gain. Relying on the rational output to screen the informed agent from the uninformed, the contract restores output (at least weakly) towards efficiency for any state of nature to prescribe an expected information rent to the agent that covers the cost of information acquisition exactly.

I extend Poblete and Spulber (2012) to study the principal's implementation of moral hazard by deterring information acquisition. Under the conditions introduced by Poblete and Spulber, the second-best contract with the presence of moral hazard is a debt contract that prescribes a debt paid to the principal, leaving the agent the claimant of the residual output. To deter information acquisition, the principal must deter the agent's opportunistic motives to acquire information off the equilibrium path, which take two forms under a debt contract. First, discovery of a sufficiently inefficient

[2]Please refer to Lewis and Sappington (1997), Crémer, Khalil, and Rochet (1998a), and Terstiege (2012) in the literature review for more details.

state of nature allows the agent to reject the contract to avoid exerting costly effort that generates only debt to the principal. Second, distinguishing a relatively efficient state of nature allows the agent to exert a positive effort to extract maximal rent based on his information.

The optimal contract is thus characterized by a downward distortion of debt from its second-best and, for a sufficiently small cost of information acquisition, by a lower equity share of output residual to the agent to restrict his ability to extract rent by acquiring information. The former implies a larger output residual, which motivates productive effort in equilibrium, whereas the latter discourages it. This results in an upward distortion of productive effort from the second-best with a sufficiently large cost of information acquisition and an ambiguous distortion otherwise. Deterrence of information acquisition is complementary to higher powered incentives when information acquisition is sufficiently costly, which is different from Crémer, Khalil, and Rochet (1998a), who did not introduce moral hazard into production when information is deterred.

The key tradeoff behind the decision to induce or to deter information acquisition, to implement adverse selection or moral hazard at the production stage, involves rent and efficiency. The agent's acquisition of information benefits the principal as it allows for more efficient production, but an information rent is given to induce hidden effort to acquire information and truthful revelation. For a sufficiently small cost of information acquisition, it is optimal to induce information acquisition and implement adverse selection because the improvement in efficiency exceeds the net information rent. For sufficiently costly information acquisition, it is optimal to deter information acquisition and implement moral hazard because the improvement in efficiency falls short of the net information rent.

Consider a firm-employee relationship for example. Inducing information acquisition to implement adverse selection is optimal if the agent is an "expert" in the field who is able to acquire productive information at a lower cost. Conversely, deterring information acquisition to implement moral hazard is optimal if the principal contracts with a "mediocre agent" who acquires productive information at a higher cost. The model also applies to investment banking. An investment bank (principal) finds it optimal to induce a funds-seeking firm (agent) to conduct costly market investigations and reveal its findings through a menu of funding options when the market is well-established and sufficiently transparent. The investment bank finds it optimal to deter the private firm from conducting costly market investigations with a single debt-with-equity-share contract if the firm is involved in a newly-established market in which past data is limited. The next question is then whether the contracts above is robust to imperfect information acquisition (e.g. there is a positive probability that the agent firm finds nothing from the market investigation) or to private knowledge of information acquiring cost (e.g. the agent employee knows if he is an expert or a mediocre).

The optimal debt-with-equity-share contract to deter information acquisition is qualitatively robust to imperfect information acquisition, as well as to private knowledge of the information acquiring cost. The main difference to perfect information acquisition with common knowledge of information acquiring cost is that the principal does not know perfectly upon offering the contract whether the agent is informed of the state of nature or not, which itself is an information advantage of the agent. The optimal contracts under these two remedies are thus designed such that the informed agent truthfully reveals being informed, and vice versa for the uninformed. The additional incentive compatibility constraints distort the contract designed to the uninformed agent towards the same direction as does the constraint to deter perfect information acquisition with common knowledge of information acquiring cost. The optimal menu contract designed to the informed agent, however, exhibits pooled output menu over some intermediate states of nature, which is absent in the optimal menu contract to induce perfect information acquisition with common knowledge of information acquiring cost. This is due to the technical resemblance of the truth telling constraints for the informed agent to the type-dependent participation constraints that generate countervailing incentives[3], although the reservation utility is assumed to be identical across states of nature.

This paper is organized as follows. The model is outlined in Section 2. Given perfect information acquisition, in Section 3, I derive the optimal menu contract when information acquisition is induced, whereas Section 4 is devoted to the optimal contract to deter information acquisition. Optimal information management and equilibrium incentive problem in the contractual relationship is discussed in Section 5, along with a couple of applications. I examine the robustness of the optimal contract with different assumptions on the information acquiring technology in Section 6. The paper is concluded in Section 7.

[3]Please refer to Lewis and Sappington (1989) for the pioneer work, and Jullien (2000) for a more general treatment.

1.1 Related Literature

Information acquisition in the environment with adverse selection has gained much attention in contract theory, and it is roughly categorized into two forms: strategic and productive information gathering. The former refers to circumstances in which information can be realized at no cost at the production stage but can be acquired at a cost ex ante to facilitate the agent's decision regarding whether to accept the contract, which affects the form of the agent's individual rationality but not truthful revelation of information. Crémer and Khalil (1992), Crémer, Khalil, and Rochet (1998b), Hoppe and Schmitz (2013b), and Szalay (2009) study this sort of information acquisition. As information would be realized at the stage of production, information acquisition is only for strategic purpose and the incentive problem at the production stage is exogenously adverse selection.

I build my propositions on the latter form of information acquisition, which corresponds to situations in which information is realized only if it is acquired at a cost. Thus, information acquisition affects both the participation and the incentive compatibility of the agent. Lewis and Sappington (1997), Crémer, Khalil, and Rochet (1998a), Kessler (1998), Krähmer and Strausz (2011), Zermeño (2011), Terstiege (2012), and Hoppe and Schmitz (2013a) fall into this category. These studies, however, either do not consider deterrence of information acquisition, or they assume that a deterministic output is a perfect measure of the agent's productive effort when information acquisition is deterred. Adverse selection endogenously arises as a consequence of inducing information acquisition, whereas moral hazard is assumed away. The interaction between deterring information acquisition and moral hazard is absent from the principal's optimization program in these papers.

A considerable literature is also devoted to inducing information acquisition on productive noise in an environment with moral hazard and a risk averse agent to explain the empirical puzzle that a higher powered incentive is given in a riskier environment.[4] Information on productive noise is assumed to be a mean-preserving imperfect signal that is unable to be communicated through a contract; truthful revelation is absent. Regardless of the level of information acquisition, the fundamental incentive problem is that of moral hazard. In this literature, a higher powered incentive in a riskier environment is attributed to inducing information acquisition, which implicitly implies a lower powered incentive if information acquisition is deterred. With risk neutrality, I show in the current paper that higher powered incentive to deter information

[4] Refer to Demski and Sappington (1987), Malcomson (2009, 2011), Raith (2008), and Zábojník (1996) for theory, and Prendergast (2002) and Shi (2011) for the empirical evidence.

acquisition may be optimal as long as the cost of information acquisition is not too small. In an appendix, I also claim that deterring a risk-averse agent from information acquisition does not necessarily rely on a lower powered incentive; it depends on the density of the state of nature.

The notion that information availability on the state of nature distinguishes adverse selection from moral hazard is also emphasized by Sobel (1993), Chu and Sappington (2009), and Poblete and Spulber (2012). Sobel (1993) compares the principal's payoff given various timing scenarios in which information becomes available: pre-contract, post-contract prior to production, and after production. Chu and Sappington (2009) develop a dynamic model in which information becomes available at an interim stage before which the incentive problem is due to hidden action, and it is due to asymmetric information thereafter. Poblete and Spulber (2012) introduce the concept of critical ratio, and relate it to the characterization of the optimal contract, under moral hazard and under adverse selection. In all three papers, however, information is not acquired by the agent, and both the timing of information availability and the underlying incentive problem are exogenous. As its contribution to the literature, the present paper elucidates the endogenous choice of the incentive problem by inducing/deterring information acquisition, which provides a refutable modification to the standard contracts.

Endogeneity of incentive problems due to productive information acquisition is also noticed by the independent work of Iossa and Martimort (2013). They study imperfect information acquisition in a similar fashion to mine in Section 6.1. There are two main differences in our models. First, regarding the agent's attitude towards risk, the agent is risk neutral yet protected by limited liability in the current paper, whereas in Iossa and Martimort, he is risk neutral but endowed with a pessimistic attitude. This difference shapes the benchmark contracts differently, i.e., debt contracts in my paper and linear contracts in Iossa and Martimort when the information structure is exogenously given. The second difference involves the timing of the contract and what the agent can do with his information. By assuming that the agent signs the contract after acquiring the information, the agent in this paper is able to use the information to either reject the contract or to accept it and use the information for production. In comparison, in Iossa and Martimort, the agent signs the contract before he decides whether to acquire the information. The agent is then unable to use the information to reject the contract even when he realizes that there would be a negative payoff. In other words, only the rent extraction motive, not the motive to avoid costly effort, generates a negative payoff. This second difference then shapes the distortion from the benchmark differently.

2 Model

A principal hires an agent to execute a project that yields publicly observable and contractible output $q(e,\theta)$, based on the agent's privately observed productive effort (e) and the state of nature (θ). Let the cost of effort be e, $q_e(e,\theta) > 0$[5], $q_\theta(e,\theta) > 0$, $q_{ee}(e,\theta) < 0$, $q_{\theta\theta}(e,\theta) \leq 0$, and $q_{e\theta}(e,\theta) > 0$ for $(e,\theta) > (0,0)$, i.e. the output function is concave in both effort and state of nature, and higher θ indicates a relatively efficient state of nature with higher total and marginal output. θ follows prior distribution $F(\theta)$ defined over $[0,\bar{\theta}]$. The principal and the agent are both risk neutral, with the principal's payoff defined as the output net of the contingent transfer specified in the contract, $u^P = q(e,\theta) - t(q(e,\theta))$, and the agent's payoff defined as the contingent transfer net of the cost of effort, $u^A = t(q(e,\theta)) - e$. The agent is protected by limited liability.[6]

Upon being offered a contract, the agent can invest effort a in information acquisition, which allows him to observe the correct signal of the state of nature with probability $a \in [0,1]$, or no signal otherwise, at a (sunk) non-monetary cost $d(\kappa,a)$, before accepting the contract, κ being the cost parameter of information acquisition. The acquired information is private to the agent, as well as his information-acquiring action, but the cost of information acquisition is common knowledge.[7] The non-monetary sunk cost of information acquisition captures the characteristic of information acquisition as "an irreversible investment of the agent's effort to distinguish one signal from another," which is unconstrained by his limited liability. I proceed with the case of perfect information acquisition, i.e. $a \in \{0,1\}$. The agent knows the realized state of nature perfectly upon exerting information acquiring effort. This corresponds to the equilibrium information acquisition with $d(\kappa,a) = \kappa a$, the agent being risk neutral in information acquisition.[8]

The cost parameter of information acquisition, κ, can be interpreted accordingly in different applications of the model. For instance, in a firm-employee relationship, it

[5]Subscripts denote partial derivatives.

[6]Contracting with a risk averse agent without limited liability is discussed in Appendix B.

[7]An extension with private knowledge of information acquiring cost is examined in Section 6.2.

[8] I discuss in Section 6.1 the implementation of imperfect information acquisition, when the optimal information acquiring effort is interior.

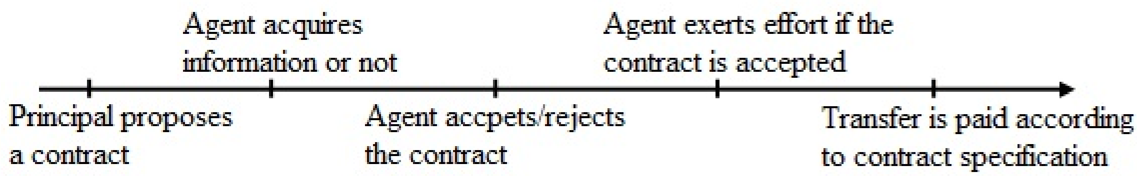

Figure 1: Time-line

represents the agent's expertise in his field. A lower cost corresponds to a higher level of expertise, as the agent is able to distinguish between productive signals at a lower cost. In investment banking, it captures the cost of market investigation (or more broadly, due diligence), which depends on market transparency or the availability of data and past experiences. A lower cost may be due to a well-established market with a high level of information transparency. In an insurance market, such cost of information acquisition may reflect the cost of conducting genetic tests or other types of health examinations, which is less costly to reach the same accuracy, compared with identifying an accident.

If the principal induces perfect information acquisition with the contract, the agent has private information on the state of nature before accepting the contract and there is no productive uncertainty. He communicates the acquired information to the principal by producing a certain level of output specified in the menu contract. The incentive problem is due to ex-ante asymmetric information, the adverse selection. If the principal deters information acquisition with the contract, the principal and the agent have symmetric information, and the publicly observed output is an imperfect measure of the agent's hidden action. The incentive problem is then moral hazard. The time-line of the game is shown in Figure 1.

To capture my main arguments, the following assumptions are made such that the moral hazard problem when information acquisition is deterred is relevant, and such assumptions do not affect the adverse selection problem when information acquisition is induced.

Assumption 1. *$q(e,0)$ is a constant, normalized to zero, e.g. $q(e,\theta) = (m(\theta) - m(0))n(e)$, where $n(0) = 0$.*

Assumption 2. *$\rho(e,\theta) \equiv \frac{f(\theta)}{1-F(\theta)} \frac{q_e(e,\theta)}{q_\theta(e,\theta)}$ is increasing in θ and $\frac{f(\theta)}{F(\theta)} \frac{q_e(e,\theta)}{q_\theta(e,\theta)}$ is decreasing in θ for $(e,\theta) > (0,0)$.*

Assumption 1 is imposed to assume out moving support in lower realization of output when θ is a stochastic variable at the production stage. Assumption 2 relies on the concept of "critical ratio" introduced by Poblete and Spulber (2012). The authors show that when the critical ratio, $\rho(e, \theta)$, is increasing in the state of nature, the optimal contract in an environment with moral hazard, risk neutrality, and limited liability takes the form of debt. They also show that this condition guarantees a second-best separating equilibrium in an environment with adverse selection. In addition, I define Condition 1 regarding the contractual form under which both players earn non-decreasing payoffs in output.

Condition 1. $0 \leq t_q(q) \leq 1$.

There are three reasons to place this condition. First, contracts that guarantee non-decreasing payoffs in output are intuitively reasonable. In addition, following Innes (1990) and Poblete and Spulber (2012), when it is impossible for the agent to acquire information, non-decreasing payoffs and limited liability make the moral hazard problem under risk neutrality relevant. When information can be acquired at no cost, this condition does not create an additional distortion in the adverse selection problem under the current model setup.

Third and most importantly for my purposes, Condition 1 is required for me to replicate the optimal contract proposed by Crémer, Khalil, and Rochet (1998a) if the principal is to induce information acquisition and implement adverse selection, whereas the optimal contract satisfying the same condition to deter information acquisition and implement moral hazard significantly differs from that proposed by Crémer et al. Allowing violations to Condition 1 to induce information acquisition, the principal is able to epsilon-implement the second-best menu contract for a sufficiently small cost of information acquisition, whereas for a higher cost of information acquisition, it predicts a different contract to induce information acquisition than that proposed by Crémer et al. This will be explained further in the later sections.

3 Inducing Information Acquisition

Applying the Revelation Principle, a feasible contract to induce information acquisition and truthful revelation consists of a menu of options, $\{t(\theta), q(\theta)\}$, in which the agent in state θ accepts the contract (is individually rational), is incentive compatible not to produce $q(\theta')$ for any $\theta' \neq \theta$, and acquires information regarding the state of nature at a cost κ before acceptance. Let $c(q(\theta'), \theta) \equiv e$, where $q(\theta') = q(e, \theta)$. Specifically, the principal's optimization program to induce information acquisition is

$$\mathscr{P}_{II}: \quad \max_{t(\theta),q(\theta)} \int_0^{\bar{\theta}} (q(\theta) - t(\theta))\, dF(\theta)$$

subject to

$$t(\theta) - c(q(\theta),\theta) \geq 0,\ \forall \theta \in [0,\bar{\theta}] \qquad (IR_\theta),$$

$$\theta \in \arg\max_{\theta'} t(\theta') - c(q(\theta'),\theta),\ \forall \theta \in [0.\bar{\theta}] \qquad (IC_\theta),$$

and

$$\int_0^{\bar{\theta}} (t(\theta) - c(q(\theta),\theta))\, dF(\theta) - \kappa \geq \max_e \int_0^{\bar{\theta}} t(q(e,\theta)) dF(\theta) - e \qquad (II).$$

At first glance, this seems to be a replication of Crémer, Khalil, and Rochet (1998a). It is if we restrict our attention to a contract with a continuous transfer that satisfies Condition 1. Specifically, the optimal contract to induce information acquisition with Condition 1 satisfied , $\mathbb{C}^{CT} = \{t^{CT}(\theta), q^{CT}(\theta)\}$, in comparison to the second-best menu, $\mathbb{C}^{SM} = \{t^{SM}(\theta), q^{SM}(\theta)\}$, is characterized in Proposition 1.

Proposition 1. *Given Condition 1, for $\kappa > \kappa^a$, $\mathbb{C}^{CT}$ exhibits a higher powered (lower powered) incentive than $\mathbb{C}^{SM}$ in sufficiently efficient (inefficient) states; $q^{CT}(\theta) \geq q^{SM}(\theta)$ for $\theta > \hat{\theta}$, with equality at $\overline{\theta}$, and $q^{CT}(\theta) \leq q^{SM}(\theta)$ for $\theta \leq \hat{\theta}$, with equality at 0, where $\hat{\theta}$ is the state of nature that the agent expected to have revealed ex-ante if he did not acquire information.*

Proof. Appendix A.1, or Crémer, Khalil, and Rochet (1998a). □

This contact implements information rent that is steeper than its second-best counterpart to motivate the agent to distinguish relatively efficient states of nature from the relatively inefficient.

There is, however, a caveat to Proposition 1. It is assumed in Proposition 1 that $t(q(e,\theta))$ is continuous and non-decreasing even over outputs that are not specified in $\mathbb{C}^{CT}$. A distinguishing feature[9] of this paper in comparison with Crémer, Khalil, and Rochet (1998a) is that in the latter paper, the state of nature is modeled as a marginal cost parameter, and the agent, with or without private information, perfectly determines the output. These modeling assumptions are not made in the current

[9] I thank an anonymous referee for reminding me of this.

model. An uninformed agent off the equilibrium path determines productive effort, which generates output stochastically. To see how the principal is able to exploit this to her advantage, consider the following contract:

$$t(q) = \begin{cases} t(\theta) & if \quad q(e,\theta) \in \mathbb{Q} \\ 0 & if \quad q(e,\theta) \notin \mathbb{Q} \end{cases}$$

and

$$q(\theta) \in \mathbb{Q},$$

$\mathbb{Q}$ denoting the set of rational numbers. With the set of state of nature being continuously distributed and the set of rational output being countable, an uninformed agent exerting any $e > 0$ generates an irrational output with probability 1. In other words, the principal is able to distinguish whether the agent acquired information by partitioning the contracted outputs into two sets, one of which is generated by an uninformed agent with probability zero. Given this proposed contract, the optimal level of effort for an uninformed agent off the equilibrium path is zero. Constraint (II) is reduced to the ex-ante individual rationality constraint of the informed agent, i.e.,

$$\int_0^{\bar{\theta}} t(\theta) - c(q(\theta),\theta) dF(\theta) - \kappa \geq 0 \qquad (II^{'}).$$

The next question is how well this contract performs compared to that in Proposition 1. At the optimal contract with continuous transfer, due to continuity, for any $q(\theta_1) \notin \mathbb{Q}$, there is an arbitrarily close $q(\theta_2) \in \mathbb{Q}$. Pooling $q(\theta_1)$ with $q(\theta_2)$ to satisfy $q(\theta) \in \mathbb{Q}$ results in only a second-order loss to the principal, yet relaxes constraint (II) to $(II^{'})$, which results in a first-order gain to the principal.

The principal's problem to induce information acquisition now becomes

$$\mathscr{P}_{II}: \quad \max_{t(\theta),q(\theta)} \int_0^{\bar{\theta}} (q(\theta) - t(\theta)) \, dF(\theta)$$

subject to

$$t(\theta) - c(q(\theta),\theta) \geq 0, \ \forall \theta \in [0, \bar{\theta}] \qquad (IR_\theta),$$

$$\theta \in \arg\max_{\theta^{'}} t(\theta^{'}) - c(q(\theta^{'}),\theta), \ \forall \theta \in [0.\bar{\theta}] \qquad (IC_\theta),$$

$$\int_0^{\bar{\theta}} t(\theta) - c(q(\theta), \theta) dF(\theta) - \kappa \geq 0 \qquad (II'),$$

and

$$q(\theta) \in \mathbb{Q} \qquad (R).$$

The optimal contract to induce information acquisition in the proposed form, $\mathbb{C}^{II} = \{t^{II}(q), q^{II}(\theta)\}$, has the following property.

Proposition 2. *Allowing Condition 1 to be violated, the optimal contract to induce information acquisition implements an output schedule arbitrarily close to the second-best for $\kappa^b \geq \kappa > \kappa^a$, and exhibits a higher powered incentive than $\mathbb{C}^{SM}$ for $\kappa > \kappa^b$; $q^{II}(\theta) \geq q^{SM}(\theta)$, with equality at $\theta = \overline{\theta}$.*

Proof. Appendix A.1. □

Utilizing the fact that an uninformed agent cannot perfectly determine output, the principal relies on the production of rational output to screen the informed agent from the uninformed. This provides room to restore the output schedule towards efficiency and meanwhile implements the expected information rent to exactly the cost of information acquisition, when information acquisition is sufficiently costly. The principal then earns a higher payoff under $\mathbb{C}^{II}$ than under $\mathbb{C}^{CT}$, as the former implements a relatively efficient output schedule and a lower expected information rent on the equilibrium path.

4 Deterring Information Acquisition

A feasible contract to deter information acquisition and to implement productive effort e prescribes a transfer that is contingent on output, $t(q)$, such that it satisfies the agent's limited liability, that both players earn payoffs that are non-decreasing in output, that the agent is incentivized to exert productive effort e, and that he does not acquire information on the state of nature before acceptance.

If deterring information acquisition is optimal, the principal offers the contract that solves the following program subject to limited liability, non-decreasing payoff, incentive compatibility, and deterring information acquisition constraints.

$$\mathscr{P}_{DI}: \quad \max_{t(q(e,\theta)),e} \int_0^{\overline{\theta}} q(e,\theta) - t(q(e,\theta)) dF(\theta)$$

subject to

$$t(q) \geq 0 \qquad (LL),$$

$$0 \leq t_q(q) \leq 1 \qquad (NDP),$$

$$e \in \arg\max_y \int_0^{\bar{\theta}} (t(q(y,\theta)) - y)\, dF(\theta) \qquad (IC),$$

$$\int_0^{\bar{\theta}} (t(q(e,\theta)) - e)\, dF(\theta) \;\; \geq \;\; \int_0^{\bar{\theta}} \mathbf{1}_{\theta \geq \tilde{\theta}}\, (t(q(e(\theta),\theta)) - e(\theta))\, dF(\theta) \; - \; \kappa \qquad (DI),$$

where $e(\theta) \in \arg\max_y t(q(y,\theta)) - y$ and $\tilde{\theta} = \max\{\theta : \; t(q(e(\theta),\theta)) - e(\theta) = 0\}$, i.e. for $\theta < \tilde{\theta}$, the agent who acquired information off the equilibrium path finds it optimal to reject the contract.[10]

Simply by the right-hand-side of (DI) one can have a glimpse of the agent's opportunistic motives to acquire information off the equilibrium path: to distinguish a sufficiently inefficient state of nature to avoid exerting costly effort that generates merely debt to the principal, and to discover a relatively efficient state of nature to exert a positive effort based on his information to extract maximal rent.

This section is devoted to discussing how deterring information acquisition interacts with moral hazard in the contractual relationship by characterizing the distortion of the optimal contract from the second-best. I focus on a tractable example assuming a specific form of contingent transfer consisting of a debt and a share of output residual in Section 4.1 and then turn to the general case in Section 4.2. I assume in both sections risk neutrality, leaving the case with a risk averse agent to Appendix B.

4.1 Debt and Equity Share

In the standard moral hazard problem with risk neutrality, limited liability, and non-decreasing payoff, Poblete and Spulber (2012) show that the optimal second-best contract is a debt contract when Assumption 2 holds, i.e. $t(q) = \max\{q(e,\theta) - \underline{q}, 0\}$, $\underline{q} \geq 0$.[11] I apply this result to examine implementation of moral hazard via deterring information acquisition. In this subsection, I focus on a simplified example in which the contract to deter information acquisition, $\mathbb{C}^{DI}$, has a contingent transfer in the

[10] This is by the envelope theorem of the informed agent's optimization problem off the equilibrium path.

[11] Please also refer to Innes (1990) for the pioneer treatment, at a stricter condition than that introduced by Poblete and Spulber (2012).

form $t^{DI}(q) = T^{DI} + \max\{s^{DI}(q(e^{DI},\theta) - \underline{q}^{DI}), 0\}$, and discuss how deterring information acquisition modifies this contract from the second-best debt contract $\mathbb{C}^{SD}$, in which $t^{SD}(q) = \max\{q(e^{SD},\theta) - \underline{q}^{SD}, 0\}$, leaving a general contractual form to the next subsection.

Lemma 1. $T^{DI} = 0$.

Proof. If (DI) is violated at the second-best, $T > 0$ does not bind (DI), as off the equilibrium path, the agent who acquires information can always accept the contract and exert any $e \geq 0$ to earn T, i.e. regardless of whether acquiring information or not, the agent's expected utility increases by T. $T < 0$ violates (LL) for $q(e,\theta) < \underline{q}$. □

For convenience of interpretation, I would phrase the simplified contract as a combination of debt ($\underline{q}$) and equity share of output residual (s) to the agent.

Assumption 3. *The production function and the cost function is well-behaved such that the first-order approach can be applied.*[12]

Given Assumption 3, (IC) can be replaced by local incentive compatibility,

$$\int_{\underline{\theta}}^{\bar{\theta}} sq_e(e,\theta)dF(\theta) - 1 = 0 \qquad (LIC'),$$

where $\underline{\theta}$ is such that $q(e,\underline{\theta}) \equiv \underline{q}$. (NDP) is expressed as

$$0 \leq s \leq 1 \qquad (NDP').$$

Lemma 2. $\tilde{q} \equiv q(e(\tilde{\theta}),\tilde{\theta})) > \underline{q}$ *and* $\tilde{\theta} > \underline{\theta}$.

Proof. Suppose that $\tilde{q} \leq \underline{q}$, and let $\theta' = \tilde{\theta} + \varepsilon$, $\varepsilon > 0$, such that $q(e(\theta'),\theta') > \tilde{q}$. As $\lim_{\varepsilon\to 0} t(q(e(\theta'),\theta')) - e(\theta') = -e(\tilde{\theta}) < 0$, along with continuity in θ, there is an arbitrarily small and positive ε such that $t(q(e(\theta'),\theta')) - e(\theta') < 0$ for $\theta' = \tilde{\theta} + \varepsilon$, a contradiction to the definition of $\tilde{\theta}$. Let θ^0 be such that $\int_{\underline{\theta}}^{\bar{\theta}} sq_e(e^*,\theta)dF(\theta) = sq_e(e^*,\theta^0)$, where e^* is the optimal effort choice of an uninformed agent, and denote $\hat{e}(\theta)$ as the solution to $sq_e(e,\theta) = 1$. Put differently, $e^* = \hat{e}(\theta^0)$ given (LIC') satisfied. Off the equilibrium path, an informed agent exerts effort $e(\theta)$ such that $sq_e(e(\theta),\theta) = 1$ if $\theta \geq \tilde{\theta}$, zero otherwise. If $\theta^0 \leq \underline{\theta}$, $q(\hat{e}(\theta^0),\theta^0) \leq q(\hat{e}(\theta^0),\underline{\theta}) = \underline{q} < \tilde{q}$. Thus, by definition of $\tilde{\theta}$, $\theta^0 < \tilde{\theta}$ for all $\theta^0 \leq \underline{\theta}$, implying that $\underline{\theta} < \tilde{\theta}$. If $\theta^0 > \underline{\theta}$, $\hat{e}(\theta^0) > \hat{e}(\underline{\theta})$; hence, $\tilde{q} > q(\hat{e}(\theta^0),\underline{\theta}) = \underline{q} > q(\hat{e}(\underline{\theta}),\underline{\theta})$, implying that $\underline{\theta} < \tilde{\theta}$. □

[12] For instance, $q(\theta,e) = \theta e^{\beta}$, and that θ follows uniform distribution on $[0,\overline{\theta}]$, with $\beta > 0$ sufficiently small, or with sufficiently large $\overline{\theta}$, in other words, when the production function is sufficiently concave, or when the most efficient state of nature is sufficiently productive.

Lemma 2 gave a preliminary hint regarding one of the agent's motives to acquire information off the equilibrium path: to distinguish a sufficiently inefficient state of nature to avoid exerting costly effort that results in a negative ex-post payoff. Along with Lemma 1, it is applied to rewrite constraint (DI) into

$$\int_{\underline{\theta}}^{\bar{\theta}} s(q(e,\theta) - \underline{q})dF(\theta) - e \geq \int_{\tilde{\theta}}^{\bar{\theta}} \left(s(q(e(\theta),\theta) - \underline{q}) - e(\theta)\right) dF(\theta) - \kappa \qquad (DI^{'}).$$

The principal's optimization program to deter a risk neutral agent from acquiring information with the simplified contract is thus reduced to

$$\mathscr{P}_{DI}^{'}: \quad \max_{s,\underline{q},e} \int_{0}^{\bar{\theta}} q(e,\theta)dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} s(q(e,\theta) - \underline{q})dF(\theta)$$

subject to

$$(NDP^{'}),\ (LIC^{'}),\ (DI^{'}).$$

The characterization of the optimal simplified contract with the binding constraint (DI) is given in Proposition 2 below.

Proposition 3. *There exists κ^s and κ^q, $\kappa^s \leq \kappa^q$, such that for $\kappa \in [\kappa^s, \kappa^q)$, the optimal contract to deter information acquisition is a debt contract with a lower debt than the second-best, $\underline{q}^{DI} < \underline{q}^{SD}$ and $s^{DI} = s^{SD} = 1$; for $\kappa < \kappa^s$, the optimal contract to deter information acquisition has $t^{DI}(q) = \max\left\{s^{DI}\left(q(e,\theta) - \underline{q}^{DI}\right), 0\right\}$, in which $\underline{q}^{DI} < \underline{q}^{SD}$ and $s^{DI} < s^{SD} = 1$. (Illustrated in Figure 2)*

Proof. Given Lemma 2, lowering the debt $\underline{q} < \underline{q}^{SD}$ increases the expected output residual more significantly on the equilibrium path than it does off the equilibrium path. Suppose that $s = 1$, for sufficiently small κ such that $\underline{q}$ is arbitrarily close to zero to satisfy constraint $(DI^{'})$, the principal earns arbitrarily close to nothing. Lowering s gives the principal a positive share of a smaller expected output. The complete proof is in Appendix A.2. □

This contract can be interpreted as a debt-with-equity-share contract when compared with the second-best debt contract. When the second-best debt contract violates constraint (DI), there are two opportunistic motives of the agent to acquire information. First, discovery of a sufficiently inefficient state of nature allows the agent to

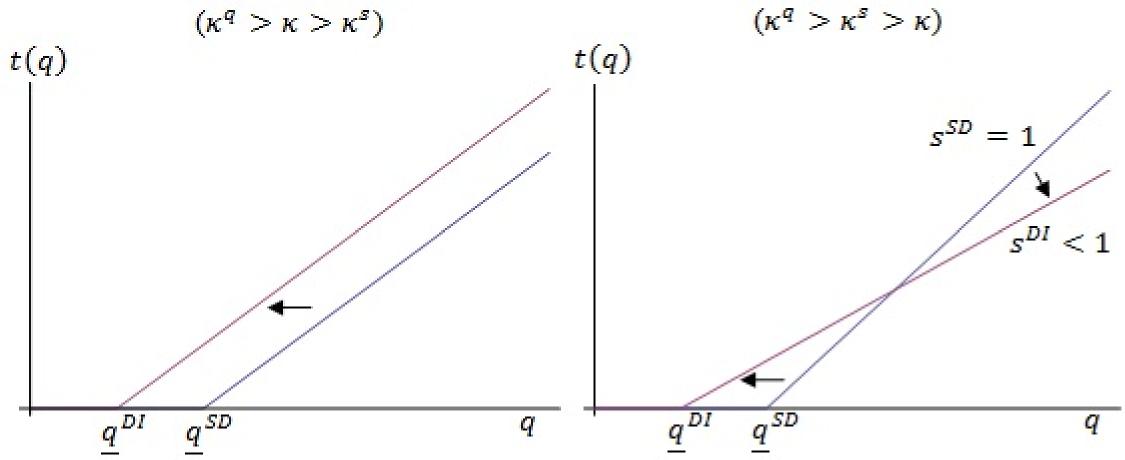

Figure 2: Debt-with-Equity-Share Contract to Deter Information Acquisition

avoid exerting costly effort that results in merely debt to the principal. Second, distinguishing a relatively efficient state of nature allows the agent to exert a positive effort based on his information to extract maximal rent. The principal is able to relax the former motive by lowering the level of debt from its second-best counterpart, which reduces the probability of an uninformed agent receiving no reward for his effort. In response to the latter motive, the principal can reduce the equity share of output residual that is granted to the agent. This weakens the agent's ability to use his information off the equilibrium path to extract rent.

With sufficiently costly information acquisition, $\kappa^s \leq \kappa < \kappa^q$, the principal deters information acquisition with a debt contract that prescribes a smaller debt than the second-best. The rent to the agent to incentivize him to remain uninformed takes the form of making him the claimant of a larger output residual. Intuitively by the principal's optimization problem, lowering the debt has a second-order effect on the principal's payoff, whereas lowering the equity share of output residual has a first-order effect. The agent is thus the sole claimant of the output beyond the lowered debt. For sufficiently costly information acquisition, the principal does not find deterring information acquisition costly to the extent that a switch from a debt contract to a debt-with-equity share contract is optimal.

A larger output residual to the agent, however, amplifies the rent extraction motive to acquire information, which becomes a significant concern to the principal when the cost of information acquisition is sufficiently small ($\kappa < \kappa^s < \kappa^q$). Lowering the debt has more than a second-order effect on the principal's payoff. Instead of granting the entire output residual to the agent, the principal finds it optimal to retrieve an equity share of the output residual, making both herself and the agent shared residual claimants. The optimal contract to deter information acquisition is thus a debt-with-equity-share contract to replace the second-best debt contract.

Another way to interpret this contract is to regard it as an option over the share of output. Comparing to the second-best debt contract, for $\kappa \in [\kappa^s, \kappa^q)$, the share is increased for any $q > \underline{q}^{DI}$ to account for the agent's motive to avoid too much effort. For $\kappa < \kappa^s$, the share is increased from the second-best for intermediate levels of output to account for the same motive and decreased from the second-best for sufficiently high output to account for the rent-extraction motive.

Lowered debt implies a larger residual to claim at any output level beyond the level of debt, in addition to a larger probability of claiming a positive output residual when the agent is uninformed. In other words, a lowered debt in the optimal contract encourages a higher powered incentive. A shared equity of output residual, on the other hand, discourages productive effort. Given a certain level of debt, shared equity implies that only a portion of output residual will be claimed by the agent. Thus, for $\kappa^s \leq \kappa < \kappa^q$, the agent who is deterred from acquiring information exerts unambiguously higher effort than he does under the second-best environment. For $\kappa < \kappa^s$, however, productive effort implemented by the debt-with-equity-share contract is ambiguous in comparison to that implemented by the second-best debt contract. In brief, for sufficiently costly information acquisition, deterring information acquisition is accompanied by a higher powered incentive, whereas for a sufficiently low cost of information acquisition, it is ambiguous in the current model whether deterrence of information acquisition is accompanied by a lower powered incentive.

Corollary 1. *For $\kappa \geq \kappa^s$, $e^{DI} \geq e^{SD}$.*

Proof. Appendix A.2. □

Having a higher powered incentive to deter information acquisition at first glance might be counter-intuitive. I think of the implemented productive effort as a result of the means to provide rent to the agent in order to deter information acquisition. For sufficiently costly information acquisition, the principal rewards the agent an expected rent that is supported by a higher expected output generated by a higher powered incentive scheme. The possible complementarity between deterring information acquisition and a higher powered incentive is absent in the related literature, e.g. Crémer, Khalil and Rochet (1998a) and Iossa and Martimort (2013). It is the joint product of the endogenous incentive problem and the agent's option to reject the contract after being informed of a sufficiently inefficient state of nature off the equilibrium path. The key departure of the current paper from Crémer et al. revolves around whether a moral hazard problem is present when information acquisition is deterred. In Crémer et al., productive uncertainty is absent even when information acquisition is deterred; contractible output is a perfect measurement for the agent's effort, as if the effort itself can also be contracted upon. In the current model, effort level is implemented with a transfer contingent on the realization of contractible output. In other words, in Crémer et al., the principal has two instruments to motivate productive effort and to deter information acquisition, whereas in this paper, the principal has only one instrument: the transfer. Iossa and Martimort (2013) shares the same view as that expressed herein that the incentive problem itself is endogenous. They, however, assumed that the agent signs the contract before the decision to acquire information, and is thus unable to use the information to reject the contract even after realizing that there will be a negative ex-post payoff. In other words, there is only the rent extraction motive discussed above, which is deterred by a lower powered incentive scheme unambiguously; the motive to avoid a negative payoff does not exist.

4.2 General Contract

The readers at this point may question the optimality of the proposed debt-with-equity-share contract with the presence of binding constraint to deter information acquisition. I respond by showing that the result of a lower debt than its second-best counterpart is indeed optimal, and a reduced share of output residual is qualitatively robust, yet in a different form of transfer, in which $s \in \{0, 1\}$ for different output intervals beyond the debt. The principal's optimization program is as follows.

$$\mathscr{P}_{DI}: \quad \max_{t(q(e,\theta)),e} \int_0^{\overline{\theta}} q(e,\theta) - t(q(e,\theta))dF(\theta)$$

subject to

$$(LL), (NDP), (IC), (DI).$$

Proposition 4. *The optimal contract to deter a risk neutral agent protected by limited liability from acquiring information has $t^{DI}(q) = 0$ for $q \leq \underline{q}^{DI} < \underline{q}^{SD}$ and $0 < t^{DI}(q) \leq q - \underline{q}^{DI}$ for $q > \underline{q}^{DI}$. In addition, the slope of transfer is either zero or one at different sets of output.*

Proof. Appendix A.3. □

The intuition discussed in the previous example prevails. Recall that in the second-best environment, the agent's opportunistic motive to acquire information is to distinguish the inefficient states of nature to avoid exerting costly effort that results in only the debt to the principal and to discover the efficient states of nature to extract maximal rent with his acquired information. A lower debt, $\underline{q}^{DI} < \underline{q}^{SD}$, is implemented to account for the former motive, and the transfer for sufficiently high realization of output is lowered to demotivate the latter. The bang-bang property of the slope of transfer is attributed to the linearity of the principal's program on the slope of transfer. It preserves the property that the contract can be regarded as an option over the share of output.

5 The Equilibrium Incentive Problem

When studying the endogenous implementation of incentive problem via information management, note that it is without loss of generality to restrict our attention to the comparison of the contract to induce with that to deter information acquisition. The notion is that every contract under which the agent has an incentive to be informed must be weakly suboptimal to the contract in Section 3, and every contract under which the agent has an incentive to remain uninformed must be weakly suboptimal to the contract in Section 4.

Lemma 3. *The optimal contract is either the one that induces information acquisition, or the one that deters information acquisition.*

Proof. Consider $\kappa > \kappa^a$ such that (II) is strictly violated under $\mathbb{C}^{SM}$. Without inducing information acquisition, $\mathbb{C}^{SM}$ implements the same outcome as $\mathbb{C}^0 = \{t^0(q(e^0, \theta))\}$, where $e^0 \in \arg\max_e \int_0^{\bar{\theta}} t^{SM}(q(e, \theta))dF(\theta) - e$ and $t^0(q) = t^{SM}(q)$ for all q. $\mathbb{C}^0$ satisfies (LL), (IC), and (DI) by construction, which must not be preferred to $\mathbb{C}^{DI}$ for the principal. Consider $\kappa < \kappa^q$ such that (DI) is strictly violated under $\mathbb{C}^{SD}$. Without deterring information acquisition, $\mathbb{C}^{SD}$ implements the same outcome as $\mathbb{C}^1 = \{t^1(q^1(\theta)), q^1(\theta)\}$, where $q^1(\theta) = q(e(\theta), \theta)$ for all $\theta \geq \tilde{\theta}$, zero otherwise, and $t^1(q^1(\theta)) = t^{SD}(q(e(\theta), \theta))$ for $\theta \geq \tilde{\theta}$, zero otherwise. $\mathbb{C}^1$ by construction satisfies (IR_θ), (IC_θ), and (II), which the principal does not prefer to $\mathbb{C}^{CT}$, let alone $\mathbb{C}^{II}$ if Condition 1 is relaxed. □

Define the principal's net value of information given Condition 1, $V(\kappa)$, as the difference between her ex ante expected utility when information acquisition is induced and that when it is deterred,

$$V(\kappa) \equiv \mathbb{E}(u^P(\mathbb{C}^{CT}, \kappa)) - \mathbb{E}(u^P(\mathbb{C}^{DI}, \kappa)).$$

It is equivalent to the expected improvement in efficiency minus the expected net information rent given to the agent to incentivize information acquisition,

$$V(\kappa) = \underbrace{\int_0^{\bar{\theta}} q^{CT}(\theta) - c(q^{CT}(\theta), \theta) dF(\theta) - \int_0^{\bar{\theta}} q(e^{DI}, \theta) - e^{DI} dF(\theta)}_{Expected\, Improvement\, in\, Efficiency}$$
$$- \underbrace{\left[\int_0^{\bar{\theta}} u^A(t^{CT}(q^{CT}(\theta)), q^{CT}(\theta), \kappa) dF(\theta) - \int_0^{\bar{\theta}} u^A(t^{DI}(q(e^{DI}, \theta)), \kappa) dF(\theta) \right]}_{Expected\, Net\, Information\, Rent}.$$

For an agent with the cost of information acquisition κ, if information is crucial, in the sense that the principal benefits more from an improvement in efficiency relative to the net information rent, to motivate the agent to acquire and use the information, the principal finds it optimal to induce information acquisition and to implement adverse selection at the production stage. Otherwise, it is optimal for her to deter information acquisition to avoid a high net information rent and to implement moral hazard at the production stage, at the expense of efficiency. The principal's information management and endogenous implementation of the incentive problem exhibits a rent-efficiency tradeoff. In a standard adverse selection problem, efficient production from the inefficient types of agent is traded off to save on information rent given to the efficient types of agent, and in the scope of information management, the efficient use of information is traded off to save on rent given to the agent obtaining such information.

Straightforward from the optimization problem of the principal, $V_\kappa(\kappa) < 0$. When inducing information acquisition, the principal's payoff, $\mathbb{E}(u^P(\mathbb{C}^{CT}, \kappa))$, is diminishing in κ for $\kappa > \kappa^a$, since constraint (II) becomes more restrictive as κ increases. When deterring information acquisition, her payoff, $\mathbb{E}(u^P(\mathbb{C}^{DI}, \kappa))$, is increasing in κ for $\kappa < \kappa^q$, because constraint (DI) is relaxed as κ increases. In addition, for $\kappa \to 0$, the principal earns second-best payoff when she induces the agent to acquire information, and she can only deter information acquisition by an extremely low-powered transfer scheme, i.e., effort is distorted far away from the efficient level. For $\kappa \to \infty$, the principal earns second-best payoff when she deters the agent from acquiring information, and if she intends to induce information acquisition, the information rent goes to infinity. Proposition 5 is thus obtained.

Proposition 5. *There exists* $0 < \kappa^I < \infty$ *such that for* $\kappa < \kappa^I$*, improvement in efficiency exceeds the net information rent, and it is optimal to induce information* *acquisition and implement adverse selection at the production stage; for $\kappa > \kappa^I$, improvement in efficiency falls short of the net information rent, and it is optimal to deter information acquisition and implement moral hazard at the production stage.*

Application: Expert and Mediocre Agent in Production. Interpreting the cost of information acquisition as the agent's expertise in this field, an "expert" is able to acquire information at a sufficiently low cost, while a "mediocre agent" is able to acquire information at a sufficiently high cost. The principal finds it optimal to induce an expert to acquire productive information and to implement adverse selection at the production stage because by acquiring this information, the improvement in efficiency is more significant than that in the net information rent. Contracting with a mediocre agent, the principal finds it optimal to deter him from acquiring information and to implement moral hazard at the production stage to avoid a significantly large information rent at the expense of efficiency.

In terms of the contractual form, if $\kappa^I \in (\kappa^a, \kappa^q)$, it is optimal to induce an agent with extremely high expertise to acquire information with the second-best menu contract for $\kappa \leq \kappa^a$, and with the menu contract arbitrarily close to the second-best for $\kappa^a < \kappa \leq \kappa^b$. As for an agent with high expertise ($\kappa^a < \kappa \leq \kappa^I$), it is optimal to induce information acquisition with a menu contract implementing a steeper information rent, $\mathbb{C}^{CT}$, in which $q^{CT}(\theta) \geq q^{SM}(\theta)$ for $\theta > \hat{\theta}$ and $q^{CT}(\theta) \leq q^{SM}(\theta)$ for $\theta \leq \hat{\theta}$, if Condition 1 is in place, or with a higher powered menu contract, $\mathbb{C}^{II}$, in which $q^{II}(\theta) \geq q^{SM}(\theta)$, if Condition 1 is relaxed. For an agent with mild expertise ($\kappa^I < \kappa < \kappa^q$), it is optimal to deter information acquisition with a debt-with-equity-share contract, $\mathbb{C}^{DI}$, in which $\underline{q}^{DI} < \underline{q}^{SD}$ and $t^{DI}(q) \leq q - \underline{q}^{DI}$ for $q \geq \underline{q}^{DI}$. Finally, deterring information acquisition with a second-best debt contract, $\mathbb{C}^{SD}$, is optimal for an agent with poor expertise ($\kappa \geq \kappa^q$). However, the level of κ^I depends on the exact functional form and the distribution of the state of nature, and is not guaranteed to be within the above-mentioned interval. If $\kappa^I \leq \kappa^a$, interval (κ^a, κ^I) is empty, and if $\kappa^I \geq \kappa^a$, interval (κ^I, κ^q) is empty. For example, given production function $q(e, \theta) = \sqrt{\theta e}$ and $\theta \sim Unif(0, \bar{\theta})$, a modified menu contract is never optimal if $\bar{\theta}$ is sufficiently low, i.e. if information on the state of nature does not improve efficiency significantly relative to the net information rent, and a debt-with-equity-share contract is never optimal if $\bar{\theta}$ is sufficiently high, where information is crucial in production.

Application: Investment Banking. One can also apply the model I present here to address the agency problems in investment banking, where an investment bank (the principal) makes decisions on funding a project executed by a private firm (the agent), the profitability of which depends on the firm's non-observable investment (human and physical capital) and stochastic market conditions. Before accepting the contract, the firm can conduct market investigation (information acquisition)[13] at a sunk cost.

The cost of market investigation may be related to the characteristics of the market where the firm participates, such as market transparency, or whether the market is a newly formed or a well-established market. If the investment bank is contracting with a firm in a well-established market with a high level of transparency, the firm can collect data and past experiences at a sufficiently low cost. It is optimal for the investment bank to offer a menu of funding options that induce the firm to conduct market investigations prior to acceptance. If the investment bank is contracting with a firm in a newly formed market or in a market with a low level of transparency, data and past experience is limited or sufficiently costly for the firm to acquire. It is optimal for the investment bank to propose a state-independent debt-with-equity-share contract, such that the firm is deterred from conducting market investigation.

I am aware of the complexity of the real investment banking industry compared with that used in this model. For example, there is more competition among investment banks and firms instead of a simple principal-agent relationship, the investment bank itself may acquire information as well, and there may also be a regulator involved. Nonetheless, this model serves as a benchmark for more sophisticated studies in which the incentive problem is optimally chosen with information management.

6 Extensions

In the main article, two assumptions are given regarding the information acquiring technology: information acquisition is perfect, with the cost to conduct it being common knowledge. In this section, I relax each of the assumptions respectively, and discuss the robustness of the contractual form derived in previous sections.

6.1 Imperfect Information Acquisition

To emphasize the difference between perfect and imperfect information acquisition, I examine here the case with an interior solution of information management, assuming that $d(\kappa, a)$ has $d_a(\kappa, a) \geq 0$, with equality at $a = 0$, $d_{aa}(\kappa, a) > 0$, $d_a(\kappa, 1) \to \infty$

[13] I restrict information acquisition to only market investigation for explanatory convenience. Information acquisition by the private firm may also include internal investigation such as management and production audit.

for all κ. That is, in equilibrium, the incentive problem at the production stage is stochastic, whose density is implemented by the contracts offered. Denote the contracts $\mathbb{C}^I = \{q^I(\theta), t^I(\theta)\}$ to an informed agent and $\mathbb{C}^U = \{t^U(q)\}$ to an uninformed agent.

Given a, if the agent observes productivity signal and is induced to reveal it truthfully, he has information advantage at the production stage and earns information rent $u^I(\theta) = t^I(\theta) - c(q^I(\theta), \theta)$. If he does not observe any signal, output is an imperfect measurement of his productive effort and he earns $u^U(\theta) \equiv t^U(q(e^U, \theta)) - e^U$, where e^U is the implemented effort by $\mathbb{C}^U$. The optimal investment in information acquisition is thus $a \in \arg\max_{a'} a' \int_0^{\overline{\theta}} u^I(\theta)dF(\theta) + (1-a') \int_0^{\overline{\theta}} u^U(\theta)dF(\theta) - d(\kappa, a')$, or by the first order condition,

$$\int_0^{\overline{\theta}} u^I(\theta)dF(\theta) - \int_0^{\overline{\theta}} u^U(\theta)dF(\theta) = d_a(\kappa, a) \qquad (A).$$

Information on productivity is not the only information advantage the agent enjoys, however. Whether the agent observes a correct signal or nothing is also his private information. The feasible contracts $\{\mathbb{C}^I, \mathbb{C}^U\}$ are designed such that an informed agent prefers $\mathbb{C}^I$ and the uninformed finds $\mathbb{C}^U$ more attractive. Respectively,

$$u^I(\theta) \geq \max_e t^U(q(e,\theta)) - e \qquad \forall \theta \in [0, \overline{\theta}] \qquad (TT_I)$$

and

$$\int_0^{\overline{\theta}} u^U(\theta)dF(\theta) \geq \max_e \int_0^{\overline{\theta}} t^I(q^I(e,\theta))dF(\theta) - e \qquad (TT_U).$$

Provided that the principal can offer a contract to the informed agent in the same fashion as that in Proposition 2, (TT_U) is reduced to the individual rationality constraint of the uninformed mediocre, which must hold given limited liability satisfied. What matters to the principal is the informed agent's motive to pretend to be uninformed by taking $\mathbb{C}^U$.

Lemma 4. *If (TT_I) is binding for some states of nature, it is binding at θ^T, $\tilde{\theta} < \theta^T \leq \overline{\theta}$, where $\tilde{\theta}$ is such that $e(\theta) \in \max_e t^U(q(e,\theta)) - c(e) = 0$ for $\theta < \tilde{\theta}$.*

Proof. Appendix A.4. □

Adjusting notation accordingly for (LIC_θ), (M), (IC), (LL), (NDP), and (R),

$\{\mathbb{C}^I, \mathbb{C}^U\}$ solves the following program to implement imperfect information acquisition,

$$\mathscr{P}_M : \quad \max_{q^I(\theta), t^I(\theta), e^U, t^U(q), a} a \int_0^{\overline{\theta}} q^I(\theta) - t^I(q^I(\theta))dF(\theta) + (1-a) \int_0^{\overline{\theta}} q(e^U, \theta) - t^U(q(e^U, \theta))dF(\theta)$$

subject to

$$(LIC_\theta),\ (M),\ (R),\ (IC),\ (LL),\ (NDP),\ (A),\ (TT_I).$$

Proposition 6. *Optimal contract $\{\mathbb{C}^I, \mathbb{C}^U\}$ with imperfect information acquisition has the following properties*

1. *If $\theta^T < \overline{\theta}$, there exists an interval (θ^a, θ^b) containing θ^T such that $q^I_\theta(\theta) = 0$ for $\theta \in (\theta^a, \theta^b)$. $q^I(\theta) = q^{SM}(\theta)$ for $\theta \geq \theta^b$, and $q^I(\theta) > q^{SM}(\theta)$ for $\theta < \theta^a$.*
2. *$t^U(q)$ has $t^U(q) = 0$ for $q \leq \underline{q}^U$ and $t^U(q) \leq q - \underline{q}^U$ for $q > \underline{q}^U$.*

Proof. Appendix A.4. □

As the agent has private information in whether a correct signal or a null signal is observed, the optimal contract in comparison to the second best[14] incorporates this dimension of truthful revelation. An informed agent in $\theta < \tilde{\theta}$ has no attempt to pretend to be uninformed and give up his rent. Thus, to induce truthful revelation of receiving a correct signal, an equity share of output residual in $t^U(q)$ is offered in equilibrium to limit an informed agent's ability to extract rent by claiming to be uninformed. $q^I(\theta)$ for $\theta < \theta^T$ is raised to give an informed agent a higher rent so that it is more costly for him to pretend uninformed, which violates monotonicity near θ^T. Pooled output schedule is then optimal for some intermediate states of nature containing θ^T.

I thus conclude the qualitative robustness of the debt-with-equity-share contract in $\mathbb{C}^U$, with a higher equity share of output residual to the principal. With imperfect information acquisition, the agent's equity share of output residual is further lowered to deter the informed agent from claiming to be uninformed. The pooled output schedule for intermediate states of nature in $\mathbb{C}^I$ is attributed to the joint effect of truthful revelation of being informed of states $\theta \in (\theta^a, \theta^b)$ and the monotonicity constraint. The former technically resembles the type-dependent participation constraints that

[14]The second best here is referred to the one with symmetric information on whether information is realized imperfectly. I find it more persuasive to compare the optimal contract to this second best instead of the one with perfect signal, as the latter includes the effect of information management and that of a possible null signal.

generate countervailing incentives. In fact, the contract designed for an uninformed agent is itself a type-dependent alternative for an informed agent.[15]

6.2 Private Cost of Information Acquisition

I have adopted the assumption of common knowledge in the cost of information acquisition. It is not surprising that this cost, interpreted as the agent's expertise, may also be the agent's private information. Consider perfect information acquisition as assumed throughout the paper except in Section 6.1. For ease of illustration, let $\kappa \in \{\kappa^L, \kappa^H\}$, $\kappa^L < \kappa^I < \kappa^H$, $\kappa = \kappa^L$ with probability k. Under common knowledge of κ, the principal finds it optimal to implement adverse selection by inducing the agent of κ^L to acquire information, and to implement moral hazard by deterring the agent of κ^H from acquiring information.

If κ is private knowledge of the agent, the principal design a pair of contract $\{\mathbb{C}^I, \mathbb{C}^U\}$, where $\mathbb{C}^I = \{q^I(\theta), t^I(\theta)\}$ is designed to induce the agent of κ^L to acquire and reveal information truthfully, and $\mathbb{C}^U = \{t^U(q)\}$ is designed to keep the agent of κ^H uninformed and motivated to exert effort, and that the agent voluntarily reveal his cost of information acquisition. In addition to the incentive compatibility, individual rationality, inducing information acquisition, and deterring information acquisition constraints in Sections 3 and 4, the pair of contracts satisfies

$$u^I(\theta) \geq \max_e t^U(q(e,\theta)) - e \qquad \forall \theta \in [0, \overline{\theta}] \qquad (TT_I)$$

and

$$\int_0^{\overline{\theta}} u^U(\theta) dF(\theta) \geq \max_e \int_0^{\overline{\theta}} t^I(q^I(e,\theta)) dF(\theta) - e \qquad (TT_U)$$

as in Section 6.1. Provided that the principal can offer a contract to the informed agent in the same fashion as that in Proposition 2, (TT_U) is reduced to the individual rationality constraint of the uninformed mediocre, which must hold given limited liability satisfied. Thus, what matters to the principal is to deter the agent who has acquired and learned information from lying to be uninformed by accepting $\mathbb{C}^U$.[16] The

[15] Please refer to Lewis and Sappington (1989) for a pioneer work and to Jullien (2000) for a general discussion of countervailing incentives. Lemma 4 and Proposition 5 here can be regarded as a justification for the presence of countervailing incentive even with type-independent reservation payoff. However, it does not perfectly coincide with Lewis and Sappington (1989) and Jullien (2000), as the "type-dependent reservation payoff" for an informed agent here depends on the principal's endogenous choice of contract to an uninformed agent.

[16] An even more realistic assumption may be that the more productive agent is also the one able to acquire information at a lower cost. Suppose the simplest possible case under this assumption:

principal's optimization program is then

$$\mathscr{P}_p: \quad \max_{q^I(\theta),t^I(\theta),e^U,t^U(q)} k\int_0^{\overline{\theta}} q^I(\theta) - t^I(q^I(\theta))dF(\theta) + (1-k)\int_0^{\overline{\theta}} q(e^U,\theta) - t^U(q(e^U,\theta))dF(\theta)$$

subject to

$$(LIC_\theta),\ (M),\ (R),\ (IC),\ (LL),\ (NDP),\ (II),\ (DI),\ (TT_I).$$

Proposition 7. *A contract to deter information acquisition with lower debt and derived in Section 4 is qualitatively robust to private knowledge of κ, and the optimal menu contract to induce information acquisition has $q^I_\theta(\theta) = 0$ for $\theta \in (\theta^c, \theta^d)$, when κ is the agent's private information.*

Proof. As shown in Appendix A.4, a binding (TT_I) further lowers the agent's equity share of output residual in $\mathbb{C}^U$ from the second-best $\mathbb{C}^{SD}$. The result predicted in Section 4 is re-enforced with asymmetric information on the cost of information acquisition. As pointed out in Section 6.1, (TT_I) technically resembles a θ-dependent reservation payoff that generates countervailing incentives, and, along with monotonicity constraint, results in pooled output schedule in intermediate states of nature for the informed agent. □

7 Conclusion

The main insights of this paper involve the treatment of the two polar incentive problems as equilibrium responses via information management, and the optimal contract to implement the equilibrium incentive problem. Model-wise, this brings the two polar incentive problems under a unified framework. In addition, this study fills a gap in the literature in which abundant analysis is focused on how existing incentive problems affect equilibrium outcomes, but little is said regarding how such incentive problems arise and how the optimal contract responds respectively to its emergence.

$\kappa(\theta) = \kappa^H$ for $\theta \in [0, m)$ and $\kappa(\theta) = \kappa^L$ for $\theta \in [m, \overline{\theta}]$, $0 < m < \overline{\theta}$. The principal's optimization problem would then be very similar to the one in this section, except that the contract designed for the informed agent and the one designed for the uninformed agent are optimized over a different set of states of nature.

The model presented in this paper is ready to be extended in several new directions. One drawback of the present model is that, given the assumed information acquiring technology, the two incentive problems are substitutes in equilibrium, which fails to explain the possible co-existence of the two incentive problems. Information acquiring effort that generates a noisy signal, which is communicated from the agent to the principal through a menu of contingent transfers, may be a more sophisticated way to model the interaction between information management and implementation of the incentive problems, yet at the expense of model complexity, as output options in the menu contract cannot be made singletons. Another caveat is that the first-order approach in the paper is assumed and may affect the generality of the result in cases where such approach is not valid. I only consider the agent to acquire information, implicitly assuming that it is impossible or infinitely costly for the principal to acquire information. Relaxing this assumption, one can incorporate into the model the principal's decision regarding whether to delegate information acquisition to the agent or whether to acquire information by herself and communicate such information to the agent. This expands the support of the endogenous incentive problem within the contractual relationship to include the possibility of an informed principal. A static contractual relationship was assumed throughout this paper, and the timing of information acquisition is exogenously given. It would be interesting to extend the model to a dynamic contracting relationship in which the timing of information acquisition is endogenously implemented and the cost of information acquisition diminishes in time as partial information may be freely observed by the agent throughout the production process.

From an empirical standpoint, I suggested the importance of identifying the cost of information acquisition as well as the essential incentive problem(s) in empirical tests on contract theory. The incentive problem within the contractual relationship is an equilibrium response, and empirical research in which it is assumed exogenously may fail to identify the true underlying incentive problem and thus generate bias conclusions on some occasions. Specifically, in a scenario in which the contractible variable depends on a stochastic and a choice variable and information on the former is acquirable at a cost, such as production, employment relationships, and investment banking, identification of the cost of information acquisition is more likely to play an important role in the analysis because it sheds light on the equilibrium incentive problem and the form of the contract. For scenarios in which information on the stochastic state of nature is almost costless to acquire, such as a buyer's preference in a trade contract after the object is produced, or situations in which information on the stochastic state of nature is extremely costly or almost impossible to acquire, such as accident insurance, assuming the source of incentive problem from the outset may benefit the researcher for its simplicity.

Appendices

A Proof of Propositions

A.1 Proof of Proposition 1 and Proposition 2

The agent's utility $u^A(t(\theta'), q(\theta'), \theta)$ satisfies the single crossing property, as $\frac{d(-\frac{u_q}{u_t})}{d\theta} = \frac{dc_q}{d\theta} = \frac{-q_{e\theta}}{q_e^2} < 0$, i.e., the marginal cost of output, relative to the marginal utility of transfer, decreases in θ. (IC_θ) can then be replaced by the local incentive compatibility constraint (LIC_θ) and the monotonicity constraint (M). The principal's optimization program to induce information acquisition is thus

$$\mathscr{P}_{II}: \quad \max_{t(\theta),q(\theta)} \mathbb{E}(u^P(q(\theta), t(\theta))) = \int_0^{\bar{\theta}} (q(\theta) - t(\theta))\, dF(\theta)$$

subject to

$$t(\theta) - c(q(\theta), \theta) \geq 0 \qquad (IR_\theta),$$

$$t_\theta(\theta) - c_q(q(\theta), \theta) q_\theta(\theta) = 0 \; \forall \theta \in [0, \bar{\theta}] \qquad (LIC_\theta),$$

$$q_\theta(\theta) \geq 0 \qquad (M),$$

$$\int_0^{\bar{\theta}} (t(\theta) - c(q(\theta), \theta))\, dF(\theta) - \kappa \geq \max_e \int_0^{\bar{\theta}} (t(q(e, \theta)) - e)\, dF(\theta) \qquad (II).$$

Subscripts stand for partial derivatives.

Let $u^A(\theta) = \max_y t(y) - c(q(y), \theta) = t(\theta) - c(q(\theta), \theta)$. $u^A_\theta(\theta) = -c_\theta(q(\theta), \theta) > 0$ by envelop theorem. Taking integral and by binding (IR_0), $u^A(\theta) = \int_0^\theta -c_\theta(q(x), x)dx$. Plug $t(\theta) = u^A(\theta) + c(q(\theta), \theta)$ into $\mathbb{E}(u^P(q(\theta), t(\theta)))$ and rearrange by integration by parts,

$$\mathbb{E}(u^P(q(\theta), t(\theta))) = \int_0^{\bar{\theta}} (q(\theta) - c(q(\theta), \theta))\, dF(\theta) - \int_0^{\bar{\theta}} \left(\frac{1 - F(\theta)}{f(\theta)} (-c_\theta(q(\theta), \theta)) \right) dF(\theta).$$

1. Proposition 1

First, restrict attention to a contract with continuous transfer to induce information acquisition. Let $u^A(\hat{\theta}) = t(\hat{\theta}) - c(q(\hat{\theta}), \hat{\theta}) = \max_e \int_0^{\bar{\theta}} (t(q(e,\theta)) - e)\, dF(\theta)$, the certainty equivalence of the right hand side of (II), then (II) can be rewritten as

$$\int_0^{\bar{\theta}} \left(\frac{\mathbf{1}_{\theta > \hat{\theta}} - F(\theta)}{f(\theta)} (-c_\theta(q(\theta), \theta)) \right) dF(\theta) \geq \kappa.$$

The principal's reduced program is thus

$$\mathscr{P}_{CT}: \quad \max_{q(\theta)} \int_0^{\bar{\theta}} \left(q(\theta) - c(q(\theta), \theta) - \frac{1 - F(\theta)}{f(\theta)} (-c_\theta(q(\theta), \theta)) \right) dF(\theta)$$

subject to

$$\int_0^{\bar{\theta}} \left(\frac{\mathbf{1}_{\theta > \hat{\theta}} - F(\theta)}{f(\theta)} (-c_\theta(q(\theta), \theta)) \right) dF(\theta) \geq \kappa \qquad (II).$$

Let λ be the Lagrange multiplier for (II), $q^{CT}(\theta)$ solves

$$\left((1 - c_q(q(\theta), \theta)) - \frac{1 - F(\theta)}{f(\theta)} (-c_{\theta q}(q(\theta), \theta)) \right) + \lambda \frac{\mathbf{1}_{\theta > \hat{\theta}} - F(\theta)}{f(\theta)} (-c_{\theta q}(q(\theta), \theta)) = 0.$$

For $\kappa < \kappa^a$, where $\kappa^a \equiv \lim_{q(\theta) \to q^{SM}(\theta)} \int_0^{\bar{\theta}} (\mathbf{1}_{\theta > \hat{\theta}} - F(\theta))(-c_\theta(q(\theta), \theta)) d\theta$, (II) slacks and the principal is able to induce information acquisition with the second-best menu contract $\mathbb{C}^{SM} = \{t^{SM}(\theta), q^{SM}(\theta)\}$, and $\lambda = 0$. Note that $\frac{1-F(\theta)}{f(\theta)}(-c_{\theta q}(q(\theta), \theta)) = \frac{\partial}{\partial q(\theta)} \left(\frac{1-F(\theta)}{f(\theta)} \frac{q_\theta(c(\theta, q(\theta)), \theta)}{q_e(c(\theta, q(\theta)), \theta)} \right) > 0$, which is decreasing in θ by Assumption 2, so the monotonicity constraint is strictly satisfied.

For $\kappa > \kappa^a$, (II) is violated given the second-best menu contract. $\lambda > 0$ in equilibrium. Along with $-c_{\theta q}(q(\theta), \theta) > 0$, the optimal contract with continuous non-decreasing transfer to induce information acquisition $\mathbb{C}^{CT} = \{t^{CT}(\theta), q^{CT}(\theta)\}$ is such that $q^{CT}(\theta) \geq q^{SM}(\theta)$ for $\theta > \hat{\theta}$, with equality at $\bar{\theta}$, and $q^{CT}(\theta) \leq q^{SM}(\theta)$ for $\theta \leq \hat{\theta}$, with equality at 0. In addition, claim that λ is increasing in κ. Implementing $q(\theta)$ is equivalent to implementing $-c_\theta(q(\theta), \theta)$ for the principal. Expressing the problem in terms of choosing $-c_\theta(q(\theta), \theta)$ instead of $q(\theta)$, the principal's objective function to induce information acquisition is concave in $-c_\theta(q(\theta), \theta)$, with constraint (II) linear in both $-c_\theta(q(\theta), \theta)$ and κ. The principal's optimal payoff to induce information acquisition is thus diminishing and concave in κ, implying that λ is increasing in κ. For intermediate κ such that $\lambda > 0$ is not too large to violate (M), the strictly separating contract is optimal. For sufficiently high κ such that λ is sufficiently large to violate (M), the contract has pooling for $\theta > \hat{\theta}$ at $q^{CT}(\overline{\theta})$ and for $\theta \leq \hat{\theta}$ at $q^{CT}(0)$, respectively.

2. Proposition 2

Proposing the contract

$$t(q) = \begin{cases} t(\theta) & if \quad q(e,\theta) \in \mathbb{Q} \\ 0 & if \quad q(e,\theta) \notin \mathbb{Q} \end{cases}$$

and

$$q(\theta) \in \mathbb{Q},$$

the principal's reduced program to induce information acquisition becomes

$$\mathscr{P}_{II}: \quad \max_{q(\theta)} \int_0^{\bar{\theta}} \left(q(\theta) - c(q(\theta),\theta) - \frac{1-F(\theta)}{f(\theta)}(-c_\theta(q(\theta),\theta)) \right) dF(\theta)$$

subject to

$$\int_0^{\bar{\theta}} \left(\frac{1-F(\theta)}{f(\theta)}(-c_\theta(q(\theta),\theta)) \right) dF(\theta) - \kappa \geq 0 \qquad (II^{'}),$$

$$q_\theta(\theta) \geq 0 \qquad (M),$$

and

$$q(\theta) \in \mathbb{Q}. \qquad (R)$$

Let $\lambda^{'}$ be the Lagrange multiplier for $(II^{'})$. Neglecting constraints(M) and (R) for now, $q^{II}(\theta)$ solves

$$\left((1 - c_q(q(\theta),\theta)) - \frac{1-F(\theta)}{f(\theta)}(-c_{\theta q}(q(\theta),\theta)) \right) + \lambda^{'} \frac{1-F(\theta)}{f(\theta)}(-c_{\theta q}(q(\theta),\theta)) = 0.$$

For a cost of information acquisition such that $(II^{'})$ slacks and (II) is violated under the second-best, $\lambda^{'} = 0$. For any solution of $(1-c_q(q(\theta),\theta))-\frac{1-F(\theta)}{f(\theta)}(-c_{\theta q}(q(\theta),\theta)) = 0$ that is not a rational number, there is an arbitrarily close rational number. The optimal contract to induce information acquisition thus has an output schedule that is arbitrarily close to the second-best. Denote this contract as $\mathbb{C}^{AS} = \{t^{AS}(q), q^{AS}(\theta)\}$. Define $\kappa^b \equiv \lim_{q(\theta)\to q^{AS}(\theta)} \int_0^{\bar{\theta}} (1-F(\theta))(-c_\theta(q(\theta),\theta))d\theta$.

For $\kappa > \kappa^b$, $(II^{'})$ is binding. First claim that λ is increasing in κ. Implementing $q(\theta)$

is equivalent to implementing $-c_\theta(q(\theta),\theta)$ for the principal. Expressing the problem in terms of choosing $-c_\theta(q(\theta),\theta)$ instead of $q(\theta)$, the principal's objective function to induce information acquisition is concave in $-c_\theta(q(\theta),\theta)$, with constraint (II') linear in both $-c_\theta(q(\theta),\theta)$ and κ. The principal's optimal payoff to induce information acquisition is thus diminishing and concave in κ, which implies that λ is increasing in κ. Next, claim that $\lambda' \leq 1$. Suppose that the cost of information acquisition increased by δ. With the optimal contract provided, the principal's equilibrium payoff dropped by $\lambda\delta$. A weakly dominated response to this increment for the principal is to increase the transfer associated with all rational output by δ, which does not violate any constraint. This results in a drop of the principal's payoff by δ. $\lambda'\delta \leq \delta$, and thus $\lambda' \leq 1$. With $0 < \lambda' \leq 1$, (M) holds as $\frac{1-F(\theta)}{f(\theta)}(-c_{\theta q}(q(\theta),\theta)) = \frac{\partial}{\partial q(\theta)}\left(\frac{1-F(\theta)}{f(\theta)}\frac{q_\theta(c(\theta,q(\theta)),\theta)}{q_e(c(\theta,q(\theta)),\theta)}\right)$ is decreasing in θ by Assumption 2. Along with $-c_{\theta q}(q(\theta),\theta) > 0$, the optimal contract to induce information acquisition $\mathbb{C}^{II} = \{t^{II}(q), q^{II}(\theta)\}$ has $q^{FM}(\theta) \geq q^{II}(\theta) \geq q^{SM}(\theta)$, with equality at $\overline{\theta}$.[17]

□

A.2 Proof of Proposition 3 and Corollary 1

$$\mathscr{P}'_{DI}: \quad \max_{s,\underline{q},e} \int_0^{\bar{\theta}} q(e,\theta)dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} s(q(e,\theta) - \underline{q})dF(\theta)$$

subject to

$$0 \leq s \leq 1 \qquad (NDP'),$$

$$\int_{\underline{\theta}}^{\bar{\theta}} sq_e(e,\theta)dF(\theta) - 1 = 0 \qquad (LIC'),$$

$$\int_{\underline{\theta}}^{\bar{\theta}} s(q(e,\theta) - \underline{q})dF(\theta) - e \geq \int_{\tilde{\theta}}^{\bar{\theta}} \left(s(q(e(\theta),\theta) - \underline{q}) - e(\theta)\right) dF(\theta) - \kappa \qquad (DI').$$

Subscripts stand for partial derivatives.

[17] $q^{FM}(\theta)$ denotes the output schedule in the first-best menu contract.

Let the Lagrange function exclusive of $(NDP^{'})$ be

$$\mathscr{L}^{'} = \int_{0}^{\bar{\theta}} q(e,\theta)dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} s(q(e,\theta)-\underline{q})dF(\theta) + \mu^{'}(\int_{\underline{\theta}}^{\bar{\theta}} sq_e(e,\theta)dF(\theta)-1)$$
$$+\phi^{'}\left(\int_{\underline{\theta}}^{\bar{\theta}} s(q(e,\theta)-\underline{q})dF(\theta) - e - \int_{\tilde{\theta}}^{\bar{\theta}} \left(s(q(e(\theta),\theta)-\underline{q}) - e(\theta)\right)dF(\theta)+\kappa\right),$$

where $\mu^{'}$ and $\phi^{'}$ are the Lagrange multipliers associated with $(LIC^{'})$ and $(DI^{'})$, respectively. The optimality conditions of the principal with respect to $\underline{q}$ and s are

$$\frac{\partial \mathscr{L}^{'}}{\partial \underline{q}} \leq 0,$$

with inequality only at $\underline{q} = 0$, and

$$\frac{\partial \mathscr{L}^{'}}{\partial s} \begin{array}{lll} > 0 & if & s=1 \\ = 0 & if & s\in(0,1) \\ < 0 & if & s=0 \end{array},$$

where

$$\frac{\partial \mathscr{L}^{'}}{\partial \underline{q}} = 1-\mu^{'}\left(\frac{q_e(e,\underline{\theta})}{q_\theta(e,\underline{\theta})}\frac{f(\underline{\theta})}{1-F(\underline{\theta})}\right)+\phi^{'}\left(\frac{F(\underline{\theta})-F(\tilde{\theta})}{1-F(\underline{\theta})}\right)$$

and

$$\frac{\partial \mathscr{L}^{'}}{\partial s} = \int_{\underline{\theta}}^{\bar{\theta}}\left(-(q(e,\theta)-\underline{q})+\mu^{'}q_e(e,\theta)\right)dF(\theta)$$
$$+\phi^{'}\left(\int_{\underline{\theta}}^{\bar{\theta}}(q(e,\theta)-\underline{q})dF(\theta)-\int_{\tilde{\theta}}^{\bar{\theta}}(q(e(\theta),\theta)-\underline{q})dF(\theta)\right).$$

As shown in Proposition 4 and in Poblete and Spulber (2012), in the second-best environment in which $\phi^{'}=0$, the optimal contract $\mathbb{C}^{SD}$ is a debt contract, with $\underline{q}^{SD}>0$ and $s^{SD}=1$. Denote the implemented effort by $\mathbb{C}^{SD}$ as e^{SD}. $\mathbb{C}^{SD}$ is sufficient to deter information acquisition if $\kappa \geq \kappa^{q} \equiv \lim_{\underline{q}\to\underline{q}^{SD}} \int_{\tilde{\theta}}^{\bar{\theta}}\left((q(e(\theta),\theta)-\underline{q})-e(\theta)\right)dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}}(q(e^{SD},\theta)-\underline{q})dF(\theta)+e^{SD}$, i.e., the level of information-acquiring cost under which constraint (DI') just binds at $\mathbb{C}^{SD}$.

For $\kappa<\kappa^{q}$, $\phi^{'}>0$. By Lemma 2, $\tilde{\theta}>\underline{\theta}$, so $F(\underline{\theta})-F(\tilde{\theta})<0$. The optimal debt contract to deter information acquisition has $0\leq \underline{q}^{DI}<\underline{q}^{SD}$. Note that $\underline{q}^{SD}>0$ and $s^{SD}=1$ implies $\frac{\partial \mathscr{L}^{'}}{\partial \underline{q}}=0$ and $\frac{\partial \mathscr{L}^{'}}{\partial s}>0$ when $\kappa\geq\kappa^{q}$. Thus, slightly lowering $\underline{q}^{DI}$

from $\underline{q}^{SD}$ has only a second order effect on the principal's payoff while lowering s^{DI} from $s^{SD} = 1$ has a first order effect. In addition, the principal's problem to deter information acquisition is concave in $\underline{q}$ and in s under certain assumptions, as will be shown in the next paragraph, and constraint (DI') is linear in κ. The principal's optimal payoff to deter information acquisition is thus increasing and concave in κ, implying that ϕ' is decreasing in κ. Thus, there exists $\kappa^s < \kappa^q$ such that for $\kappa > \kappa^s$, the principal is able to deter information acquisition with $s^{DI} = s^{SD} = 1$. Define $\kappa^s \equiv \lim_{s \to 1} \int_{\tilde{\theta}}^{\bar{\theta}} \left(s(q(e(\theta), \theta) - \underline{q}^s) - e(\theta) \right) dF(\theta) - \int_{\underline{\theta}^s}^{\bar{\theta}} s(q(e^s, \theta) - \underline{q}^s) dF(\theta) + e^s$, with superscript s denoting the level of choice variables at which $\frac{\partial \mathscr{L}'}{\partial s} \geq 0$ is just binding. Let $\Omega(s) \equiv -\int_{\tilde{\theta}}^{\bar{\theta}} (q(e(\theta), \theta) - \underline{q}) dF(\theta)$, then $\Omega_s(s) = (\tilde{q} - \underline{q}) f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial s} < 0$ given Lemma 2. Thus, for $\kappa < \kappa^s$, the optimal contract to deter information acquisition has $s^{DI} < s^{SD} = 1$, solving $\left. \frac{\partial \mathscr{L}'}{\partial s} \right|_{\underline{q} = \underline{q}^{DI}} = 0$.

The solution is optimal if $\frac{\partial^2 \mathscr{L}'}{\partial \underline{q}^2} < 0$ and $\frac{\partial^2 \mathscr{L}'}{\partial s^2} < 0$. For $\frac{\partial \rho(e, \underline{\theta})}{\partial \underline{\theta}} \frac{\partial \underline{\theta}}{\partial \underline{q}} = \frac{\partial \rho(e, \underline{\theta})}{\partial \underline{\theta}} \frac{1}{q_\theta(e, \underline{\theta})} > 0$, $\frac{\partial^2 \mathscr{L}'}{\partial \underline{q}^2} < 0$ if $\left(f(\underline{\theta}) \frac{\partial \underline{\theta}}{\partial \underline{q}} - f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial \underline{q}} \right) (1 - F(\underline{\theta})) + \left(F(\underline{\theta}) - F(\tilde{\theta}) \right) f(\underline{\theta}) \frac{\partial \underline{\theta}}{\partial \underline{q}} = (1 - F(\tilde{\theta})) \frac{f(\underline{\theta})}{q_\theta(e, \underline{\theta})} - (1 - F(\underline{\theta})) \frac{f(\tilde{\theta})}{q_\theta(e(\tilde{\theta}), \tilde{\theta})} \leq 0$. Multiply both sides of the inequality by $\frac{q_\theta(e, \underline{\theta}) q_\theta(e(\tilde{\theta}), \tilde{\theta})}{f(\underline{\theta}) f(\tilde{\theta})} > 0$ yields the sufficient condition $\frac{1 - F(\tilde{\theta})}{f(\tilde{\theta})} q_\theta(e(\tilde{\theta}), \tilde{\theta}) - \frac{1 - F(\underline{\theta})}{f(\underline{\theta})} q_\theta(e, \underline{\theta}) \leq 0$. If the monotone hazard rate holds, $\frac{1 - F(\tilde{\theta})}{f(\tilde{\theta})} \leq \frac{1 - F(\underline{\theta})}{f(\underline{\theta})}$ as $\tilde{\theta} > \underline{\theta}$ by Lemma 2. Since $q(e, \theta)$ is increasing and concave in θ, $\theta = q^{-1}(e, q)$ is increasing and convex in q, so $q_\theta(e(\tilde{\theta}), \tilde{\theta}) < q_\theta(e, \underline{\theta})$ given $\tilde{q} > \underline{q}$ by Lemma 2. Hence, $\frac{\partial^2 \mathscr{L}'}{\partial \underline{q}^2} < 0$ if $\frac{1 - F(\theta)}{f(\theta)}$ is non-increasing and $q(e, \theta)$ is concave. $\frac{\partial^2 \mathscr{L}'}{\partial s^2} = \phi' \Omega_s(s) < 0$.

For $\kappa > \kappa^s$, $e^{DI} \in \arg\max_e \int_{\underline{\theta}}^{\bar{\theta}} \left(q(e, \theta) - \underline{q} \right) F(\theta) - e$, where $\underline{q} \equiv q(e, \underline{\theta})$, or equivalently, $\underline{\theta} = q^{-1}(e, \underline{q})$. The first order derivative with respect to e has $\int_{q^{-1}(e, \underline{q})}^{\bar{\theta}} q_e(e, \theta) dF(\theta) - 1$, which is decreasing in $\underline{q}$ as $-q_e(e, \underline{\theta}) f(\underline{\theta}) \frac{\partial q^{-1}(e, \underline{q})}{\partial \underline{q}} = -\frac{q_e(e, \underline{\theta})}{q_\theta(e, \underline{\theta})} f(\underline{\theta}) < 0$. Thus, with $\underline{q}^{DI} < \underline{q}^{SD}$, $e^{DI} > e^{SD}$. For $\kappa < \kappa^s$, $e^{DI} \in \arg\max_e \int_{\underline{\theta}}^{\bar{\theta}} s \left(q(e, \theta) - \underline{q} \right) F(\theta) - e$. The first order derivative with respect to e has $\int_{q^{-1}(e, \underline{q})}^{\bar{\theta}} s q_e(e, \theta) dF(\theta) - 1$, which is increasing in s as $\int_{q^{-1}(e, \underline{q})}^{\bar{\theta}} q_e(e, \theta) dF(\theta) > 0$. e^{DI} is increasing in s but diminishing in $\underline{q}$. The sign of $e^{DI} - e^{SD}$ is thus ambiguous for $\kappa < \kappa^s$.

□

A.3 Proof of Proposition 4

Suppose that the first-order approach is applicable (Assumption 3), the principal's optimization program is

$$\mathscr{P}_{DI}: \quad \max_{t(q(e,\theta)),e} \int_0^{\overline{\theta}} \left(q(e,\theta) - t(q(e,\theta))\right) dF(\theta)$$

subject to

$$t(q(e,\theta)) \geq 0 \qquad (LL),$$

$$0 \leq t_q(q(e,\theta)) \leq 1 \qquad (NDP),$$

$$\int_0^{\overline{\theta}} t_q(q(e,\theta))q_e(e,\theta)dF(\theta) = 1 \qquad (LIC),$$

$$\int_0^{\overline{\theta}} t(q(e,\theta))dF(\theta) - e \geq \int_0^{\overline{\theta}} \mathbf{1}_{\theta \geq \tilde{\theta}} \left(t(q(e(\theta),\theta)) - e(\theta)\right) dF(\theta) - \kappa \qquad (DI).$$

Subscripts represent partial derivatives. Let μ, and ϕ be the Lagrange multipliers associated to (LIC), and (DI), respectively.

With limited liability, $t(q(e,\theta)) = \int_0^{\theta} t_q(q(e,x))q_\theta(e,x)dx$, and by integration by parts, $\int_0^{\overline{\theta}} t(q(e,\theta))dF(\theta) = \int_0^{\overline{\theta}}(1-F(\theta))t_q(q(e,\theta))q_\theta(e,\theta)d\theta$. By the envelope theorem of an informed agent off the equilibrium path and integration by parts, $t(q(e(\theta),\theta)) - c(e(\theta)) = \int_{\tilde{\theta}}^{\theta} t_q(q(e(x),x))q_\theta(e(x),x)dx$, and $\int_{\tilde{\theta}}^{\overline{\theta}} \left(t(q(e(\theta),\theta)) - e(\theta)\right) dF(\theta) = \int_{\tilde{\theta}}^{\overline{\theta}}(1 - F(\theta))t_q(q(e(\theta),\theta))q_\theta(e(\theta),\theta)d\theta$. The (point-wise) Lagrange function of the principal's problem to deter an agent from acquiring information, excluding (LL) and (NDP), is then written as

$$\begin{aligned}\mathscr{L} = t_q(q(e,\theta)) &\big(-(1-F(\theta))q_\theta(e,\theta) + \mu q_e(e,\theta)f(\theta) + \phi((1-F(\theta))q_\theta(e,\theta) \\ &-\phi \mathbf{1}_{\theta' \geq \tilde{\theta}}(1-F(\theta'))q_\theta(e(\theta'),\theta'))\big) + q(e,\theta)f(\theta) - \mu + \phi(e+\kappa),\end{aligned}$$

where θ' is such that $q(e,\theta) \equiv q(e(\theta'),\theta')$. If κ is sufficiently large that $\phi = 0$, $t_q(q(e,\theta)) = 1$ if $\mu \geq \frac{1-F(\theta)}{f(\theta)}\frac{q_\theta(e,\theta)}{q_e(e,\theta)} \equiv \frac{1}{\rho(e,\theta)}$, $t_q(q(e,\theta)) = 0$ otherwise, since $\mathscr{L}$ is linear in $t_q(q(e,\theta))$. As $\frac{1}{\rho(e,\theta)}$ is decreasing in θ by Assumption 2, the second-best contract is in the form of debt, where $t^{SD}(q) = 0$ for $q \leq \underline{q}^{SD}$, and $t^{SD}(q) = q - \underline{q}^{SD}$ otherwise.

Let the solution to$\mathscr{P}_{DI}$ be $t^{DI}(q)$. If κ is sufficiently small that $\phi > 0$, for $q < \tilde{q} \equiv q(e(\tilde{\theta}),\tilde{\theta})$, i.e. where $\theta' < \tilde{\theta}$, $t_q^{DI}(q) = 1$ if $\mu > \frac{1-\phi}{\rho(e,\theta)}$, $t_q^{DI}(q) = 0$ otherwise. Thus, for $q < \tilde{q}$, $t_q^{DI}(q) = \max\{q - \underline{q}^{DI}, 0\}$, where $\underline{q}^{DI} < \underline{q}^{SD}$. For $q \geq \tilde{q}$, i.e. where $\theta' \geq \tilde{\theta}$, $t_q^{DI}(q) = 1$ if $\mu > \frac{1-\phi}{\rho(e,\theta)} + \phi\frac{1-F(\theta')}{f(\theta)}\frac{q_\theta(e(\theta'),\theta')}{q_e(e,\theta)} = \frac{1-\phi}{\rho(e,\theta)} + \frac{\phi}{\rho(e,\theta)}\left(\frac{(1-F(\theta'))q_\theta(e(\theta'),\theta')}{(1-F(\theta))q_\theta(e,\theta)}\right)$, $t_q^{DI} = 0$ otherwise. Since $\frac{\phi}{\rho(e,\theta)}\left(\frac{(1-F(\theta'))q_\theta(e(\theta'),\theta')}{(1-F(\theta))q_\theta(e,\theta)}\right) > 0$, $t^{DI}(q) < q - \underline{q}^{DI}$ for $q \geq \tilde{q}$.

□

A.4 Proof of Lemma 4 and Proposition 6

$u^I(\theta) = \int_0^\theta -\hat{c}_\theta(q^I(x), x)dx + u^I(0)$. The principal's optimization program is then

$$\mathscr{P}_M : \quad \max_{q^I(\theta), e^U, t^U(q), a} a \left(\int_0^{\bar{\theta}} q^I(\theta) - c(q^I(\theta), \theta) - \frac{1-F(\theta)}{f(\theta)}(-c_\theta(q^I(\theta), \theta))dF(\theta) - u^I(0) \right)$$
$$+ (1-a) \left(\int_0^{\bar{\theta}} q(e^U, \theta) - t^U(q(e^U, \theta))dF(\theta) \right)$$

subject to

$$t^U(q(e^U, \theta) \geq 0 \qquad (LL),$$
$$0 \leq t^U_q(q) \leq 1 \qquad (NDP),$$
$$q^I_\theta(\theta) \geq 0 \qquad (M),$$
$$q^I(\theta) \in \mathbb{Q} \qquad (R)$$
$$\int_0^{\bar{\theta}} t^U_q(q(e^U, \theta))q_e(e^U, \theta)dF(\theta) = 1 \qquad (LIC_U),$$
$$\int_0^{\bar{\theta}} u^I(\theta)dF(\theta) - \int_0^{\bar{\theta}} u^U(\theta)dF(\theta) = d_a(\kappa, a) \qquad (A),$$
$$u^I(\theta) \geq \max_e t^U(q(e, \theta)) - e \ \forall \theta \in [0, \bar{\theta}] \qquad (TT_I)$$

Subscripts in the functions stand for derivatives. Let μ, ϕ, $\lambda^I(\theta)$ be the Lagrange multipliers for (LIC_U), (A), (TT_I), respectively.

$q^I(\theta)$ solves the point-wise optimization condition

$$\left(a - ac_q(q(\theta), \theta) - (a - \phi)\frac{1-F(\theta)}{f(\theta)}(-c_{\theta q}(q^I(\theta), \theta)) \right)$$
$$+ \frac{1}{f(\theta)} \int_\theta^{\bar{\theta}} \lambda^I(x)dx(-c_{\theta q}(q^I(\theta), \theta)) = 0$$

and, by similar method as Appendix A.3, $t^U_q(q) = 1$ if

$$\mu > \frac{1 - a + \phi}{\rho(e, \theta)} + \frac{\int_{\theta'}^{\bar{\theta}} \lambda^I(\theta)q_\theta(e(\theta), \theta)d\theta}{q_e(e, \theta)f(\theta)}.$$

Show Lemma 4. Let $\{\hat{\mathbb{C}}^I, \hat{\mathbb{C}}^U\}$ be the optimal contract excluding (TT_I), in which

$t^U_q(q) = 1$ if

$$\mu > \frac{1-a+\phi}{\rho(e,\theta)}.$$

Claim that $1-a+\phi \geq 0$. Suppose that in addition to the optimal contracts, that the principal increases the transfer to the uninformed agent by δ and adjust a downward by η to bind (A), which does not violate any constraint excluding (TT_I). Downward adjustment of a has a second order effect yet increment of transfer has a first order effect. The principal's indirect objective function is then changed by $(-1+a-\phi)\delta \leq 0$ as she is moving from the optimal solution to the suboptimal. As $1-a+\phi \geq 0$, the optimal contingent transfer excluding (TT_I) to an uninformed agent is a debt contract. Thus, given implemented productive effort, there exist $\tilde{\theta}$ such that $e(\theta) \in \max_e t^U(q(e,\theta)) - c(e) = 0$ for $\theta < \tilde{\theta}$. Along with individual rationality of the informed agent, (TT_I) is strictly satisfied for $\theta < \tilde{\theta}$. Hence, if $\hat{u}^I(\theta)$ is sufficiently convex such that (TT_I) is violated for some $\theta \in [\theta_1, \theta_2]$, it is where $\theta_1 > \tilde{\theta}$ and $\theta_2 \leq \overline{\theta}$. To deter an informed agent in states $\theta \in (\theta_1, \theta_2)$ from lying to be uninformed, there exists θ^T such that $q^I(\theta)$ for $\theta < \theta^T$ are raised to increase the rent in these states. As (TT_I) is strictly satisfied in $\theta \leq \tilde{\theta}$, $\tilde{\theta} < \theta^T \leq \overline{\theta}$.

Given Lemma 4, $q^I(\theta)$ solves

$$\left(a - ac_q(q(\theta),\theta) - (a-\phi)\frac{1-F(\theta)}{f(\theta)}(-c_{\theta q}(q^I(\theta),\theta))\right) + \frac{\lambda^I(\theta^T)}{f(\theta)}\mathbf{1}_{\theta\leq\theta^T}(-c_{\theta q}(q^I(\theta),\theta)) = 0$$

and $t^U_q(q) = 1$ if

$$\mu > \frac{1-a+\phi}{\rho_1(e,\theta)} + \lambda^I(\theta^T)\frac{q_\theta(e(\theta'),\theta')}{q_e(e,\theta)f(\theta)}\mathbf{1}_{\theta'\in[\tilde{\theta},\theta^T]}.$$

Show part 1 in Proposition 6. As $-c_{\theta q}(q^I(\theta),\theta) = \frac{\partial}{\partial q(\theta)}\left(\frac{q_\theta(h(\theta,q(\theta)),\theta)}{q_e(h(\theta,q(\theta)),\theta)}\right) > 0$, binding (TT_I) distorts $q^I(\theta)$ upward from $q^{SM}(\theta)$ for $\theta \leq \theta^T$ to prevent an informed agent from pretending to be uninformed, implied by $\frac{\lambda^I(\theta^T)}{f(\theta)}\mathbf{1}_{\theta\leq\theta^T}(-c_{\theta q}(q^I(\theta),\theta)) \geq 0$. If $\theta^T < \overline{\theta}$, monotonicity must be violated near θ^T as a result. Optimal $q^I(\theta)$ thus have $q^I_\theta(\theta) = 0$ for $\theta \in (\theta^a, \theta^b)$, where $\theta^T \in (\theta^a, \theta^b)$. Show part 2 in Proposition 6. For $q \in [\tilde{q}, q^T]$, where $\tilde{q} \equiv q(e(\tilde{\theta}), \tilde{\theta})$ and $q^T \equiv q(e(\theta^T), \theta^T)$, $t^U(q)$ is lowered in the sense that $t^U_q(q) = 1$ for $q(e,\theta) > q_1 > \underline{q}^U$, as $\lambda^I(\theta^T)\frac{q_\theta(e(\theta'),\theta')}{q_e(e,\theta)f(\theta)} > 0$, which violates monotonicity near $\tilde{q}$. Hence, there exist an interval (q^a, q^b) containing $\tilde{q}$, such that $t^U_q(q) = 0$ for $q(e,\theta) \in (q^a, q^b)$. Thus, $t^U(q) \leq q - \underline{q}^U$ for $q > \underline{q}^U$, share of output residual to the agent is reduced.

$\square$

B Deterrence of Information Acquisition with a Risk Averse Agent

Suppose that the agent is risk averse in the realization of transfer, in the sense that $u^A = v(t(q(e,\theta)) - e$, where $v_t(t) > 0$ and $v_{tt}(t) < 0$. The (IC) constraint can be replaced by the local incentive compatibility constraint

$$\int_0^{\bar{\theta}} \left(v_t(t) t_q(q) q_e(e,\theta) - 1\right) dF(\theta) = 0 \qquad (LIC_a)$$

if $v(t)$ is sufficiently concave, and transfer is non-decreasing, $t_q(q) \geq 0$.[18] We assume that the former two hold, along with the following assumption for the second-best contract to be monotonically non-decreasing.

Assumption 4. *$v(t)$ has non-increasing absolute risk aversion, i.e.* $\frac{\partial\left(-\frac{v_{tt}(t)}{v_t(t)}\right)}{\partial t} \leq 0.$

Replacing (IC) by (LIC_a), the principal's optimization program to deter a risk averse agent from information acquisition is

$$\mathscr{P}_a: \quad \max_{t(q),e} \int_0^{\bar{\theta}} \left(q(e,\theta) - t(q(e,\theta))\right) dF(\theta)$$

subject to

$$(IR),\ (LIC_a),\ (DI).$$

How the binding constraint (DI) distort the optimal (non-monotonic) contract, $t^{DI}(q)$, from the second best, $t^{SB}(q)$ is characterized in the following lemma.

Lemma 5. *Implementing e^*,*

1. *for $q(e^*,\theta) < q(e(\tilde{\theta}),\tilde{\theta})$, $t^{DI}(q) > t^{SB}(q)$;*
2. *for $q(e^*,\theta) \geq q(e(\tilde{\theta}),\tilde{\theta})$, $t^{DI}(q) > t^{SB}(q)$ if $f(\theta) > f(\theta')$, and $t^{DI}(q) \leq t^{SB}(q)$ if $f(\theta) \leq f(\theta')$ with equality holds at $f(\theta) = f(\theta')$, where θ' is such that $q(e(\theta'),\theta') \equiv q(e^*,\theta)$;*
3. *there is a downward gap of $t^{DI}(q)$ at $q(e(\tilde{\theta}),\tilde{\theta})$ from the left*

[18] This is straightforward from the second order derivative of the agent's optimization problem.

Proof. Let $\theta' > 0$ is such that $q(e^*, \theta) \equiv q(e(\theta'), \theta')$. For sufficiently small $\theta > 0$, $e(\theta) < e^*$ and $q(e(\theta), \theta) < q(e^*, \theta)$, for sufficiently large θ, $e(\theta) > e^*$ and $q(e(\theta), \theta) > q(e^*, \theta)$, and by $q_{e\theta} > 0$, $\theta' \in (0, \overline{\theta})$ exists. By the first order condition of the principal's point-wise optimization problem with respect to $t(q)$,

$$\frac{1}{v_t(t(q(e^*, \theta))} = \lambda^{IR} + \mu^a \frac{v_{tt}(t(q(e^*, \theta))}{v_t(t(q(e^*, \theta))} t_q(q(e^*, \theta) q_e(e^*, \theta) + \phi^a \left(1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)}\right),$$

where λ^{IR}, μ^a, and ϕ^a are the Lagrange multipliers associated with constraints (IR), (LIC_a), and (DI), respectively. If κ is sufficiently small that $\phi^a > 0$, $1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)} = 1$ for $\theta' < \tilde{\theta}$, i.e., for $q(e^*, \theta) \equiv q(e(\theta'), \theta') < q(e(\tilde{\theta}), \tilde{\theta})$, hence part 1. For $\theta' \geq \tilde{\theta}$, i.e., for $q(e^*, \theta) \equiv q(e(\theta'), \theta') > q(e(\tilde{\theta}), \tilde{\theta})$, $0 < 1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)} < 1$ for $f(\theta) > f(\theta')$, and $1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)} \leq 0$ for $f(\theta) \leq f(\theta')$, with equality holds at $f(\theta) = f(\theta')$, hence part 2. As $1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)} = 1$ for $\theta' < \tilde{\theta}$ and $1 - \mathbf{1}_{\theta' \geq \tilde{\theta}} \frac{f(\theta')}{f(\theta)} < 1$ for $\theta' \geq \tilde{\theta}$, part 3 is straightforward. □

Part 1 in Lemma 5 is intuitive: one motive for the agent to acquire information is to distinguish a sufficiently inefficient state of nature to avoid exerting effort at a loss. Thus, to counter such opportunistic motive, the principal increases the transfer for sufficiently inefficient states of nature, reducing the loss subject to those states. It can also be understood as a reward for not acquiring information to avoid loss in the most inefficient states of nature, as $q(e^*, \theta) < q(e(\tilde{\theta}), \tilde{\theta})$ would have been avoided if the agent had acquired information.

Part 2 captures the other opportunistic motive for the agent to acquire information off the equilibrium path: to discover a relatively efficient state of nature to extract maximum rent. It would be clearer if we think of states of nature as discrete states, such that the density is the probability distribution. The principal is unable to judge directly whether a certain realization of output is produced by an uninformed or an informed agent. If an output level is more likely to be realized by an agent who opportunistically acquired information, $f(\theta') > f(\theta)$, it is optimal for the principal to punish the agent for such realization relative to the second-best contract, and if it is more likely to be realized by an agent who did not acquire information, $f(\theta) > f(\theta')$, it is then optimal for the principal to reward the agent for such realization more than the second-best would have. This is as if the principal deters information acquisition by exposing the agent with a higher risk (and thus a higher risk premium).

Lemma 5 is derived without imposing monotonicity on the transfer scheme. Part 3 indicates that, even if the second-best transfer is monotonically increasing, the binding constraint to deter information acquisition creates non-monotonicity to the optimal contract. Thus, imposing non-decreasing transfers, there are some non-contingencies of transfer on outputs at least near $q(e(\tilde{\theta}),\tilde{\theta})$.

Corollary 2. *If* $f_\theta(\theta) = 0\,\forall\theta \in [0,\bar{\theta}]$, $t^{DI}(q) = t^{SB}(q)$ *for* $q(e^*,\theta) \geq q(e(\tilde{\theta}),\tilde{\theta})$*; if* $f_\theta(\theta) > 0\,\forall\theta \in [0,\bar{\theta}]$, $t^{DI}(q) < t^{SB}(q)$ *for* $q(e(\theta^0),\theta^0) > q(e^*,\theta) \geq q(e(\tilde{\theta}),\tilde{\theta})$ *and* $t^{DI}(q) \geq t^{SB}(q)$ *for* $q(e^*,\theta) \geq q(e(\theta^0),\theta^0)$*; if* $f_\theta(\theta) < 0\,\forall\theta \in [0,\bar{\theta}]$, $t^{DI}(q) > t^{SB}(q)$ *for* $q(e(\theta^0),\theta^0) > q(e^*,\theta) \geq q(e(\tilde{\theta}),\tilde{\theta})$ *and* $t^{DI}(q) \leq t^{SB}(q)$ *for* $q(e^*,\theta) \geq q(e(\theta^0),\theta^0)$*, where* θ^0 *is such that* $e^* = e(\theta^0)$.

The corollary indicates that, in the case of accepting the contract, if the density of the state of nature is increasing (decreasing), the principal rewards the agent less (more) than what he would have been rewarded under the second-best contract upon observing an intermediate level of output; otherwise, she rewards him more (less) than what he would have been rewarded in the second-best. The optimal contract to deter information acquisition offers a higher powered (lower powered) incentive than offered in the second-best if the density of state of nature is increasing (decreasing). This serves as a complement to the literature on information acquisition with the presence of moral hazard mentioned in the literature review, which attributes a higher powered incentive to inducing information acquisition that generates a mean-preserving signal of the random noise. I argue that deterring information acquisition does not necessarily rely on a lower-powered incentive, depending on the density of the state of nature. The first case in Corollary 2 corresponds to an example with a uniformly distributed state of nature. Given which, the principal rewards the agent what he would have been rewarded under the second-best contract if output is sufficiently high ($q(e^*,\theta) \geq q(e(\tilde{\theta}),\tilde{\theta})$). Because it is equally likely that a certain level of output is generated by an uninformed agent as by an informed agent, the distortion of transfer from the second-best does not provide the agent additional incentive to remain uninformed.

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