Information Advantage and Minimum Wage

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Abstract

I study the effects of minimum wage policy on employment contract with the presence of asymmetric information. The key channel is on how minimum wage acts jointly with incentive compatibility (truthful revelation) given different levels of information advantage upon signing the employment contract. Such information advantage is captured by the timing of information arrival, prior to or posterior to employment contracting. Minimum wage manipulates the rent-efficiency tradeoff with ex-ante information arrival, and it introduces binding incentive compatibility with ex-post information arrival. Implications on whether the minimum wage legislation improves productive efficiency, whether it benefits the low-skilled workers, and whether it reduces inequality in the society are studied.

Keywords: Minimum Wage, Contract, Asymmetric Information.
JEL Code: J41, J38, D86, D82

1 Introduction

Recognizing that the labor market is composed of employment contracts, I study impacts of the minimum wage policy in a contract theoretical framework with asymmetric information. Information structure at the contracting stage plays a role in the policy evaluation of minimum wages, as it relates to the fundamental tradeoff in the contracting relationship with the minimum wage legislation. The effects of minimum

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wage legislation on productive efficiency and on welfare distribution, as well as how these effects relate to the employee's information advantage upon signing the contract is characterized in this paper.

In a contractual relationship with the employee having private information on his ability measured by cost of production, the optimal employment contract is a menu of options, each consisting of a pair of output from the task and associated wage. How a binding minimum wage legislation affects this contract depends on how it shapes the tradeoff behind the optimal contract, which depends on the timing of information arrival. If the employee's private information arrives before signing the contract, e.g. the employee is well aware of his own ability, the employee has information advantage upon contracting. The optimal employment contract is a result of the rent-efficiency tradeoff. Efficiency is traded off to reduce information rent required for binding incentive compatibility. An effective minimum wage acts as a counter force that manipulates this tradeoff by imposing a lower bound on the distorted efficiency.

If the employee's private information arrives after signing the contract but before conducting the task, e.g. a fresh graduate who is imperfectly informed of his match to the task until encountering one, the employee does not have information advantage upon contracting. The optimal contract exhibits no tradeoff of efficiency with slacking incentive compatibility. Incentive is induced by an ex-post exploitative contract that the low-skilled employee who has high cost of production is unable to reject due to late information arrival. An effective minimum wage introduces a new tradeoff to this employment contract. It mitigates the employer's ability to lower the monetary wage to the high-cost employee even if the latter does not have information advantage when signing this contract. A higher wage to such employee violates incentive compatibility that were otherwise slacking. A binding minimum wage not only acts as a binding constraint itself, but indirectly introduces a binding incentive compatibility constraint to the contracting problem.

With sufficiently low minimum wage such that separating employment contract is optimal, binding minimum wage results in binding incentive compatibility even when the employee does not have information advantage upon contracting, and it tightens incentive compatibility when the employee has information advantage. In the former scenario, production by the high-cost employee exhibits inefficient upward distortion. In the latter scenario, the binding minimum wage restores output by the high-cost employee towards efficiency, as it countervails the rent-efficiency tradeoff. Socially optimal binding minimum wage exists only when the employee has information advantage when signing the contract.

In terms of welfare and monetary wage inequality, binding minimum wage along with a more restrictive incentive compatibility reduces inequality in monetary wage but enlarges inequality in cost of production from the upward distortion of output. The magnitude of the enlarged inequality in cost of production is higher than that of the reduced wage inequality. Welfare inequality is thus more severe with a binding minimum wage. Inequality of welfare is a result of binding incentive compatibility, which occurs after signing the contract when information is asymmetric, arriving early or late. Such inequality result does not qualitatively depend on the timing of private information arrival. Distribution of the welfare inequality does.

With information advantage upon contracting, the high-cost employee is well informed to reject an ex-post exploitative contract. The optimal contract to induce participation is such that the high-cost employee earns his reservation utility. With binding minimum wage, incentive compatibility is satisfied solely by a spillover information rent to the high-skilled employee who has low cost of production. Without information advantage upon contracting, the high-cost employee is not perfectly informed to reject an ex-post exploitative contract. The optimal contract to induce participation is such that the employee expects to earn his reservation utility when signing the contract. With binding minimum wage, incentive compatibility is satisfied by a spillover rent to the low-cost employee as well as by further lowering the negative information rent to the high-cost employee. Raising minimum wage only improves welfare of the low-cost employee, and whether it hurts the employee who earns the minimum wage depends on the employee's information advantage when signing the contract.

With intermediate minimum wage such that it is still optimal to employ both types of employee, an additional form of inefficiency occurs: failure of screening. With sufficiently high minimum wage, it is not feasible to confine the employment contract to the minimum wage legislation and meanwhile screen the low-cost employee from the high-cost employee. Binding individual rationality is not relaxed with a sufficiently high minimum wage. This implies that the high-cost employee earns his reservation utility with information advantage upon contracting, whereas without information advantage upon contracting, he suffers from a higher loss.

1.1 Related Literature

The impact of minimum wage on the labor market outcome has been in heated scientific debate for 20 years since the conflicting empirical evidence of Card and Krueger

(1994) and Neumark and Wascher (2000). The former challenged the conventional understanding with empirical evidence, while the latter provided evidence that supports otherwise.[1] Interest in providing a theoretical explanation to the mixed evidence has arose ever since.

Manning (2003) and Burdett and Mortensen (1998) studied the employment impact of minimum wage when the employers have monopsonistic power attributed to search frictions. Also in a search and matching model, Flinn (2006) showed that an increase in minimum wage can be welfare improving with endogenous contact rates. Kaas and Madden (2008, 2010) showed that minimum wage is welfare-improving in an oligopsonistic labor market, with the former emphasizing its role on reducing the hold-up problem and the latter focusing on its impact on differentiated non-wage characteristics. Strobl and Walsh (2007) illustrated the difficulty in evaluating the overall effect of minimum wage in a monopsony when both employment and labor hours are accounted for, and Strobl and Walsh (2011) recognized such difficulty in a competitive market and derived conditions under which the number of workers and working hours increase or decrease in response to a raise in minimum wage. Pries and Rogerson (2005) regarded hiring as a matching process with symmetric information and evaluated the minimum wage policy by its effect on worker turnover. Angerhausen, Bayer and Hehenkamp (2010) argued that minimum wage reduces strategic unemployment to signal privately known reservation wage. Danziger and Danziger (2015) proposed a graduated minimum wage schedule for the government to impose which results in a Pareto improvement over the optimal income tax.

Complementary to the literature that emphasizes the impacts of minimum wage on the labor market as a whole, I focus on the impacts of minimum wage at the micro level within a vertical contractual relationship. Instead of the employment debate, I emphasize how the minimum wage legislation alters the incentive compatible employment contract and its implications on efficiency of production and employee welfare.

Employment relationship is governed by contracts, but only a few research on minimum wage takes the contract theoretical approach. De Fraja (1999), Jewitt, Kadan, and Swinkels (2008), Kadan and Swinkels (2013), and Fahn (2017) also recognized that

[1] For a thorough survey of the long-lasting debate please refer to Card and Krueger (1995), Brown (1999), Neumark and Wascher (2008), and MacLeod (2011). More recently, Totty (2017) addressed with a factor model the concern of unobserved heterogeneity on the issue of minimum wage and employment. Jardim, Long, Plotnick, van Inwegen, Vigdor, and Wething (2017) provided a recent evidence on the employment effect of Seattle minimum wage legislation, identifying target labors by wage level instead of by industry.

the labor market is composed of employment contracts and that the response of the employment relationship to the minimum wage policy is reflected in the contractual design. De Fraja (1999) focused on the contracted wage and unemployment with the presence of the minimum wage policy in an environment with asymmetric information, without recognizing the timing of information advantage as a relevant factor.[2] Jewitt, Kadan, and Swinkels (2008) and Kadan and Swinkels (2013) studied the incentive effect of bounded payments (including minimum wage) in an environment with moral hazard and show the optimality of an option-like contract. The latter further derive conditions on informativeness under which an increases in minimum wage makes it more costly for the employer to induce productive effort. Equilibrium productive effort is distorted downward due to an increase in minimum wage, opposite to the prediction of my paper. Fahn (2017) analyzed the incentive impact of a minimum wage in a relational contracting framework when productive effort and output are non-verifiable. He provided a different possible explanation for efficiency improvement in response to the minimum wage, as well as a different welfare prediction.

Different from the above literature, I emphasize on how the employee's information advantage upon employment contracting shapes the interaction between the minimum wage legislation and incentive compatibility (truthful revelation). Not only asymmetric information, but the arrival timing of asymmetric information, is crucial to the potential impact of the minimum wage policy.

2 Employment Contracting Model

Consider an employer-employee relationship in a labor market composed of a unit mass of labor. The employer has full bargaining power to propose a take-it-or-leave-it employment contract to a randomly matched potential employee. The employee is hired to execute a task which generates contractible output q and yields the employer revenue $R(q)$, at a cost θq to the employee. Output q is not restricted to physical output. It is a general job description of the position. A more complicated, difficult task generates higher revenue to the employer at a higher cost to the employee. Let $R(q)$ be increasing and concave in q and satisfy Inada's condition, i.e. $R(0) = 0$, $R'(q) > 0$, $R''(q) < 0$, and $\lim_{q \to 0} R'(q) = \infty$. The employee's productivity is measured by his marginal cost of production, $\theta \in \left\{\underline{\theta}, \overline{\theta}\right\}$, with $0 < \underline{\theta} < \overline{\theta} < \infty$. I will refer the

[2] Laffont and Martimort (2004: 120-121) on the other hand discussed briefly how limited liability shaped the incentive compatible contract when the agent does not have information advantage upon contracting. Discussion on the spillover effect, wage and welfare dispersion, and the role of timing of information advantage are left out.

employee of type $\underline{\theta}$ as the low-cost or the high-skilled employee, and that of type $\overline{\theta}$ as the high-cost or the low-skilled employee. The employee's marginal cost of production is his private information, and it is common knowledge that θ follows the distribution function $F(\theta)$, with $F(\underline{\theta})$ denoted as σ, i.e. σ of the unit mass of employees are high-skilled.

Timing of information arrival determines whether the employee has private information on his ability upon employment contracting. If private information on the employee's ability arrives after the employment contract is signed and before the task is executed, the employment contract is phrased as an ex-ante contract. At the time of ex-ante contracting, the employee has no information advantage in the sense that he does not know more than the employer does. If private information on the employee's ability arrives before the employment contract is signed, the employment contract is phrased as an ex-post contract. At the time of ex-post contracting, the employee has information advantage regarding his ability in the sense that he knows more than the employer does. I would think of the ex-ante employment contract applying to a freshly graduated employee who has not yet discover his own ability in this industry until the training stage between contracting and actual execution of the task, while the ex-post employment contract applying to an employee who has good knowledge of his ability in this industry prior to employment contracting.

Applying the revelation principle, the employment contract is a menu of options. Each option consists of a pair of output from the task and associated wage, i.e. $\mathbb{C} = \left\{q(\hat{\theta}), w(\hat{\theta})\right\}$, where $\hat{\theta} \in \left\{\underline{\theta}, \overline{\theta}\right\}$ denoting the employee's report of his ability. With the employment of a θ-type employee, the employer earns $V(q, w) = R(q(\hat{\theta})) - w(\hat{\theta})$, and the employee earns $U(q, w, \theta) = w(\hat{\theta}) - \theta q(\hat{\theta})$. If the employment contract is rejected, each earns reservation payoff zero. The employee is protected by minimum wage m, so that legit wage satisfies $w(\theta) \geq m$ for each θ. To avoid cluster, denote $q(\underline{\theta})$ as $\underline{q}$, $q(\overline{\theta})$ as $\overline{q}$, $w(\underline{\theta})$ as $\underline{w}$, and $w(\overline{\theta})$ as $\overline{w}$.

2.1 Ex-Ante and Ex-Post Employment Contract

The main difference between the two timings of private information arrival is on whether the employee has information advantage at the stage of contracting. This is crucial when the employee decides whether to accept the employment contract. Arrival of private information before or after signing the contract implies different individual rationality constraints. With ex-ante contracting, the employee does not have information advantage until right before execution of the task. The later-to-be-truth-telling employee has incentive to take the employment contract if it satisfies the ex-ante individual rationality constraint

$$\sigma\left(\underline{w}-\underline{\theta}\underline{q}\right)+(1-\sigma)\left(\overline{w}-\overline{\theta}\overline{q}\right)\geq 0. \tag{1}$$

With ex-post contracting, the employee has information advantage at the stage of contracting. The employee has incentive to take the employment contract if the ex-post individual rationality constraints are satisfied for both the low-cost employee and the high-cost employee,

$$\underline{w}-\underline{\theta}\underline{q}\geq 0 \tag{2}$$

and

$$\overline{w}-\overline{\theta}\overline{q}\geq 0. \tag{3}$$

For information is asymmetric after the signing of the employment contract and before the production takes place, incentive compatibility (truth-revealing) constraints remain the same with or without information advantage upon contracting, i.e.

$$\underline{w}-\underline{\theta}\underline{q}\geq\overline{w}-\underline{\theta}\overline{q} \tag{4}$$

for the low-cost employee and

$$\overline{w}-\overline{\theta}\overline{q}\geq\underline{w}-\overline{\theta}\underline{q} \tag{5}$$

for the high-cost employee. As standard in contract theory, an employment contract is incentive compatible for the high-cost employee if it satisfies both (4) and monotonicity

$$\underline{q}\geq\overline{q} \tag{6}$$

derived from (4) and (5). The employment contract is said to be incentive feasible if it is individually rational and incentive compatible.

With the minimum wage policy, the employment contract is legit if the wage paid to each type of the employee is no lower than the minimum wage. The incentive feasible contract is legit if it satisfies the minimum wage constraints

$$\underline{w}\geq m \tag{7}$$

and

$$\overline{w}\geq m. \tag{8}$$

Unless otherwise specified, throughout the rest of the paper a minimum wage is referred to as an effective minimum wage, defined as what follows.

Definition 1. The level of minimum wage is effective if at least one of the minimum wage constraints is binding in equilibrium. It is otherwise ineffective.

Given incentive compatibility, the contracted wage to the low-cost employee is no lower than that to the high-cost. This holds for both ex-ante and ex-post contracting. Lemma 1 is then straightforward.

Lemma 1. *Regardless of ex-ante or ex-post contracting, at most (8) is the relevant minimum wage constraint.*

Proof. From (4) and (6), $\underline{w} - \overline{w} \geq \underline{\theta}\left(\underline{q} - \overline{q}\right) \geq 0$, so (7) is satisfied if (4), (6), and (8) hold. □

If the employee receives his private information after signing the contract, the employer's ex-ante contracting problem given Lemma 1 is to

$$\max_{q,w} \sigma\left(R(\underline{q}) - \underline{w}\right) + (1-\sigma)\left(R(\overline{q}) - \overline{w}\right)$$

subject to (1), (4), (5), and (8). Define the employee's welfare measured by the information rent as $\underline{u} \equiv \underline{w} - \underline{\theta}\underline{q}$ and $\overline{u} \equiv \overline{w} - \overline{\theta}\overline{q}$. The ex-ante contracting problem is reduced to

$$\mathscr{P}^a: \qquad \max_{q,u} \sigma\left(R(\underline{q}) - \underline{\theta}\underline{q} - \underline{u}\right) + (1-\sigma)\left(R(\overline{q}) - \overline{\theta}\overline{q} - \overline{u}\right)$$

subject to

$$\sigma\underline{u} + (1-\sigma)\overline{u} \geq 0,$$

$$\underline{u} - \overline{u} \geq \left(\overline{\theta} - \underline{\theta}\right)\overline{q},$$

$$\underline{q} \geq \overline{q},$$

$$\overline{u} + \overline{\theta}\overline{q} \geq m.$$

If the employee receives his private information before signing the contract, the employer's ex-post contracting problem given Lemma 1 is to

$$\max_{q,w} \sigma\left(R(\underline{q}) - \underline{w}\right) + (1-\sigma)\left(R(\overline{q}) - \overline{w}\right)$$

subject to (2), (3), (4), (5), and (8). The contracting problem can be expressed in terms of the employee's welfare $\underline{u} \equiv \underline{w} - \underline{\theta}\underline{q}$ and $\overline{u} \equiv \overline{w} - \overline{\theta}\overline{q}$ and reduced to

$$\mathscr{P}^p : \qquad \max_{q,u} \sigma \left(R(\underline{q}) - \underline{\theta}\underline{q} - \underline{u} \right) + (1-\sigma) \left(R(\overline{q}) - \overline{\theta}\overline{q} - \overline{u} \right)$$

subject to

$$\overline{u} \geq 0,$$

$$\underline{u} - \overline{u} \geq \left(\overline{\theta} - \underline{\theta} \right) \overline{q},$$

$$\underline{q} \geq \overline{q},$$

$$\overline{u} + \overline{\theta}\overline{q} \geq m.$$

With an effective minimum wage, denote the optimal ex-ante contract solving $\mathscr{P}^a$ as $\mathbb{C}^a = \left\{ \left(\underline{q}^a, \underline{w}^a \right), \left(\overline{q}^a, \overline{w}^a \right) \right\}$, which implements information rent $\underline{u}^a$ and $\overline{u}^a$, and denote the optimal ex-post contract solving $\mathscr{P}^p$ as $\mathbb{C}^p = \left\{ \left(\underline{q}^p, \underline{w}^p \right), \left(\overline{q}^p, \overline{w}^p \right) \right\}$, which implements information rent $\underline{u}^p$ and $\overline{u}^p$. As a benchmark, the optimal ex-ante contract and the optimal ex-post contract without an effective minimum wage are denoted as $\mathbb{C}^0 = \left\{ \left(\underline{q}^0, \underline{w}^0 \right), \left(\overline{q}^0, \overline{w}^0 \right) \right\}$, implementing $\underline{u}^0$ and $\overline{u}^0$, and $\mathbb{C}^1 = \left\{ \left(\underline{q}^1, \underline{w}^1 \right), \left(\overline{q}^1, \overline{w}^1 \right) \right\}$, implementing $\underline{u}^1$ and $\overline{u}^1$, respectively. Also define the efficient level of output from the high-cost employee as $\overline{q}^e \in \arg\max_q R(q) - \overline{\theta}q$ and that from the low-cost employee as $\underline{q}^e \in \arg\max_q R(q) - \underline{\theta}q$. A detailed solution to the model, as well as all proofs, are left in Appendix A. The main text is devoted to the discussion of the contractual effects from an increase in the minimum wage. Specifically, how productive efficiency and welfare implemented by the optimal employment contract differs in response to the imposition of a higher minimum wage.

3 Information Advantage and Minimum Wage

3.1 Separating Employment Contract

I first analyze the interaction of the minimum wage legislation and the employee's information advantage at the time of contracting when a sufficiently low minimum wage is raised without resulting in equilibrium unemployment[3] nor pooling of the contract. In contract theoretical terms, the sufficiently low minimum wage does not result in violation to monotonicity (6). Specifically, an effective minimum wage is sufficiently low if it is lower than $m_1 \equiv \lim_{\overline{q} \to \underline{q}^e} \left(((1-\sigma)\overline{\theta} + \sigma\underline{\theta})\overline{q} \right)$ without information advantage upon contracting, and if it is lower than $m_3 \equiv \lim_{\overline{q} \to \underline{q}^e} \left(\overline{\theta}\overline{q} \right)$ with information advantage upon contracting.

The employer designs the contract to satisfy the minimum wage constraint (8) by raising output $\overline{q}$ and/or by raising rent $\overline{u}$ for the high-cost employee. The former comes with a second-order loss relative to the contract without effective minimum wage regulation, while the latter is subject to a first-order loss. With a sufficiently low minimum wage, it is conjectured that the low skilled employee is contracted to produce a higher output (or to conduct a more difficult task) without improvement of rent. This conjecture is not as intriguing as the mechanism behind it. Specifically, in an employment relationship with asymmetric information, the key question would be how the effective minimum wage interacts with the employee's incentive in terms of information revelation, and this interaction is related to the timing of information arrival.

Lemma 2. *Without information advantage upon contracting, an effective minimum wage results in binding incentive compatibility (4). With information advantage upon contracting, incentive compatibility (4) is binding with or without effective minimum wage.*

Proof. Appendix A1. □

If the employee has no information advantage at the time of contracting and the minimum wage is ineffective, information asymmetry does not create an incentive problem in production, because by then the employment contract is already signed. The employer is able to design a contract that induces efficient output and exploits the high-cost employee, who is not perfectly informed of his type when signing the contract. An effective minimum wage manipulates this exploitation by raising the wage of the high-cost employee. This higher wage encourages the low-cost employee to take the option designed for the high-cost employee. Incentive compatibility (4) becomes binding with an effective minimum wage. The optimal employment contract then exhibits a tradeoff between efficiency and minimum wage.

[3]This is resulted from the assumed bargaining power of the employer to propose a take-it-or-leave-it employment contract. Please refer to Manning (2003) for a thorough study on the minimum wage policy in an environment where the employer has wage setting power.

If the employee has information advantage upon contracting, incentive compatibility is binding with or without an effective minimum wage. A binding minimum wage posts a lower bound on the contracted output for the high-cost employee, which results in a more restrictive incentive compatibility (4). An effective minimum wage serves as a counter force that manipulates the usual rent-efficiency tradeoff.

The next question is, how the tradeoff between efficiency and minimum wage without early information advantage and how the manipulated rent-efficiency tradeoff with early information advantage shape the optimal employment contract. Proposition 1 marks the productive effect, and Proposition 2 and 3 mark the distributional effect.

Proposition 1. *An increase in sufficiently low minimum wage results in an inefficient upward distortion of output from the high-cost employee if he does not enjoy the information advantage upon contracting, $\overline{q}^a > \overline{q}^0 = \overline{q}^e$ with $\overline{q}^a$ increasing in m. It restores efficiency from the high-cost employee if he enjoys information advantage upon contracting, $\overline{q}^e \geq \overline{q}^p > \overline{q}^1$ with $\overline{q}^p$ increasing in m. An increase in intermediate minimum wage results in an inefficient upward distortion of output from the high-cost employee regardless of his information advantage upon contracting, $\overline{q}^a > \overline{q}^e$ and $\overline{q}^p > \overline{q}^e$. Productive activity of the low-cost employee is not distorted regardless of his information advantage upon contracting, $\underline{q}^a = \underline{q}^p = \underline{q}^e$.*

Proof. Appendix A2. □

An effective minimum wage introduces incentive for the low-cost employee to take the minimum wage option that is designed for the high-cost employee. For incentive compatibility, the employer raises the output from the high-cost employee so that such option is no more attractive to the low-cost employee. This creates a tradeoff of efficiency. Without information advantage upon contracting, this is the only tradeoff the employer faces when minimum wage is increased. Inefficiently high output from the high-cost employee is a result of binding incentive compatibility (4) triggered by the binding minimum wage. With information advantage upon contracting, this tradeoff of efficiency acts as a counter force to the usual rent-efficiency tradeoff. It enforces a lower bound to the employer's ability to reduce information rent to the low-cost employee by reducing output from the high-cost employee. Improvement of efficiency with a sufficiently low minimum wage is resulted from these counteracting tradeoffs. With an intermediate minimum wage, the effect of binding minimum wage exceeds the rent-efficiency tradeoff, the contract then has inefficiently high output from the high-cost employee. The following corollary is then straightforward.

Corollary 1. *There is a socially optimal and effective minimum wage only if the employee has information advantage upon contracting.*[4]

Empirical evidence had mixed findings regarding the effect of minimum wage on labor productivity and on labor efforts. Riley and Rosazza Bondibene (2017) found higher labor productivity after the introduction of National Minimum Wage of UK due to an increase in total factor productivity instead of a rise in capital intensity nor reduction of the workforce. This result is consistent to the prediction of Proposition 1, which provides one possible reason for such increase in total factor productivity. Stewart and Swaffield (2008) found a lagged reduction in working hours due to the U.K. minimum wage. Zavodny (2000), using individual-level data of the U.S., found a higher working hours as a result of a higher increase in wage to the minimum wage for affected teenagers who remained employed relative to the unaffected. Evidence on working hours, however, does not point to direct consistency or contradiction to Proposition 1. Change of working hours may be associated with a change of job description, and how the latter changed (or not) is unclear in these papers.

A more restrictive incentive compatibility indirectly driven by the binding minimum wage also has a distributional effect on the employment contract. Welfare inequality measured by the difference in information rent $\underline{u} - \overline{u}$ is required to induce truthful information revelation. With a higher minimum wage, the employer's ability to implement a lower welfare inequality is bounded below through a higher output from the high-cost employee, captured by the more restrictive incentive compatibility (4). Welfare inequality is thus more severe. The risen welfare inequality is qualitatively irrelevant to the employee's information advantage upon contracting. This is due to the fact that welfare inequality is crucial to incentive compatibility, which occurs after the contract is signed and and before production takes place, by then the employee has received his private information. Monetary inequality measured by the difference in wage $\underline{w} - \overline{w}$, however, is lowered. Intuitively, a higher minimum wage reduces the monetary inequality as it raises the wage to the high-cost employee at

[4] Fahn (2017) found an increase in productive effort and efficiency in an employment relationship governed by relational contracts. Improvement of efficiency is due to the fact that a binding minimum wage serves as a commitment for future wages if employed, which improves the employees' incentive to exert non-verifiable effort. Kaas and Madden (2008) studied the existence of minimum wage to induce efficient investment by the employers in a model with oligopsonistic wage competition. Efficiency-improving minimum wage exists because it alleviates the hold-up problem under wage competition. Flinn (2006) showed that an increase in minimum wage can be welfare improving in a search and matching model with endogenous contact rates. Welfare improvement is possible as a higher minimum wage increases the effective bargaining power of the employees. I provide an alternative explanation for efficiency improvement: the employer's bounded ability to implement a lower information rent via distortion of output when the employee has information advantage upon contracting.

a larger magnitude, but it amplifies welfare inequality through a higher difference in cost of production as indicated in the previous proposition. This is summarized in Proposition 2.

Proposition 2. *An increase in sufficiently low minimum wage reduces wage inequality but amplifies welfare inequality, regardless of the employee's information advantage upon contracting, i.e.* $\underline{w}^a - \overline{w}^a < \underline{w}^0 - \overline{w}^0$, $\underline{w}^p - \overline{w}^p < \underline{w}^1 - \overline{w}^1$, *and both* $\underline{w}^a - \overline{w}^a$ *and* $\underline{w}^p - \overline{w}^p$ *are diminishing in* m, *while* $\underline{u}^a - \overline{u}^a > \underline{u}^0 - \overline{u}^0$, $\underline{u}^p - \overline{u}^p > \underline{u}^1 - \overline{u}^1$, *and both* $\underline{u}^a - \overline{u}^a$ *and* $\underline{u}^p - \overline{u}^p$ *are increasing in* m.

Proof. Appendix A2. □

Reduced wage inequality from a higher minimum wage is consistent to several empirical evidence. To name a few, Card and Krueger (1995) documented a narrowing effect on wage distribution due to a higher minimum wage in the early 90s; Card and DiNardo (2002) suspected that the rise in wage inequality in the earlier 80s is primarily due to the fall in the real value of the minimum wage; Neumark, Schweitzer and Wascher (2004) reported both contemporaneous and cumulative narrowing effects on wage distribution over initial wage distribution. More recently, Autor, Manning, and Smith (2016) reassessed the effect of minimum wages on US earnings inequality over three decades and found reduced inequality in the lower tail of the wage distribution. Nevertheless, the causation between wage inequality (and more broadly earned income inequality) and the minimum wage policy is still an open empirical question. To further complicate the analysis, Proposition 2 suggests that empirical findings of a narrowed wage distribution after a raise of minimum wage does not necessarily imply a lower welfare inequality in the labor market.

Proposition 3 provides a detailed look into the distribution of the welfare inequality.

Proposition 3. *The low-cost employee enjoys a positive spillover rent from an increase of sufficiently low minimum wage,* $\underline{u}^a > \underline{u}^0$ *and* $\underline{u}^p > \underline{u}^1$. *Information advantage upon contracting allows the high-cost employee to avoid welfare loss from the minimum wage legislation,* $\overline{u}^a < \overline{u}^0 < 0$ *while* $\overline{u}^p = \overline{u}^1 = 0$.

Proof. Appendix A2. □

The low-cost employee enjoys a positive spillover effect that is resulted from the more restrictive incentive compatibility (4) in response to an increase in the binding minimum wage. Satisfaction of the binding minimum wage for the high-cost employee (8) introduces incentive for the low-cost employee to take the minimum wage option unless a higher information rent is granted for truthful revelation.[5] An increased minimum wage manipulates the rent-efficiency tradeoff towards his advantage. With efficiency unaffected, the spillover rent is reflected solely in a higher monetary wage. Monetary spillover effect is found in several empirical papers on employees who earn wages higher than the binding minimum, e.g. Card and Krueger (1995), Neumark, Schweitzer and Wascher (2004), and Autor, Manning, and Smith (2016).

The high-cost employee earns a higher monetary wage at the expense of a higher disutility of production from the increase of minimum wage. The relative sizes of the two is related to the timing of information arrival. Without information advantage upon contracting, the high-cost employee is unable to avoid signing a contract that exploits him with an inefficiently high output ex-post without sufficient monetary payoff to match his reservation utility. Theoretically, a larger dispersion of information rent and the binding ex-ante individual rationality (1) imply that the low-cost employee enjoys a higher positive information rent and that the high-cost employee suffers from a lower negative information rent (or a larger information loss). With information advantage upon contracting, the high-cost employee has the information to reject an ex-post exploitative contract. To induce participation, the employer raises the monetary payoff in the contract to offset the higher cost of production in response to a higher minimum wage. Theoretically, a larger dispersion of information rent and the binding ex-post individual rationality (3) imply that the low-cost employee enjoys a higher positive information rent and that the high-cost employee earns his reservation utility. The conventional argument that the high-cost employee is hurt by the more restrictive minimum wage legislation can thus be explained by his lack of information advantage upon contracting to protect himself against an ex-post unattractive offer.

3.2 Pooling Employment Contract

With a sufficiently low minimum wage, the efficiency effect in the previous subsection stems from the introduction of a new tradeoff when private information arrives after signing the contract or from the manipulation of rent-efficiency tradeoff when private information arrives before signing the contract. A sufficiently high minimum wage (conditional on optimality to contract) accompanies with an additional form of inefficient allocation: failure of screening contracts. The employer designs the contract to satisfy the minimum wage constraint (8) by an upward distortion of output $\overline{q}$. With a

[5] Grossman (1983) in her efficiency wage model also recognized wage differential as a key channel for the spillover effect. The fundamental reason to maintain the wage differential in her model is to motivate effort, while it is to induce truthful revelation in the current paper.

sufficiently high minimum wage, such upward distortion is so severe that monotonicity (6) is binding in equilibrium. The incentive of the low-cost employee and that of the high-cost employee is incompatible for any separating employment contract. The optimal contract exhibits pooling. Specifically, the minimum wage is sufficiently high if it is higher than $m_1 \equiv \lim_{\overline{q} \to \underline{q}^e} \left(((1-\sigma)\overline{\theta} + \sigma\underline{\theta})\overline{q}\right)$ without information advantage upon contracting, and if it is higher than $m_3 \equiv \lim_{\overline{q} \to \underline{q}^e} \left(\overline{\theta}\overline{q}\right)$ with information advantage upon contracting.

Proposition 4. *At a sufficiently high minimum wage (conditional on optimality to contract), the optimal contract fails to screen the low-cost employee from the high-cost employee. For minimum wage $m > m_1$, it is optimal to propose a pooling contract with $\underline{q}^a = \overline{q}^a > \underline{q}^e$ and $\underline{w}^a = \overline{w}^a = m$ when information arrives after signing the contract. For minimum wage $m > m_3$, it is optimal to propose a pooling contract with $\underline{q}^p = \overline{q}^p > \underline{q}^e$ and $\underline{w}^p = \overline{w}^p = m$ when the employee has information advantage upon contracting. The employee's information advantage upon contracting, however, relaxes such failure of screening in the sense that the critical level of minimum wage is higher, $m_3 > m_1$.*

Proof. Appendix A3. □

Inefficiency is relatively relaxed with the employee's information advantage upon contracting. The rent-efficiency tradeoff provides the employer with larger room for adjustment of output produced by the high-cost employee to satisfy binding minimum wage legislation without violation to monotonicity. Failure of screening thus occurs at a higher level of minimum wage.

Indicated by Proposition 3, a sufficiently low minimum wage fails to improve welfare of the high-cost employee. A sufficiently high minimum wage, according to Proposition 4, results in a pooled, highly upward-distorted output. Conceding information rent to the high-cost employee with such a high cost of production is extremely costly for the employer. It is far from optimal to propose a contract that implement an improved welfare of the high-cost employee, regardless of his information advantage upon signing the contract.

Proposition 5. *It is sub-optimal to grant a positive expected information rent to the high-cost employee even with the presence of minimum wage legislation.*

Proof. Appendix A3. □

4 Conclusion

In an employment relationship governed by contracts, a sufficiently low yet effective minimum wage serves as a counter force to the rent-efficiency tradeoff when the employee has information advantage on his productivity when signing the contract. Such information advantage is thus essential for the existence of social welfare improving minimum wage legislation. In addition, a binding minimum wage makes incentive compatibility more restrictive, which results in a larger inequality in information rent between the low-cost and the high-cost employee. The employee's information advantage upon contracting protects the high-cost employee from being exploited by a contract following a more restrictive minimum wage legislation. The amplified inequality of information rent solely reflects a spillover effect when asymmetric information arrives before signing the contract. Without information advantage upon contracting, the high-cost employee fails to reject an ex-post exploitative contract and is worse off with a more restrictive minimum wage. A sufficiently high minimum wage is not only subject to inefficient production but also failure to screen the low-cost employee from the high-cost employee. Information advantage upon contract, however, relaxes this effect.

The scope of this paper is limited to the interaction of information asymmetry and the minimum wage policy within a contractual relationship. A generalization to the labor market as a whole that consists of many related contractual units provides a more thorough understanding of the widely debated topic, at the cost of complexity. In addition, only a full commitment of the entire principal-agent relationship is considered. In reality, the employment contracts are mostly signed for a shorter term and are updated and renegotiated throughout the tenure. With only short-term commitment, the information revelation strategy can be very different from truthful revelation with probability one. Information asymmetry at every period to sign a short-term contract is hence endogenous. How the dynamic information asymmetry interacts with the minimum wage policy when only short-term commitment is possible is a potentially interesting future research.

Appendix

A Proof of Lemmas and Propositions in Section 3

The ex-ante contracting problem of the employer is to

$$\mathscr{P}^a : \qquad \max_{q,u} \sigma \left(R(\underline{q}) - \underline{\theta}\underline{q} - \underline{u}\right) + (1-\sigma)\left(R(\overline{q}) - \overline{\theta}\overline{q} - \overline{u}\right)$$

subject to

$$\sigma\underline{u} + (1-\sigma)\overline{u} \geq 0, \tag{9}$$

$$\underline{u} - \overline{u} \geq \left(\overline{\theta} - \underline{\theta}\right)\overline{q}, \tag{10}$$

$$\underline{q} \geq \overline{q}, \tag{11}$$

$$\overline{u} + \overline{\theta}\overline{q} \geq m. \tag{12}$$

Let the Lagrange function be

$$\begin{aligned}\mathscr{L}^a =& \sigma\left(R(\underline{q}) - \underline{\theta}\underline{q} - \underline{u}\right) + (1-\sigma)\left(R(\overline{q}) - \overline{\theta}\overline{q} - \overline{u}\right)\\ &+ \lambda^a\left(\sigma\underline{u} + (1-\sigma)\overline{u}\right) + \mu^a\left(\underline{u} - \overline{u} - \left(\overline{\theta} - \underline{\theta}\right)\overline{q}\right) + \nu^a\left(\underline{q} - \overline{q}\right) + \phi^a\left(\overline{u} + \overline{\theta}\overline{q} - m\right),\end{aligned}$$

where λ^a, μ^a, ν^a, ϕ^a are the Lagrange multipliers for (9), (10), (11), and (12) respectively. All are non-negative by construction. The optimality conditions with respect to q and u are

$$\sigma\left(R_q(\underline{q}) - \underline{\theta}\right) + \nu^a = 0 \tag{13}$$

$$(1-\sigma)\left(R_q(\overline{q}) - \overline{\theta}\right) - \mu^a\left(\overline{\theta} - \underline{\theta}\right) - \nu^a + \phi^a\overline{\theta} = 0 \tag{14}$$

$$-\sigma + \lambda^a\sigma + \mu^a = 0 \tag{15}$$

$$-(1-\sigma) + \lambda^a(1-\sigma) - \mu^a + \phi^a = 0 \tag{16}$$

From (15) and (16), $\mu^a = \sigma(1-\lambda^a)$ and $\phi^a = 1-\lambda^a$. There are three scenarios to be considered regarding the level of Lagrange multipliers λ^a, μ^a, and ϕ^a.

$$\begin{cases} \phi^a = 0 & \Leftrightarrow & \lambda^a = 1,\ \mu^a = 0\\ 0 < \phi^a < 1 & \Leftrightarrow & 0 < \lambda^a < 1,\ \mu^a = \sigma(1-\lambda^a)\\ \phi^a = 1 & \Leftrightarrow & \lambda^a = 0,\ \mu^a = \sigma \end{cases}$$

Note that ϕ^a is no larger than one. If (12) is violated in the sense that $\overline{u}+\overline{\theta}\overline{q}-m=-\delta$, $\delta>0$, a suboptimal way for the employer to satisfy the minimum wage law without violating any of the constraints is to raise the wage to both types of the employee by δ, which results in a loss of the employer's payoff by the amount δ. Any contract such that $\phi^a>1$ is thus dominated by this suboptimal contract.

The ex-post contracting problem of the employer is to

$$\mathscr{P}^p: \qquad \max_{q,u} \sigma\left(R(\underline{q})-\underline{\theta}\underline{q}-\underline{u}\right)+(1-\sigma)\left(R(\overline{q})-\overline{\theta}\overline{q}-\overline{u}\right)$$

subject to

$$\overline{u}\geq 0, \tag{17}$$

$$\underline{u}-\overline{u}\geq\left(\overline{\theta}-\underline{\theta}\right)\overline{q}, \tag{18}$$

$$\underline{q}\geq\overline{q}, \tag{19}$$

$$\overline{u}+\overline{\theta}\overline{q}\geq m. \tag{20}$$

Let the Lagrange function be

$$\begin{aligned}\mathscr{L}^p=&\sigma\left(R(\underline{q})-\underline{\theta}\underline{q}-\underline{u}\right)+(1-\sigma)\left(R(\overline{q})-\overline{\theta}\overline{q}-\overline{u}\right)\\&+\lambda^p\overline{u}+\mu^p\left(\underline{u}-\overline{u}-\left(\overline{\theta}-\underline{\theta}\right)\overline{q}\right)+\nu^p\left(\underline{q}-\overline{q}\right)+\phi^p\left(\overline{u}+\overline{\theta}\overline{q}-m\right),\end{aligned}$$

where λ^p, μ^p, ν^p, ϕ^p are the Lagrange multipliers for (17), (18), (19), and (20) respectively. All are non-negative by construction. The optimality conditions with respect to q and u are

$$\sigma\left(R_q(\underline{q})-\underline{\theta}\right)+\nu^p=0 \tag{21}$$

$$(1-\sigma)\left(R_q(\overline{q})-\overline{\theta}\right)-\mu^p\left(\overline{\theta}-\underline{\theta}\right)-\nu^p+\phi^p\overline{\theta}=0 \tag{22}$$

$$-\sigma+\mu^p=0 \tag{23}$$

$$-(1-\sigma)+\lambda^p-\mu^p+\phi^p=0 \tag{24}$$

From (23) and (24), $\mu^p=\sigma$ and $\phi^p=1-\lambda^p$. There are three scenarios to be considered regarding the level of Lagrange multipliers λ^p and ϕ^p.

$$\begin{cases} \phi^p = 0 & \Leftrightarrow & \lambda^p = 1 \\ 0 < \phi^p < 1 & \Leftrightarrow & 0 < \lambda^p < 1 \\ \phi^p = 1 & \Leftrightarrow & \lambda^p = 0 \end{cases}$$

Note that ϕ^p is no larger than one. If (20) is violated in the sense that $\overline{u}+\overline{\theta}\overline{q}-m = -\delta$, $\delta > 0$, a suboptimal way for the employer to satisfy the minimum wage law without violating any of the constraints is to raise the wage to both types of the employee by δ, which results in a loss of the employer's payoff by the amount δ. Any contract such that $\phi^p > 1$ is thus dominated by this suboptimal contract.

For later reference, denote the efficient level of output produced by the low-cost employee and the high-cost employee respectively as $\underline{q}^e$ and $\overline{q}^e$, which solves $\max_q R(q)-\theta q$ for $\theta \in \{\underline{\theta}, \overline{\theta}\}$ respectively.

A.1 Lemma 2

With ex-ante contracting, $\mu^a = \sigma(1-\lambda^a)$ and $\phi^a = 1-\lambda^a$ from (15) and (16). For ineffective minimum wage m such that (12) slacks, $\phi^a = 0$, the problem is a typical contracting problem with ex-ante participation constraint. By the sum of (15) and (16), $\lambda^a = 1$ and thus $\mu^a = 0$. The latter implies slacking incentive compatibility (10). For effective minimum wage m such that (12) is binding, $0 < \phi^a < 1$, incentive compatibility (10) is binding. To see this, suppose that $\mu^a = 0$, and by (15) $\lambda^a = 1$. With $0 < \phi^a < 1$ and $\lambda^a = 1$, (16) is violated at $\mu^a = 0$, a contradiction to the optimization problem. With ex-post contracting, $\mu^p = \sigma$ and $\phi^p = 1-\lambda^p$ from (23) and (24). The former implies binding incentive compatibility (18) for all $\phi^p \geq 0$, i.e. regardless of binding minimum wage (20) or not.

A.2 Proposition 1 to Proposition 3

First consider the optimal contract without information advantage upon contracting, i.e. the contract solving $\mathscr{P}^a$.

For ineffective minimum wage m such that the minimum wage constraint (12) slacks, $\phi^a = 0$ and the problem is a typical contracting problem with ex-ante participation constraint. By the sum of (15) and (16), $\lambda^a = 1$ and thus $\mu^a = 0$. The optimal employment contract $\mathbb{C}^0 = \left\{\left(\underline{q}^0, \underline{w}^0\right), \left(\overline{q}^0, \overline{w}^0\right)\right\}$ has both low-cost and high-cost employee producing efficiently, i.e. $\underline{q}^0 = \underline{q}^e$, $\overline{q}^0 = \overline{q}^e$ solved from (13) and (14). It implements the employee's welfare dispersion $\underline{u}^0-\overline{u}^0=\left(\overline{\theta}-\underline{\theta}\right)\overline{q}^e$ derived from binding incentive compatibility (10) with wage dispersion $\underline{w}^0-\overline{w}^0=\underline{\theta}\left(\underline{q}^e-\overline{q}^e\right)$. The low-cost employee enjoys a positive information rent $\underline{u}^0\equiv\underline{w}^0-\underline{\theta}\underline{q}^e=(1-\sigma)\left(\overline{\theta}-\underline{\theta}\right)\overline{q}^e>0$ under the contract, while the high-cost employee suffers from a negative information rent $\overline{u}^0\equiv\overline{w}^0-\overline{\theta}\overline{q}^e=-\sigma\left(\overline{\theta}-\underline{\theta}\right)\overline{q}^e<0$ from the binding ex-ante individual rationality and incentive compatibility constraints.

Prove that equilibrium level of ϕ^a and $\overline{q}$ are both increasing in m. Denote the maximum of ineffective minimum wage as $m_0\equiv\lim_{\overline{q}\to\overline{q}^e}\left(((1-\sigma)\overline{\theta}+\sigma\underline{\theta})\overline{q}\right)$. For $m>m_0$, (12) is violated under $\mathbb{C}^0$. From (14) and concave $R(\cdot)$, ϕ^a is increasing in $\overline{q}$. For any $0<\phi^a<1$, $0<\lambda^a<1$ and $0<\mu^a<\sigma$, so (9) and (10) are binding, which implies that $\overline{u}<\overline{u}^0$. $\overline{q}$ is thus increasing in m to satisfy (12). Hence, ϕ^a is increasing in m.

For m sufficiently large such that $\phi^a=1$, $\overline{q}\to\infty$ by (14) given $\nu^a=0$. Monotonicity (11) is strictly violated. There thus exists $m_1\equiv\lim_{\overline{q}\to\underline{q}^e}\left(((1-\sigma)\overline{\theta}+\sigma\underline{\theta})\overline{q}\right)$ such that for any $m\geq m_1$, $1>\phi^a\geq\phi_1$ and the optimal contract exhibits pooling: $\underline{q}=\overline{q}$. The threshold Lagrange multipliers ϕ_1 and λ_1 solves (14) and (16) at $\overline{q}=\underline{q}^e$ and $\mu^a=\sigma(1-\lambda^a)$, i.e. $\phi_1=\frac{(1-\sigma)(\overline{\theta}-\underline{\theta})}{(1-\sigma)(\overline{\theta}-\underline{\theta})+\underline{\theta}}$ and $\lambda_1=\frac{\underline{\theta}}{\sigma\underline{\theta}+(1-\sigma)\overline{\theta}}$. For $m\in(m_0,m_1)$, $0<\phi^a<\phi_1$, $0<\lambda^a<1$, and $\nu^a=0$. Denote the optimal contract when $m\in(m_0,m_1)$ as $\mathbb{C}^a=\left\{\left(\underline{q}^a,\underline{w}^a\right),\left(\overline{q}^a,\overline{w}^a\right)\right\}$.

By (16), $-\mu^a+\phi^a>0$, so $\overline{q}^a>\overline{q}^e$ by (14) and $\overline{q}^a$ increases in m. The high-cost employee is contracted to produce an inefficiently high output. By (13), the low-cost employee produces efficiently, $\underline{q}^a=\underline{q}^e$. From (10), the employee's welfare dispersion increases to $\underline{u}^a-\overline{u}^a=\left(\overline{\theta}-\underline{\theta}\right)\overline{q}^a>\underline{u}^0-\overline{u}^0$ and wage dispersion decreases to $\underline{w}^a-\overline{w}^a=\underline{\theta}\left(\underline{q}^e-\overline{q}^a\right)<\underline{w}^0-\overline{w}^0$. With binding individual rationality (9) and incentive compatibility (10), this increase in welfare inequality is resulted from an increase in the welfare of the low-cost employee, $\underline{u}^a>\underline{u}^0$, and a decrease in that of the high-cost employee, $\overline{u}^a<\overline{u}^0$. Since $\overline{q}^a$ increases in m, $\underline{u}^a-\overline{u}^a$ increases in m and $\underline{w}^a-\overline{w}^a$ decreases in m. Precisely, solved from the binding constraints (12), (9), and (10), $\overline{q}^a=\frac{m}{\sigma\underline{\theta}+(1-\sigma)\overline{\theta}}$, $\underline{u}^a=m\cdot\frac{(1-\sigma)(\overline{\theta}-\underline{\theta})}{\sigma\underline{\theta}+(1-\sigma)\overline{\theta}}$, and $\overline{u}^a=m\cdot\frac{\sigma(\underline{\theta}-\overline{\theta})}{\sigma\underline{\theta}+(1-\sigma)\overline{\theta}}$.

Now consider the optimal contract with information advantage upon contracting, i.e. the contract solving $\mathscr{P}^p$.

For ineffective minimum wage m such that the minimum wage constraint (20) slacks, $\phi^p=0$ and the problem is a typical contracting problem with ex-post participation constraint. By (23) and (24), $\lambda^p=1$ and $\mu^p=\sigma$. The optimal employment contract $\mathbb{C}^1=\left\{\left(\underline{q}^1,\underline{w}^1\right),\left(\overline{q}^1,\overline{w}^1\right)\right\}$ has the low-cost employee produces efficiently and the high-cost employee under-produce, i.e. $\underline{q}^1=\underline{q}^e$ solved from (21) and $\overline{q}^1<\overline{q}^e$ solves

$R_q(q) = \overline{\theta} + \frac{\sigma}{1-\sigma}\left(\overline{\theta} - \underline{\theta}\right)$ from (22). It implements the employee's welfare dispersion $\underline{u}^1 - \overline{u}^1 = \left(\overline{\theta} - \underline{\theta}\right)\overline{q}^1$ derived from the binding incentive compatibility (18) with wage dispersion $\underline{w}^1 - \overline{w}^1 = \underline{\theta}\left(\underline{q}^e - \overline{q}^1\right)$. The low-cost employee enjoys a positive information rent $\underline{u}^1 \equiv \underline{w}^1 - \underline{\theta}\underline{q}^e = \left(\overline{\theta} - \underline{\theta}\right)\overline{q}^1 > 0$ under the contract, while the high-cost employee receives his reservation payoff zero, $\overline{u}^1 \equiv \overline{w}^1 - \overline{\theta}\overline{q}^1 = 0$ from the binding individual rationality constraint of the high-cost employee and the binding incentive compatibility constraint.

Prove that equilibrium level of ϕ^p and $\overline{q}$ are both increasing in m. Denote the maximum of ineffective minimum wage as $m_2 \equiv \lim_{\overline{q}\to\overline{q}^1}\left(\overline{\theta}\overline{q}\right)$. For $m > m_2$, (20) is violated under $\mathbb{C}^1$. From (22) and concave $R(\cdot)$, ϕ^p is increasing in $\overline{q}$. For any $0 < \phi^p < 1$, $0 < \lambda^p < 1$, so (17) is binding and $\overline{u} = 0$. $\overline{q}$ is thus increasing in m to satisfy (20). Hence, ϕ^p is increasing in m.

For m sufficiently large such that $\phi^p = 1$, $\overline{q} \to \infty$ by (22) given $\nu^p = 0$. Monotonicity (19) is strictly violated. There thus exists $m_3 \equiv \lim_{\overline{q}\to\underline{q}^e}\left(\overline{\theta}\overline{q}\right)$ such that for any $m \geq m_3$, $1 > \phi^p \geq \phi_3$ and the optimal contract exhibits pooling: $\underline{q} = \overline{q}$. The threshold Lagrange multiplier ϕ_3 solves (22) at $\overline{q} = \underline{q}^e$ and $\mu^p = \sigma$, i.e. $\phi_3 = \frac{\overline{\theta}-\underline{\theta}}{\overline{\theta}}$. For $m \in (m_2, m_3)$, $0 < \phi^p < \phi_3$, $0 < \lambda^p < 1$, and $\nu^p = 0$. Denote the optimal contract when $m \in (m_2, m_3)$ as $\mathbb{C}^p = \left\{\left(\underline{q}^p, \underline{w}^p\right), \left(\overline{q}^p, \overline{w}^p\right)\right\}$.

By (21), the low-cost employee produces efficiently, $\underline{q}^a = \underline{q}^e$. By (22), $\overline{q}^1 < \overline{q}^p < \overline{q}^e$ if $0 < \phi^p < \frac{\sigma\left(\overline{\theta}-\underline{\theta}\right)}{\overline{\theta}}$, $\overline{q}^p > \overline{q}^e$ if $\frac{\sigma\left(\overline{\theta}-\underline{\theta}\right)}{\overline{\theta}} < \phi^p < \phi_3$, and $\overline{q}^p = \overline{q}^e$ if $\phi^p = \frac{\sigma\left(\overline{\theta}-\underline{\theta}\right)}{\overline{\theta}}$. That is, the socially optimal level of minimum wage $m^* = \overline{\theta}\overline{q}^e \in (m_2, m_3)$ and the associated Lagrange multiplier $\phi^* = \frac{\sigma\left(\overline{\theta}-\underline{\theta}\right)}{\overline{\theta}} \in (0, \phi_3)$ exist. Output produced by the high-cost employee is restored towards the efficient level in response to an increase of sufficiently low minimum wage $m \in (m_2, m^*]$, and the high-cost employee is contracted to produce an inefficiently high output for minimum wage $m \in (m^*, m_3)$. From (18), the employee's welfare dispersion increases to $\underline{u}^p - \overline{u}^p = \left(\overline{\theta} - \underline{\theta}\right)\overline{q}^p > \underline{u}^1 - \overline{u}^1$ and wage dispersion decreases to $\underline{w}^p - \overline{w}^p = \underline{\theta}\left(\underline{q}^e - \overline{q}^p\right) < \underline{w}^1 - \overline{w}^1$. With binding individual rationality (17) and incentive compatibility (18), the increase in welfare inequality is resulted from an increase in the low-cost employee's welfare $\underline{u}^p > \underline{u}^1$, with that of the high-cost employee remaining the same, $\overline{u}^p = \overline{u}^1$. Since $\overline{q}^p$ increases in m, $\underline{u}^p - \overline{u}^p$ increases in m and $\underline{w}^p - \overline{w}^p$ decreases in m. Precisely, solved from the binding constraints (20) and (18), $\overline{q}^p = \frac{m}{\overline{\theta}}$ and $\underline{u}^p = m \cdot \frac{\overline{\theta}-\underline{\theta}}{\overline{\theta}}$.

A.3 Proposition 4 and Proposition 5

From Appendix A.2, the optimal ex-ante contract exhibits pooling for $m \geq m_1$ and the optimal ex-post contract exhibits pooling for $m \geq m_3$, where $m_1 = \lim_{\overline{q} \to \underline{q}^e} \left(((1-\sigma)\overline{\theta} + \sigma\underline{\theta})\overline{q} \right)$ and $m_3 \equiv \lim_{\overline{q} \to \underline{q}^e} \left(\overline{\theta}\overline{q} \right)$. It is straightforward that $m_3 > m_1$ for any $\sigma \in (0, 1]$. With information advantage upon contracting, the incentive compatible employment contract exhibits pooling at a higher critical level of minimum wage.

Without information advantage upon contracting, for $m \geq m_1$ and $\phi < 1$, denote the optimal pooling contract as $\hat{\mathbb{C}} = \{\hat{q}, \hat{w}\}$. By (13), (14), and concave $R(\cdot)$, $\underline{q} = \overline{q} = \hat{q} > \underline{q}^e$, and by binding ((10)), $\hat{w} = m$. Both types of employee produces inefficiently high output and earn the minimum wage. For m sufficiently large that $\phi = 1$, $\lambda = 0$ and $\mu = \sigma$. By the sum of (13) and (14), $R_q(q) = 0$. This implies $\hat{q} \to \infty$ if $\phi = 1$, given which ((12)) slacks if $\overline{u}$ is finite and ((9)) does not slack if $\overline{u} \to -\infty$. The former contradicts to $\phi = 1$ and the latter contradicts to $\lambda = 0$. The optimal contract does not have slacked ((9)). Precisely, $\hat{w} = m$ by binding ((10)) and $\hat{q} = \frac{m}{\sigma\underline{\theta} + (1-\sigma)\overline{\theta}}$ by binding ((9)).

With information advantage upon contracting, for $m \geq m_3$ and $\phi < 1$, denote the optimal pooling contract as $\tilde{\mathbb{C}} = \{\tilde{q}, \tilde{w}\}$. By (21), (22) and concave $R(\cdot)$, $\underline{q} = \overline{q} = \tilde{q} > \underline{q}^e$, and by binding ((18)), $\tilde{w} = m$. Both types of employee produces inefficiently high output and earn the minimum wage. If it is optimal to propose the pooling contract to both types of employee, for m sufficiently large such that $\phi = 1$, $\lambda = 0$. By the sum of (21) and (22), $R_q(q) = 0$. This implies $\hat{q} \to \infty$ if $\phi = 1$, given which ((20)) slacks as $\overline{u} > 0$, a contradiction to $\phi = 1$. The optimal contract does not have slacked ((17)). Precisely, $\tilde{w} = m$ by binding ((18)) and $\tilde{q} = \frac{m}{\overline{\theta}}$ by binding ((17)). With binding ((17)), it is optimal not to exclude the high-cost employee as long as contracting with whom yields a non-negative surplus. It is optimal to propose the pooling contract to both types of employee instead of screening out the high-cost employee if $R(\tilde{q}) - \overline{\theta}\tilde{q} \geq 0$, which holds if $\overline{\theta} \leq \frac{m}{R^{-1}(m)}$.

□

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