Screening with Privacy on (Im)persistency

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Abstract

I study how private information on the evolution of preference interacts with the dynamic screening contracts in a principal-agent framework with short-term commitment. Privacy on the evolution of preference preserves the agent's future information advantage, even following truthful revelation of preference. This relaxes the ratchet effects if the consumer's initial preference is skewedly distributed and the evolution of preference is distributed sufficiently evenly, while it strengthens the ratchet effects if otherwise. Through its relaxation or strengthening of the ratchet effects, privacy on (im)persistency implies an improvement or distortion in the equilibrium revelation of preference. I also characterize the respective implications on the optimal contracts. Such privacy is not welcomed by all types of the agent, for it redistributes the information rent among different types. Privacy on the evolution of preference sustains in equilibrium, as it is not optimal for the principal to perfectly screen the (im)persistency *per se*.

Keywords: privacy on evolving preference, ratchet effect, dynamic screening, incentive compatible contracts
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1 Introduction

A consumer may have evolving preference for goods that are demanded periodically without long-term commitment. Not only is the consumer's preference his private information, how his preference evolves may also be his private information. For instance, the consumer's past consumption pattern with other sellers or his previous search records reflect whether he tends to be one with persistent preference or not. In the digital era where rich data on past activities that infer the consumer's evolution of preference can be detailedly documented, lack of privacy protection may lead to leak of information that hints on the consumer's (im)persistency.[1] Under some privacy protection protocol that prevents the seller to obtain or share such information, the evolution of preference remains privately known to the consumer himself. I study how this privacy on (im)persistency interacts with the traditional screening problem in a contract theoretical framework with asymmetric information between a seller-principal and a consumer-agent.

Consider a seller contracting with a consumer on the delivery of a product, when they can commit only to short-term contracts in each of the two periods: current and future. The consumer's preference (valuation of the product) in each period is his private information realized at the beginning of such period. Lack of privacy protection, whether the consumer is persistent or impersistent in preference is common knowledge. This, for instance, can be a result of a technology that the seller invested to track the consumer's previous shopping and search pattern elsewhere. By truthfully revealing his current preference, the consumer allows the seller to infer his future preference perfectly through her knowledge of the consumer's evolution of preference. The consumer's future information advantage is completely extracted unless he lies with a positive probability in the current period. On the contrary, if the consumer is under some privacy protection to easily hide his tracks, the evolution of preference remains privately known. The seller is unable to infer the consumer's future preference with incomplete information on the (im)persistency, even if the consumer truthfully reveals his current preference with probability one. Information on the evolution of preference may seem redundant in the sense that it does not carry any additional piece of information that preferences in each period jointly do. Privacy on this seemingly redundant information, however, preserves the asymmetric information structure in the future. It is as if an implicit commitment of the seller not to exploit the current

[1] The terms evolution of preference and (im)persistency are used interchangeably throughout this paper.

information against the consumer in the future.

On the ground of this observation, I characterize the optimal contract with possibly mixed revelation strategies to study how and through what channel privacy on the evolution of preference affects the consumer's incentive to reveal the payoff-relevant information on preference, and how such privacy affects consumption efficiency and welfare among different types of consumer.

Privacy on the evolution of preference preserves future information asymmetry, which affects to what extent the consumer's current revelation of preference manipulates the seller's future belief and the consumer's future information rent. Abusing the term in the literature of dynamic screening with limited commitment, privacy on (im)persistency manipulates the ratchet effects. How such privacy shapes the optimal contracts to screen the consumer's preference depends on how the ratchet effects are manipulated. With commonly known evolution of preference, the magnitude of the ratchet effects depends on the distribution of the consumer's initial preference; with privately known evolution of preference, the magnitude of the ratchet effects depends on the distribution of (im)persistency. Implications of privacy on (im)persistency are then divided into the following three scenarios.

If the distribution of the consumer's initial preference is not too skewed and his (im)persistency follows a sufficiently even distribution, ratchet effects are mild regardless of privacy on the evolution of preference. The seller optimally induces the consumer to truthfully reveal his current preference with probability one, with or without privacy on the evolution of preference. Given completely truthful revelation, privacy on (im)persistency preserves future information asymmetry, which restores the future rent-efficiency tradeoff that were absent with commonly known evolution of preference. This reduces the potential gain of future information rent from lying and further relaxes the ratchet effects. Without distortion on revelation, privacy on the evolution of preference has no effect on the current consumption; due to the restored rent-efficiency tradeoff, it distorts the future consumption. The preserved future information asymmetry and the relaxed ratchet effects allow the seller to implement a lower intertemporal information rent for the consumer who persistently has a high valuation and a higher intertemporal information rent for the consumer who has impersistent preference. The consumer of different (im)persistency have conflicting opinions about privacy protection.

If the consumer's initial preference follows a sufficiently skewed distribution and the distribution of (im)persistency is sufficiently even, privacy on (im)persistency improves revelation of preference-related information. With commonly known evolution of preference, the consumer anticipates a significant gain of future information rent by mimicking a similar preference type at a small current loss. Ratchet effects are binding such that it is optimal for the seller to induce the consumer to take a mixed revelation strategy. With privacy on the evolution of preference, the seller cannot distinguish consumers of different (im)persistency who have the same current preference. The consumer anticipates a much lower gain of future information rent by lying in the current period. Privacy on (im)persistency relaxes the ratchet effects, to an extent that completely truthful revelation is optimally induced. Mixed revelation due to binding ratchet effects is implemented with a steeper distortion of current consumption. Improvement of revelation due to privacy on (im)persistency accompanies improvement of current consumption efficiency. In addition, with commonly known evolution of preference, the seller is able to exploit the impersistent consumer who anticipates a positive future information rent, by extracting his potential rent earlier. With privacy on (im)persistency, the seller cannot distinguish the persistent consumer from the impersistent who has the same current preference. This protects the impersistent consumer against exploitation. Along with the improvement of current consumption efficiency, the impersistent consumer is better off when his impersistency is privately known.

If the distribution of the consumer's initial preference is not too skewed and his (im)persistency follows a sufficiently skewed distribution, privacy on (im)persistency distorts revelation of preference-related information. Ratchet effects are mild with commonly known evolution of preference, as the consumer anticipates a relatively small gain of future information rent by mimicking a very different preference type at a large current loss. The seller induces completely truthful revelation of current preference. With privacy on evolution of preference that is skewedly distributed, the seller's Bayesian updated belief on the consumer's future preference is highly sensitive to the consumer's current revelation, so is the future rent extraction. Value of mis-reporting the current preference for future gain in information rent is high for the consumer whose evolution of preference is in the majority.[2] Privacy on (im)persistency strengthens the ratchet effect for the consumer whose evolution of preference is in the majority, to an extent that the consumer is induced to take a mixed revelation strategy.[3] Mixed

[2]The consumer's evolution of preference is said to be in the majority if such type realizes with probability more than a half.

[3]For the consumer whose evolution of preference is in the minority, the seller cannot distinguish him from the majority. Binding incentive compatibility for the majority implies slacking incentive compatibility for the minority. The minority type of consumer stays under the radar and truthfully reveals his current preference with certainty.

revelation due to the binding ratchet effect is implemented with a steeper distortion of current consumption. Meanwhile, the restored future rent-efficiency tradeoff due to preservation of future information asymmetry distorts future consumption. Privacy on (im)persistency results in distortion of consumption efficiency in both periods. Preservation of the future information asymmetry improves the welfare of the impersistent consumer who anticipates a positive future information rent, while the consumption distortion accompanies a lower intertemporal information rent to the consumer who persistently has a high valuation. The consumer of different (im)persistency disagree on privacy protection.

Information on (im)persistency can be regarded as an early arrival of future preference-related information, which the seller may have incentive to offer a more complicated menu of separating contracts to screen the evolution type. It is, however, not feasible for the seller to propose such menu of separating contracts to induce the consumer to truthfully reveal his (im)persistency with certainty. Privacy on (im)persistency thus remains asymmetrically known in equilibrium. Intuitively, the impersistent consumer is indistinguishable from different preference types of the persistent consumer in each period. Without long-term commitment, to induce the impersistent consumer to truthfully reveal his impersistency, it requires the same information rent to different preference types of the persistent consumer in each period. The same information rent must be implemented by the same consumption plan. Otherwise, there is room for the consumer to mis-report both his current preference and his evolution of preference.

1.1 Literature Review

I study the impact of private information on the evolution of preference in a dynamic screening framework with short-term commitment. In the literature of dynamic screening with limited commitment, seminal papers by Laffont and Tirole (1987 and 1988) found that pooling occurs due to ratchet effect, when privately known types are persistent and it is common knowledge that types are persistent.[4] Bester and Strausz (2001) and Doval and Skreta (2019) developed concepts à la revelation principle.[5] Skreta (2006) proposed that sequential price posting is an optimal mechanism in a general environment even when the Bester-Strausz revelation principle does not ap-

[4]Sun (2011) examined non-optimality of a non-partitioned continuation equilibrium as a follow-up.

[5]Bester and Strausz (2001) showed in a discrete-type model that one can restrict to a direct revelation mechanism which induced mixed revelation strategies. Bester and Strausz (2007) proposed a noisy communication device along with the contract in replacement of direct revelation. Doval and Skreta (2019) identified canonical mechanisms that takes into account the principal's posterior belief satisfying sequential rationality.

ply. The focus in these papers are mainly the role of limited commitment on the characteristics and the performance of contracts.

As for factors that shape contracts with limited commitment, Shin and Strausz (2014) studied in a production relationship that future private information on productivity of capital input granted through ownership allocation improves truthful information revelation earlier on the consistent productivity of labor input. Fiocco and Strausz (2015) analyzed how strategic delegation with mandated weights on firm's profit and consumer's welfare reduces ratchet effect and improve dynamic efficiency of short-term regulatory contracts. Deb and Said (2015) incorporated a second cohort of agents who only arrive in the future and found that their presence reduces the principal's ability to extract rents via the dynamic contract with limited commitment. Strategic delay of contract occurs in equilibrium as an endogenous commitment to future terms of contract. Accompanying the previous literature, I observe that information asymmetry on the evolution of preference preserves future information asymmetry even with full information revelation. This prevents the principal from completely extracting future information rent, which manipulates the ratchet effect. Applying the Bester-Strausz revelation principle, the optimal revelation strategy and the optimal contract are characterized.

Loginova and Taylor (2008) shared similar idea to this paper that a consumer would strategically use his private information on future preference. In a price-posting model, their focus was on how a buyer strategically rejects a posted price to conceal or to signal information, with a benchmark comparison to a non-strategic buyer. As a result, the seller posts an equilibrium price that preempts information transmission. In this paper, the focus is on truth-revealing incentives and the intertemporal distortion of contracts in a dynamic screening framework, with a benchmark comparison to commonly known pattern of preference.

With full commitment, Baron and Besanko (1984), Courty and Li (2001), Pavan, Segal, and Toikka (2014), and Krähmer and Strausz (2015) provided classical treatments of dynamic contracts with asymmetric information. Boleslavsky and Said (2013) and Skrzypacz and Toikka (2015) studied sequential screening with full commitment when the stochastic sequence of preference shocks as well as the conditional density of the stochastic sequence are the agent's the private information. The contract is designed to screen the parameters of the conditional density of the stochastic sequence, as well as the realization of the stochastic sequence periodically based on which the consumer's preference evolves. Evolution of preference in the current paper differs from the stochastic evolution of preference in the above literature in terms of its role on preserving future information asymmetry.

The results in this paper also have implications on privacy protection, which has been actively debated in the digital era. Lack of a sound definition, the literature on privacy diverged in topics[6] but is heavily studied in the effects of protection and disclosure of personal information on the market and organizational outcomes. In Fudenberg and Tirole (1998), Villas-Boas (2004), Acquisti and Varian (2005), and Conitzer, Taylor, and Wagman (2012), privacy decision was modeled as an investment in technology or an action to identify the consumer, to track or to untrack the consumer's past actions and choices. Consumer's anonymity affects the seller's ability to utilize the information on previous consumption that reveals persistent preference type. With multiple sellers of correlated goods, Taylor (2004) and Calzolari and Pavan (2006) further examined trade of such preference-revealing information among the sellers. The current paper distinguishes itself from the above literature in the sense that the consumer's chosen option in the earlier contract is identified and remembered in the future at no cost. The key discussion is on whether and how privacy protection on the seemingly redundant information facilitates the revelation of crucial information.

2 Model

A seller-principal randomly matches with a consumer-agent to contract on the delivery of a non-durable product that is demanded for two periods, current and future. They are able to commit only to a short-term contract, so that in each period $t = 1, 2$, the seller proposes a take-it-or-leave-it contract to the consumer who then accepts or rejects the short-term contract. The seller specifies consumption output q_t and associated payment p_t in the contract proposed at period t, denoted as $\mathbb{C}_t = \{(q_t, p_t)\}$. The consumption $q_t \geq 0$ can be the frequency of gym lessons per month, the magazine subscription detail (e-access, hard-copy, other associated membership benefits...etc.), or terms on specific usage of charity donations in the above examples. The associated payment p_t is the total transfer payment, including membership fee and usage fee (if separated). It costs $c(q_t) = \frac{1}{2} \cdot q_t^2$ for the seller to supply q_t in period t.

The consumer may have evolving preference defined as the change of privately known taste. His preference at period t is measured by the marginal value of the product in such period, denoted as $v_t \in \{v_l, v_h\}$ with $0 < v_l < v_h < \infty$. A consumer is said to be persistent (denoted as evolution type P) if he has persistent preference

[6] Please refer to Acquisti, Taylor, and Wagman (2016) for a survey on the Economic literature of privacy.

$v_1 = v_2$, while he is said to be impersistent (denoted as evolution type I) if his preference switches within the two periods $v_1 \neq v_2$. (Im)persistency can be viewed as the consumer's intrinsic personality, which is perfectly known by himself. For ease of expression, I phrase the persistent consumer with high valuation and that with low valuation as the persistent-high type and the persistent-low type respectively. The impersistent consumer who has a higher valuation in the earlier period ($v_1 = v_h > v_2 = v_l$) and that who has a higher valuation in the later period ($v_1 = v_l < v_2 = v_h$) are phrased as the descending type and the ascending type respectively.

The consumer receives a private and perfect signal on his evolution of preference and his first-period preference prior to the first-period contracting.[7] The prior distribution of the consumer's first-period preference and that of his evolution of preference are common knowledge. With probability $\sigma \in (0, 1)$ the consumer values the product highly in the first period, and with probability $\phi \in (0, 1)$ the consumer is persistent. One can consider that there is a continuum of consumers of mass one, in which $\phi \cdot \sigma$ (respectively $\phi \cdot (1 - \sigma)$) of them have persistently high (resp. low) valuation over the product and $(1 - \phi) \cdot \sigma$ (resp. $(1 - \phi) \cdot (1 - \sigma)$) of them perceive a higher valuation over the product at an earlier (resp. a later) date.[8] For ease of description, the persistent (resp. impersistent) consumer is said to be in the minority of evolution type if $\phi < 1 - \phi$ (resp. $\phi > 1 - \phi$), and said to be in the majority if otherwise.

At the beginning of each period t, the consumer receives a private and perfect signal on his period-t preference prior to contracting. The seller proposes a take-it-or-leave-it contract $\mathbb{C}_t = \{(q_{ti}, p_{ti})\}$, $t = 1, 2$ and $i = l, h$, which consists of options for the consumer to choose from upon acceptance, with $i \in \{l, h\}$ indicative of the consumer's message on his preference.[9] If the consumer accepts the contract and take option (q_{ti}, p_{ti}), production is executed and trade occurs at the end of this period. The consumer's (im)persistency is then reflected in the change of preferred option in the contract at different periods. If the consumer rejects the contract, both players earn reservation payoff zero.

The consumer with preference type $v_i \in \{v_l, v_h\}$ in period t earns rent $u_{ti} =$

[7] Information on the evolution of preference seems redundant in the sense that it does not carry any additional piece of information that preference in each period jointly do. However, the redundant information is non-negligible as an early signal of the consumer's future taste.

[8] To neglect the uninteresting corner solution that some types of the consumer are induced to consume zero output in equilibrium, assume that $\max\{\sigma, \phi, 1 - \phi\} \cdot v_h \leq v_l$.

[9] The contract is assumed to induce a mixed revelation strategy on the preference type, without screening the evolution type. In a later discussion in Section 5, I show that the information on the evolution type remains asymmetrically known in equilibrium, even when screening (im)persistency *per se* is considered.

$v_i \cdot q_{ti} - p_{ti}$ if he takes the option (q_{ti}, p_{ti}), $t = 1, 2$ and $i = l, h$. The seller earns profit $\pi_{ti} = p_{ti} - c(q_{ti})$ in period t when the consumer takes the option (q_{ti}, p_{ti}), $t = 1, 2$ and $i = l, h$. Let there be no discount between the payoffs in two periods to abstract from the situation where the players have exogenous preference over the first or the second period.[10] For future reference, denote the efficient consumption output when contracting with a consumer of valuation v_i in period t as $q_i^* \in \arg\max_q v_i \cdot q - c(q)$. I adopt the perfect Bayesian equilibrium solution concept throughout.

It is worthwhile to briefly discuss the intuition on the role of asymmetric information on the evolution of preference and its implication on privacy protection. For example, the consumer's intrinsic personality, whether he tends to be persistent or not, can be inferred from his past interactions with other firms. With the popularity of online shopping, search engine, and social media platform...etc., rich data on past activities that infer intrinsic personality can be very detailed.[11] If such information can be costlessly acquired without privacy protection, this corresponds to the case in Section 3.1 where the evolution type is common knowledge. The consumer, by truthfully revealing the first-period preference to the seller, lost his information advantage in the second period. For instance, the seller knows that a persistent consumer will be willing to make a high payment in the second period when he is induced to truthfully reveal his high valuation in the first period. Alternatively, if the consumer is under some privacy protection against documentation or share of personal information, or that it is sufficiently costly to acquire information on the consumer's past activities that infer his intrinsic personality, the consumer's evolution type is his private information. Even when the first period preference is truthfully revealed, the consumer in the second period still maintains some information advantage due to his private information on the evolution of preference. For instance, the seller only knows that with probability ϕ the consumer who truthfully revealed a high valuation is still willing to make a high payment in the second period. Asymmetric information on the consumer's evolution of preference is a potential source of the consumer's information advantage in the future.

[10]Robustness of the results can be easily shown when the second-period payoff is discounted. Asymmetric information on the evolution of preference can play a more important role when the first-period payoff is discounted.

[11]Please refer to the related literature on privacy listed in the previous section. Bergemann, Bonatti, and Smolin (2018) also studied theoretically the contracts for data, with practical implications.

3 Contracts

3.1 Commonly Known Evolution of Preference

As a benchmark, consider only in this section that the consumer's evolution of preference is common knowledge. Applying the revelation principle à la Bester and Strausz (2001), the seller proposes the first-period contract $\mathbb{C}_1^k = \{(q_{1i}^k, p_{1i}^k)\}$ to induce the consumer of evolution type $k = P, I$ and preference type $v_i = v_l, v_h$ to truthfully reveal his first-period taste with probability n_{ki}. Following the type-k consumer's choice of first-period option (q_{1i}^k, p_{1i}^k), the seller's Bayesian updated belief of consumer having high valuation in the second period is denoted as $\nu_k(v_i)$. Given such belief, the seller proposes the second-period contract $\mathbb{C}_2^{ki} = \{(q_{2j}^{ki}, p_{2j}^{ki})\}$. To the extreme, following the chosen option (q_{1h}^k, p_{1h}^k) in a truth revealing first-period contract, the (im)persistent consumer is believed to have a high valuation in the second period with probability $\nu_P(v_h) = 1$ $(\nu_I(v_h) = 0)$. With truthful revelation in the first period, the consumer is left with no information advantage in the second period.

The seller's problem can be separated into three stages: 1) a second-period contracting problem given belief $\nu_k(v_i)$, 2) a first-period contracting problem that is incentive compatible and individually rational to induce an arbitrary revelation strategy n_{ki}, and 3) the optimal revelation strategy to implement that maximizes the seller's intertemporal payoff.

By backward induction, the conventional revelation principle applies to the seller's second-period contracting problem with belief $\nu_k(v_i)$. Following the consumer's first-period choice of (q_{1i}, p_{1i}), the seller proposes a standard contract $\mathbb{C}_2^{ki} = \{(q_{2j}^{ki}, p_{2j}^{ki})\}$, $j = l, h$, implementing payoff $u_{2j}^{ki} = v_j \cdot q_{2j}^{ki} - p_{2j}^{ki}$ that

$$\mathscr{P}_2^{ki}: \quad \max_{q_2, u_2} \nu_k(v_i) \cdot (v_h \cdot q_{2h} - c(q_{2h}) - u_{2h}) + (1 - \nu_k(v_i)) \cdot (v_l \cdot q_{2l} - c(q_{2l}) - u_{2l})$$

subject to individual rationality $u_{2j} \geq 0$ for both $j = l, h$, and incentive compatibility $(v_h - v_l) \cdot q_{2h} \geq u_{2h} - u_{2l} \geq (v_h - v_l) \cdot q_{2l}$. For later reference, denote $\pi_2^{ki}(n_k)$ as the seller's second-period expected payoff as a solution to the above contracting problem, given the consumer's first-period revelation strategy $n_k = (n_{kh}, n_{kl})$.

Anticipating the second-period contract, it is individually rational[12] for the persis-

[12] I restrict to contracts to induce full participation. Off the equilibrium path following the first-period rejection, the seller believes that with probability one the consumer has a second-period valuation of v_h. This belief is weakly consistent, and is most beneficial to the seller as it allows her to implement a lower rent to the consumer on the equilibrium path. It is also reasonable for the seller to believe so, because a consumer with $v_2 = v_l$ by rejecting the first-period contract would not earn

tent consumer to accept the first-period contract if

$$\begin{aligned} u_{1h} + u_{2h}^{Ph} &\geq 0 \qquad (IR_h^P), \\ u_{1l} + u_{2l}^{Pl} &\geq 0 \qquad (IR_l^P). \end{aligned}$$

It is incentive compatible for the persistent consumer to reveal his true preference with probability $n_{Pi} > 0$ in the first period if

$$\begin{aligned} u_{1h} + u_{2h}^{Ph} &\geq u_{1l} + (v_h - v_l) \cdot q_{1l} + u_{2h}^{Pl} \qquad (IC_h^P), \\ u_{1l} + u_{2l}^{Pl} &\geq u_{1h} - (v_h - v_l) \cdot q_{1h} + u_{2l}^{Ph} \qquad (IC_l^P), \end{aligned}$$

with equality if $n_{Pi} \neq 1$ is implemented. Meanwhile, it is individually rational for the impersistent consumer to accept the first-period contract if

$$\begin{aligned} u_{1h} + u_{2l}^{Ih} &\geq 0 \qquad (IR_h^I), \\ u_{1l} + u_{2h}^{Il} &\geq 0 \qquad (IR_l^I). \end{aligned}$$

It is incentive compatible for the impersistent consumer to reveal his true preference with probability $n_{Ii} > 0$ in the first period if

$$\begin{aligned} u_{1h} + u_{2l}^{Ih} &\geq u_{1l} + (v_h - v_l) \cdot q_{1l} + u_{2l}^{Il} \qquad (IC_h^I), \\ u_{1l} + u_{2h}^{Il} &\geq u_{1h} - (v_h - v_l) \cdot q_{1h} + u_{2h}^{Ih} \qquad (IC_l^I), \end{aligned}$$

with equality if $n_{Pi} \neq 1$ is implemented.

Denote $\rho^k = \sigma \cdot n_{kh} + (1 - \sigma) \cdot (1 - n_{kl})$ as the probability that option (q_{1h}, p_{1h}) is chosen by the consumer of evolution type k. The seller proposes the first-period contract $\mathbb{C}_1^k = \{(q_{1i}^k, p_{1i}^k)\}$, $i = l, h$, implementing payoff $u_{1i}^k = v_i \cdot q_{1i}^k - p_{1i}^k$ that

$$\mathscr{P}_1^k: \quad \max_{q_1, u_1} \rho^k \cdot (v_h \cdot q_{1h} - c(q_{1h}) - u_{1h}) + (1 - \rho^k) \cdot (v_l \cdot q_{1l} - c(q_{1l}) - u_{1l})$$

subject to (IR_i^k) and (IC_i^k) for both $i = l, h$. Let $\pi_1^k(n_k)$ be the seller's first-period expected payoff as a solution to the above contracting problem, given the consumer's revelation strategy. Anticipating the contracts $\mathbb{C}_t^k$, the seller induces revelation strategies n_k that maximizes her intertemporal payoff $\pi_1^k(n_k) + \rho^k \cdot \pi_2^{kh}(n_k) + (1 - \rho^k) \cdot \pi_2^{kl}(n_k)$ and is consistent to incentive compatibility.

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a higher payoff than he does under the full participation contracts. Inducing rejection is then payoff equivalent to the first-period contract with $q_{1l} = 0$, $q_{1h} = v_h$, $t_{1l} = 0$, and $t_{1h} = v_h^2$. This contract is incentive compatible and individually rational in the first period for all types. I do not rule out the possibility of this contract, so if this contract is not optimal, neither is rejection.

Condition 1. Preference and the distribution of initial preference satisfy

1. $\frac{1}{1-\sigma} \geq \frac{v_l}{v_h - v_l}$
2. $\frac{1}{1-\sigma} \geq \frac{v_l}{v_h - v_l}$ when $\frac{\sigma}{1-\sigma} \cdot \frac{v_l}{v_h - v_l} \leq 1$
3. $\frac{\sigma}{1-\sigma} \geq \frac{v_l}{v_h - v_l}$

Lemma 1. *The first-period contract with a consumer of evolution type $k = P, I$ satisfies binding (IR_l^k) and (IC_h^k). Constraint (IC_l^k) is binding only when Condition 1 is violated.*

Proof. Appendix A.1. □

Implied by the slacking incentive compatibilities in Lemma 1 given Condition 1, the consumer who has a low valuation in the first period reveals his preference truthfully with probability one, regardless of his evolution type. Condition 1 resembles the slacking monotonicity constraint in standard contracting. Satisfaction of the traditional monotonicity[13] is not sufficient for us to neglect the incentive compatibility of the consumer who has a low valuation in the first period. The anticipated future information rent depends on the first-period revelation, so incentive compatibility is more restrictive than the traditional.

Intuitively, there is a ratchet effect for the persistent-high type and the ascending type of consumer to mis-lead the seller's future belief. Contracting with a persistent consumer, a higher information rent to the persistent-high type of consumer to cope with the binding ratchet effect may introduce the persistent-low type to lie to take the first-period benefit and reject the unfavorable second-period contract. Contracting with an impersistent consumer, the ascending type has incentive to lie about his first-period preference in order to protect his future information advantage. Satisfaction of Condition 1 rules out the binding incentive for the persistent-low type to take the instant benefit and run, and it implies a non-binding ratchet effect for the ascending type.

Evolution of preference being commonly known, to what extent the consumer's revelation manipulates the seller's future belief and hence the consumer's future information rent depends on the difference in the consumer's taste and the distribution of the initial preference, so is Condition 1. It holds when i) the consumer's preference in each period is sufficiently diverse, and/or ii) the distribution of the consumer's initial taste is not too skewed towards low valuation.

[13] This would be translated to $v_h \geq v_l$ in this model.

Lemma 2. *Satisfaction of Condition 1, the optimal first-period contracts induce* $n^*_{Ih} \leq n^*_{ki} = 1$ *for all* $(k,i) \neq (I,h)$ *with consumption* $\hat{q}^k_{1h} = q^*_h$ *and* $\hat{q}^P_{1l} \leq \hat{q}^I_{1l} < q^*_l$*; equality holds except given Condition 1-3. Following completely truthful revelation of first-period preference, the optimal second-period contract has a single option with efficient consumption given the seller's degenerate belief,* $\hat{q}^{Pl}_2 = \hat{q}^{Ih}_2 = q^*_l$ *and* $\hat{q}^{Ph}_2 = \hat{q}^{Il}_2 = q^*_h$*. Following* $n^*_{Ih} < 1$*, the optimal second-period contract to the impersistent consumer has* $\hat{q}^{Il}_{2h} = q^*_h$ *and* $\hat{q}^{Il}_{2l} < \hat{q}^{Ih}_2 = q^*_l$*. The contracts implement intertemporal information rent* $\hat{u}_{Ph} = (v_h - v_l) \cdot (\hat{q}^P_{1l} + q^*_l)$ *to the persistent-high type,* $\hat{u}_{Pl} = 0$ *to the persistent-low type,* $\hat{u}_{Ih} = (v_h - v_l) \cdot \hat{q}^I_{1l} - \hat{u}^{Il}_{2h} \geq 0$ *to the descending type, and* $\hat{u}_{Il} = 0$ *to the ascending type of consumer.*

Proof. Appendix A.1. □

Contracting with a persistent consumer, it is incentive compatible for the persistent-high type of consumer to truthfully reveal his first-period preference with a positive probability only if he receives in advance the potential future information rent had he possessed information advantage in the second period by mis-reporting his preference in the first period. Satisfaction of Condition 1-1 implies that the persistent-low type of consumer does not find such information rent attractive enough to mimic the persistent-high type. The optimal first-period contract thus induces completely truthful revelation, with efficient consumption for the consumer who reveals high valuation and downward distorted consumption for whom revealing low valuation, due to the conventional rent-efficiency tradeoff. This is summarized in Lemma 2.

Violation to Condition 1-1 implies that the persistent-low type has incentive to lie to take the first-period benefit and reject the future contract given completely truthful revelation. There are two channels for the seller to cope with such take-the-money-and-run incentive[14]: consumption distortion and revelation distortion. To induce the persistent-low type to reveal his true preference with a positive probability, the seller distorts the first-period consumption upwards for the consumer who reveals high valuation, and further downwards for whom revealing low valuation. Efficiency loss from such steeper consumption distortion welcomes the implementation of mixed revelation. This is summarized in Lemma 3 below.

Contracting with an impersistent consumer, the standard static information rent is sufficient to induce the descending type of consumer to truthfully reveal his first-period preference with a positive probability, anticipating a zero second-period information rent regardless. Satisfaction of Condition 1-2 or 1-3 implies that the ascending type of

[14]Please excuse me to adopt the term from Shin and Strausz (2014).

consumer does not find the potential future information advantage valuable enough to outweigh the first-period loss from mimicking the descending type. However, this does not guarantee the optimality of completely truthful revelation in the first period. The ascending type of consumer is willing to accept an exploitative first-period contract implementing a negative interim information rent, had he anticipated that rejection leads to a forgone future information rent. This is possible only if the descending type is induced to lie with a positive probability such that the ascending type has a positive future information rent given truthful revelation. This exploitative contract is attractive to the seller in the sense that she can not only exploit the ascending type but also reduce the information rent yielded to the descending type by incentive compatibility. It is at a cost of revelation efficiency, as it takes more than a marginal distortion in the revelation strategy. An exploitative contract is optimal only if $\frac{\sigma}{1-\sigma} \cdot \frac{v_l}{v_h - v_l}$ is sufficiently large such that the seller can implement a small distortion in the descending type's revelation strategy to generate a sufficiently high future information rent for the ascending type to be exploited. Otherwise, the optimal first-period contract induces completely truthful revelation without exploitation, which exhibits the conventional rent-efficiency tradeoff. This is summarized in Lemma 2.

When Conditions 1-2 and 1-3 are violated, the ascending type has incentive to lie to protect his future information advantage as the value of such advantage is sufficiently high to outweigh the first-period loss of lying. There are two channels for the seller to cope with such binding ratchet effect: consumption distortion and revelation distortion. To induce the ascending type to reveal his true preference with a positive probability, the seller distorts the first-period consumption upwards for the consumer who reveals high valuation, and further downwards for whom revealing low valuation. Efficiency loss from such steeper consumption distortion welcomes the implementation of mixed revelation. This is summarized in Lemma 3 below.

Lemma 3. *Violation to Condition 1, the optimal first-period contracts induce mixed revelation of first-period preference with steeper consumption distortion* $\tilde{q}_{1h}^k \geq q_h^*$ *and* $\tilde{q}_{1l}^k \leq \hat{q}_{1l}^k$.[15] *Following mixed revelation of first-period preference, the optimal second-period contract is a menu of options given the seller's non-degenerate belief, with* $\tilde{q}_{2h}^{ki} = q_h^*$ *and* $\tilde{q}_{2l}^{ki} \leq q_l^*$ *for any* (k, i). *The contracts implement intertemporal information rent* $\tilde{u}_{Ph} = (v_h - v_l) \cdot (\tilde{q}_{1l}^P + \tilde{q}_{2l}^{Pl})$ *to the persistent-high type,* $\tilde{u}_{Pl} = 0$ *to the persistent-low type,* $\tilde{u}_{Ih} = (v_h - v_l) \cdot \tilde{q}_{1l}^I - \tilde{u}_{2h}^{Il} \geq 0$ *to the descending type, and* $\tilde{u}_{Il} = 0$ *to the ascending type of consumer.*

[15] Strict inequality holds when (IC_l^k) is binding.

Proof. Appendix A.1. □

With commonly known evolution of preference, information on the second-period preference is indirectly revealed through completely truthful revelation in the first period. If completely truthful revelation is optimally induced, the seller has degenerate belief on the consumer's second-period taste, so the optimal second-period contract has a single option with efficient consumption and zero information rent. Violation to Condition 1, mixed revelation is implemented in the first period. Asymmetric information on the second-period preference remains. The seller has non-degenerate belief on the consumer's second-period taste, so the optimal second-period contract is a menu of options which exhibits the static rent-efficiency tradeoff. The consumer with a high valuation in the first period enjoys a positive information rent. The ascending type of consumer has his future information rent completely extracted in the first period either through the induced truthful revelation or the exploitation with mixed revelation.

3.2 Privately Known Evolution of Preference

With private information on the evolution of preference, we focus on the contracts that implement mixed revelation of preference à la Bester and Strausz (2001). The seller proposes the first-period contract $\mathbb{C}_1 = \{(q_{1i}, p_{1i})\}$ to induce the consumer of evolution type $k = P, I$ and preference type $v_i = v_l, v_h$ to truthfully reveal his first-period preference with probability m_{ki}. Following the consumer's choice of first-period option (q_{1i}, p_{1i}), the seller's Bayesian updated belief of consumer having high valuation in the second period is denoted as $\mu(v_i)$. Given such belief, the seller proposes the second-period contract $\mathbb{C}_2^i = \{(q_{2j}^i, p_{2j}^i)\}$.[16] To the extreme, following the chosen option (q_{1h}, p_{1h}) in a truth revealing first-period contract, the seller believes that with probability $\mu(v_h) = \phi$ the consumer has a high valuation in the second period as well, and following the choice of option (q_{1l}, p_{1l}), the seller believes that with probability $\mu(v_l) = 1 - \phi$ the consumer has a high valuation in the second period instead. The asymmetric information on the evolution of preference preserves the consumer's second-period information advantage even with completely truthful first-period rev-

[16] I focus on first-period contracts to screen only the preference types, based on the intuition that (im)persistency is simply an early arrival of future information that is not yet productive in the first period. Anticipating screening in the second period by backward induction, consumers of different evolution types are only separated by possibly different revelation strategies. This intuition is confirmed in Section 5, where I show that separating contracts to induce truthful revelation of evolution types exhibit pooling on evolution types.

elation. It is as if a commitment device that the seller will not use the information revealed in the first period to completely extract the consumer's future rent.

The seller's problem can be separated into three stages: 1) a second-period contracting problem given belief $\mu(v_i)$, 2) a first-period contracting problem that is incentive compatible and individually rational to induce an arbitrary revelation strategy m_{ki}, and 3) the optimal revelation strategy to implement that maximizes the seller's intertemporal payoff.

By backward induction, conventional revelation principle applies to the seller's second-period contracting problem with belief $\mu(v_i)$. Following the consumer's first-period choice of (q_{1i}, p_{1i}), the seller proposes a standard contract $\mathbb{C}_2^i = \{(q_{2j}^i, p_{2j}^i)\}$, $j = l, h$, implementing payoff $u_{2j}^i = v_j \cdot q_{2j}^i - p_{2j}^i$ that

$$\mathscr{P}_2^i: \quad \max_{q_2, u_2} \mu(v_i) \cdot (v_h \cdot q_{2h} - c(q_{2h}) - u_{2h}) + (1 - \mu(v_i)) \cdot (v_l \cdot q_{2l} - c(q_{2l}) - u_{2l})$$

subject to individual rationality $u_{2j} \geq 0$ for both $j = l, h$, and incentive compatibility $(v_h - v_l) \cdot q_{2h} \geq u_{2h} - u_{2l} \geq (v_h - v_l) \cdot q_{2l}$. For later reference, denote $\pi_2^i(m)$ as the seller's second-period expected payoff as a solution to the above contracting problem, given the consumer's first-period revelation strategy $m = (m_{Ph}, m_{Pl}, m_{Ih}, m_{Il})$.

Anticipating the second-period contract, it is individually rational for the (im)persistent consumer to accept the first-period contract if

$$\begin{aligned} u_{1h} + u_{2h}^h &\geq 0 && (IR_{Ph}), \\ u_{1l} + u_{2l}^l &\geq 0 && (IR_{Pl}), \\ u_{1h} + u_{2l}^h &\geq 0 && (IR_{Ih}), \\ u_{1l} + u_{2h}^l &\geq 0 && (IR_{Il}). \end{aligned}$$

It is incentive compatible for the (im)persistent consumer to reveal his true preference with probability $m_{ki} > 0$ in the first period if

$$\begin{aligned} u_{1h} + u_{2h}^h &\geq u_{1l} + (v_h - v_l) \cdot q_{1l} + u_{2h}^l && (IC_{Ph}), \\ u_{1l} + u_{2l}^l &\geq u_{1h} - (v_h - v_l) \cdot q_{1h} + u_{2l}^h && (IC_{Pl}), \\ u_{1h} + u_{2l}^h &\geq u_{1l} + (v_h - v_l) \cdot q_{1l} + u_{2l}^l && (IC_{Ih}), \\ u_{1l} + u_{2h}^l &\geq u_{1h} - (v_h - v_l) \cdot q_{1h} + u_{2h}^h && (IC_{Il}), \end{aligned}$$

with equality if $m_{ki} \neq 1$ is implemented.

Denote $\rho = \sigma \cdot \phi \cdot m_{Ph} + \sigma \cdot (1-\phi) \cdot m_{Ih} + (1-\sigma) \cdot \phi \cdot (1-m_{Pl}) + (1-\sigma) \cdot (1-\phi) \cdot (1-m_{Il})$ as the probability that option (q_{1h}, p_{1h}) is chosen by the consumer of any type. The seller proposes the first-period contract $\mathbb{C}_1 = \{(q_{1i}, p_{1i})\}$, $i = l, h$, implementing payoff $u_{1i} = v_i \cdot q_{1i} - p_{1i}$ that

$$\mathscr{P}_1: \quad \max_{q_1, u_1} \rho \cdot (v_h \cdot q_{1h} - c(q_{1h}) - u_{1h}) + (1-\rho) \cdot (v_l \cdot q_{1l} - c(q_{1l}) - u_{1l})$$

subject to (IR_{ki}) and (IC_{ki}) for both $k = P, I$ and $i = l, h$. Let $\pi_1(m)$ be the seller's first-period expected payoff as a solution to the above contracting problem, given the consumer's revelation strategy. Anticipating the contracts $\mathbb{C}_t$, the seller induces revelation strategies m that maximizes her intertemporal payoff $\pi_1(m) + \rho \cdot \pi_2^h(m) + (1-\rho) \cdot \pi_2^l(m)$ and is consistent to incentive compatibility.

Condition 2. The distribution of evolution type and that of initial preference satisfy

1. $\frac{1}{1-\sigma} \geq \frac{\phi}{1-\phi} - \frac{1-\phi}{\phi}$ when $\frac{1}{1-\sigma} \geq 2 \cdot \left(\frac{\sigma}{\phi} + \frac{1-\sigma}{1-\phi}\right)$ if $\phi > 1 - \phi$
2. $\frac{1}{1-\sigma+\sigma\cdot\phi} \geq \frac{\phi}{1-\phi} - \frac{1-\phi}{\phi}$ when $\frac{1}{1-\sigma} < 2 \cdot \left(\frac{\sigma}{\phi} + \frac{1-\sigma}{1-\phi}\right)$ if $\phi > 1 - \phi$
3. $\frac{1}{1-\sigma} \geq \frac{1-\phi}{\phi} - \frac{\phi}{1-\phi}$ if $1 - \phi > \phi$

Lemma 4. *The first-period contract satisfies binding (IR_{Pl}). The relevant incentive compatibility constraints are those of the majority of evolution type. The contract satisfies binding (IC_{Ph}) when $\phi > 1-\phi$, or binding (IC_{Ih}) when $1-\phi > \phi$. Constraint (IC_{Pl}) or (IC_{Il}) is binding when $\phi > 1 - \phi$ or when $1 - \phi > \phi$ respectively only if Condition 2 is violated.*

Proof. Appendix A2. □

Implied by the slacking incentive compatibilities in Lemma 4, the consumer who is in the minority of evolution type reveals his first-period preference truthfully with probability one. Intuitively, the seller is unable to distinguish the consumer's evolution of preference and anticipates a lower probability to trade with the minority. If the majority is persistent, the seller believes that the consumer who has revealed v_l in the first period remains as a low valuation type in the second period with a higher probability than is the consumer who has revealed v_h. The second-period information rent would be higher following a first-period revelation of v_l. This implies a more restrictive incentive compatibility for the persistent consumer. If the majority is impersistent, the seller believes that the consumer who has revealed v_l in the first period becomes a high valuation type in the second period with a higher probability than is the consumer who has revealed v_h. The second-period information rent would be higher following a first-period revelation of v_h. This implies a more restrictive incentive compatibility for the impersistent consumer. Asymmetric information on the evolution of preference allows the minority of evolution type to stay under the radar and enjoy a spillover information rent above the minimum requirement for incentive compatibility. Privacy on the evolution of preference weakens the ratchet effect of the minority type to an extent that it is non-binding. The minority type thus fully reveals his preference in the first period if he has private information on the evolution of preference.

Among the majority of evolution type, slacking incentive compatibilities in Lemma 4 given Condition 2 implies that the consumer who has a low valuation in the first period reveals his preference truthfully with probability one. Condition 2 resembles the slacking monotonicity constraint in standard contracting. Satisfaction of the traditional monotonicity is not sufficient for us to neglect the incentive compatibility of the consumer who has a low valuation in the first period. The anticipated future information rent depends on the first-period revelation, so incentive compatibility is more restrictive than the traditional.

If the majority is persistent, there is a binding ratchet effect for the persistent-high type of consumer to mis-lead the seller's future belief. A higher information rent to the persistent-high type to cope with the ratchet effect may introduce the persistent-low type to lie to take the first-period benefit, anticipating a zero second-period information rent regardless of revelation. Satisfaction of Condition 2-1 and 2-2 rules out the binding incentive for the persistent-low type to take the instant benefit. If the majority is impersistent, the ascending type of consumer has incentive to manipulate the seller's future belief through the first-period revelation in his own favor. This ratchet effect for the ascending type is non-binding if Condition 2-3 is satisfied.

Evolution of preference being privately known, to what extent the consumer's revelation manipulates the seller's future belief and hence the consumer's future information rent depends on the distribution of evolution type and the distribution of the initial preference, so is Condition 2. It holds when i) the distribution of evolution type is sufficiently even, and/or ii) the distribution of the consumer's initial taste is not too skewed towards low valuation.

Lemma 5. *Satisfaction of Condition 2, the optimal first-period contract* $\mathbb{C}_1^a$ *induces* $m^*_{Ph} \leq m^*_{ki} = 1$[17] *for all* $(k,i) \neq (P,h)$ *if* $\phi > 1-\phi$ *and* $m^*_{ki} = 1$ *for all* (k,i) *if*

[17] Equality holds except given Condition 2-2.

$1-\phi > \phi$, with consumption $q^a_{1h} = q^*_h$ and $q^a_{1l} < q^*_l$. Following revelation of first-period preference, completely truthful or not, the optimal second-period contract $\mathbb{C}^{ai}_2$ is a menu of options given the seller's non-degenerate belief, with $q^{ai}_{2h} = q^*_h$ and $q^{ai}_{2l} < q^*_l$, with $q^{al}_{2l} \gtreqless q^{ah}_{2l}$ if $\phi \gtreqless 1-\phi$. The contracts implement intertemporal information rent $u^a_{Ph} = (v_h - v_l) \cdot (q^a_{1l} + \max\{q^{al}_{2l}, q^{ah}_{2l}\})$ to the persistent-high type, $u^a_{Pl} = 0$ to the persistent-low type, $u^a_{Ih} = (v_h - v_l) \cdot (q^a_{1l} + \max\{q^{al}_{2l} - q^{ah}_{2l}, 0\})$ to the descending type, and $u^a_{Il} = (v_h - v_l) \cdot q^{al}_{2l}$ to the ascending type of consumer.

Proof. Appendix A.2. □

When the majority is persistent, it is incentive compatible for the persistent-high type of consumer to truthfully reveal his preference with a positive probability only if he receives in advance the potential future gain of information rent from misreporting his first-period preference. Satisfaction of Condition 2-1 or 2-2 implies that the persistent-low type of consumer does not find such additional benefit attractive enough to bear the cost of lying in the first period. Inducing the persistent-high type to take a mixed revelation strategy manipulates the seller's future belief and reduces such potential gain of information rent. If Condition 2-1 holds, the probability of the consumer having a low valuation in the second period is sufficiently high. The marginal reduction in the potential gain of information rent is sufficiently low that it is optimal for the seller to induce completely truthful revelation. If Condition 2-2 holds instead, it is optimal for the seller to induce the persistent-high type of consumer to take a mixed revelation strategy to reduce the future gain of information rent from lying, which reduces the first-period information rent. Truthful or mixed revelation induced, the optimal first-period contract consists of efficient consumption for the consumer who reveals high valuation and downward distorted consumption for whom revealing low valuation, due to a rent-efficiency tradeoff in terms of consumption and of mixed revelation under Condition 2-2. This is summarized in Lemma 5.

When Conditions 2-1 and 2-2 are violated, the persistent-low type of consumer has incentive to lie to take the additional benefit in the first period, anticipating a zero second-period information rent regardless. The seller relies on consumption distortion and revelation distortion to cope with such incentive. To induce the persistent-low type to reveal his true preference with a positive probability, the seller distorts the first-period consumption upwards for the consumer who reveals high valuation, and further downwards for whom revealing low valuation. Efficiency loss from such steeper consumption distortion welcomes the implementation of mixed revelation for the persistent consumer, with the impersistent consumer revealing his first-period preference truthfully. This is summarized in Lemma 6 below.

When the majority is impersistent, the standard static information rent is sufficient to induce the descending type of consumer to truthfully reveal his first-period preference with a positive probability, anticipating a zero second-period information rent regardless. Satisfaction of Condition 2-3 implies that the ascending type of consumer does not find the potential future information advantage valuable enough to bear the cost of lying in the first period. The optimal first-period contract induces completely truthful revelation, with efficient consumption for the consumer who reveals high valuation and downward distorted consumption for whom revealing low valuation, due to the conventional rent-efficiency tradeoff. This is summarized in Lemma 5.

When Condition 2-3 is violated, the ascending type of consumer finds the potential future information advantage sufficiently valuable to outweigh the first-period loss of mis-reporting. The seller relies on consumption distortion and revelation distortion to cope with such binding ratchet effect. To induce the ascending type to reveal his true preference with a positive probability, the seller distorts the first-period consumption upwards for the consumer who reveals high valuation, and further downwards for whom revealing low valuation. Efficiency loss from such steeper consumption distortion welcomes the implementation of mixed revelation for the impersistent consumer, with the persistent consumer revealing his first-period preference truthfully. This is summarized in Lemma 6 below.

Lemma 6. *Violation to Condition 2, the optimal first-period contract $\mathbb{C}_1^b$ induces completely truthful revelation from the minority type and mixed revelation from the majority type. The first-period contract has steeper consumption distortion with $q_{1h}^b \geq q_h^*$ and $q_{1l}^b \leq q_{1l}^a$.*[18] *Following revelation of first-period preference, the optimal second-period contract $\mathbb{C}_2^{bi}$ is a menu of options given the seller's non-degenerate belief, with $q_{2h}^{bi} = q_h^*$ and $q_{2l}^{bi} < q_l^*$, with $q_{2l}^{bl} \gtreqless q_{2l}^{bh}$ if $\phi \gtreqless 1-\phi$. The contracts implement intertemporal information rent $u_{Ph}^b = (v_h - v_l) \cdot (q_{1l}^b + \max\{q_{2l}^{bl}, q_{2l}^{bh}\})$ to the persistent-high type, $u_{Pl}^b = 0$ to the persistent-low type, $u_{Ih}^a = (v_h - v_l) \cdot (q_{1l}^b + \max\{q_{2l}^{bl} - q_{2l}^{bh}, 0\})$ to the descending type, and $u_{Il}^a = (v_h - v_l) \cdot q_{2l}^{bl}$ to the ascending type of consumer.*

Proof. Appendix A.2. □

With privacy on the evolution of preference, the second-period preference is asymmetrically known even with completely truthful revelation of first-period preference. The seller has non-degenerate belief on the consumer's second-period taste, so the

[18]Strict inequality holds when (IC_{Pl}) is binding if $\phi > 1-\phi$ or when (IC_{Il}) is binding if $1-\phi > \phi$.

optimal second-period contract is a menu of options which exhibits the static rent-efficiency tradeoff, Condition 2 satisfied or not. The persistent-high type and the ascending type of consumer enjoys his second-period information rent even when he is induced to reveal his first-period preference truthfully. If the majority is persistent (impersistent), the descending type (persistent-high type) of consumer enjoys a first-period information rent partly due to his private information on the high valuation and partly attributed to being in the minority. In addition, the impersistent consumer is protected from first-period rent exploitation regardless of the implemented revelation strategy. The seller in the first period alone is unable to distinguish the ascending type from the persistent-low type and the descending type from the persistent-high type of consumer. The contract to induce participation of the persistent-low type implies zero rent exploited from the ascending type, which by incentive compatibility, implies no exploitation from the descending type either.

4 Implications of Privacy on (Im)persistency

Difference in the contracts with commonly known and privately known evolution of preference marks the maximum effects of privacy protection of the consumer's traits. If the distribution of initial preference is not too skewed and the distribution of (im)persistency is sufficiently even, privacy on the evolution of preference only affects the contract to induce completely truthful revelation of preference, but not the optimality of completely truthful revelation.

Proposition 1. Contracts with Truthful Revelation. *When Conditions 1-1 and 1-2, Conditions 2-1 and 2-3 are satisfied, completely truthful revelation of preference is induced with or without privacy on the evolution of preference. Privacy on (im)persistency has no effect on the first-period consumption but distorts the second-period consumption efficiency. The persistent consumer is weakly worse off; the impersistent consumer is weakly better off, strictly better off if the majority of consumer is persistent.*

Proof. This is implied by Lemmata 1, 2, 4 and 5. □

When the distribution of initial preference is not too skewed and the distribution of (im)persistency is sufficiently even, truthful revelation has a marginally small effect on the potential future information rent, with or without privacy on the evolution of preference. The ratchet effects are sufficiently mild that neither first-period revelation nor consumption is distorted beyond the conventional rent-efficiency tradeoff. Truthful revelation in the first period induces a degenerate belief of the seller with commonly known evolution of preference, whereas with privacy on (im)persistency, second-period preference remains privately known by the consumer following truthful revelation in the first period. The seller in the latter scenario finds it optimal to distort the second-period consumption to reduce the second-period information rent à la static rent-efficiency tradeoff. Privacy on (im)persistency results in future consumption distortion.

Privacy on (im)persistency preserves the consumer's future information advantage, especially when he has a high valuation in the second period. This directly improves the welfare of the ascending type of consumer, who is free from complete rent extraction in the second period following truthful revelation in the first. If the impersistent consumer is in the minority, the descending type also benefits from the seller's inability to distinguish him from the persistent counterpart, earning a spillover information rent beyond the minimum to induce truthful revelation in the first period. However, such information asymmetry restores the second-period rent-efficiency tradeoff and relaxes the ratchet effect of the persistent-high type[19], who is worse off from a lower first-period information rent in advance to induce truthful revelation, as well as a distorted second-period information rent. Privacy on the evolution of preference results in redistribution of intertemporal information rent to the consumer of different evolution types. This echos the conflicting opinions for privacy protection even among the consumers.[20]

If the distribution of initial preference is sufficiently skewed and the distribution of the evolution type is sufficiently even, privacy on the evolution of preference improves the first-period revelation of preference.

Proposition 2. Improved Revelation of Preference. *With violation to Condition 1-1 and 1-2 but satisfaction of Conditions 2-1 and 2-3, mixed revelation is induced*

[19]This is implied by the more relaxed (IC_{Ph}) than (IC_h^P) given truthful revelation.

[20]As a brief comparison with the literature, Schumacher (2016) also pointed out cross-subsidization between agents in an insurance model with dynamic inconsistency. Cross-subsidization in their model is between the naive and the sophisticated insurees when a long-term contract can be committed, and such cross-subsidization is absent when a spot contract is offered, as opposed to the prediction of Proposition 1. In the literature on contracting with dynamic inconsistency, e.g. Eliaz and Spiegler (2006) (2008), Heidhues and Kőszegi (2010), and Yilmaz (2015) to list a few, both inconsistency and misperception of inconsistency are necessary for a welfare distortion or improvement. The welfare discussion here does not rely on misperception of the evolution of preference. In fact, the (im)persistent consumer is so sophisticated that he is able to utilize his private information on the evolution of preference. It is this information advantage on which the welfare result stands. Conitzer, Taylor, and Wagman (2012) found that it is always beneficial for the consumer to maintain anonymous as a commitment for the seller not to post an unfavorable behavior-based price in the future. Differently implied in Proposition 1, an impersistent consumer would like to have his evolution type anonymous while a persistent consumer may prefer otherwise.

with commonly known evolution of preference, while completely truthful revelation is induced with privately known evolution of preference.

Proof. This is implied by Lemmata 1, 3, 4 and 5. □

When the distribution of initial preference is sufficiently skewed, the ratchet effect with commonly known persistency is so strong that it takes a large in-advance information rent to induce the persistent-high type of consumer to truthfully reveal his preference in the first period, which motivates the persistent-low type to take the instant benefit by lying with probability one. The strong ratchet effect is also present with commonly known impersistency. The ascending type of consumer has a larger future information rent forgone than his loss from mimicking the descending type in the first period. To cope with such binding ratchet effects, the seller induces mixed revelation strategies. With privately known evolution of preference distributed sufficiently evenly, the seller cannot identify the consumer's evolution type, and her Bayesian-updated belief has a sufficiently even distribution over the consumer's preference. The value of lying for future gain in information rent is thus lower for both the persistent-high type and the ascending type of consumer. The ratchet effects are relaxed, so is the incentive for the persistent-low type to take the instant benefit. The seller optimally induces completely truthful revelation. Privacy on the evolution of preference relaxes the ratchet effects, which improves equilibrium revelation of preference-related information.

Relaxation of the ratchet effects from privacy on (im)persistency also improves the first-period consumption efficiency. Binding incentive compatibilities with commonly known evolution of preference is accompanied with steeper consumption distortion in the first period. Privacy on (im)persistency relaxes the ratchet effects such that the upward incentive compatibilities are non-binding. This restores consumption efficiency for the consumer who has a high valuation in the first period and reduces consumption distortion for the consumer who has a low valuation in the first period. The effect on the second-period consumption, however, is ambiguous, depending on the equilibrium revelation strategy with commonly known evolution of preference.

Improved first-period consumption efficiency accompanies an increase in the information rent to the descending type of consumer. On top of that, private information on the evolution of preference protects the impersistent consumer against an exploitative contract. The ascending type is free from rent exploitation when the seller is unable to distinguish him from the persistent-low type, so is the descending type by incentive compatibility. The impersistent consumer is better off from privacy on (im)persistency that improves the revelation of preference as well. With an ambiguous effect on the second-period consumption, the welfare effect on the persistent-high type of consumer is ambiguous.

If the distribution of initial preference is not too skewed and the distribution of the evolution type is sufficiently skewed, privacy on the evolution of preference distorts the first-period revelation of preference.

Proposition 3. Distorted Revelation of Preference. *With satisfaction of Conditions 1-1 and 1-2 but violation to Conditions 2-1 and 2-3, completely truthful revelation is induced with commonly known evolution of preference, while mixed revelation is induced for the majority of evolution type with privately known evolution of preference.*

Proof. This is implied by Lemmata 1, 2, 4 and 6. □

When the distribution of initial preference is not too skewed, the ratchet effect with commonly known evolution of preference is sufficiently mild that truthful revelation is optimally induced. With privately known evolution of preference, the consumer whose evolution type is in the minority truthfully reveals his preference in the first period as the seller is unable to distinguish him from the majority. If Condition 2-2 is satisfied, the ratchet effect is insufficiently strong that the upward incentive compatibility is slacking for the consumer with low valuation in the first period. However, it is insufficiently mild either that the seller finds it optimal to induce the persistent-high type of consumer to take a mixed revelation strategy, as the marginal gain of future information rent from lying is sufficiently high when the latter truthfully reveals his first-period preference with a higher probability.[21] Privacy on the evolution of preference distorts the equilibrium revelation of preference-related information for the consumer who persistently has a high valuation.

With sufficiently skewed distribution of the evolution type, Condition 2 is violated. If the majority is persistent, following the first-period revelation of low valuation, the seller believes that with a very high probability the consumer persistently has a low valuation in the second period. Difference in future rent extraction following different first-period revelation is sufficiently large. The value of lying for future gain in information rent is so high for the persistent-high type of consumer that it takes a large in-advance information rent to induce him to truthfully reveal his first-period preference. This instant benefit is attractive for the persistent-low type of consumer to take by lying with probability one. If the majority is impersistent, following the first-period

[21] This is reflected in the size of $\frac{\partial(u_{2h}^{l}-u_{2h}^{h})}{\partial m_{Ph}}$ in the seller's revelation implementation problem.

revelation of high valuation, the seller believes that with a very high probability the consumer impersistently has a low valuation in the second period. Difference in future rent extraction following different first-period revelation is sufficiently large. The value of lying for future gain in information rent is sufficiently high for the ascending type of consumer to outweigh the current loss of lying. Privacy on the evolution of preference strengthens the ratchet effects. The seller optimally induces the majority of evolution type to take a mixed revelation strategy. Equilibrium revelation of preference-related information is distorted with privacy on (im)persistency.

Tighter ratchet effects from privacy on the evolution of preference also distorts consumption efficiency. With commonly known (im)persistency, truthful revelation is induced with static rent-efficiency tradeoff in the first period. Following truthful revelation, the seller has a degenerate belief in the second period and offers efficient consumption. Privacy on (im)persistency strengthens the ratchet effects such that in the first period, the upward incentive compatibility of the majority type of consumer is binding. The first-period contract has steeper consumption distortion. Following truthful or mixed revelation, the seller has a non-degenerate belief in the second period and proposes a contract that exhibits the static rent-efficiency tradeoff. Privacy on (im)persistency weakly distorts the second-period consumption.

Privacy on (im)persistency preserves the consumer's future information advantage, which directly improves the welfare of the ascending type of consumer. The preservation of future information advantage, however, restores the second-period rent-efficiency tradeoff and results in a binding ratchet effect in the first period when Condition 2 is violated. These two effects jointly make the persistent-high type of consumer worse off. The descending type of consumer suffers from the binding ratchet effect when the majority is impersistent; his welfare effect is ambiguous when the majority is persistent, depending on whether the binding ratchet effect or the spillover information rent is stronger in magnitude. With mixed revelation optimally induced, it remains that privacy on the evolution of preference results in redistribution of intertemporal information rent to the consumer of different evolution types.

Numerical Examples. To see how the above propositions depend on the distribution of preference type and that of evolution type, consider some numerical examples with $v_h = 7$ and $v_l = 4$. Condition 1-1 and 1-2 become $\frac{1}{1-\sigma} \geq \frac{4}{3}$ and $\frac{\sigma}{1-\sigma} \leq \frac{3}{4}$. Both of them hold for $\sigma \in [\frac{1}{4}, \frac{3}{7}]$; the former is violated for $\sigma < \frac{1}{4}$ and the latter is violated for $\sigma > \frac{3}{7}$. If the majority of consumer is impersistent, at $\sigma = \frac{1}{4}$ such that Condition 1 holds at the lowest $\frac{1}{1-\sigma}$, Condition 2-3 becomes $\frac{4}{3} \geq \frac{1-\phi}{\phi} - \frac{\phi}{1-\phi}$, which holds for

$\phi \in [\frac{5-\sqrt{13}}{4}, \frac{1}{2}]$. If $\sigma \in [\frac{1}{4}, \frac{3}{7}]$ and $\phi \in [\frac{5-\sqrt{13}}{4}, \frac{1}{2}]$, the scenario in Proposition 1 holds. If the majority of consumer is impersistent, at $\sigma = \frac{3}{7}$ such that Condition 1 holds at the highest $\frac{1}{1-\sigma}$, Condition 2-3 becomes $\frac{7}{4} \geq \frac{1-\phi}{\phi} - \frac{\phi}{1-\phi}$, which holds for $\phi \in [\frac{15-\sqrt{113}}{14}, \frac{1}{2}]$. If $\sigma \in [\frac{1}{4}, \frac{3}{7}]$ and $\phi < \frac{15-\sqrt{113}}{14}$, the scenario in Proposition 3 holds. If $\sigma = \phi \geq \frac{1}{2}$, Condition 1-2 is violated. Condition 2-1 becomes $\frac{1}{1-\sigma} \geq \frac{\phi}{1-\phi} - \frac{1-\phi}{\phi}$ when $\frac{1}{1-\sigma} \geq 4$, which is satisfied for $\sigma = \phi \in [\frac{3}{4}, 1)$. If $\sigma = \phi \in [\frac{3}{4}, 1)$, the scenario in Proposition 2 holds.

5 Screening (Im)persistency

The previous analysis is based on the implicit assumption that the consumer's evolution of preference *per se* is not screened by a menu of separating contracts. Inducing the consumer to reveal his preference and his (im)persistency in the first-period is equivalent to inducing the consumer to reveal his second-period preference early in the first period. The following proposition indicates that in equilibrium, privacy on the evolution of preference remains asymmetrically known between the seller and the consumer.

Proposition 4. *It is sub-optimal to offer a menu of separating contracts to screen the evolution of preference per se such that the evolution type is completely revealed in equilibrium.*[22]

Proof. Appendix A.3. □

Suppose that the first-period menu of separating contracts is offered to induce truthful revelation of both the evolution type and the preference type, each with a positive probability. If it is optimal to induce one type of the consumer to truthfully reveal his (im)persistency with probability one, the opposite type, in terms of different (im)persistency and first-period preference, must be induced to take a mixed revelation of preference. Otherwise, at least one type of the consumer has incentive to mimic the type that the seller believes to be highly unlikely. To illustrate the idea, consider for example a contract to induce the ascending type to truthfully reveal his impersistency with certainty. If the persistent-high type of consumer is induced to truthfully reveal his high valuation in the first period, following the revelation of "persistent-low type,"

[22] Proposition 4 does not rule out the possibility of a menu of separating contracts to imperfectly screen the evolution type. It simply implies that truthful revelation of (im)persistency with probability one is not optimal, so that the contracts with privately known evolution of preference is qualitatively robust.

the seller believes with probability one that the consumer has a low valuation in the second period, because this message must not be revealed by an ascending type (who truthfully reveals his impersistency) nor by a persistent-high type (who truthfully reveals his first-period taste). If this message were in fact revealed by the consumer who has a high valuation in the second period, he would have earned the maximum second-period rent. The ascending type of consumer thus has incentive to mimic the persistent-low type with probability one. The same idea applies to other types of consumer, which is shown in Lemma A in Appendix A.3.

If the seller is to offer a menu of separating contracts that induces all types of consumer to truthfully reveal his evolution of preference with probability one, by the argument above, this is incentive compatible only with each type of the consumer taking a completely mixed revelation of first-period preference. The consumer is indifferent between revelation of preference types, so the menu of contracts induces the consumer to truthfully reveal his evolution type with a positive probability only if it implements pooled per-period information rents. Otherwise, the consumer has incentive to mimic the opposite type, to lie on both the evolution type and the preference type with probability one. Intuitively, the impersistent consumer is indistinguishable from different preference types of the persistent consumer in each period. In order to deter the impersistent consumer from lying with probability one, it requires an intertemporal information rent equal to the summation of the per-period information rents to different preference types of the persistent counterpart. For instance, to deter the ascending type from mimicking the persistent-low type with certainty, the intertemporal information rent to the ascending type is implemented as the sum of the first-period rent to the persistent-low type and the second-period rent to the persistent-high type. Without commitment to the future in the first period, equality of intertemporal information rent is implemented by equality of per-period information rent. If the same information rent is implemented by a menu of separating contracts with different levels of consumption, the consumer with high valuation in the first period has incentive to mimic the one with low valuation by choosing the contract that has a higher consumption intended for the consumer with low valuation. The incentive compatible contracts thus have the same level of consumption for the persistent and the impersistent consumer. The optimal menu of contracts is pooled with respect to the consumer's evolution type.

6 Conclusion

Private information on the evolution of preference preserves asymmetry on future preference-related information. With short-term commitment, this secures the consumer's future information advantage and manipulates the ratchet effect. How privacy on the evolution of preference shapes the optimal contracts to screen the consumer's preference depends on how the ratchet effects are manipulated. Different factors play a crucial role in such manipulation under different information structures. With commonly known evolution of preference, the ratchet effects are relatively mild when the distribution of the consumer's initial preference is not too skewed; with privately known evolution of preference, the ratchet effects are relatively mild when the distribution of (im)persistency is sufficiently even.

When both conditions hold, completely truthful revelation on preference is optimally induced with or without privacy on (im)persistency. Such privacy has no first-period effects on consumption and revelation, but it does redistribute the consumer's information rent due to its effect on the second-period information structure and hence the relaxation of first-period incentive compatibility. When the former condition holds while the latter is violated, privacy on the evolution of preference strengthens the ratchet effects to an extent that the upward incentive compatibility constraint becomes binding for the consumer whose evolution type is in the majority. Revelation is distorted along with steeper consumption distortion. Privacy on (im)persistency distorts truthful revelation of preference-related information if the distribution of the consumer's initial preference is not too skewed and the distribution of (im)persistency is sufficiently skewed. When the former condition is violated while the latter holds, privacy on the evolution of preference relaxes the ratchet effects to an extent that the upward incentive compatibility constraints become non-binding. Revelation and consumption efficiency in the first period are improved. Privacy on (im)persistency improves truthful revelation of preference-related information if the consumers have sufficiently skewed distribution of initial preference and the distribution of (im)persistency is sufficiently even.

To demonstrate in a simple model how the privacy on seemingly redundant information improves or distorts the incentive to reveal payoff-relevant information, some potentially interesting research questions were left out. The seller was assumed to be the principal in this paper. Allowing the consumer to have some bargaining power, we will be able to study how the privacy on the evolution of preference affects the informed player's incentive to actively reveal his preference-related information. Generalization of the model beyond a binary type space is also of interest. The consumer's private knowledge of the evolution of preference can be generalized to his private information on the function of future preference. For instance, suppose that the consumer's current preference is v_1 which will possibly differ in the future indicated by the function $v_2 = f(v_1)$. The consumer has private information not only on the realization of v_1 but also on how his preference will evolve represented by the function $f(\cdot)$. In practice, privacy on the consumer's evolution of preference is related to the use of technology to keep track of the consumer's past activities, as well as the legal restrictions on the use of tracking technology. It was assumed exogenous in this paper. A natural extension is to study the seller's decision to invest in such tracking technology, and her endogenous choice of privacy protection in response to the legal restrictions.

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A Appendix: Proof

A.1 Proof of Lemma 1 to Lemma 3 in Section 3.1

The consumer with first-period preference v_i, $i = l, h$, and evolution of preference $k = P, I$ reveals truthfully his first-period taste with probability n_{ki}. The seller's Bayesian updated belief of a high valuation type in the second period following the persistent consumer's choice of first-period option (q_{1h}, p_{1h}) and (q_{1l}, p_{1l}) are

$$\nu_P(v_h) = \frac{\sigma \cdot n_{Ph}}{\sigma \cdot n_{Ph} + (1-\sigma) \cdot (1-n_{Pl})}$$

and

$$\nu_P(v_l) = \frac{\sigma \cdot (1-n_{Ph})}{\sigma \cdot (1-n_{Ph}) + (1-\sigma) \cdot n_{Pl}}$$

respectively. Her Bayesian updated belief of a high valuation type in the second period following the impersistent consumer's choice of first-period option (q_{1h}, p_{1h}) and (q_{1l}, p_{1l}) are

$$\nu_I(v_h) = \frac{(1-\sigma)\cdot(1-n_{Il})}{\sigma\cdot n_{Ih} + (1-\sigma)\cdot(1-n_{Il})}$$

and

$$\nu_I(v_l) = \frac{(1-\sigma)\cdot n_{Il}}{\sigma\cdot(1-n_{Ih}) + (1-\sigma)\cdot n_{Il}}$$

respectively. Following the consumer's first-period choice of (q_{1i}, p_{1i}), the seller proposes the contract $\mathbb{C}_2^{ki} = \{(q_{2j}^{ki}, p_{2j}^{ki})\}$, $j = l, h$, implementing payoff $u_{2j}^{ki} = v_j \cdot q_{2j}^{ki} - p_{2j}^{ki}$ that

$$\mathscr{P}_2^{ki}: \quad \max_{q_2, u_2} \nu_k(v_i)\cdot(v_h\cdot q_{2h} - c(q_{2h}) - u_{2h}) + (1-\nu_k(v_i))\cdot(v_l\cdot q_{2l} - c(q_{2l}) - u_{2l})$$

subject to $u_{2j} \geq 0$ for both $j = l, h$, and $(v_h - v_l)\cdot q_{2h} \geq u_{2h} - u_{2l} \geq (v_h - v_l)\cdot q_{2l}$.

By backward induction and the standard methods in static contract theory, the optimal contract $\mathbb{C}_2^{ki}$ has $q_{2h}^{ki} = \arg\max_q v_h \cdot q - c(q) = q_h^*$ and $q_{2l}^{ki} = \arg\max_q (1 - \nu_k(v_i))\cdot(v_l\cdot q - c(q)) - \nu_k(v_i)\cdot(v_h - v_l)\cdot q \leq q_l^*$ if the option (q_{1i}, p_{1i}) was taken in the first period, which implements $u_{2h}^{ki} = (v_h - v_l)\cdot q_{2l}^{ki}$ and $u_{2l}^{ki} = 0$. With quadratic $c(q) = \frac{1}{2}\cdot q^2$, $q_{2h}^k = q_h^* = v_h$ and $q_{2l}^{ki} = \max\{v_l - \frac{\nu_k(v_i)}{1-\nu_k(v_i)}\cdot(v_h - v_l), 0\}$. Second-period information rents to the persistent consumer satisfy $u_{2h}^{Pl} \gtreqless u_{2h}^{Ph}$ if $q_{2l}^{Pl} \gtreqless q_{2l}^{Ph}$, which holds if $\frac{\nu_P(v_l)}{1-\nu_P(v_l)} \lesseqgtr \frac{\nu_P(v_h)}{1-\nu_P(v_h)}$, or equivalently, $\frac{\sigma\cdot(1-n_{Ph})}{(1-\sigma)\cdot n_{Pl}} \lesseqgtr \frac{\sigma\cdot n_{Ph}}{(1-\sigma)\cdot(1-n_{Pl})}$. Second-period information rents to the impersistent consumer satisfy $u_{2h}^{Il} \gtreqless u_{2h}^{Ih}$ if $q_{2l}^{Il} \gtreqless q_{2l}^{Ih}$, which holds if $\frac{\nu_I(v_l)}{1-\nu_I(v_l)} \lesseqgtr \frac{\nu_I(v_h)}{1-\nu_I(v_h)}$, or equivalently, $\frac{(1-\sigma)\cdot n_{Il}}{\sigma\cdot(1-n_{Ih})} \lesseqgtr \frac{(1-\sigma)\cdot(1-n_{Il})}{\sigma\cdot n_{Ih}}$.

Anticipating the second-period contract $\mathbb{C}_2^{ki}$, the seller proposes the first-period contract $\mathbb{C}_1^k = \{(q_{1i}^k, p_{1i}^k)\}$, $i = l, h$, implementing payoff $u_{1i}^k = v_i \cdot q_{1i}^k - p_{1i}^k$ that solves

$$\mathscr{P}_1^P: \quad \max_{q_1, u_1} \rho^P\cdot(v_h\cdot q_{1h} - c(q_{1h}) - u_{1h}) + (1-\rho^P)\cdot(v_l\cdot q_{1l} - c(q_{1l}) - u_{1l})$$

subject to

$$u_{1h} + u_{2h}^{Ph} \geq 0 \qquad (IR_h^P)$$
$$u_{1l} \geq 0 \qquad (IR_l^P)$$
$$u_{1h} - u_{1l} \geq (v_h - v_l)\cdot q_{1l} + u_{2h}^{Pl} - u_{2h}^{Ph} \qquad (IC_h^P)$$
$$u_{1h} - u_{1l} \leq (v_h - v_l)\cdot q_{1h} \qquad (IC_l^P)$$

with a persistent consumer, and

$$\mathscr{P}_1^I: \quad \max_{q_1, u_1} \rho^I\cdot(v_h\cdot q_{1h} - c(q_{1h}) - u_{1h}) + (1-\rho^I)\cdot(v_l\cdot q_{1l} - c(q_{1l}) - u_{1l})$$

subject to

$$u_{1h} \geq 0 \qquad (IR_h^I)$$
$$u_{1l} + u_{2h}^{Il} \geq 0 \qquad (IR_l^I)$$
$$u_{1h} - u_{1l} \geq (v_h - v_l) \cdot q_{1l} \qquad (IC_h^I)$$
$$u_{1h} - u_{1l} \leq (v_h - v_l) \cdot q_{1h} + u_{2h}^{Il} - u_{2h}^{Ih} \qquad (IC_l^I)$$

with an impersistent consumer. ρ^k denotes the probability that the consumer of evolution type $k = P, I$ reveals a high valuation in the first period.

Contract with a persistent consumer.

Rearranged from (IC_h^P), $u_{1h} + u_{2h}^{Ph} \geq (v_h - v_l) \cdot q_{1l} + u_{2h}^{Pl} + u_{1l} \geq 0$, (IR_h^P) is jointly implied by (IR_l^P), $u_{2h}^{Pl} \geq 0$, and $q_{1l} \geq 0$. For any pair of (u_{1h}, u_{1l}) satisfying all constraints in problem $\mathscr{P}_1^P$ with $u_{1l} > 0$, there is a sufficiently small $\varepsilon > 0$ such that $(u_{1h}-\varepsilon, u_{1l}-\varepsilon)$ is individually rational and incentive compatible. The principal prefers $(u_{1h} - \varepsilon, u_{1l} - \varepsilon)$ to (u_{1h}, u_{1l}). The optimal contract to the persistent consumer has $u_{1l}^P = 0$ from binding (IR_l^P).

The optimal contract has at least (IC_h^P) binding. To see this, if the optimal contract has both (IC_h^P) and (IC_l^P) slacking, the consumer reveals his true preference with certainty. The first-period contract implements efficient consumption output $(q_{1h}, q_{1l}) = (q_h^*, q_l^*)$, $u_{1l} = 0$, and $u_{1h} > (v_h - v_l) \cdot q_l^* + u_{2h}^{Pl} - u_{2h}^{Ph}$. There is a sufficiently small $\varepsilon > 0$ such that $u_{1h} - \varepsilon$ is incentive feasible and preferred by the principal. This contradicts to optimality. If the optimal contract has binding (IC_l^P) and slacking (IC_h^P), the persistent-high type reveals his preference with probability one. The first-period contract implements consumption output $(q_{1h}, q_{1l}) = (q_l^*, q_l^*)$, which is a solution to $\max_{q_1} \rho^P \cdot (v_h \cdot q_{1h} - c(q_{1h}) - (v_h - v_l) \cdot q_{1h}) + (1 - \rho^P) \cdot (v_l \cdot q_{1l} - c(q_{1l}))$, with implemented rent $u_{1l} = 0$ and $u_{1h} = (v_h - v_l) \cdot q_l^*$. This violates (IC_h^P) as $u_{2h}^{Pl} - u_{2h}^{Ph} = (v_h - v_l) \cdot (q_l^* - q_{2l}^{Ph}) > 0$ with $n_{Ph} = 1$, a contradiction to incentive compatibility.

If (IC_l^P) is slacking, the persistent-low type reveals his preference with certainty $n_{Pl} = 1$, so $\rho^P = \sigma \cdot n_{Ph}$. With binding (IR_l^P) and (IC_h^P), the optimal contract $\mathbb{C}_1^P$ has $q_{1h}^P = \arg\max_q v_h \cdot q - c(q) = q_h^*$ and $q_{1l}^P = v_l - \frac{\sigma \cdot n_{Ph}}{1-\sigma \cdot n_{Ph}} \cdot (v_h - v_l) < q_l^*$, implementing $u_{1l}^P = 0$ and $u_{1h}^P = (v_h - v_l) \cdot q_{1l}^P + u_{2h}^{Pl} - u_{2h}^{Ph}$. With $n_{Ph} \leq n_{Pl} = 1$, $\nu_P(v_h) = 1$ and $\nu_P(v_l) = \frac{\sigma \cdot (1-n_{Ph})}{\sigma \cdot (1-n_{Ph}) + (1-\sigma)}$. The second period contract following revelation of v_l has $q_{2h}^{Pl} = \arg\max_q v_h \cdot q - c(q) = q_h^*$ and $q_{2l}^{Pl} = v_l - \frac{\sigma \cdot (1-n_{Ph})}{1-\sigma} \cdot (v_h - v_l) < q_l^*$, which implements $u_{2h}^{Pl} = (v_h - v_l) \cdot q_{2l}^{Pl}$ and $u_{2l}^{Pl} = 0$. The second period contract following revelation of v_h has a single option $q_2^{Ph} = \arg\max_q v_h \cdot q - c(q) = q_h^*$, which implements $u_{2h}^{Ph} = 0$ and $u_{2l}^{Ph} = 0$ off the equilibrium path.

The contracts and revelation strategy strictly satisfies (IC_l^P) if $(v_h - v_l) \cdot q_h^* \geq (v_h - v_l) \cdot q_{1l}^P + u_{2h}^{Pl} - u_{2h}^{Ph}$, which holds if $v_h \geq v_l - \frac{\sigma \cdot n_{Ph}}{1-\sigma \cdot n_{Ph}} \cdot (v_h - v_l) + v_l - \frac{\sigma \cdot (1-n_{Ph})}{1-\sigma} \cdot (v_h - v_l)$, or equivalently, $\frac{1}{1-\sigma \cdot n_{Ph}} + \frac{\sigma \cdot (1-n_{Ph})}{1-\sigma} \geq \frac{v_l}{v_h - v_l}$. The inequality holds for all n_{Ph} if the minimum of the LHS (at $1 - \sigma \cdot n_{Ph} = \sqrt{1-\sigma}$) is higher than the RHS, which holds if $\frac{2 \cdot \sqrt{1-\sigma}}{1-\sigma} - 1 \geq \frac{v_l}{v_h - v_l}$. The inequality holds at $n_{Ph} = 1$ if $\frac{1}{1-\sigma} \geq \frac{v_l}{v_h - v_l}$, which is a more relaxed condition as $\frac{2 \cdot \sqrt{1-\sigma} - (1-\sigma)}{1-\sigma} < \frac{1}{1-\sigma}$. If $\frac{1}{1-\sigma} < \frac{v_l}{v_h - v_l}$, $(v_h - v_l) \cdot q_h^* < (v_h - v_l) \cdot q_{1l}^P + u_{2h}^{Pl} - u_{2h}^{Ph}$ for some n_{Ph}, at which both (IC_h^P) and (IC_l^P) are binding.

If $\frac{1}{1-\sigma} \geq \frac{v_l}{v_h - v_l}$, labeled as Condition 1-1, the principal implements $n_{Ph}^* \leq 1$ that maximizes her intertemporal payoff, where $\sigma \cdot (v_h \cdot q_h^* - c(q_h^*) - u_{1h}^P + v_h \cdot q_h^* - c(q_h^*) - u_{2h}^{Ph}) - \sigma \cdot n_{Ph} \cdot \frac{\partial u_{2h}^{Pl}}{\partial n_{Ph}} \geq \sigma \cdot (v_l \cdot q_{1l}^P - c(q_{1l}^P) + v_h \cdot q_h^* - c(q_h^*) - u_{2h}^{Pl})$ by the envelope theorem of $\mathscr{P}_t^P$, with equality at $n_{Ph} \neq 1$. This can be reduced to $v_h \cdot q_h^* - c(q_h^*) - (v_h - v_l) \cdot q_{1l}^P - (v_l \cdot q_{1l}^P - c(\hat{q}_{1l}^P)) \geq n_{Ph} \cdot \frac{\partial u_{2h}^{Pl}}{\partial n_{Ph}}$. The difference in the seller's payoff between a truth revealing and a lying persistent-high type is higher than or equal to the weighted increment in the gain of second-period information rent from lying when inducing the persistent-high type to reveal his true preference with a higher probability. Given the contract above, this is rearranged to $(v_h - v_l)^2 \cdot \frac{1}{2 \cdot (1-\sigma \cdot n_{Ph})^2} \geq n_{Ph} \cdot \frac{\sigma}{1-\sigma} \cdot (v_h - v_l)^2$. At $n_{Ph} = 1$, the inequality strictly holds as $2 \cdot \sigma^2 - 2 \cdot \sigma + 1 > 0$ for any σ. It is optimal for the seller to implement $n_{Ph}^* = 1$ given Condition 1-1.

At $n_{Pi}^* = 1$ with Condition 1-1 satisfied, the optimal contract $\hat{\mathbb{C}}_1^P$ has $\hat{q}_{1h}^P = q_h^*$ and $\hat{q}_{1l}^P = v_l - \frac{\sigma}{1-\sigma} \cdot (v_h - v_l) < q_l^*$, implementing $\hat{u}_{1l}^P = 0$ and $\hat{u}_{1h}^P = (v_h - v_l) \cdot \hat{q}_{1l}^P + \hat{u}_{2h}^{Pl} - \hat{u}_{2h}^{Ph}$. The second-period contract $\hat{\mathbb{C}}_2^{Pl}$ following revelation of v_l has single option of $\hat{q}_2^{Pl} = q_l^*$, which implements $\hat{u}_{2h}^{Pl} = (v_h - v_l) \cdot q_l^*$ off the equilibrium path and $\hat{u}_{2l}^{Pl} = 0$. The second-period contract $\hat{\mathbb{C}}_2^{Ph}$ following revelation of v_h has a single option $\hat{q}_2^{Ph} = q_h^*$, which implements $\hat{u}_{2h}^{Ph} = 0$ and $\hat{u}_{2l}^{Ph} = 0$ off the equilibrium path. The optimal contracts implement intertemporal information rent $\hat{u}_{Pi}$ to the persistent consumer with $v_1 = v_i$ such that $\hat{u}_{Ph} = \hat{u}_{1h}^P + \hat{u}_{2h}^{Ph} = (v_h - v_l) \cdot (\hat{q}_{1l}^P + q_l^*)$ and $\hat{u}_{Pl} = \hat{u}_{1l}^P + \hat{u}_{2l}^{Pl} = 0$.

When Condition 1-1 is violated and (IR_l^P), (IC_h^P), and (IC_l^P) are all binding, with $\rho^P = \sigma \cdot n_{Ph} + (1-\sigma) \cdot (1 - n_{Pl})$, the optimal contract $\tilde{\mathbb{C}}_1^P$ solves the reduced problem

$$\mathscr{P}_1^P: \quad \max_{q_1, u_1} \rho^P \cdot (v_h \cdot q_{1h} - c(q_{1h})) + (1-\rho^P) \cdot (v_l \cdot q_{1l} - c(q_{1l})) - \rho^P \cdot ((v_h - v_l) \cdot q_{1l} + u_{2h}^{Pl} - u_{2h}^{Ph})$$

subject to $(v_h - v_l) \cdot q_{1h} = (v_h - v_l) \cdot q_{1l} + u_{2h}^{Pl} - u_{2h}^{Ph}$. The optimal contract has $\tilde{q}_{1h}^P > q_h^*$ as a solution to $\rho^P \cdot (v_h - q_{1h}) + \lambda^P \cdot (v_h - v_l) = 0$, where $\lambda^P > 0$ denotes the equilibrium Lagrange multiplier associated with binding (IC_l^P). It has $\tilde{q}_{1l}^P < \hat{q}_{1l}^P$ as a solution to

$(1-\rho^P)\cdot(v_l-q_{1l})-\rho^P\cdot(v_h-v_l)-\lambda^P\cdot(v_h-v_l)=0$. First-period information rents $\tilde{u}^P_{1l}=0$ and $\tilde{u}^P_{1h}=(v_h-v_l)\cdot\tilde{q}^P_{1l}+\tilde{u}^{Pl}_{2h}-\tilde{u}^{Ph}_{2h}$ are implemented.

The principal implements $n^*_{Ph}\leq 1$ that maximizes her intertemporal payoff, where $\sigma\cdot(v_h\cdot\tilde{q}^P_{1h}-c(\tilde{q}^P_{1h})-\tilde{u}^P_{1h}+v_h\cdot q^*_h-c(q^*_h)-\tilde{u}^{Ph}_{2h})-(\sigma\cdot n_{Ph}+(1-\sigma)\cdot(1-n_{Pl})+\lambda^P)\cdot\frac{\partial(\tilde{u}^{Pl}_{2h}-\tilde{u}^{Ph}_{2h})}{\partial n_{Ph}}\geq\sigma\cdot(v_l\cdot\tilde{q}^P_{1l}-c(\tilde{q}^P_{1l})+v_h\cdot q^*_h-c(q^*_h)-\tilde{u}^{Pl}_{2h})$ by the envelope theorem of $\mathscr{P}^P_t$, with equality at $n_{Ph}\neq 1$. This can be reduced to $v_h\cdot\tilde{q}^P_{1h}-c(\tilde{q}^P_{1h})-(v_h-v_l)\cdot\tilde{q}^P_{1l}-(v_l\cdot\tilde{q}^P_{1l}-c(\tilde{q}^P_{1l}))\geq\frac{\sigma\cdot n_{Ph}+(1-\sigma)\cdot(1-n_{Pl})+\lambda^P}{\sigma}\cdot\frac{\partial(\tilde{u}^{Pl}_{2h}-\tilde{u}^{Ph}_{2h})}{\partial n_{Ph}}$, where $\frac{\partial(\tilde{u}^{Pl}_{2h}-\tilde{u}^{Ph}_{2h})}{\partial n_{Ph}}>0$. The difference in the seller's payoff between a truth revealing and a lying persistent-high type is higher than or equal to the weighted increment in the gain of second-period information rent from lying when inducing the persistent-high type to reveal his true preference with a higher probability. The principal implements $n^*_{Pl}\leq 1$ that maximizes her intertemporal payoff, where $(1-\sigma)\cdot(v_l\cdot\tilde{q}^P_{1l}-c(\tilde{q}^P_{1l})+v_l\cdot\tilde{q}^{Pl}_{2l}-c(\tilde{q}^{Pl}_{2l}))-(\sigma\cdot n_{Ph}+(1-\sigma)\cdot(1-n_{Pl})+\lambda^P)\cdot\frac{\partial(\tilde{u}^{Pl}_{2h}-\tilde{u}^{Ph}_{2h})}{\partial n_{Pl}}\geq(1-\sigma)\cdot(v_h\cdot\tilde{q}^P_{1h}-c(\tilde{q}^P_{1h})-(v_h-v_l)\cdot\tilde{q}^P_{1l}-\tilde{u}^{Pl}_{2h}+\tilde{u}^{Ph}_{2h}+v_l\cdot\tilde{q}^{Ph}_{2l}-c(\tilde{q}^{Ph}_{2l}))$ by the envelope theorem of $\mathscr{P}^P_t$, with equality at $n_{Pl}\neq 1$. This can be reduced to $v_h\cdot\tilde{q}^P_{1l}-c(\tilde{q}^P_{1l})-(v_h\cdot\tilde{q}^P_{1h}-c(\tilde{q}^P_{1h}))+\tilde{u}^{Pl}_{2h}-\tilde{u}^{Ph}_{2h}+v_l\cdot\tilde{q}^{Pl}_{2l}-c(\tilde{q}^{Pl}_{2l})-(v_l\cdot\tilde{q}^{Ph}_{2l}-c(\tilde{q}^{Ph}_{2l}))\geq\frac{\sigma\cdot n_{Ph}+(1-\sigma)\cdot(1-n_{Pl})+\lambda^P}{1-\sigma}\cdot\frac{\partial(\tilde{u}^{Pl}_{2h}-\tilde{u}^{Ph}_{2h})}{\partial n_{Pl}}$, where $\frac{\partial(\tilde{u}^{Pl}_{2h}-\tilde{u}^{Ph}_{2h})}{\partial n_{Pl}}>0$. The difference in the seller's payoff between a truth revealing and a lying persistent-low type is higher than or equal to the weighted increment in the gain of second-period information rent from lying when inducing the persistent-low type to reveal his true preference with a higher probability.

At $n^*_{Pi}\leq 1$ with violation to Condition 1-1, the optimal contract $\tilde{\mathbb{C}}^P_1$ has $\tilde{q}^P_{1h}>q^*_h$ such that $\rho^P\cdot(v_h-q_{1h})+\lambda^P\cdot(v_h-v_l)=0$, where $\rho^P=\sigma\cdot n^*_{Ph}+(1-\sigma)\cdot(1-n^*_{Pl})$, and $\tilde{q}^P_{1l}<\hat{q}^P_{1l}$ such that $(1-\rho^P)\cdot(v_l-q_{1l})-\rho^P\cdot(v_h-v_l)-\lambda^P\cdot(v_h-v_l)=0$. The second-period contracts $\tilde{\mathbb{C}}^{Pi}_2$ following revelation of v_i have $\tilde{q}^{Pi}_{2h}=q^*_h$ and $\tilde{q}^{Ph}_{2l}=\max\{v_l-\frac{\sigma\cdot n^*_{Ph}}{(1-\sigma)\cdot(1-n^*_{Pl})}\cdot(v_h-v_l),0\}<\tilde{q}^{Pl}_{2l}=v_l-\frac{\sigma\cdot(1-n^*_{Ph})}{(1-\sigma)\cdot n^*_{Pl}}\cdot(v_h-v_l)\leq q^*_l$. The contracts implement intertemporal information rent $\tilde{u}_{Pi}$ to the persistent consumer with $v_1=v_i$ such that $\tilde{u}_{Ph}=\tilde{u}^P_{1h}+\tilde{u}^{Ph}_{2h}=(v_h-v_l)\cdot(\tilde{q}^P_{1l}+\tilde{q}^{Pl}_{2l})$ and $\tilde{u}_{Pl}=\tilde{u}^P_{1l}+\tilde{u}^{Pl}_{2l}=0$.

Contract with an impersistent consumer.

The optimal contract has at least (IC^I_h) binding. To see this, if the optimal contract has both (IC^I_h) and (IC^I_l) slacking, the consumer reveals his true preference with certainty. The second-period contract given the seller's updated belief has $u^{Il}_{2h}=0$ and $u^{Ih}_{2h}=(v_h-v_l)\cdot q^*_l$. Slacking (IC^I_h) and satisfaction of (IR^I_l) imply $u_{1h}>(v_h-v_l)\cdot q_{1l}+u_{1l}\geq 0$, so (IR^I_h) is slacking. For any pair of (u_{1h},u_{1l}) strictly satisfying all constraints in problem $\mathscr{P}^I_1$, there is a sufficiently small $\varepsilon>0$ such that $(u_{1h}-\varepsilon,u_{1l}-\varepsilon)$ is individually rational and incentive compatible. The principal prefers $(u_{1h}-\varepsilon, u_{1l}-\varepsilon)$ to (u_{1h}, u_{1l}), so (IR_l^I) is binding. The first-period contract implements efficient consumption output $(q_{1h}, q_{1l}) = (q_h^*, q_l^*)$, $u_{1l}=0$, and $u_{1h} > (v_h - v_l)\cdot q_l^* > 0$. There is a sufficiently small $\varepsilon > 0$ such that $u_{1h}-\varepsilon$ is incentive feasible and preferred by the principal. This contradicts to optimality. If the optimal contract has binding (IC_l^I) and slacking (IC_h^I), the descending type reveals his preference with probability one. The second-period contract given the seller's updated belief has $u_{2h}^{Il}=0$ and $u_{2h}^{Ih} = (v_h - v_l)\cdot q_{2l}^{Ih}$. The first-period contract solves $\max_{q_1, u_{1l}} \rho^I \cdot (v_h \cdot q_{1h} - c(q_{1h})) + (1-\rho^I)\cdot(v_l \cdot q_{1l} - c(q_{1l})) + u_{1l} - \rho^I \cdot ((v_h - v_l)\cdot q_{1h} - u_{2h}^{Ih})$ subject to (IR_i^I). Neglecting (IR_i^I), the solution to such problem has $q_{1h} = q_{1l} = q_l^*$. Binding (IC_l^I) and satisfaction of (IR_l^I) imply $u_{1h} = (v_h - v_l)\cdot q_l^* - u_{2h}^{Ih} + u_{1l} > 0$, so (IR_h^I) is slacking. The contract that solves the above problem thus implements $u_{1l}=0$. However, this contract violates (IC_h^I) as $u_{2h}^{Il} - u_{2h}^{Ih} = -(v_h - v_l)\cdot q_{2l}^{Ih} < 0$. The optimal contract must not have slacking (IC_h^I).

The optimal contract must not have both (IR_i^I) slacking. For any pair of (u_{1h}, u_{1l}) that satisfies both (IC_i^I) and strictly satisfies both (IR_i^I), there is a sufficiently small $\varepsilon > 0$ such that $(u_{1h}-\varepsilon, u_{1l}-\varepsilon)$ is individually rational with no effect on incentive compatibility. The principal prefers $(u_{1h}-\varepsilon, u_{1l}-\varepsilon)$ to (u_{1h}, u_{1l}). If (IR_l^I) is binding, $u_{1h} = (v_h - v_l)\cdot q_{1l} - u_{2h}^{Il}$ by binding (IC_h^I), which satisfies (IR_h^I) if and only if $q_{1l} \geq q_{2l}^{Il}$; if (IR_h^I) is binding, $u_{1l} = -(v_h - v_l)\cdot q_{1l}$ by binding (IC_h^I), which satisfies (IR_l^I) if and only if $q_{1l} \leq q_{2l}^{Il}$. When (IC_l^I) is slacking, the contract implements $u_{1h} - u_{1l} = (v_h - v_l)\cdot q_{1l} < (v_h - v_l)\cdot q_{1h} + u_{2h}^{Il} - u_{2h}^{Ih}$ with $u_{2h}^{Il} = (v_h - v_l)\cdot q_{2l}^{Il} < u_{2h}^{Ih} = (v_h - v_l)\cdot q_l^*$. If (IR_l^I) is slacking, the contract has $q_{1h} = q_{1l} = q_h^*$ as a solution to $\max_{q_1} \rho^I \cdot (v_h \cdot q_{1h} - c(q_{1h})) + (1-\rho^I)\cdot(v_l \cdot q_{1l} - c(q_{1l})) + (1-\rho^I)\cdot(v_h - v_l)\cdot q_{1l}$, which violates both (IR_l^I) and (IC_l^I). If (IC_l^I) is slacking, (IR_l^I) must be binding. When both (IC_i^I) are binding, the contract implements $u_{1h} - u_{1l} = (v_h - v_l)\cdot q_{1l} = (v_h - v_l)\cdot q_{1h} + u_{2h}^{Il} - u_{2h}^{Ih}$. If (IR_h^I) is binding, the contract solves the reduced problem $\max_{q_1} \rho^I \cdot (v_h \cdot q_{1h} - c(q_{1h})) + (1-\rho^I)\cdot(v_l \cdot q_{1l} - c(q_{1l})) + (1-\rho^I)\cdot(v_h - v_l)\cdot q_{1l} + \lambda^I \cdot ((v_h - v_l)\cdot q_{1h} + u_{2h}^{Il} - u_{2h}^{Ih} - (v_h - v_l)\cdot q_{1l})$, where $\lambda^I > 0$ denotes the equilibrium Lagrange multiplier associated with binding (IC_l^I). Solution to this problem has $q_{1h} > q_h^*$. Binding (IC_l^I) and (IR_h^I) imply $u_{1l} + u_{2h}^{Il} = -(v_h - v_l)\cdot q_{1h} + u_{2h}^{Ih}$. This satisfies (IR_l^I) only if $q_{1h} < q_{2l}^{Ih}$, which contradicts to $q_{1h} > q_h^*$. If both (IC_i^I) are binding, (IR_h^I) must be slacking. Therefore, the optimal contract has (IR_l^I) binding.

If (IC_l^I) is slacking, the ascending type reveals his preference with certainty $n_{Il} = 1$, so $\rho^I = \sigma \cdot n_{Ih}$. With binding (IR_l^I) and (IC_h^I), the optimal contract $\mathbb{C}_1^I$ has $q_{1h}^I = \arg\max_q v_h \cdot q - c(q) = q_h^*$ and $q_{1l}^I = \arg\max_q (1 - \sigma \cdot n_{Ih})\cdot(v_l \cdot q - c(q)) - \sigma \cdot n_{Ih} \cdot (v_h - v_l)\cdot q + \eta \cdot ((v_h - v_l)\cdot q - u_{2h}^{Il}) < q_l^*$, implementing $u_{1l}^I = -u_{2h}^{Il}$ and

$u^I_{1h} = (v_h - v_l) \cdot q^I_{1l} - u^{Il}_{2h} \geq 0$, where $\eta \geq 0$ denotes the equilibrium Lagrange multiplier associated with (IR^I_h). The contract must have $q^I_{1l} > q^{Il}_{2l}$ with $\eta = 0$ or $q^I_{1l} = q^{Il}_{2l}$ with $\eta > 0$. With $n_{Ih} \leq n_{Il} = 1$, $\nu_I(v_h) = 0$ and $\nu_I(v_l) = \frac{(1-\sigma)}{\sigma \cdot (1-n_{Ih}) + (1-\sigma)}$. The second period contract following revelation of v_l has $q^{Il}_{2h} = \arg\max_q v_h \cdot q - c(q) = q^*_h$ and $q^{Il}_{2l} = \max\{v_l - \frac{1-\sigma}{\sigma \cdot (1-n_{Ih})} \cdot (v_h - v_l), 0\} < q^*_l$, which implements $u^{Il}_{2h} = (v_h - v_l) \cdot q^{Il}_{2l}$ and $u^{Il}_{2l} = 0$. The second period contract following revelation of v_h has a single option $q^{Ih}_2 = \arg\max_q v_l \cdot q - c(q) = q^*_l$, which implements $u^{Ih}_{2h} = (v_h - v_l) \cdot q^*_l$ and $u^{Ih}_{2l} = 0$.

The contracts and revelation strategy strictly satisfies (IC^I_l) if $(v_h - v_l) \cdot q^*_h + u^{Il}_{2h} - u^{Ih}_{2h} \geq (v_h - v_l) \cdot q^I_{1l}$, which holds if $v_h - \frac{1-\sigma}{\sigma \cdot (1-n_{Ih})} \cdot (v_h - v_l) \geq v_l - \frac{\sigma \cdot n_{Ih} - \eta}{1 - \sigma \cdot n_{Ih}} \cdot (v_h - v_l)$ given $1 - n_{Ih} > \frac{1-\sigma}{\sigma} \cdot \frac{v_h - v_l}{v_l}$ such that $u^{Il}_{2h} > 0$ or $v_h - v_l \geq v_l - \frac{\sigma}{1-\sigma} \cdot (v_h - v_l)$ otherwise. If $\frac{1-\sigma}{\sigma} \cdot \frac{v_h - v_l}{v_l} \geq 1$, only the latter is relevant, which can be rearranged to $\frac{1}{1-\sigma} \geq \frac{v_l}{v_h - v_l}$. This is labeled as Condition 1-2. The former can be rearranged to $\frac{1}{1-\sigma \cdot n_{Ih}} \geq \frac{1-\sigma}{\sigma \cdot (1-n_{Ih})}$ with slacking (IR^I_h), or strictly holds given $q^I_{1l} = q^{Il}_{2l}$ with binding (IR^I_i). With slacking (IR^I_h), this can be further rearranged to $1 - n_{Ih} \geq (\frac{1-\sigma}{\sigma})^2$, which is consistent to $1 - n_{Ih} > \frac{1-\sigma}{\sigma} \cdot \frac{v_h - v_l}{v_l}$ if $\frac{v_h - v_l}{v_l} \geq \frac{1-\sigma}{\sigma}$. This is rearranged to $\frac{\sigma}{1-\sigma} \geq \frac{v_l}{v_h - v_l}$ and labeled as Condition 1-3. If Condition 1-2 or 1-3 is violated, $(v_h - v_l) \cdot q^*_h + u^{Il}_{2h} - u^{Ih}_{2h} < (v_h - v_l) \cdot q^I_{1l}$ for some n_{Ih}, at which both (IC^I_h) and (IC^I_l) are binding.

Given the above conditions, the principal implements $n^*_{Ih} \leq 1$ that maximizes her intertemporal payoff, where $\sigma \cdot (v_h \cdot q^*_h - c(q^*_h) - u^I_{1h} + v_l \cdot q^*_l - c(q^*_l)) + (1 - \eta) \cdot \frac{\partial u^{Il}_{2h}}{\partial n_{Ih}} \geq \sigma \cdot (v_l \cdot q^I_{1l} - c(q^I_{1l}) - u^I_{1l} + v_l \cdot q^{Il}_{2l} - c(q^{Il}_{2l}))$ by the envelope theorem of $\mathscr{P}^I_t$, with equality at $n_{Ih} \neq 1$. This can be reduced to $v_h \cdot q^*_h - c(q^*_h) + v_l \cdot q^*_l - c(q^*_l) - (v_h \cdot q^I_{1l} - c(q^I_{1l}) + v_l \cdot q^{Il}_{2l} - c(q^{Il}_{2l})) \geq -\frac{1-\eta}{\sigma} \cdot \frac{\partial u^{Il}_{2h}}{\partial n_{Ih}}$. The difference in the seller's payoff between a truth revealing and a lying descending type is greater than or equal to the weighted decrement in the exploited rent by inducing the descending type to reveal his true preference with a higher probability. It is optimal for the seller to implement $n^*_{Ih} = 1$ as $v_h \cdot q^*_h - c(q^*_h) + v_l \cdot q^*_l - c(q^*_l) > v_h \cdot q^I_{1l} - c(q^I_{1l})$ or to implement $1 - n^*_{Ih} > \frac{1-\sigma}{\sigma} \cdot \frac{v_h - v_l}{v_l}$ such that the above equality holds. Truthful revelation $n^*_{Ih} = 1$ is optimal if $\frac{\sigma}{1-\sigma} \cdot \frac{v_l}{v_h - v_l} \leq 1$ such that it takes a large downward distortion in $n_{Ih} < 1$ to generate an insufficiently high marginal exploitation $-\frac{\partial u^{Il}_{2h}}{\partial n_{Ih}} = (v_h - v_l)^2 \cdot \frac{1-\sigma}{\sigma} \cdot \frac{1}{(1-n_{Ih})^2} < \frac{\sigma}{1-\sigma} \cdot v_l^2$. Mixed revelation $n^*_{Ih} < 1$ is optimal if otherwise.

At $n^*_{Ih} \leq 1$ with Condition 1-2 or 1-3 satisfied, the optimal contract $\hat{\mathbb{C}}^I_1$ has $\hat{q}^I_{1h} = q^*_h$ and $\hat{q}^I_{1l} = v_l - \frac{\sigma \cdot n^*_{Ih} - \eta}{1 - \sigma \cdot n^*_{Ih}} \cdot (v_h - v_l) < q^*_l$, implementing $\hat{u}^I_{1l} = -\hat{u}^{Il}_{2h}$ and $\hat{u}^I_{1h} = (v_h - v_l) \cdot \hat{q}^I_{1l} - \hat{u}^{Il}_{2h}$. The second-period contract $\hat{\mathbb{C}}^{Il}_2$ following revelation of v_l has $\hat{q}^{Il}_{2h} = \arg\max_q v_h \cdot q - c(q) = q^*_h$ and $\hat{q}^{Il}_{2l} = \max\{v_l - \frac{1-\sigma}{\sigma \cdot (1-n^*_{Ih})} \cdot (v_h - v_l), 0\} < q^*_l$, which implements $\hat{u}^{Il}_{2h} = (v_h - v_l) \cdot \hat{q}^{Il}_{2l}$ and $\hat{u}^{Il}_{2l} = 0$. The second-period contract $\hat{\mathbb{C}}^{Ih}_2$ following revelation of v_h has a single option $\hat{q}^{Ih}_2 = \arg\max_q v_l \cdot q - c(q) = q^*_l$, which implements $\hat{u}^{Ih}_{2h} = (v_h - v_l) \cdot q^*_l$ off the equilibrium path and $\hat{u}^{Ih}_{2l} = 0$. The optimal contracts implement intertemporal information rent $\hat{u}_{Ii}$ to the impersistent consumer with $v_1 = v_i$ such that $\hat{u}_{Ih} = \hat{u}^I_{1h} + \hat{u}^{Ih}_{2l} = (v_h - v_l) \cdot \hat{q}^I_{1l} - \hat{u}^{Il}_{2h}$ and $\hat{u}_{Il} = \hat{u}^I_{1l} + \hat{u}^{Il}_{2h} = 0$.

When Condition 1 is violated and (IR^I_l), (IC^I_h), and (IC^I_l) are all binding, with $\rho^I = \sigma \cdot n_{Ih} + (1-\sigma) \cdot (1 - n_{Il})$, the optimal contract $\tilde{\mathbb{C}}^I_1$ solves the reduced problem

$$\mathscr{P}^I_1 : \quad \max_{q_1} \rho^I \cdot (v_h \cdot q_{1h} - c(q_{1h})) + (1-\rho^I) \cdot (v_l \cdot q_{1l} - c(q_{1l})) - \rho^I \cdot (v_h - v_l) \cdot q_{1l} - u^{Il}_{2h}$$

subject to $(v_h - v_l) \cdot q_{1h} + u^{Il}_{2h} - u^{Ih}_{2h} = (v_h - v_l) \cdot q_{1l}$. The optimal contract has $\tilde{q}^I_{1h} > q^*_h$ as a solution to $\rho^I \cdot (v_h - q_{1h}) + \lambda^I \cdot (v_h - v_l) = 0$, where $\lambda^I > 0$ denotes the equilibrium Lagrange multiplier associated with binding (IC^I_l). It has $\tilde{q}^I_{1l} < \hat{q}^I_{1l}$ as a solution to $(1-\rho^I) \cdot (v_l - q_{1l}) - \rho^I \cdot (v_h - v_l) - \lambda^I \cdot (v_h - v_l) = 0$. First-period information rents $\tilde{u}^I_{1l} = -\tilde{u}^{Il}_{2h}$ and $\tilde{u}^I_{1h} = (v_h - v_l) \cdot \tilde{q}^I_{1l} - \tilde{u}^{Il}_{2h}$ are implemented. With both incentive compatibility constraints binding, $\tilde{u}^I_{1h} = (v_h - v_l) \cdot \tilde{q}^I_{1l} - \tilde{u}^{Il}_{2h} = (v_h - v_l) \cdot \tilde{q}^I_{1h} - \tilde{u}^{Ih}_{2h} > 0$, so (IR^I_h) strictly holds.

The principal implements $n^*_{Ih} \leq 1$ that maximizes her intertemporal payoff, where $\sigma \cdot (v_h \cdot \tilde{q}^I_{1h} - c(\tilde{q}^I_{1h}) - \tilde{u}^I_{1h} + v_l \cdot q^*_l - c(q^*_l)) + \frac{\partial \tilde{u}^{Il}_{2h}}{\partial n_{Ih}} + \lambda^I \cdot \frac{\partial(\tilde{u}^{Il}_{2h} - \tilde{u}^{Ih}_{2h})}{\partial n_{Ih}} \geq \sigma \cdot (v_l \cdot \tilde{q}^I_{1l} - c(\tilde{q}^I_{1l}) - \tilde{u}^I_{1l} + v_l \cdot \tilde{q}^{Il}_{2l} - c(\tilde{q}^{Il}_{2l}))$ by the envelope theorem of $\mathscr{P}^I_t$, with equality at $n_{Ih} \neq 1$. This can be reduced to $v_h \cdot \tilde{q}^I_{1h} - c(\tilde{q}^I_{1h}) + v_l \cdot q^*_l - c(q^*_l)) - (v_h \cdot \tilde{q}^I_{1l} - c(\tilde{q}^I_{1l}) + v_l \cdot \tilde{q}^{Il}_{2l} - c(\tilde{q}^{Il}_{2l})) \geq -\frac{1}{\sigma} \cdot \frac{\partial \tilde{u}^{Il}_{2h}}{\partial n_{Ih}} - \frac{\lambda^I}{\sigma} \cdot \frac{\partial(\tilde{u}^{Il}_{2h} - \tilde{u}^{Ih}_{2h})}{\partial n_{Ih}}$, where $\frac{\partial \tilde{u}^{Il}_{2h}}{\partial n_{Ih}} \leq 0$ and $\frac{\partial(\tilde{u}^{Il}_{2h} - \tilde{u}^{Ih}_{2h})}{\partial n_{Ih}} < 0$ as $\tilde{u}^{Il}_{2h} = (v_h - v_l) \cdot \tilde{q}^{Il}_{2l}$ and $\tilde{u}^{Il}_{2h} - \tilde{u}^{Ih}_{2h} = (v_h - v_l) \cdot (\tilde{q}^{Il}_{2l} - \tilde{q}^{Ih}_{2l})$. The difference in the seller's payoff between a truth revealing and a lying descending type is higher than or equal to the sum of weighted decrement in the exploited rent and weighted increment in the gain of second-period information rent from lying when inducing the descending type to reveal his true preference with a higher probability. The principal implements $n^*_{Il} \leq 1$ that maximizes her intertemporal payoff, where $(1-\sigma) \cdot (v_l \cdot \tilde{q}^I_{1l} - c(\tilde{q}^I_{1l}) - \tilde{u}^I_{1l} + v_h \cdot q^*_h - c(q^*_h) - \tilde{u}^{Il}_{2h}) + \frac{\partial \tilde{u}^{Il}_{2h}}{\partial n_{Il}} + \lambda^I \cdot \frac{\partial(\tilde{u}^{Il}_{2h} - \tilde{u}^{Ih}_{2h})}{\partial n_{Il}} \geq (1-\sigma) \cdot (v_h \cdot \tilde{q}^I_{1h} - c(\tilde{q}^I_{1h}) - \tilde{u}^I_{1h} + v_h \cdot q^*_h - c(q^*_h) - \tilde{u}^{Ih}_{2h})$ by the envelope theorem of $\mathscr{P}^I_t$, with equality at $n_{Il} \neq 1$. This can be reduced to $v_h \cdot \tilde{q}^I_{1l} - c(\tilde{q}^I_{1l}) - (v_h \cdot \tilde{q}^I_{1h} - c(\tilde{q}^I_{1h})) - (v_h - v_l) \cdot (\tilde{q}^{Il}_{2l} - \tilde{q}^{Ih}_{2l}) \geq -\frac{1}{1-\sigma} \cdot \frac{\partial \tilde{u}^{Il}_{2h}}{\partial n_{Il}} - \frac{\lambda^I}{1-\sigma} \cdot \frac{\partial(\tilde{u}^{Il}_{2h} - \tilde{u}^{Ih}_{2h})}{\partial n_{Il}}$, where $\frac{\partial \tilde{u}^{Il}_{2h}}{\partial n_{Il}} \leq 0$ and $\frac{\partial(\tilde{u}^{Il}_{2h} - \tilde{u}^{Ih}_{2h})}{\partial n_{Il}} < 0$. The difference in the seller's payoff between a truth revealing and a lying ascending type is higher than or equal to the sum of weighted decrement in the exploited rent and weighted increment in the gain of second-period information rent from lying when inducing the descending type to reveal his true preference with a higher probability.

At $n^*_{Ii} \leq 1$ with violation to Condition 1, the contract $\tilde{\mathbb{C}}^I_1$ has $\tilde{q}^I_{1h} > q^*_h$ such that $\rho^I \cdot (v_h - q_{1h}) + \lambda^I \cdot (v_h - v_l) = 0$, where $\rho^I = \sigma \cdot n^*_{Ih} + (1-\sigma) \cdot (1 - n^*_{Il})$, and $\tilde{q}^I_{1l} < \hat{q}^I_{1l}$ such that $(1-\rho^I) \cdot (v_l - q_{1l}) - (\rho^I + \lambda^I) \cdot (v_h - v_l) = 0$. The second-period contract $\tilde{\mathbb{C}}^{Ii}_2$ following revelation of v_i have $\tilde{q}^{Ii}_{2h} = q^*_h$ and $\tilde{q}^{Il}_{2l} = \max\{v_l - \frac{(1-\sigma)\cdot n^*_{Il}}{\sigma\cdot(1-n^*_{Ih})} \cdot (v_h - v_l), 0\} < \tilde{q}^{Ih}_{2l} = v_l - \frac{(1-\sigma)\cdot(1-n^*_{Il})}{\sigma \cdot n^*_{Ih}} \cdot (v_h - v_l) \leq q^*_l$. The contracts implement intertemporal information rent $\tilde{u}_{Ii}$ to the impersistent consumer with $v_1 = v_i$ such that $\tilde{u}_{Ih} = \tilde{u}^I_{1h} + \tilde{u}^{Ih}_{2l} = (v_h - v_l) \cdot \tilde{q}^I_{1l} - \tilde{u}^{Il}_{2h}$ and $\tilde{u}_{Il} = \tilde{u}^I_{1l} + \tilde{u}^{Il}_{2h} = 0$.

A.2 Proof of Lemma 4 to Lemma 6 in Section 3.2

The consumer with first-period preference v_i, $i = l, h$, and evolution of preference $k = P, I$ reveals truthfully his first-period taste with probability m_{ki}. The seller's Bayesian updated belief of a high valuation type in the second period following the consumer's first-period choice of option (q_{1h}, p_{1h}) and (q_{1l}, p_{1l}) are

$$\mu(v_h) = \frac{\sigma \cdot \phi \cdot m_{Ph} + (1-\sigma) \cdot (1-\phi) \cdot (1 - m_{Il})}{\sigma \cdot \phi \cdot m_{Ph} + (1-\sigma) \cdot (1-\phi) \cdot (1-m_{Il}) + \sigma \cdot (1-\phi) \cdot m_{Ih} + (1-\sigma) \cdot \phi \cdot (1 - m_{Pl})}$$

and

$$\mu(v_l) = \frac{\sigma \cdot \phi \cdot (1 - m_{Ph}) + (1-\sigma) \cdot (1-\phi) \cdot m_{Il}}{\sigma \cdot \phi \cdot (1-m_{Ph}) + (1-\sigma) \cdot (1-\phi) \cdot m_{Il} + \sigma \cdot (1-\phi) \cdot (1-m_{Ih}) + (1-\sigma) \cdot \phi \cdot m_{Pl}}$$

respectively. Following the consumer's first-period choice of (q_{1i}, p_{1i}), the seller proposes the contract $\mathbb{C}^i_2 = \{(q^i_{2j}, p^i_{2j})\}$, $j = l, h$, implementing payoff $u^i_{2j} = v_j \cdot q^i_{2j} - p^i_{2j}$ that

$$\mathscr{P}^i_2: \quad \max_{q_2, u_2} \mu(v_i) \cdot (v_h \cdot q_{2h} - c(q_{2h}) - u_{2h}) + (1 - \mu(v_i)) \cdot (v_l \cdot q_{2l} - c(q_{2l}) - u_{2l})$$

subject to $u_{2j} \geq 0$ for both $j = l, h$, and $(v_h - v_l) \cdot q_{2h} \geq u_{2h} - u_{2l} \geq (v_h - v_l) \cdot q_{2l}$.

By backward induction and the standard methods in static contract theory, the optimal contract $\mathbb{C}^i_2$ has $q^i_{2h} = \arg\max_q v_h \cdot q - c(q) = q^*_h$ and $q^i_{2l} = \arg\max_q (1 - \mu(v_i)) \cdot (v_l \cdot q - c(q)) - \mu(v_i) \cdot (v_h - v_l) \cdot q < q^*_l$ if the option (q_{1i}, p_{1i}) was taken in the first period, which implements $u^i_{2h} = (v_h - v_l) \cdot q^i_{2l}$ and $u^i_{2l} = 0$. With quadratic cost, $q^i_{2h} = q^*_h = v_h$ and $q^i_{2l} = \max\{v_l - \frac{\mu(v_i)}{1-\mu(v_i)} \cdot (v_h - v_l), 0\}$. Second-period information rent to the consumer satisfies $u^l_{2h} \gtreqless u^h_{2h}$ if $q^l_{2l} \gtreqless q^h_{2l}$, which holds if $\frac{\mu(v_l)}{1-\mu(v_l)} \lesseqgtr \frac{\mu(v_h)}{1-\mu(v_h)}$, or equivalently, $\frac{\sigma\cdot\phi\cdot(1-m_{Ph})+(1-\sigma)\cdot(1-\phi)\cdot m_{Il}}{\sigma\cdot(1-\phi)\cdot(1-m_{Ih})+(1-\sigma)\cdot\phi\cdot m_{Pl}} \lesseqgtr \frac{\sigma\cdot\phi\cdot m_{Ph}+(1-\sigma)\cdot(1-\phi)\cdot(1-m_{Il})}{\sigma\cdot(1-\phi)\cdot m_{Ih}+(1-\sigma)\cdot\phi\cdot(1-m_{Pl})}$.

Anticipating the second-period contract $\mathbb{C}^i_2$, the seller proposes the first-period contract $\mathbb{C}_1 = \{(q_{1i}, p_{1i})\}$, $i = l, h$, implementing payoff $u_{1i} = v_i \cdot q_{1i} - p_{1i}$ that solves

$$\mathscr{P}_1 : \quad \max_{q_1, u_1} \rho \cdot (v_h \cdot q_{1h} - c(q_{1h}) - u_{1h}) + (1 - \rho) \cdot (v_l \cdot q_{1l} - c(q_{1l}) - u_{1l})$$

subject to

$$u_{1h} + u_{2h}^h \geq 0 \qquad (IR_{Ph})$$

$$u_{1l} \geq 0 \qquad (IR_{Pl})$$

$$u_{1h} \geq 0 \qquad (IR_{Ih})$$

$$u_{1l} + u_{2h}^l \geq 0 \qquad (IR_{Il})$$

$$u_{1h} - u_{1l} \geq (v_h - v_l) \cdot q_{1l} + u_{2h}^l - u_{2h}^h \qquad (IC_{Ph})$$

$$u_{1h} - u_{1l} \leq (v_h - v_l) \cdot q_{1h} \qquad (IC_{Pl})$$

$$u_{1h} - u_{1l} \geq (v_h - v_l) \cdot q_{1l} \qquad (IC_{Ih})$$

$$u_{1h} - u_{1l} \leq (v_h - v_l) \cdot q_{1h} + u_{2h}^l - u_{2h}^h \qquad (IC_{Il})$$

Relevant Constraints.

With $u_{2h}^i \geq 0$ and $u_{2l}^i = 0$ implemented by $\mathbb{C}_2^i$, (IR_{Ph}) and (IR_{Il}) are implied by (IR_{Ih}) and (IR_{Pl}). Rearranged from (IC_{Ih}), $u_{1h} \geq (v_h - v_l) \cdot q_{1l} + u_{1l} \geq 0$ by (IR_{Pl}), so (IR_{Ih}) is implied. For any pair of (u_{1h}, u_{1l}) satisfying all constraints in problem $\mathscr{P}_1$ with $u_{1l} > 0$, there is a sufficiently small $\varepsilon > 0$ such that $(u_{1h} - \varepsilon, u_{1l} - \varepsilon)$ is individually rational and incentive compatible. The principal prefers $(u_{1h} - \varepsilon, u_{1l} - \varepsilon)$ to (u_{1h}, u_{1l}). The optimal contract has $u_{1l}^a = 0$ from binding (IR_{Pl}).

Incentive compatibility of the impersistent consumer is slacking if $u_{2h}^l - u_{2h}^h > 0$, given which $m_{Ii} = 1$. The seller's belief is consistent to $u_{2h}^l - u_{2h}^h > 0$ if $\frac{\sigma \cdot \phi \cdot (1 - m_{Ph}) + (1-\sigma) \cdot (1-\phi)}{(1-\sigma) \cdot \phi \cdot m_{Pl}} < \frac{\sigma \cdot \phi \cdot m_{Ph}}{\sigma \cdot (1-\phi) + (1-\sigma) \cdot \phi \cdot (1 - m_{Pl})}$ such that $q_{2l}^l - q_{2l}^h > 0$. This inequality fails to hold for any m_{Pi} if the minimum of the LHS is higher than the maximum of the RHS, i.e. if $1 - \phi > \phi$. Incentive compatibility of the persistent consumer is slacking if $u_{2h}^l - u_{2h}^h < 0$, given which $m_{Pi} = 1$. The seller's belief is consistent to $u_{2h}^l - u_{2h}^h < 0$ if $\frac{(1-\sigma) \cdot (1-\phi) \cdot m_{Il}}{\sigma \cdot (1-\phi) \cdot (1 - m_{Ih}) + (1-\sigma) \cdot \phi} > \frac{\sigma \cdot \phi + (1-\sigma) \cdot (1-\phi) \cdot (1 - m_{Il})}{\sigma \cdot (1-\phi) \cdot m_{Ih}}$ such that $q_{2l}^l - q_{2l}^h < 0$. This inequality fails to hold for any m_{Ii} if the maximum of the LHS is lower than the minimum of the RHS, i.e. if $1 - \phi < \phi$. If $1 - \phi > \phi$, $u_{2h}^l - u_{2h}^h < 0$ and (IC_{Pi}) are slacking for both $i = l, h$, so the persistent consumer reveals his preference with probability $m_{Pi} = 1$; if $1 - \phi < \phi$, $u_{2h}^l - u_{2h}^h > 0$ and (IC_{Ii}) are slacking for both $i = l, h$, so the impersistent consumer reveals his preference with probability $m_{Ii} = 1$.

If $\phi > 1 - \phi$ and $(v_h - v_l) \cdot q_{1h} \geq (v_h - v_l) \cdot q_{1l} + u_{2h}^l - u_{2h}^h$, the optimal contract has at least (IC_{Ph}) binding. To see this, if the optimal contract has both (IC_{Ph}) and (IC_{Pl}) slacking, the consumer reveals his true preference with certainty. The first-period contract implements efficient consumption output $(q_{1h}, q_{1l}) = (q_h^*, q_l^*)$, $u_{1l} = 0$, and $u_{1h} > (v_h - v_l) \cdot q_l^* + u_{2h}^l - u_{2h}^h$. There is a sufficiently small $\varepsilon > 0$ such that $u_{1h} - \varepsilon$ is incentive feasible and preferred by the principal. This contradicts to optimality. If the optimal contract has binding (IC_{Pl}) and slacking (IC_{Ph}), the persistent-high type reveals his preference with probability one. The first-period contract implements consumption output $(q_{1h}, q_{1l}) = (q_l^*, q_l^*)$, which is a solution to $\max_{q_1} \rho \cdot (v_h \cdot q_{1h} - c(q_{1h}) - (v_h - v_l) \cdot q_{1h}) + (1 - \rho) \cdot (v_l \cdot q_{1l} - c(q_{1l}))$, with implemented rent $u_{1l} = 0$ and $u_{1h} = (v_h - v_l) \cdot q_l^*$. This violates (IC_{Ph}) as $(v_h - v_l) \cdot q_l^* < (v_h - v_l) \cdot q_l^* + u_{2h}^l - u_{2h}^h$ when $\phi > 1 - \phi$.

If $1 - \phi > \phi$ and $(v_h - v_l) \cdot q_{1h} + u_{2h}^l - u_{2h}^h \geq (v_h - v_l) \cdot q_{1l}$, the optimal contract has at least (IC_{Ih}) binding. To see this, if the optimal contract has both (IC_{Ih}) and (IC_{Il}) slacking, the consumer reveals his true preference with certainty. The first-period contract implements efficient consumption output $(q_{1h}, q_{1l}) = (q_h^*, q_l^*)$, $u_{1l} = 0$, and $u_{1h} > (v_h - v_l) \cdot q_l^*$. There is a sufficiently small $\varepsilon > 0$ such that $u_{1h} - \varepsilon$ is incentive feasible and preferred by the principal. This contradicts to optimality. If the optimal contract has binding (IC_{Il}) and slacking (IC_{Ih}), the descending type reveals his preference with probability one. The first-period contract implements consumption output $(q_{1h}, q_{1l}) = (q_l^*, q_l^*)$, which is a solution to $\max_{q_1} \rho \cdot (v_h \cdot q_{1h} - c(q_{1h}) - (v_h - v_l) \cdot q_{1h} - u_{2h}^l + u_{2h}^h) + (1 - \rho) \cdot (v_l \cdot q_{1l} - c(q_{1l}))$, with implemented rent $u_{1l} = 0$ and $u_{1h} = (v_h - v_l) \cdot q_l^* + u_{2h}^l - u_{2h}^h$. This violates (IC_{Ih}) as $(v_h - v_l) \cdot q_l^* + u_{2h}^l - u_{2h}^h < (v_h - v_l) \cdot q_l^*$ when $1 - \phi > \phi$.

Contract and Revelation.

Scenario 1. $\phi > 1 - \phi$**.** If $\phi > 1 - \phi$ and (IC_{Pl}) is slacking, $m_{Ii} = m_{Pl} = 1$ so $\rho = \sigma \cdot \phi \cdot m_{Ph} + \sigma \cdot (1 - \phi)$. With binding (IR_{Pl}) and (IC_{Ph}), the optimal contract $\mathbb{C}_1^a$ has $q_{1h}^a = \arg\max_q v_h \cdot q - c(q) = q_h^*$ and $q_{1l}^a = v_l - \frac{\sigma \cdot \phi \cdot m_{Ph} + \sigma \cdot (1-\phi)}{\sigma \cdot \phi \cdot (1 - m_{Ph}) + 1 - \sigma} \cdot (v_h - v_l) < q_l^*$, implementing $u_{1l}^a = 0$ and $u_{1h}^a = (v_h - v_l) \cdot q_{1l}^a + u_{2h}^l - u_{2h}^h$. With $m_{Ph} < m_{Pl}^* = m_{Ii}^* = 1$, $\mu(v_h) = \frac{\sigma \cdot \phi \cdot m_{Ph}}{\sigma \cdot \phi \cdot m_{Ph} + \sigma \cdot (1-\phi)}$ and $\mu(v_l) = \frac{\sigma \cdot \phi \cdot (1 - m_{Ph}) + (1-\sigma) \cdot (1-\phi)}{\sigma \cdot \phi \cdot (1 - m_{Ph}) + (1 - \sigma)}$. The second-period contract following first-period revelation of v_i has $q_{2h}^i = \arg\max_q v_h \cdot q - c(q) = q_h^*$ and $q_{2l}^h = v_l - \frac{\phi \cdot m_{Ph}}{1-\phi} \cdot (v_h - v_l) < q_{2l}^l = v_l - \frac{\sigma \cdot \phi \cdot (1 - m_{Ph}) + (1-\sigma) \cdot (1-\phi)}{(1-\sigma) \cdot \phi} \cdot (v_h - v_l) < q_l^*$, which implements $u_{2h}^i = (v_h - v_l) \cdot q_{2l}^i$ and $u_{2l}^i = 0$.

The contracts and revelation strategy satisfies (IC_{Pl}) if $(v_h - v_l) \cdot q_{1h}^a \geq (v_h - v_l) \cdot q_{1l}^a + u_{2h}^l - u_{2h}^h$, which holds if $1 + \frac{\sigma \cdot \phi \cdot m_{Ph} + \sigma \cdot (1-\phi)}{\sigma \cdot \phi \cdot (1 - m_{Ph}) + 1 - \sigma} \geq \frac{\phi \cdot m_{Ph}}{1-\phi} - \frac{\sigma \cdot \phi \cdot (1 - m_{Ph}) + (1-\sigma) \cdot (1-\phi)}{(1-\sigma) \cdot \phi}$. The inequality holds for all m_{Ph} if the minimum of the LHS is higher than the maximum of the RHS, i.e. if $\frac{1}{1 - \sigma + \sigma \cdot \phi} \geq \frac{\phi}{1-\phi} - \frac{1-\phi}{\phi}$. The inequality holds at $m_{Ph} = 1$ if $\frac{1}{1-\sigma} \geq$

$\frac{\phi}{1-\phi} - \frac{1-\phi}{\phi}$. If $\frac{1}{1-\sigma} < \frac{\phi}{1-\phi} - \frac{1-\phi}{\phi}$, $(v_h - v_l) \cdot q_{1h}^a < (v_h - v_l) \cdot q_{1l}^a + u_{2h}^l - u_{2h}^h$ for some m_{Ph}, at which both (IC_{Ph}) and (IC_{Pl}) are binding.

Given the above conditions, the principal implements $m_{Ph}^* \leq 1$ that maximizes her intertemporal payoff, where $\sigma \cdot \phi \cdot (v_h \cdot q_h^* - c(q_h^*) - u_{1h}^a + v_h \cdot q_h^* - c(q_h^*) - u_{2h}^h) - \sigma \cdot (\phi \cdot m_{Ph} + 1 - \phi) \cdot \frac{\partial(u_{2h}^l - u_{2h}^h)}{\partial m_{Ph}} \geq \sigma \cdot \phi \cdot (v_l \cdot q_{1l}^a - c(q_{1l}^a) + v_h \cdot q_h^* - c(q_h^*) - u_{2h}^l)$ by the envelope theorem of $\mathscr{P}_t$, with equality at $m_{Ph} \neq 1$. This can be reduced to $v_h \cdot q_h^* - c(q_h^*) - (v_h - v_l) \cdot q_{1l}^a - (v_l \cdot q_{1l}^a - c(q_{1l}^a)) \geq \frac{\phi \cdot m_{Ph} + 1 - \phi}{\phi} \cdot \frac{\partial(u_{2h}^l - u_{2h}^h)}{\partial m_{Ph}}$. The difference in the seller's payoff between a truth revealing and a lying persistent-high type is greater than or equal to the weighted increment in the second-period gain of information rent from lying when inducing the persistent-high type to reveal his true preference with a higher probability. Given the contract above, this is rearranged to $(v_h - v_l)^2 \cdot \frac{1}{2} \cdot \frac{1}{(\sigma \cdot \phi \cdot (1 - m_{Ph}) + 1 - \sigma)^2} \geq \frac{\phi \cdot m_{Ph} + 1 - \phi}{\phi} \cdot \left(\frac{\sigma}{1-\sigma} + \frac{\phi}{1-\phi}\right) \cdot (v_h - v_l)^2$. It is optimal for the seller to implement $m_{Ph}^* = 1$ if $\frac{1}{2} \cdot \frac{1}{(1-\sigma)^2} \geq \frac{1}{\phi} \cdot \left(\frac{\sigma}{1-\sigma} + \frac{\phi}{1-\phi}\right)$, which is rearranged to $\frac{1}{1-\sigma} \geq 2 \cdot \left(\frac{\sigma}{\phi} + \frac{1-\sigma}{1-\phi}\right)$. Label $\frac{1}{1-\sigma} \geq \frac{\phi}{1-\phi} - \frac{1-\phi}{\phi}$ when $\frac{1}{1-\sigma} \geq 2 \cdot \left(\frac{\sigma}{\phi} + \frac{1-\sigma}{1-\phi}\right)$ as Condition 2-1. It is optimal for the seller to implement $m_{Ph}^* < 1$ if $\frac{1}{1-\sigma} < 2 \cdot \left(\frac{\sigma}{\phi} + \frac{1-\sigma}{1-\phi}\right)$. Label $\frac{1}{1-\sigma+\sigma\cdot\phi} \geq \frac{\phi}{1-\phi} - \frac{1-\phi}{\phi}$ when $\frac{1}{1-\sigma} < 2 \cdot \left(\frac{\sigma}{\phi} + \frac{1-\sigma}{1-\phi}\right)$ as Condition 2-2.

At $m_{Ph}^* \leq 1$ and $m_{ki}^* = 1$ for $(k, i) \neq (P, h)$ with Condition 2 satisfied, the optimal contract $\mathbb{C}_1^a$ has $q_{1h}^a = q_h^*$ and $q_{1l}^a = v_l - \frac{\sigma \cdot \phi \cdot m_{Ph}^* + \sigma \cdot (1-\phi)}{\sigma \cdot \phi \cdot (1 - m_{Ph}^*) + 1 - \sigma} \cdot (v_h - v_l) < q_l^*$, and the optimal contracts $\mathbb{C}_2^{ai}$ following revelation of v_i have $q_{2h}^{ai} = q_h^*$ and $q_{2l}^{ah} = v_l - \frac{\phi \cdot m_{Ph}^*}{1-\phi} \cdot (v_h - v_l) < q_{2l}^{al} = v_l - \frac{\sigma \cdot \phi \cdot (1 - m_{Ph}^*) + (1-\sigma) \cdot (1-\phi)}{(1-\sigma) \cdot \phi} \cdot (v_h - v_l) < q_l^*$. The contracts implement intertemporal information rent u_{ki}^a to the consumer of evolution type k and $v_1 = v_i$ such that $u_{Ph}^a = u_{1h}^a + u_{2h}^{ah} = (v_h - v_l) \cdot (q_{1l}^a + q_{2l}^{al})$, $u_{Pl}^a = u_{1l}^a + u_{2l}^{al} = 0$, $u_{Ih}^a = u_{1h}^a + u_{2l}^{ah} = (v_h - v_l) \cdot (q_{1l}^a + q_{2l}^{al} - q_{2l}^{ah})$, and $u_{Il}^a = u_{1l}^a + u_{2h}^{al} = (v_h - v_l) \cdot q_{2l}^{al}$.

When Condition 2 is violated and (IC_{Pl}), (IC_{Ph}), and (IR_{Pl}) are all binding, with $\rho = \sigma \cdot \phi \cdot m_{Ph} + (1-\sigma) \cdot \phi \cdot (1 - m_{Pl}) + \sigma \cdot (1 - \phi)$, the optimal contract $\mathbb{C}_1^b$ solves the reduced problem

$$\mathscr{P}_1 : \quad \max_{q_1, u_1} \rho \cdot (v_h \cdot q_{1h} - c(q_{1h})) + (1-\rho) \cdot (v_l \cdot q_{1l} - c(q_{1l})) - \rho \cdot ((v_h - v_l) \cdot q_{1l} + u_{2h}^l - u_{2h}^h)$$

subject to $(v_h - v_l) \cdot q_{1h} = (v_h - v_l) \cdot q_{1l} + u_{2h}^l - u_{2h}^h$. The optimal contract has $q_{1h}^b > q_h^*$ as a solution to $\rho \cdot (v_h - q_{1h}) + \lambda^b \cdot (v_h - v_l) = 0$, where $\lambda^b > 0$ denotes the equilibrium Lagrange multiplier associated with binding (IC_{Pl}). It has $q_{1l}^b < q_{1l}^a$ as a solution to $(1-\rho) \cdot (v_l - q_{1l}) - \rho \cdot (v_h - v_l) - \lambda^b \cdot (v_h - v_l) = 0$. First-period information rents $u_{1l}^b = 0$ and $u_{1h}^b = (v_h - v_l) \cdot q_{1l}^b + u_{2h}^l - u_{2h}^h$ are implemented, where $u_{2h}^l - u_{2h}^h = \left(\frac{\sigma \cdot \phi \cdot m_{Ph}}{\sigma \cdot (1-\phi) + (1-\sigma) \cdot \phi \cdot (1-m_{Pl})} - \frac{\sigma \cdot \phi \cdot (1-m_{Ph}) + (1-\sigma) \cdot (1-\phi)}{(1-\sigma) \cdot \phi \cdot m_{Pl}} \right) \cdot (v_h - v_l)^2$.

The principal implements $m_{Ph}^* \leq 1$ that maximizes her intertemporal payoff, where $\sigma \cdot \phi \cdot (v_h \cdot q_{1h}^b - c(q_{1h}^b) - u_{1h}^b + v_h \cdot q_h^* - c(q_h^*) - u_{2h}^h) - (\sigma \cdot \phi \cdot m_{Ph} + \sigma \cdot (1-\phi) + (1-\sigma) \cdot \phi \cdot (1-m_{Pl}) + \lambda^b) \cdot \frac{\partial (u_{2h}^l - u_{2h}^h)}{\partial m_{Ph}} \geq \sigma \cdot \phi \cdot (v_l \cdot q_{1l}^b - c(q_{1l}^b) + v_h \cdot q_h^* - c(q_h^*) - u_{2h}^l)$ by the envelope theorem of $\mathscr{P}_t$, with equality at $m_{Ph} \neq 1$. This can be reduced to $v_h \cdot q_{1h}^b - c(q_{1h}^b) - (v_h \cdot q_{1l}^b - c(q_{1l}^b)) \geq \frac{\sigma \cdot \phi \cdot m_{Ph} + \sigma \cdot (1-\phi) + (1-\sigma) \cdot \phi \cdot (1-m_{Pl}) + \lambda^b}{\sigma \cdot \phi} \cdot \frac{\partial (u_{2h}^l - u_{2h}^h)}{\partial m_{Ph}}$, where $\frac{\partial (u_{2h}^l - u_{2h}^h)}{\partial m_{Ph}} > 0$. The difference in the seller's payoff between a truth revealing and a lying persistent-high type is higher than or equal to the weighted increment in the gain of second-period information rent from lying when inducing the persistent-high type to reveal his true preference with a higher probability. The principal implements $m_{Pl}^* \leq 1$ that maximizes her intertemporal payoff, where $(1-\sigma) \cdot \phi \cdot (v_l \cdot q_{1l}^b - c(q_{1l}^b) + v_l \cdot q_{2l}^l - c(q_{2l}^l)) - (\sigma \cdot \phi \cdot m_{Ph} + \sigma \cdot (1-\phi) + (1-\sigma) \cdot \phi \cdot (1-m_{Pl}) + \lambda^b) \cdot \frac{\partial (u_{2h}^l - u_{2h}^h)}{\partial m_{Pl}} \geq (1-\sigma) \cdot \phi \cdot (v_h \cdot q_{1h}^b - c(q_{1h}^b) - u_{1h}^b + v_l \cdot q_{2l}^h - c(q_{2l}^h))$ by the envelope theorem of $\mathscr{P}_t^P$, with equality at $m_{Pl} \neq 1$. This can be reduced to $v_h \cdot q_{1l}^b - c(q_{1l}^b) - (v_h \cdot q_{1h}^b - c(q_{1h}^b)) + u_{2h}^l - u_{2h}^h + v_l \cdot q_{2l}^l - c(q_{2l}^l) - (v_l \cdot q_{2l}^h - c(q_{2l}^h)) \geq \frac{\sigma \cdot \phi \cdot m_{Ph} + \sigma \cdot (1-\phi) + (1-\sigma) \cdot \phi \cdot (1-m_{Pl}) + \lambda^b}{(1-\sigma) \cdot \phi} \cdot \frac{\partial (u_{2h}^l - u_{2h}^h)}{\partial m_{Pl}}$, where $\frac{\partial (u_{2h}^l - u_{2h}^h)}{\partial m_{Pl}} > 0$. The difference in the seller's payoff between a truth revealing and a lying persistent-low type is higher than or equal to the weighted increment in the gain of second-period information rent from lying when inducing the persistent-low type to reveal his true preference with a higher probability.

At $m_{Pi}^* \leq 1$ and $m_{Ii}^* = 1$ with violation to Condition 2, the optimal contract $\mathbb{C}_1^b$ has $q_{1h}^b > q_h^*$ such that $\rho^* \cdot (v_h - q_{1h}) + \lambda^b \cdot (v_h - v_l) = 0$, where $\rho^* = \sigma \cdot \phi \cdot m_{Ph}^* + (1-\sigma) \cdot \phi \cdot (1 - m_{Pl}^*) + \sigma \cdot (1-\phi)$, and $q_{1l}^b < q_{1l}^a$ such that $(1-\rho^*) \cdot (v_l - q_{1l}) - \rho^* \cdot (v_h - v_l) - \lambda^b \cdot (v_h - v_l) = 0$. The optimal contracts $\mathbb{C}_2^{bi}$ following revelation of v_i have $q_{2h}^{bi} = q_h^*$ and $q_{2l}^{bh} = v_l - \frac{\sigma \cdot \phi \cdot m_{Ph}^*}{\sigma \cdot (1-\phi) + (1-\sigma) \cdot \phi \cdot (1-m_{Pl}^*)} \cdot (v_h - v_l) < q_{2l}^{bl} = v_l - \frac{\sigma \cdot \phi \cdot (1-m_{Ph}^*) + (1-\sigma) \cdot (1-\phi)}{(1-\sigma) \cdot \phi \cdot m_{Pl}^*} \cdot (v_h - v_l) < q_l^*$. The contracts implement intertemporal information rent u_{ki}^b to the consumer of evolution type k and $v_1 = v_i$ such that $u_{Ph}^b = u_{1h}^b + u_{2h}^{bh} = (v_h - v_l) \cdot (q_{1l}^b + q_{2l}^{bl})$, $u_{Pl}^b = u_{1l}^b + u_{2l}^{bl} = 0$, $u_{Ih}^b = u_{1h}^b + u_{2l}^{bh} = (v_h - v_l) \cdot (q_{1l}^b + q_{2l}^{bl} - q_{2l}^{bh})$, and $u_{Il}^b = u_{1l}^b + u_{2h}^{bl} = (v_h - v_l) \cdot q_{2l}^{bl}$.

Scenario 2. $1 - \phi > \phi$**.** If $1 - \phi > \phi$ and (IC_{Il}) is slacking, $m_{Pi} = m_{Il} = 1$ so $\rho = \sigma \cdot \phi + \sigma \cdot (1-\phi) \cdot m_{Ih}$. With binding (IR_{Pl}) and (IC_{Ih}), the optimal contract $\mathbb{C}_1^a$ has $q_{1h}^a = \arg\max_q v_h \cdot q - c(q) = q_h^*$ and $q_{1l}^a = v_l - \frac{\sigma \cdot \phi + \sigma \cdot (1-\phi) \cdot m_{Ih}}{1 - \sigma + \sigma \cdot (1-\phi) \cdot (1-m_{Ih})} \cdot (v_h - v_l) < q_l^*$, implementing $u_{1l}^a = 0$ and $u_{1h}^a = (v_h - v_l) \cdot q_{1l}^a$. With $m_{Ih} < m_{Il}^* = m_{Pi}^* = 1$, $\mu(v_h) = \frac{\phi}{\phi + (1-\phi) \cdot m_{Ih}}$ and $\mu(v_l) = \frac{(1-\sigma) \cdot (1-\phi)}{(1-\sigma) + \sigma \cdot (1-\phi) \cdot (1-m_{Ih})}$. The second period contract following revelation of v_i has $q_{2h}^i = \arg\max_q v_h \cdot q - c(q) = q_h^*$ and $q_{2l}^l = v_l - \frac{(1-\sigma) \cdot (1-\phi)}{(1-\sigma) \cdot \phi + \sigma \cdot (1-\phi) \cdot (1-m_{Ih})} \cdot (v_h - v_l) < q_{2l}^h = v_l - \frac{\phi}{(1-\phi) \cdot m_{Ih}} \cdot (v_h - v_l) < q_l^*$, which implements $u_{2h}^i = (v_h - v_l) \cdot q_{2l}^i$

and $u^i_{2l} = 0$.

The contracts and revelation strategy satisfies (IC_{Il}) if $(v_h - v_l) \cdot q^a_{1h} + u^l_{2h} - u^h_{2h} \geq (v_h - v_l) \cdot q^a_{1l}$, which holds if $1 + \frac{\sigma \cdot \phi + \sigma \cdot (1-\phi) \cdot m_{Ih}}{1-\sigma+\sigma \cdot (1-\phi) \cdot (1-m_{Ih})} \geq \frac{(1-\sigma)\cdot(1-\phi)}{(1-\sigma)\cdot\phi+\sigma\cdot(1-\phi)\cdot(1-m_{Ih})} - \frac{\phi}{(1-\phi)\cdot m_{Ih}}$. The inequality holds for all m_{Ih} if the minimum of the LHS is higher than the maximum of the RHS, i.e. if $\frac{1}{1-\sigma+\sigma\cdot(1-\phi)} \geq \frac{1-\phi}{\phi} - \frac{\phi}{1-\phi}$. The inequality holds at $m_{Ih} = 1$ if $\frac{1}{1-\sigma} \geq \frac{1-\phi}{\phi} - \frac{\phi}{1-\phi}$, labeled as Condition 2-3.

Given the above conditions, the principal implements $m^*_{Ih} \leq 1$ that maximizes her intertemporal payoff, where $v_h \cdot q^*_h - c(q^*_h) - u^a_{1h} + v_l \cdot q^h_{2l} - c(q^h_{2l}) \geq v_l \cdot q^a_{1l} - c(q^a_{1l}) + v_l \cdot q^l_{2l} - c(q^l_{2l})$ by the envelope theorem of $\mathscr{P}_t$, with equality at $m_{Ih} \neq 1$. The seller's payoff when contracting with a truth revealing descending type is higher than or equal to that with a lying descending type. This can be reduced to $v_h \cdot q^*_h - c(q^*_h) - (v_h - v_l) \cdot q^a_{1l} + v_l \cdot q^h_{2l} - c(q^h_{2l}) \geq v_l \cdot q^a_{1l} - c(q^a_{1l}) + v_l \cdot q^l_{2l} - c(q^l_{2l})$, which strictly holds as $v_h \cdot q^*_h - c(q^*_h) + v_l \cdot q^h_{2l} - c(q^h_{2l}) > v_h \cdot q^a_{1l} - c(q^a_{1l}) + v_l \cdot q^l_{2l} - c(q^l_{2l})$ when $1 - \phi > \phi$. It is optimal for the seller to implement $m^*_{Ih} = 1$ given Condition 2-3.

At $m^*_{ki} = 1$ with Condition 2 satisfied, the optimal contract $\mathbb{C}^a_1$ has $q^a_{1h} = q^*_h$ and $q^a_{1l} = v_l - \frac{\sigma}{1-\sigma} \cdot (v_h - v_l) \cdot q < q^*_l$, and the optimal contracts $\mathbb{C}^{ai}_2$ following revelation of v_i have $q^{ai}_{2h} = q^*_h$ and $q^{al}_{2l} = v_l - \frac{1-\phi}{\phi} \cdot (v_h - v_l) \cdot q < q^{ah}_{2l} = v_l - \frac{\phi}{1-\phi} \cdot (v_h - v_l) < q^*_l$. The contracts implement intertemporal information rent u^a_{ki} to the consumer of evolution type k and $v_1 = v_i$ such that $u^a_{Ph} = u^a_{1h} + u^{ah}_{2h} = (v_h - v_l) \cdot (q^a_{1l} + q^{ah}_{2l})$, $u^a_{Pl} = u^a_{1l} + u^{al}_{2l} = 0$, $u^a_{Ih} = u^a_{1h} + u^{ah}_{2l} = (v_h - v_l) \cdot q^a_{1l}$, and $u^a_{Il} = u^a_{1l} + u^{al}_{2h} = (v_h - v_l) \cdot q^{al}_{2l}$.

When Condition 2 is violated and (IC_{Il}), (IC_{Ih}), and (IR_{Pl}) are all binding, with $\rho = \sigma \cdot \phi + \sigma \cdot (1-\phi) \cdot m_{Ih} + (1-\sigma) \cdot (1-\phi) \cdot (1 - m_{Il})$, the optimal contract $\mathbb{C}^b_1$ solves the reduced problem

$$\mathscr{P}_1: \quad \max_{q_1, u_1} \rho \cdot (v_h \cdot q_{1h} - c(q_{1h})) + (1-\rho) \cdot (v_l \cdot q_{1l} - c(q_{1l})) - \rho \cdot ((v_h - v_l) \cdot q_{1l})$$

subject to $(v_h - v_l) \cdot q_{1h} + u^l_{2h} - u^h_{2h} = (v_h - v_l) \cdot q_{1l}$. The optimal contract has $q^b_{1h} > q^*_h$ as a solution to $\rho \cdot (v_h - q_{1h}) + \lambda^b \cdot (v_h - v_l) = 0$, where $\lambda^b > 0$ denotes the equilibrium Lagrange multiplier associated with binding (IC_{Il}). It has $q^b_{1l} < q^a_{1l}$ as a solution to $(1-\rho) \cdot (v_l - q_{1l}) - \rho \cdot (v_h - v_l) - \lambda^b \cdot (v_h - v_l) = 0$. First-period information rents $u^b_{1l} = 0$ and $u^b_{1h} = (v_h - v_l) \cdot q^b_{1l}$ are implemented.

The principal implements $m^*_{Ih} \leq 1$ that maximizes her intertemporal payoff, where $\sigma \cdot (1-\phi) \cdot (v_h \cdot q^b_{1h} - c(q^b_{1h}) - u^b_{1h} + v_l \cdot q^h_{2l} - c(q^h_{2l})) + \lambda^b \cdot \frac{\partial(u^l_{2h} - u^h_{2h})}{\partial m_{Ih}} \geq \sigma \cdot (1-\phi) \cdot (v_l \cdot q^b_{1l} - c(q^b_{1l}) + v_l \cdot q^l_{2l} - c(q^l_{2l}))$ by the envelope theorem of $\mathscr{P}_t$, with equality at $m_{Ih} \neq 1$. This can be reduced to $v_h \cdot q_{1h}^b - c(q_{1h}^b) - (v_h \cdot q_{1l}^b - c(q_{1l}^b)) + v_l \cdot q_{2l}^h - c(q_{2l}^h) - (v_l \cdot q_{2l}^l - c(q_{2l}^l)) \geq -\frac{\lambda^b}{\sigma \cdot (1-\phi)} \cdot \frac{\partial (u_{2h}^l - u_{2h}^h)}{\partial m_{Ih}}$, where $\frac{\partial (u_{2h}^l - u_{2h}^h)}{\partial m_{Ih}} < 0$ as $u_{2h}^l - u_{2h}^h = -\frac{(1-\sigma)\cdot(1-\phi)\cdot m_{Il}}{\sigma\cdot(1-\phi)\cdot(1-m_{Ih})+(1-\sigma)\cdot\phi} \cdot (v_h - v_l)^2$. The difference in the seller's payoff between a truth revealing and a lying descending type is higher than or equal to the weighted increment in the gain of second-period information rent from lying when inducing the descending type to reveal his true preference with a higher probability. The principal implements $m_{Il}^* \leq 1$ that maximizes her intertemporal payoff, where $(1-\sigma)\cdot(1-\phi)\cdot(v_l \cdot q_{1l}^b - c(q_{1l}^b) + v_h \cdot q_h^* - c(q_h^*) - u_{2h}^l) + \lambda^b \cdot \frac{\partial (u_{2h}^l - u_{2h}^h)}{\partial m_{Il}} \geq (1-\sigma)\cdot(1-\phi)\cdot(v_h \cdot q_{1h}^b - c(q_{1h}^b) - u_{1h}^b + v_h \cdot q_h^* - c(q_h^*) - u_{2h}^h)$ by the envelope theorem of $\mathscr{P}_t$, with equality at $m_{Il} \neq 1$. This can be reduced to $v_h \cdot q_{1l}^b - c(q_{1l}^b) - (v_h \cdot q_{1h}^b - c(q_{1h}^b)) - (v_h - v_l)\cdot(q_{2l}^l - q_l^*) \geq -\frac{\lambda^I}{(1-\sigma)\cdot(1-\phi)} \cdot \frac{\partial (u_{2h}^l - u_{2h}^h)}{\partial m_{Il}}$, where $\frac{\partial (u_{2h}^l - u_{2h}^h)}{\partial m_{Il}} < 0$. The difference in the seller's payoff between a truth revealing and a lying ascending type is higher than or equal to the weighted increment in the gain of second-period information rent from lying when inducing the ascending type to reveal his true preference with a higher probability.

At $m_{Ii}^* \leq 1$ and $m_{Pi}^* = 1$ with violation to Condition 2, the contract $\mathbb{C}_1^b$ has $q_{1h}^b > q_h^*$ such that $\rho^* \cdot (v_h - q_{1h}) + \lambda^b \cdot (v_h - v_l) = 0$, where $\rho^* = \sigma \cdot \phi + \sigma \cdot (1-\phi) \cdot m_{Ih}^*$, and $q_{1l}^b < q_{1l}^a$ such that $(1-\rho^*)\cdot(v_l - q_{1l}) - \rho^* \cdot (v_h - v_l) - \lambda^b \cdot (v_h - v_l) = 0$. The optimal contracts $\mathbb{C}_2^{bi}$ following revelation of v_i have $q_{2h}^{bi} = q_h^*$ and $q_{2l}^{bl} = v_l - \frac{(1-\sigma)\cdot(1-\phi)}{\sigma\cdot(1-\phi)\cdot(1-m_{Ih}^*)+(1-\sigma)\cdot\phi} \cdot (v_h - v_l) < q_{2l}^{bh} = v_l - \frac{\phi}{(1-\phi)\cdot m_{Ih}^*} \cdot (v_h - v_l) < q_l^*$. The contracts implement intertemporal information rent u_{ki}^b to the consumer of evolution type k and $v_1 = v_i$ such that $u_{Ph}^b = u_{1h}^b + u_{2h}^{bh} = (v_h - v_l)\cdot(q_{1l}^b + q_{2l}^{bh})$, $u_{Pl}^b = u_{1l}^b + u_{2l}^{bl} = 0$, $u_{Ih}^b = u_{1h}^b + u_{2l}^{bh} = (v_h - v_l)\cdot q_{1l}^b$, and $u_{Il}^b = u_{1l}^b + u_{2h}^{bl} = (v_h - v_l)\cdot q_{2l}^{bl}$.

A.3 Proof of Proposition 4 in Section 5

The consumer truthfully reveals his evolution type with probability r_{ki} and truthfully reveals his first-period preference type with probability m_{ki}. Denote $\omega(k, v_i)$ as the seller's Bayesian updated second-period belief of a high valuation type following revelation (k, v_i) in the first period. By backward induction and the standard methods in static contract theory, the second-period contract $\mathbb{C}_2^{ki}$ following revelation (k, v_i) has $q_{2h}^{ki} = \arg\max_q v_h \cdot q - c(q) = q_h^*$ and $q_{2l}^{ki} = \arg\max_q (1-\omega(k, v_i))\cdot(v_l \cdot q - c(q)) - \omega(k, v_i)\cdot(v_h - v_l)\cdot q \leq q_l^*$, which implements second-period rent $u_{2h}^{ki} = (v_h - v_l)\cdot q_{2l}^{ki}$ and $u_{2l}^{ki} = 0$. The first-period contracts $\{\mathbb{C}_1^P, \mathbb{C}_1^I\}$ to the consumer has consumption q_{1i}^k and implements first-period rent $u_{1i}^k = v_i \cdot q_{1i}^k - p_{1i}^k$. Anticipating $\mathbb{C}_2^{ki}$, the first-period menu of contracts $\{\mathbb{C}_1^P, \mathbb{C}_1^I\}$ is incentive compatible to induce $r_{ki} > 0$ and $m_{ki} > 0$ if

$$u_{1h}^P + u_{2h}^{Ph} \geq \begin{cases} u_{1l}^P + (v_h - v_l) \cdot q_{1l}^P + u_{2h}^{Pl} & (Ph-1) \\ u_{1h}^I + u_{2h}^{Ih} & (Ph-2) \\ u_{1l}^I + (v_h - v_l) \cdot q_{1l}^I + u_{2h}^{Il} & (Ph-3) \end{cases}$$

$$u_{1l}^P \geq \begin{cases} u_{1h}^P - (v_h - v_l) \cdot q_{1h}^P & (Pl-1) \\ u_{1l}^I & (Pl-2) \\ u_{1h}^I - (v_h - v_l) \cdot q_{1h}^I & (Pl-3) \end{cases}$$

$$u_{1h}^I \geq \begin{cases} u_{1l}^I + (v_h - v_l) \cdot q_{1l}^I & (Ih-1) \\ u_{1h}^P & (Ih-2) \\ u_{1l}^P + (v_h - v_l) \cdot q_{1l}^P & (Ih-3) \end{cases}$$

$$u_{1l}^I + u_{2h}^{Il} \geq \begin{cases} u_{1h}^I - (v_h - v_l) \cdot q_{1h}^I + u_{2h}^{Ih} & (Il-1) \\ u_{1l}^P + u_{2h}^{Pl} & (Il-2) \\ u_{1h}^P - (v_h - v_l) \cdot q_{1h}^P + u_{2h}^{Ph} & (Il-3) \end{cases}$$

The first line indicates incentive compatibility to truthfully reveal the preference type with a positive probability, conditional on truthful revelation of the evolution type. The second line indicates incentive compatibility to truthfully reveal the evolution type with a positive probability, conditional on truthful revelation of the preference type. The third line indicates incentive compatibility to truthfully reveal both dimensions of types with a positive probability.

Lemma A. *If the seller implements truthful revelation of persistency $r_{Pi} = 1$, the persistent consumer is induced to truthfully reveal his preference and the impersistent consumer is induced to take a mixed revelation of preference, $m_{Pi} = 1$ and $m_{Ii} \neq 1$ for both $i = h, l$ with binding $(Ih-1)$ and $(Il-1)$. If the seller implements truthful revelation of impersistency $r_{Ii} = 1$, the impersistent consumer is induced to truthfully reveal his preference and the persistent consumer is induced to take a mixed revelation of preference, $m_{Ii} = 1$ and $m_{Pi} \neq 1$ for both $i = h, l$ with binding $(Ph-1)$ and $(Pl-1)$.*

Proof. Suppose that $r_{Ph} = 1$ is implemented. The seller believes in the second period that whoever reveals to be impersistent is certainly not a persistent-high type. If $m_{Il} = 1$, following the revelation of (I, v_h) in the first period, the seller believes that with probability one the consumer has $v_2 = v_l$, so $u_{2h}^{Ih} = (v_h - v_l) \cdot q_l^* > u_{2h}^{ki}$ for all $(k, i) \neq (I, h)$ had this be a lying ascending type. By $(Ph-2)$, $u_{1h}^P - u_{1h}^I \geq u_{2h}^{Ih} - u_{2h}^{Ph}$, and by $(Ih-2)$, $u_{1h}^P - u_{1h}^I \leq 0$. These two contradict each other with $u_{2h}^{Ih} > u_{2h}^{Ph}$. The first-period contract is incentive compatible only if $m_{Il} \neq 1$, which is implementable with binding $(Il-1)$. Suppose that $r_{Pl}=1$ is implemented. The seller believes in the second period that whoever reveals to be impersistent is certainly not a persistent-low type. If $m_{Ih}=1$, following the revelation of (I,v_l) in the first period, the seller believes that with probability one the consumer has $v_2=v_h$, so $u_{2h}^{Il}=0<u_{2h}^{ki}$ for all $(k,i)\neq(I,l)$. By $(Pl-2)$, $u_{1l}^{P}-u_{1l}^{I}\geq 0$, and by $(Il-2)$, $u_{1l}^{P}-u_{1l}^{I}\leq u_{2h}^{Il}-u_{2h}^{Pl}$. These two contradict each other with $u_{2h}^{Il}<u_{2h}^{Pl}$. The first-period contract is incentive compatible only if $m_{Ih}\neq 1$, which is implementable with binding $(Ih-1)$.

If $r_{Pi}=1$ for both preference types is implemented, binding $(Ih-1)$ and $(Il-1)$ imply $u_{1h}^{I}-u_{1l}^{I}=(v_h-v_l)\cdot q_{1l}^{I}=(v_h-v_l)\cdot q_{1h}^{I}+u_{2h}^{Il}-u_{2h}^{Ih}$. Summation of these with $(Ph-1)$ and $(Pl-1)$ finds

$$\begin{aligned}(v_h-v_l)\cdot q_{1h}^{P}+(v_h-v_l)\cdot q_{1h}^{I}+u_{2h}^{Il}-u_{2h}^{Ih}&\\ \geq u_{1h}^{P}-u_{1l}^{P}+u_{1h}^{I}-u_{1l}^{I}&\\ \geq (v_h-v_l)\cdot q_{1l}^{P}+(v_h-v_l)\cdot q_{1l}^{I}+u_{2h}^{Pl}-u_{2h}^{Ph}&.\end{aligned}$$

Combination of $(ki-3)$ finds

$$\begin{aligned}(v_h-v_l)\cdot q_{1h}^{I}+(v_h-v_l)\cdot q_{1h}^{P}+u_{2h}^{Il}-u_{2h}^{Ph}&\\ \geq u_{1h}^{P}-u_{1l}^{I}+u_{1h}^{I}-u_{1l}^{P}&\\ \geq (v_h-v_l)\cdot q_{1l}^{P}+(v_h-v_l)\cdot q_{1l}^{I}+u_{2h}^{Il}-u_{2h}^{Ph}&.\end{aligned}$$

Constraints $(Ih-2)$ and $(Ph-2)$ both hold only if $u_{2h}^{Ph}\geq u_{2h}^{Ih}$, so the combination of $(Pl-3)$ and $(Il-3)$ implies slacking $(Pl-1)$. Constraints $(Pl-2)$ and $(Il-2)$ both hold only if $u_{2h}^{Il}\geq u_{2h}^{Pl}$, so the combination of $(Ph-3)$ and $(Ih-3)$ implies slacking $(Ph-1)$. The persistent consumer truthfully reveals his preference with certainty if truthful revelation of persistency is induced.

Suppose that $r_{Ih}=1$ is implemented. The seller believes in the second period that whoever reveals to be persistent is certainly not a descending type. If $m_{Pl}=1$, following the revelation of (P,v_h) in the first period, the seller believes that with probability one the consumer has $v_2=v_h$, so $u_{2h}^{Ph}=0<u_{2h}^{ki}$ for all $(k,i)\neq(P,h)$. By $(Ph-2)$, $u_{1h}^{P}-u_{1h}^{I}\geq u_{2h}^{Ih}-u_{2h}^{Ph}$, and by $(Ih-2)$, $u_{1h}^{P}-u_{1h}^{I}\leq 0$. These two contradict each other with $u_{2h}^{Ph}<u_{2h}^{Ih}$. The first-period contract is incentive compatible only if $m_{Pl}\neq 1$, which is implementable with binding $(Pl-1)$. Suppose that $r_{Il}=1$ is implemented. The seller believes in the second period that whoever reveals to be persistent is certainly not an ascending type. If $m_{Ph}=1$, following the revelation of

(P, v_l) in the first period, the seller believes that with probability one the consumer has $v_2 = v_l$, so $u_{2h}^{Pl} = (v_h - v_l) \cdot q_l^* > u_{2h}^{ki}$ for all $(k, i) \neq (P, l)$ had this be a lying persistent-high type. By $(Pl - 2)$, $u_{1l}^P - u_{1l}^I \geq 0$, and by $(Il - 2)$, $u_{1l}^P - u_{1l}^I \leq u_{2h}^{Il} - u_{2h}^{Pl}$. These two contradict each other with $u_{2h}^{Pl} > u_{2h}^{Il}$. The first-period contract is incentive compatible only if $m_{Ph} \neq 1$, which is implementable with binding $(Ph - 1)$.

If $r_{Ii} = 1$ for both preference types is implemented, binding $(Ph - 1)$ and $(Pl - 1)$ imply $u_{1h}^P - u_{1l}^P = (v_h - v_l) \cdot q_{1l}^P + u_{2h}^{Pl} - u_{2h}^{Ph} = (v_h - v_l) \cdot q_{1h}^P$. Summation of these with $(Ih - 1)$ and $(Il - 1)$ finds

$$
\begin{aligned}
&(v_h - v_l) \cdot q_{1h}^P + (v_h - v_l) \cdot q_{1h}^I + u_{2h}^{Il} - u_{2h}^{Ih} \\
&\qquad \geq u_{1h}^I - u_{1l}^I + u_{1h}^P - u_{1l}^P \\
&\qquad\qquad \geq (v_h - v_l) \cdot q_{1l}^I + (v_h - v_l) \cdot q_{1l}^P + u_{2h}^{Pl} - u_{2h}^{Ph}.
\end{aligned}
$$

Combination of $(ki - 3)$ finds

$$
\begin{aligned}
&(v_h - v_l) \cdot q_{1h}^I + (v_h - v_l) \cdot q_{1h}^P + u_{2h}^{Il} - u_{2h}^{Ph} \\
&\qquad \geq u_{1h}^P - u_{1l}^I + u_{1h}^I - u_{1l}^P \\
&\qquad\qquad \geq (v_h - v_l) \cdot q_{1l}^P + (v_h - v_l) \cdot q_{1l}^I + u_{2h}^{Il} - u_{2h}^{Ph}.
\end{aligned}
$$

Constraints $(Ih - 2)$ and $(Ph - 2)$ both hold only if $u_{2h}^{Ph} \geq u_{2h}^{Ih}$, so the combination of $(Pl - 3)$ and $(Il - 3)$ implies slacking $(Il - 1)$. Constraints $(Pl - 2)$ and $(Il - 2)$ both hold only if $u_{2h}^{Il} \geq u_{2h}^{Pl}$, so the combination of $(Ph - 3)$ and $(Ih - 3)$ implies slacking $(Ih - 1)$. The impersistent consumer truthfully reveals his preference with certainty if truthful revelation of impersistency is induced. □

Truthful Revelation of (Im)persistency.

If the seller implements revelation strategy $r_{ki} = 1$ for all (k, i) so that her second-period Bayesian updated belief is completely degenerate on the evolution type. By Lemma A, the consumer takes a completely mixed revelation of preference $m_{ki} \neq 1$ with binding $(ki - 1)$ for all (k, i). Summation of the binding $(ki - 1)$ finds

$$
\begin{aligned}
&(v_h - v_l) \cdot q_{1h}^P + (v_h - v_l) \cdot q_{1h}^I + u_{2h}^{Il} - u_{2h}^{Ih} \\
&\qquad = u_{1h}^P - u_{1l}^P + u_{1h}^I - u_{1l}^I \\
&\qquad\qquad = (v_h - v_l) \cdot q_{1l}^I + (v_h - v_l) \cdot q_{1l}^P + u_{2h}^{Pl} - u_{2h}^{Ph}
\end{aligned}
$$

Combination of $(ki-3)$ finds

$$
\begin{aligned}
(v_h - v_l)\cdot q_{1h}^I + (v_h - v_l)\cdot q_{1h}^P + u_{2h}^{Il} - u_{2h}^{Ph} & \\
\geq u_{1h}^P - u_{1l}^I + u_{1h}^I - u_{1l}^P & \\
\geq (v_h - v_l)\cdot q_{1l}^P + (v_h - v_l)\cdot q_{1l}^I + u_{2h}^{Il} - u_{2h}^{Ph}, &
\end{aligned}
$$

which is satisfied only if $u_{2h}^{Ph} \leq u_{2h}^{Ih}$ and $u_{2h}^{Il} \leq u_{2h}^{Pl}$. By $(Ph-2)$ and $(Ih-2)$, $0 \geq u_{1h}^P - u_{1h}^I \geq u_{2h}^{Ih} - u_{2h}^{Ph}$ is satisfied only if $u_{2h}^{Ph} \geq u_{2h}^{Ih}$, and by $(Pl-2)$ and $(Il-2)$, $u_{2h}^{Il} - u_{2h}^{Pl} \geq u_{1l}^P - u_{1l}^I \geq 0$ is satisfied only if $u_{2h}^{Il} \geq u_{2h}^{Pl}$. All of $(ki-2)$ and $(ki-3)$ hold only if $u_{2h}^{Ph} = u_{2h}^{Ih}$ and $u_{2h}^{Il} = u_{2h}^{Pl}$, which along with $(ki-2)$ implies $u_{1h}^P = u_{1h}^I$ and $u_{1l}^P = u_{1l}^I$.

Denote $u_{1i}^P = u_{1i}^I = u'_{1i}$ and $u_{2j}^{Pi} = u_{2j}^{Ii} = u'^{i}_{2j}$. By backward induction and the standard methods in static contract theory $q_{2h}^{Pi} = q_{2h}^{Ii} = q_h^*$ and $u_{2h}^{ki} = (v_h - v_l)\cdot q_{2l}^{ki}$, so $u_{2j}^{Pi} = u_{2j}^{Ii}$ is implemented by $q_{2l}^{Pi} = q_{2l}^{Ii}$. Combining $(Ph-1)$ and $(Ph-3)$, $(Pl-1)$ and $(Pl-3)$, $(Ih-1)$ and $(Ih-3)$, and $(Il-1)$ and $(Il-3)$ respectively reads the following incentive compatibility constraints,

$$
\begin{aligned}
u'_{1h} - u'_{1l} &\geq (v_h - v_l)\cdot \max\{q_{1l}^P, q_{1l}^I\} + u'^{l}_{2h} - u'^{h}_{2h} && (1)\\
u'_{1h} - u'_{1l} &\leq (v_h - v_l)\cdot \min\{q_{1h}^P, q_{1h}^I\} && (2)\\
u'_{1h} - u'_{1l} &\geq (v_h - v_l)\cdot \max\{q_{1l}^P, q_{1l}^I\} && (3)\\
u'_{1h} - u'_{1l} &\leq (v_h - v_l)\cdot \min\{q_{1h}^P, q_{1h}^I\} + u'^{l}_{2h} - u'^{h}_{2h} && (4)
\end{aligned}
$$

If $u'^{l}_{2h} - u'^{h}_{2h} > 0$, (1) and (2) both hold if and only if $(v_h - v_l)\cdot \min\{q_{1h}^P, q_{1h}^I\} \geq (v_h - v_l)\cdot \max\{q_{1l}^P, q_{1l}^I\} + u'^{l}_{2h} - u'^{h}_{2h}$. The arguments in Section A.2 qualitatively applies here that at least (1) is binding. The optimal contract has $\min\{q_{1h}^P, q_{1h}^I\} \geq q_h^*$ and $\max\{q_{1h}^P, q_{1h}^I\} = q_h^*$ if $q_{1h}^P \neq q_{1h}^I$, which hold only if $q_{1h}^P = q_{1h}^I$. It has $\max\{q_{1l}^P, q_{1l}^I\} < q_l^*$ and $\min\{q_{1l}^P, q_{1l}^I\} = q_l^*$ if $q_{1l}^P \neq q_{1l}^I$, which contradict each other unless $q_{1l}^P = q_{1l}^I$. If $u'^{l}_{2h} - u'^{h}_{2h} < 0$, (3) and (4) both hold if and only if $(v_h - v_l)\cdot \min\{q_{1h}^P, q_{1h}^I\} + u'^{l}_{2h} - u'^{h}_{2h} \geq (v_h - v_l)\cdot \max\{q_{1l}^P, q_{1l}^I\}$. The arguments in Section A.2 qualitatively applies here that at least (3) is binding. The optimal contract has $\min\{q_{1h}^P, q_{1h}^I\} \geq q_h^*$ and $\max\{q_{1h}^P, q_{1h}^I\} = q_h^*$ if $q_{1h}^P \neq q_{1h}^I$, which hold only if $q_{1h}^P = q_{1h}^I$. It has $\max\{q_{1l}^P, q_{1l}^I\} < q_l^*$ and $\min\{q_{1l}^P, q_{1l}^I\} = q_l^*$ if $q_{1l}^P \neq q_{1l}^I$, which contradict each other unless $q_{1l}^P = q_{1l}^I$. The optimal contracts must have $q_{1i}^P = q_{1i}^I$. With pooled per-period information rent, the consumption is pooled on the evolution type as well. The optimal contracts have $\mathbb{C}_1^P = \mathbb{C}_1^I$ and $\mathbb{C}_2^{Pi} = \mathbb{C}_2^{Ii}$.

□