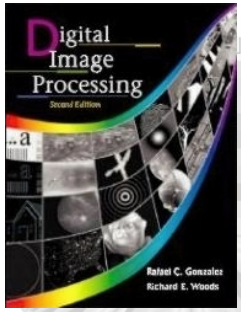

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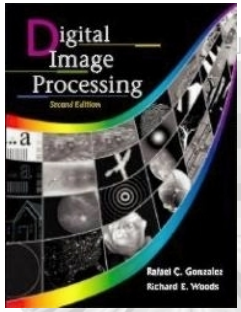
Image Compression

eg. Fax machine

Run Length Coding

data pair：（run length , grey level）



512

	0	0	0	·····	·····	·····	·····	·····	0	0	(512,0)
	0	0	0	·····	·····	·····	·····	·····	0	0	(512,0)
	0	0	100	100	100	·····	·····	·····	0	0	
(2,	0	0	100	100	100	·····	·····	·····	0	0	
(2,	0	0	0	·····	·····	·····	·····	·····	0	0	
											(512,0)

512

Before： 512*512*8

After：(512+4)*(9+8)～516*8*2

Def：Compression technique

information processing with less data represention

Information lossless ⇨ Input = Output

Information lossy ⇨ Input ≠ Output

8.1 if n_1 and n_2 denote the number of info in two sets (image)

Relation data redundancy

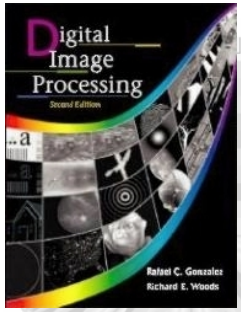
$$R_D = 1 - \frac{1}{C_R} \quad (0<R_D<1) \qquad C_R = \frac{n_1}{n_2}$$

compression

$n_2 = n_1, C_R = 1, R_D = 0 \rightarrow$ rate

if $n_2 << n_1, C_R \rightarrow \infty, R_D = 1 \rightarrow$ no redundancy

high redundancy



Redundancy

1.Coding Redundancy

data $\rightarrow$ histogram

$$P_r(r_k) = \frac{n_k}{n}$$

grey level r_k occurs with probability $P_r(r_k)$

$l(r_k)$ ：the number of bits used to represent r_k

the average number of bits required to represent each pixel is

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k) P_r(r_k)$$

Concept：

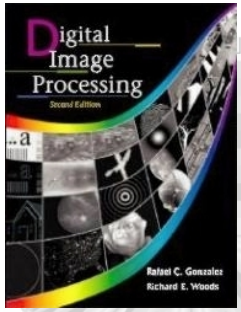
1.variable length coding

2.assign lower bits to the more probability grey level

The total bits of the conpressed M*N image is

$$M \times N \times L_{avg}$$

eg. P.413 (TABLE 8.1)



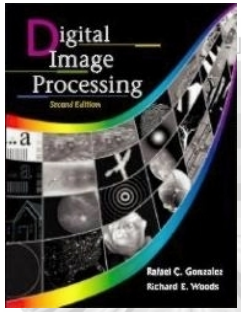
2.Interpixel Redundancy

3.Psychovisual Redundancy

Quantization

Re-Scale

IGS Algorithms：is used to solve false contour problem.



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r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_0 = 0$	0.19	000	3	11	2
$r_1 = 1/7$	0.25	001	3	01	2
$r_2 = 2/7$	0.21	010	3	10	2
$r_3 = 3/7$	0.16	011	3	001	3
$r_4 = 4/7$	0.08	100	3	0001	4
$r_5 = 5/7$	0.06	101	3	00001	5
$r_6 = 6/7$	0.03	110	3	000001	6
$r_7 = 1$	0.02	111	3	000000	6

TABLE 8.1
Example of variable-length coding.

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Image Compression

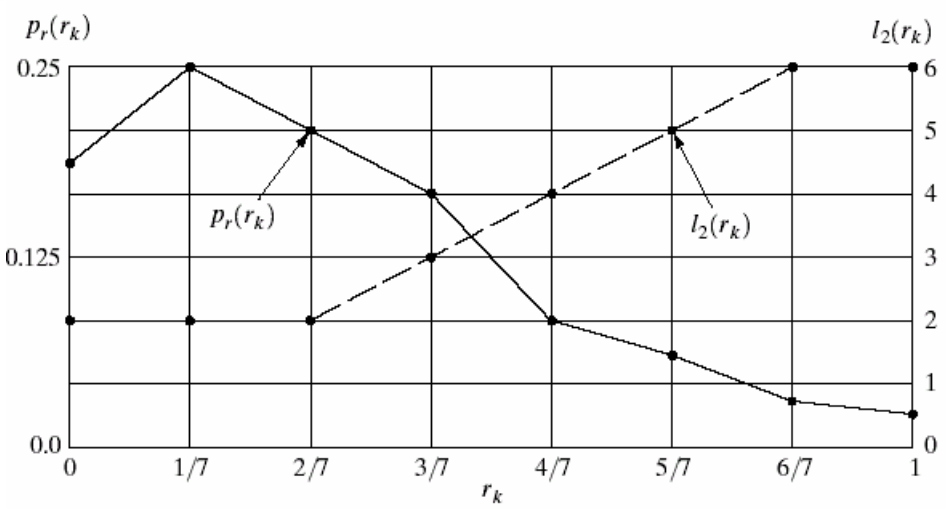
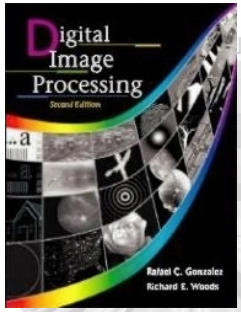


FIGURE 8.1 Graphic representation of the fundamental basis of data compression through variable-length coding.



2.Interpixel Redundancy

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Quantization

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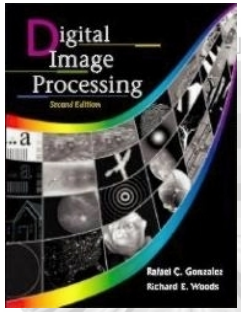

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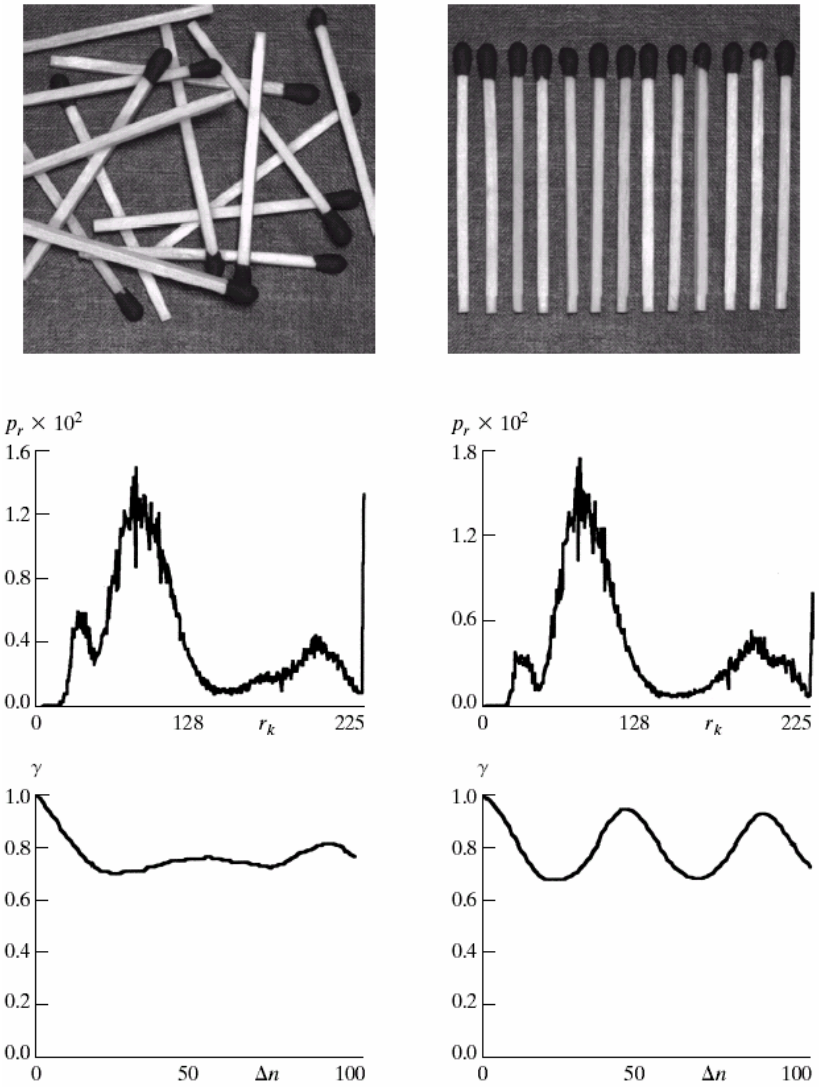
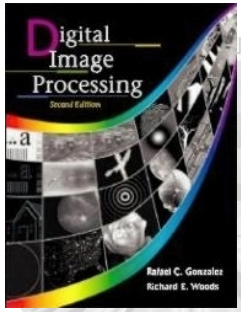


a b
c d
e f

FIGURE 8.2 Two images and their gray-level histograms and normalized autocorrelation coefficients along one line.



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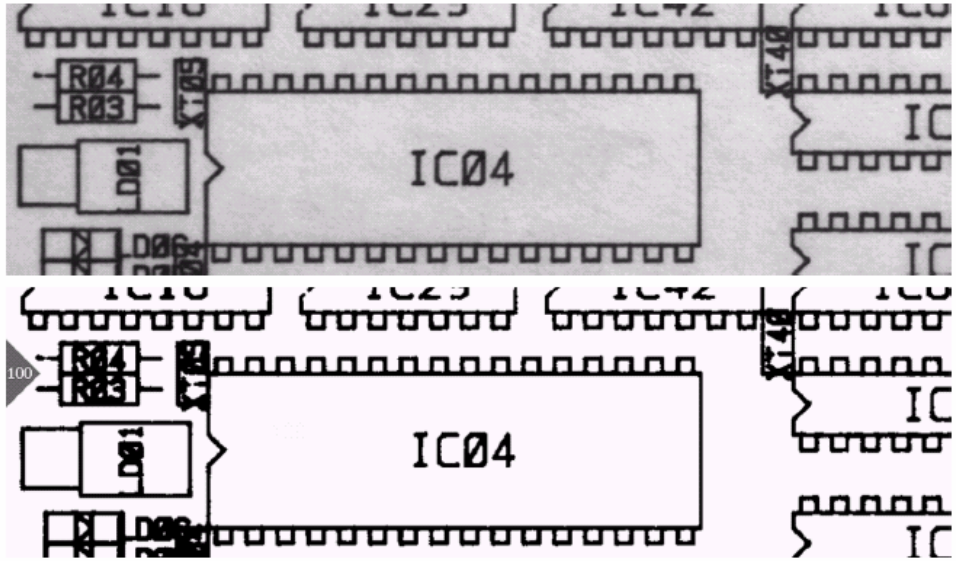
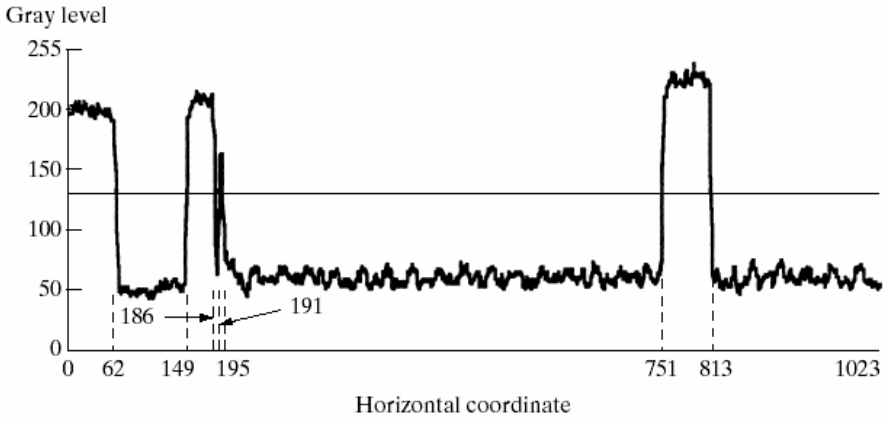
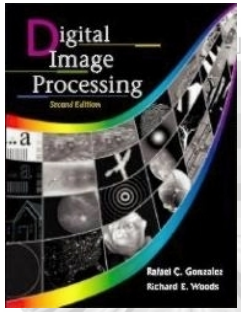




Line 100: (1, 63) (0, 87) (1, 37) (0, 5) (1, 4) (0, 556) (1, 62) (0, 210)

a
b
c
d

FIGURE 8.3
Illustration of run-length coding: (a) original image. (b) Binary image with line 100 marked. (c) Line profile and binarization threshold. (d) Run-length code.



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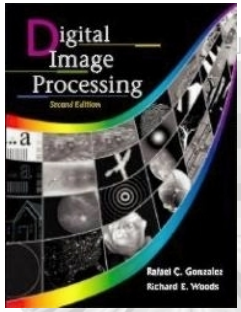
a b c

FIGURE 8.4
(a) Original image. (b) Uniform quantization to 16 levels. (c) IGS quantization to 16 levels.



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Pixel	Gray Level	Sum	IGS Code
$i-1$	N/A	0000 0000	N/A
i	0110 1100	0110 1100	0110
$i+1$	1000 1011	1001 0111	1001
$i+2$	1000 0111	1000 1110	1000
$i+3$	1111 0100	1111 0100	1111

TABLE 8.2
IGS quantization procedure.

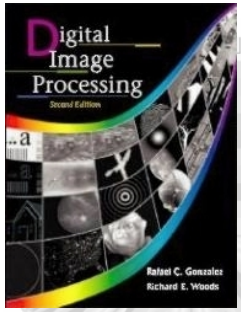

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TABLE 8.3
Rating scale of the Television Allocations Study Organization. (Frendendall and Behrend.)

Value	Rating	Description
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.

Fidelity Criteria

$\hat{f}$

rmse (root – mean – square error)

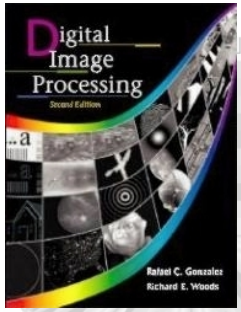
$$e(x,y) = \hat{f}(x,y) - f(x,y)$$

$$e_{rmse} = \left[\frac{1}{M} \times \frac{1}{N} \sum_{x=1}^{M} \sum_{y=1}^{N} e^2(x,y) \right]^{\frac{1}{2}}$$

$$SNR_{ms} = \frac{\sum_{x=1}^{M} \sum_{y=1}^{N} \hat{f}^2(x,y)}{\sum_{x=1}^{M} \sum_{y=1}^{N} \left(\hat{f}(x,y) - f(x,y) \right)^2}$$

8.4 Error-Free Compression (lossless)

- 一. Variable length Coding
  - 1. Huffman Coding
    - 1. 求Histogram
    - 2. 從大到小排序
    - 3. 最小的兩機率相加往上 推(再排序)
    - 4. 按原徑循序assign回去
  - 2. Arithmetic Coding



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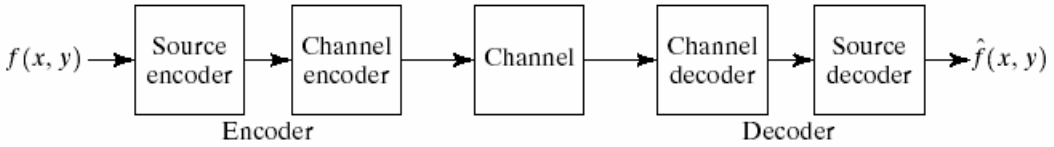
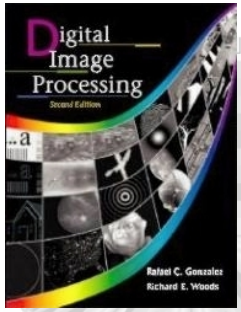


FIGURE 8.5 A general compression system model.



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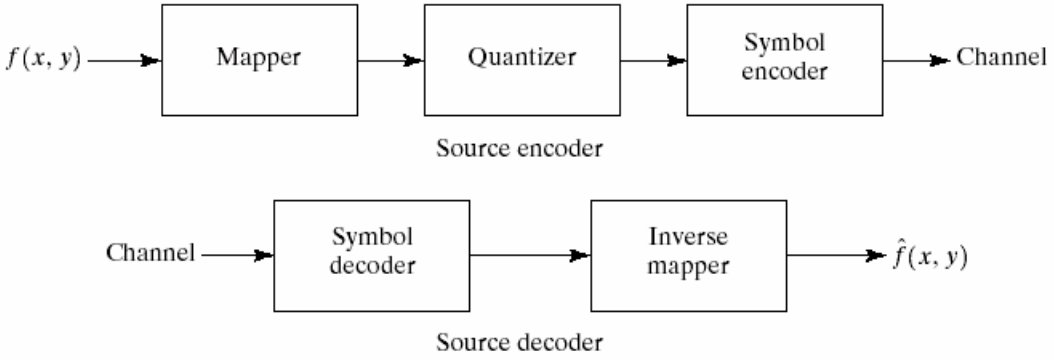
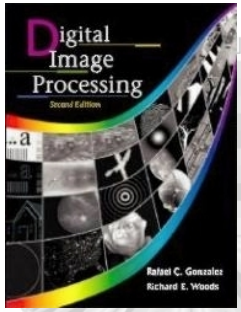


a
b

FIGURE 8.6 (a) Source encoder and (b) source decoder model.



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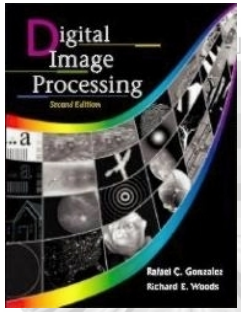
Ensemble $(A, \mathbf{z})$
$A = \{a_j\}$
$\mathbf{z} = [P(a_1), P(a_2), \ldots, P(a_J)]^T$

$Q = [q_{kj}]$

Ensemble $(B, \mathbf{v})$
$B = \{b_k\}$
$\mathbf{v} = [P(b_1), P(b_2), \ldots, P(b_K)]^T$

FIGURE 8.7 A simple information system.



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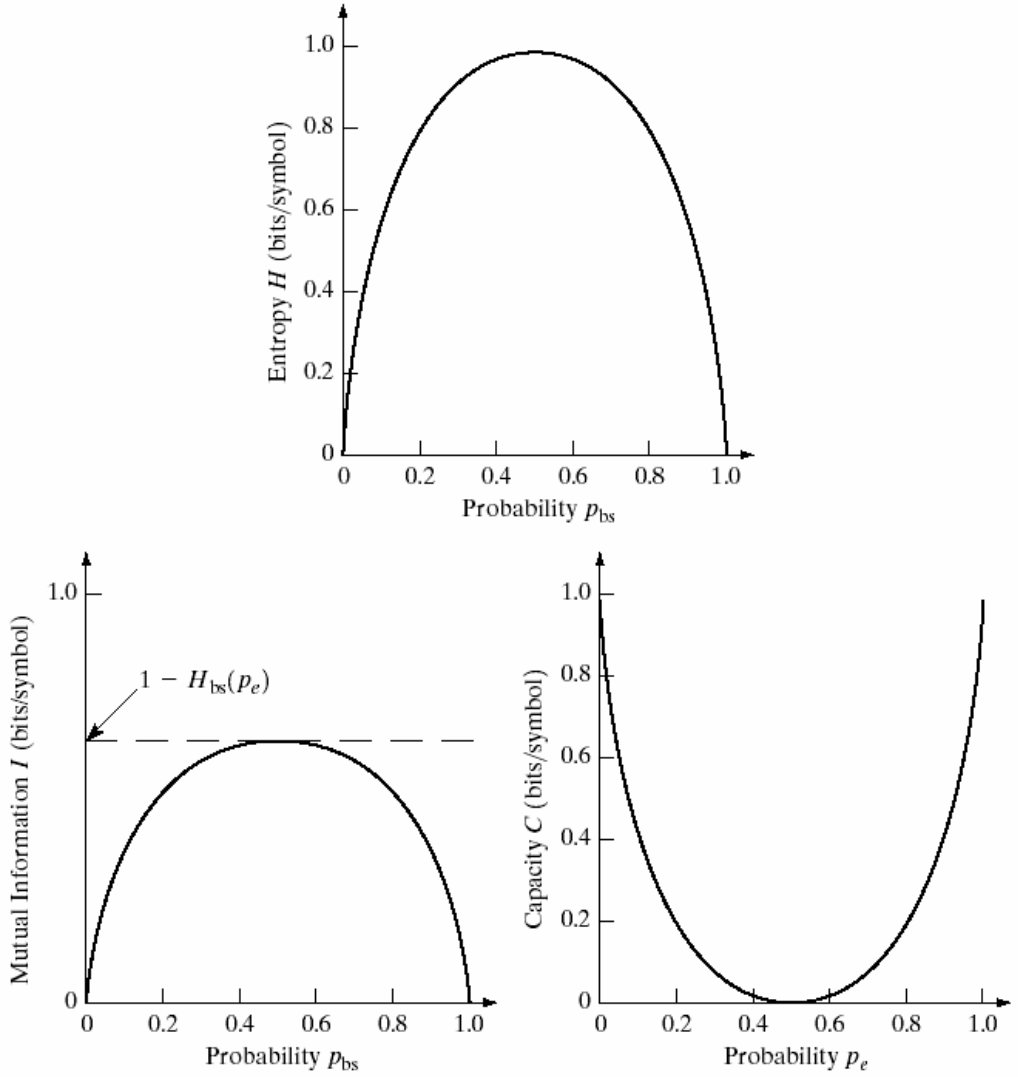
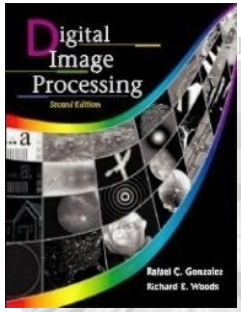


a
b c

FIGURE 8.8 Three binary information functions: (a) the binary entropy function; (b) the mutual information of a binary symmetric channel (BSC); (c) the capacity of the BSC.



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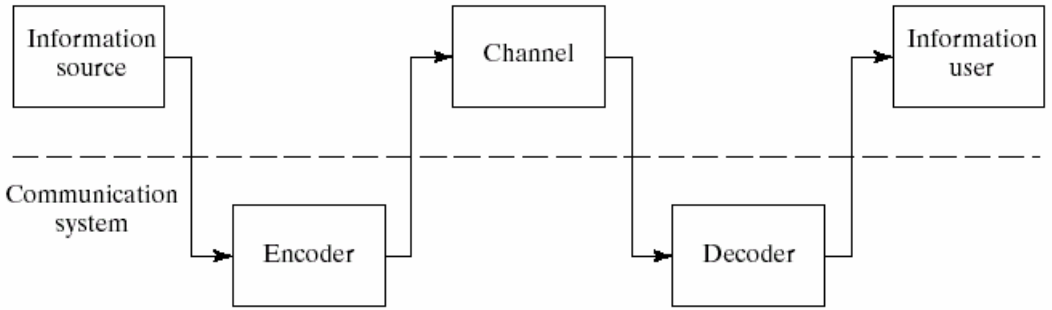

FIGURE 8.9 A communication system model.

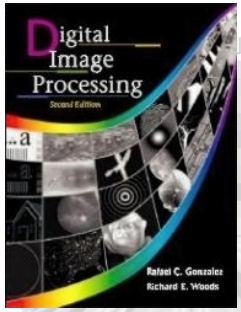

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TABLE 8.4
Extension coding example.

α_i	Source Symbols	$P(\alpha_i)$ Eq. (8.3-14)	$I(\alpha_i)$ Eq. (8.3-1)	$l(\alpha_i)$ Eq. (8.3-16)	Code Word	Code Length
First Extension						
α_1	a_1	2/3	0.59	1	0	1
α_2	a_2	1/3	1.58	2	1	1
Second Extension						
α_1	a_1a_1	4/9	1.17	2	0	1
α_2	a_1a_2	2/9	2.17	3	10	2
α_3	a_2a_1	2/9	2.17	3	110	3
α_4	a_2a_2	1/9	3.17	4	111	3

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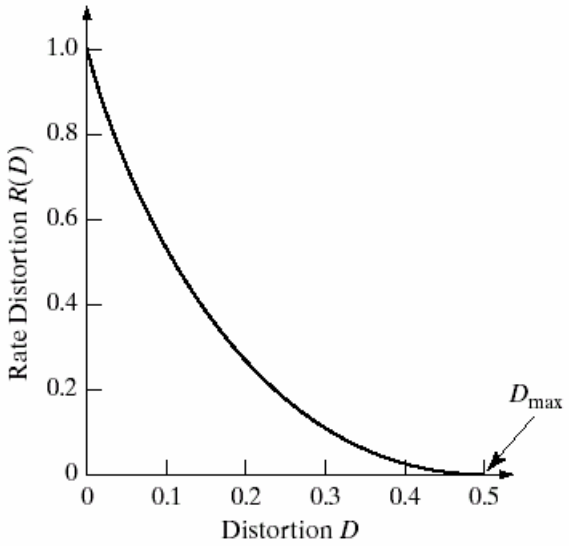
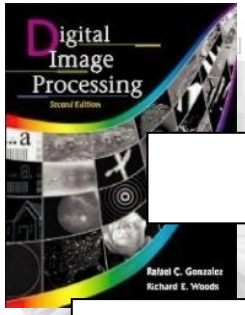
FIGURE 8.10 The rate distortion function for a binary symmetric source.





Huffman coding

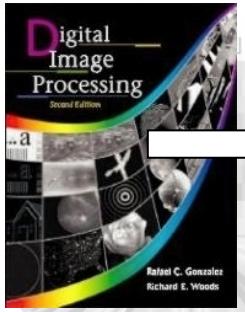
1)Get Symbol and Probability from original source

Symbol	a1	a2	a3	a4	a5	a6
Probability	0.1	0.4	0.06	0.1	0.04	0.3

2)Sort Symbol by Probability then compound the bottom two probability (write in Source reduction) until the

reduced source with two symbols

	Original source	Source reduction			
Symbol	Probability	1	2	3	4
a2	0.4	0.4	0.4	0.4	0.6
a6	0.3	0.3	0.3	0.3	0.4
a1	0.1	0.1	0.2	0.3	
a4	0.1	0.1	0.1		
a3	0.06	0.1			
a5	0.04				



3) Coding each reduced sources are back to the original source.

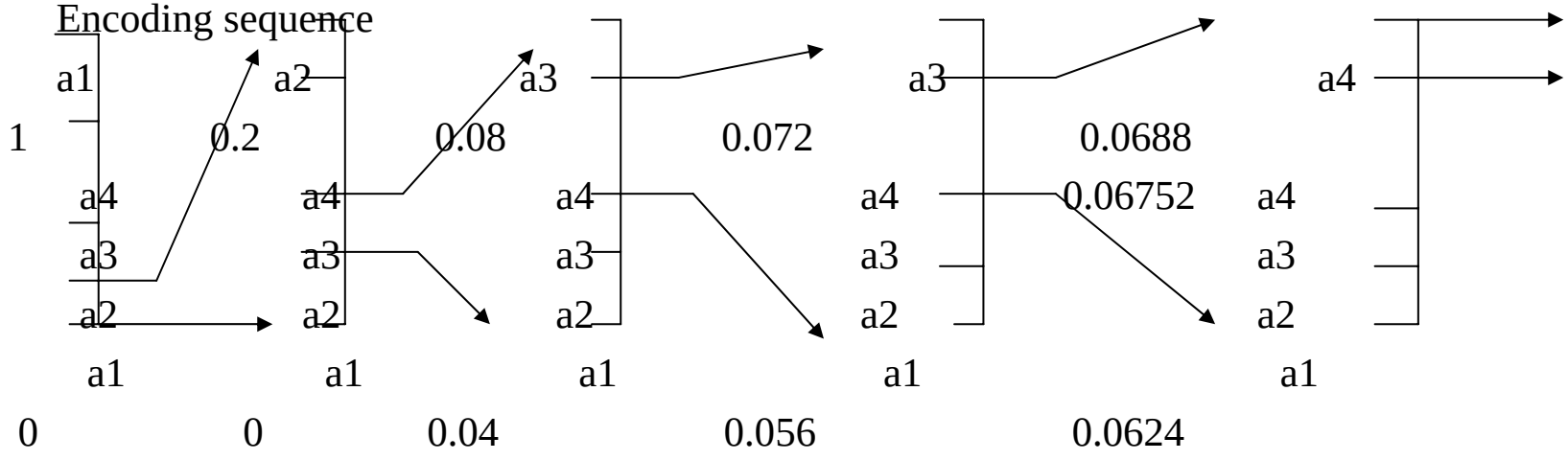
The code is using binary (0 1)

Arithmetic coding

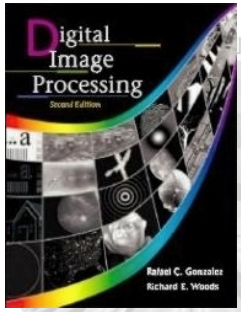
1)Get the Probability and Initial Subinterval of the Source Symbol

Source Symbol	Probability	Initial Subinterval
a1	0.2	[0.0,0.2)
a2	0.2	[0.2,0.4)
a3	0.4	[0.4,0.8)
a4	0.2	[0.8,1.0)

2)Use the Initial Subinterval of a_n to get the next encoding range



3)Any number within the range [0.06752,0.0688) can be used



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Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6
a_6	0.3	0.3	0.3	0.3	0.4
a_1	0.1	0.1	0.2	0.3	
a_4	0.1	0.1	0.1		
a_3	0.06	0.1			
a_5	0.04				

FIGURE 8.11 Huffman source reductions.

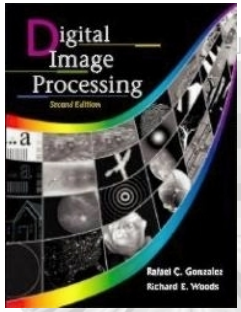

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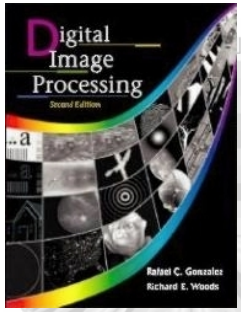
FIGURE 8.12 Huffman code assignment procedure.

Original source			Source reduction							
Sym.	Prob.	Code	1		2		3		4	
a_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_6	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
a_1	0.1	011	0.1	011	0.2	010	0.3	01		
a_4	0.1	0100	0.1	0100	0.1	011				
a_3	0.06	01010	0.1	0101						
a_5	0.04	01011								



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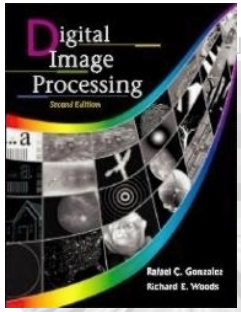
Source symbol	Probability	Binary Code	Huffman	Truncated Huffman	B_2-Code	Binary Shift	Huffman Shift
Block 1							
a_1	0.2	00000	10	11	C00	000	10
a_2	0.1	00001	110	011	C01	001	11
a_3	0.1	00010	111	0000	C10	010	110
a_4	0.06	00011	0101	0101	C11	011	100
a_5	0.05	00100	00000	00010	C00C00	100	101
a_6	0.05	00101	00001	00011	C00C01	101	1110
a_7	0.05	00110	00010	00100	C00C10	110	1111
Block 2							
a_8	0.04	00111	00011	00101	C00C11	111 000	00 10
a_9	0.04	01000	00110	00110	C01C00	111 001	00 11
a_{10}	0.04	01001	00111	00111	C01C01	111 010	00 110
a_{11}	0.04	01010	00100	01000	C01C10	111 011	00 100
a_{12}	0.03	01011	01001	01001	C01C11	111 100	00 101
a_{13}	0.03	01100	01110	100000	C10C00	111 101	00 1110
a_{14}	0.03	01101	01111	100001	C10C01	111 110	00 1111
Block 3							
a_{15}	0.03	01110	01100	100010	C10C10	111 111 000	00 00 10
a_{16}	0.02	01111	010000	100011	C10C11	111 111 001	00 00 11
a_{17}	0.02	10000	010001	100100	C11C00	111 111 010	00 00 110
a_{18}	0.02	10001	001010	100101	C11C01	111 111 011	00 00 100
a_{19}	0.02	10010	001011	100110	C11C10	111 111 100	00 00 101
a_{20}	0.02	10011	011010	100111	C11C11	111 111 101	00 00 1110
a_{21}	0.01	10100	011011	101000	C00C00C00	111 111 110	00 00 1111
Entropy	4.0						
Average length		5.0	4.05	4.24	4.65	4.59	4.13

TABLE 8.5
Variable-length codes.



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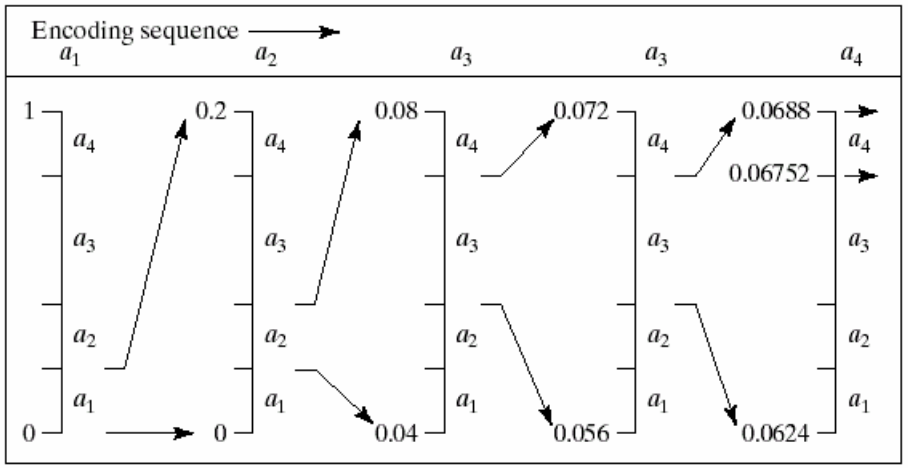
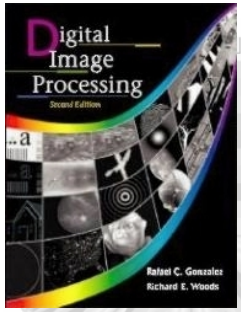


FIGURE 8.13
Arithmetic coding procedure.



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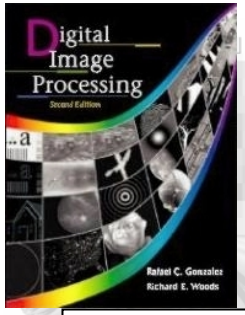
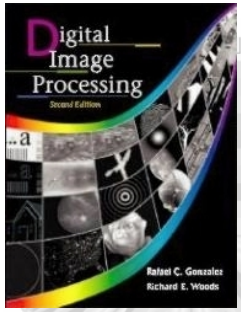
Source Symbol	Probability	Initial Subinterval
a_1	0.2	[0.0, 0.2)
a_2	0.2	[0.2, 0.4)
a_3	0.4	[0.4, 0.8)
a_4	0.2	[0.8, 1.0)

TABLE 8.6
Arithmetic coding example.



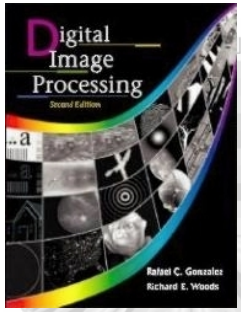
8.4.2 LZW Coding

LZW壓縮技術是由Lempel、Ziv和Welch 三個人發明的 ，這種壓縮技術是藉由將經常出現的string編碼來減少Bit數，在壓縮時不會犧牲掉畫面的品質，但是不能設定壓縮的比值。



LZW 編碼演算法是對資料的string做編碼。 這些string與原始影像資料中像素值的序列一致。它透過創造一個string table，其中含有string以及和它相對應的代碼所表示的顏色。在讀取資料時同時update這個string table。

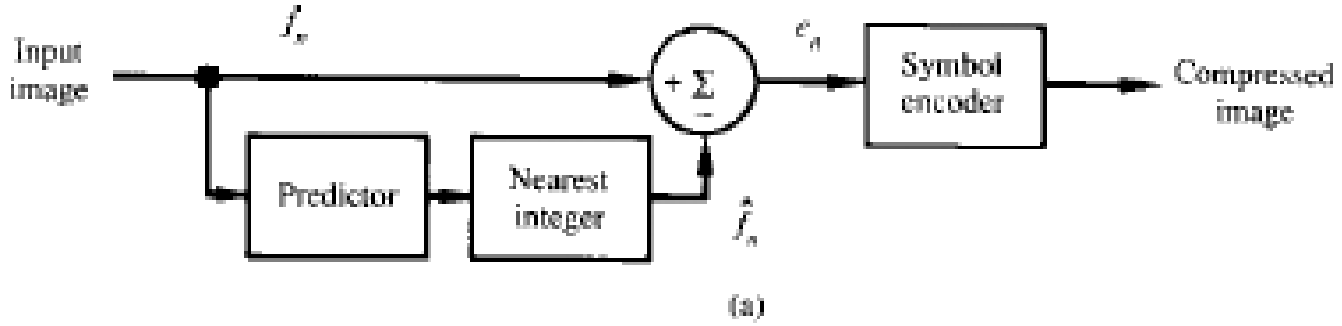
在編碼過程中當遇到一個新string時就分派一個新代碼來表示並將其記錄在string table中。如果遇到一個string已經在string table之中，就用與那個string相對應的代碼來代表那個string的值。



LZW演算法適用於原始資料變化頻繁但有很多重覆出現部分的情況之下。

其應用有：

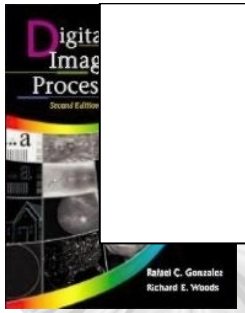
PDF、GIF、TIF….



(a)

$$e_n = f_n - \hat{f}_n$$

$$\hat{f}_n = round[\sum_{i=1}^{m} \alpha_i f_{n-i}]$$



m=1 first order

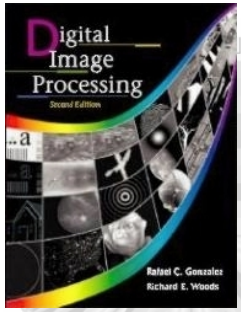
m=2 second order

一階 $round = [\alpha_1 f_{n-1}]$

二階 $round = [\alpha_1 f_{n-1} + \alpha_2 f_{n-2}]$

傳送時只要將影像的第一個值、誤差值還有α值傳送給對方即可。

Decode $f_n = e_n + f_n = e_n + f_n \; f_{n-1}$



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Currently Recognized Sequence	Pixel Being Processed	Encoded Output	Dictionary Location (Code Word)	Dictionary Entry
	39			
39	39	39	256	39-39
39	126	39	257	39-126
126	126	126	258	126-126
126	39	126	259	126-39
39	39			
39-39	126	256	260	39-39-126
126	126			
126-126	39	258	261	126-126-39
39	39			
39-39	126			
39-39-126	126	260	262	39-39-126-126
126	39			
126-39	39	259	263	126-39-39
39	126			
39-126	126	257	264	39-126-126
126		126		

TABLE 8.7
LZW coding example.

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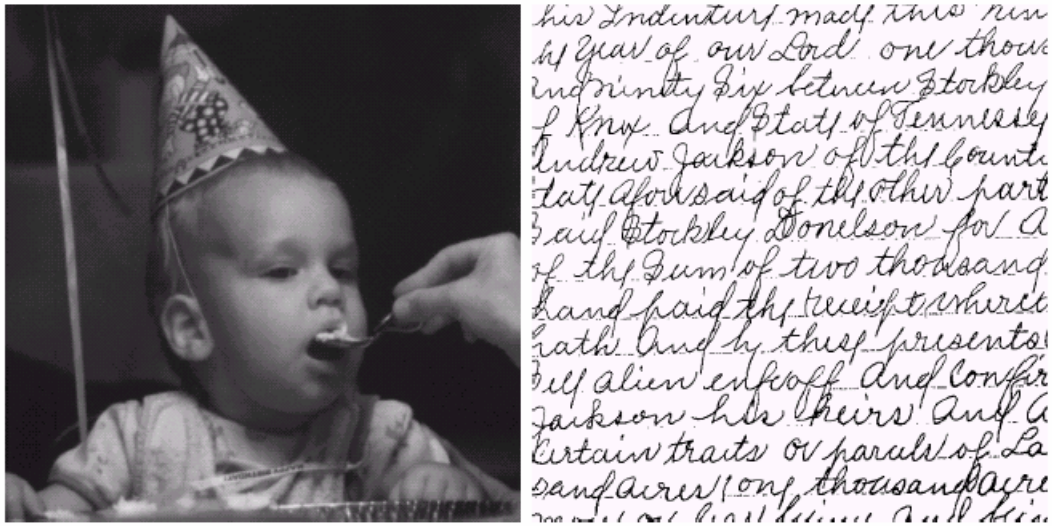

a b

FIGURE 8.14 A 1024 × 1024 (a) 8-bit monochrome image and (b) binary image.

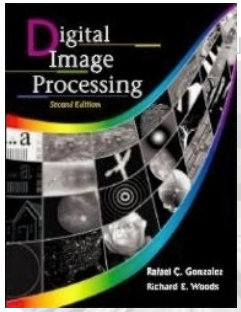

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FIGURE 8.15 The four most significant binary (left column) and Gray-coded (right column) bit planes of the image in Fig. 8.14(a).

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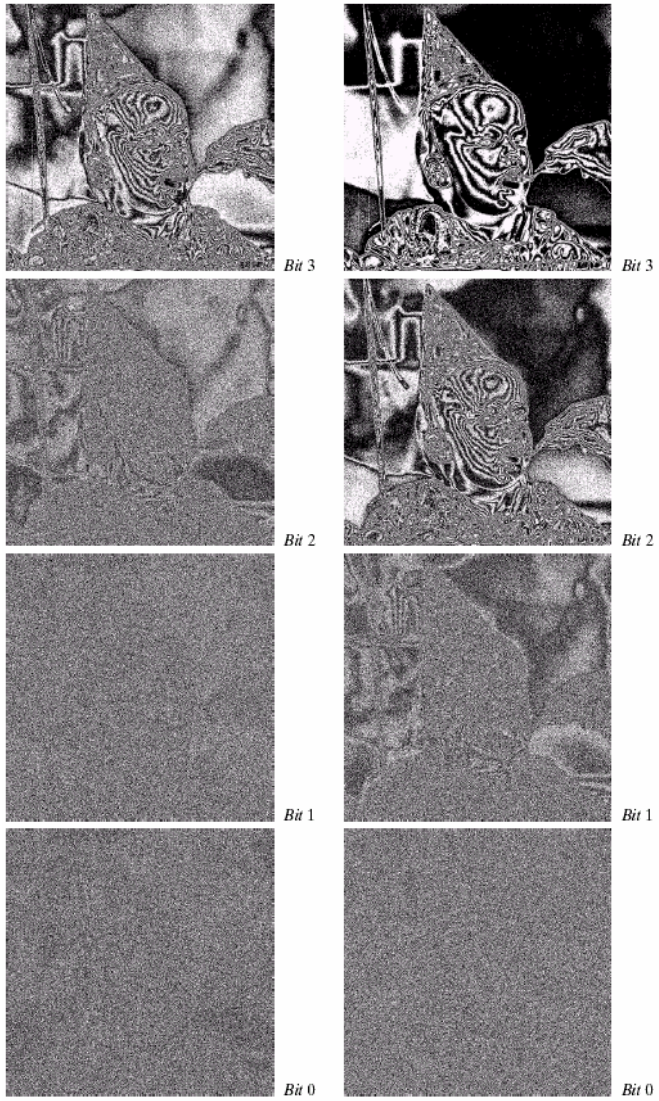
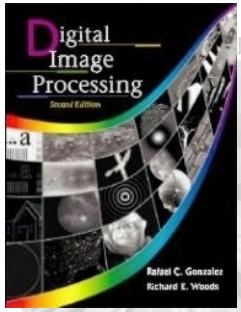


FIGURE 8.16 The four least significant binary (left column) and Gray-coded (right column) bit planes of the image in Fig. 8.14(a).



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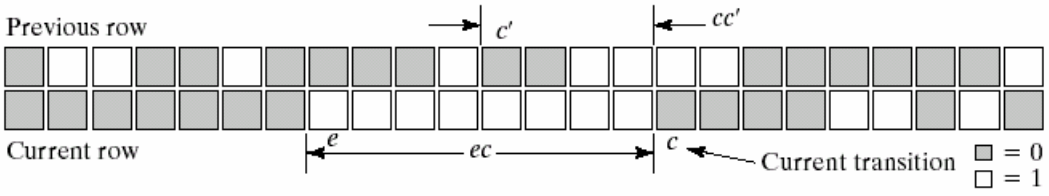
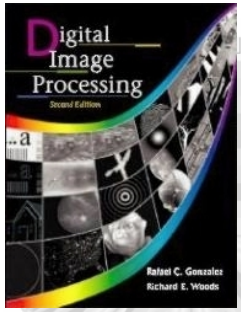


a
b

FIGURE 8.17 A relative address coding (RAC) illustration.

Distance measured	Distance	Code	Distance range	Code $h(d)$
cc'	0	0	1 – 4	0 xx
ec or cc' (left)	1	100	5 – 20	10 xxxx
cc' (right)	1	101	21 – 84	110 xxxxxx
ec	$d(d>1)$	111 $h(d)$	85 – 340	1110 xxxxxxxx
cc' (c' to left)	$d(d>1)$	1100 $h(d)$	341 – 364	11110 xxxxxxxxx
cc' (c' to right)	$d(d>1)$	1101 $h(d)$	1365 – 5460	111110 xxxxxxxxxx



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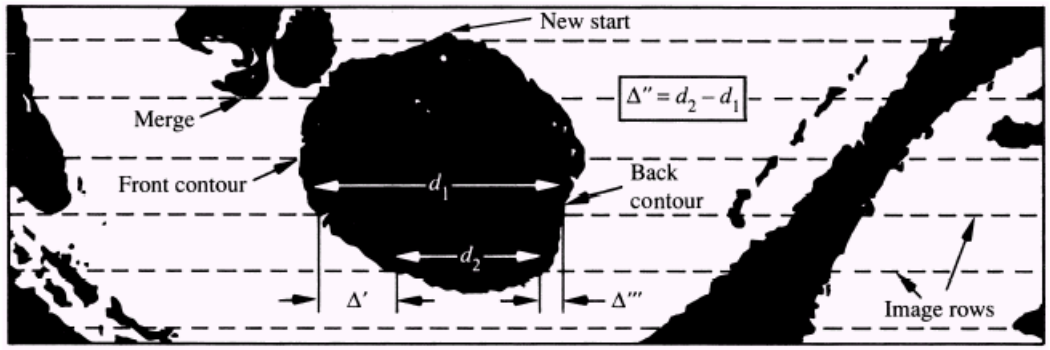
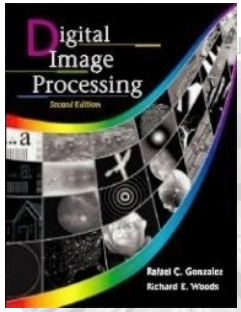


FIGURE 8.18 Parameters of the PDQ algorithm.



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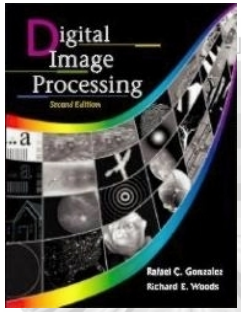
Method	Bit-plane code rate (bits/pixel) 7	6	5	4	3	2	1	0	Code Rate	Compression Ratio
Binary Bit-Plane Coding										
CBC (4 × 4)	0.14	0.24	0.60	0.79	0.99	—	—	—	5.75	1.4:1
RLC	0.09	0.19	0.51	0.68	0.87	1.00	1.00	1.00	5.33	1.5:1
PDQ	0.07	0.18	0.79	—	—	—	—	—	6.04	1.3:1
DDC	0.07	0.18	0.79	—	—	—	—	—	6.03	1.3:1
RAC	0.06	0.15	0.62	0.91	—	—	—	—	5.17	1.4:1
Gray Bit-Plane Coding										
CBC (4 × 4)	0.14	0.18	0.48	0.40	0.61	0.98	—	—	4.80	1.7:1
RLC	0.09	0.13	0.40	0.33	0.51	0.85	1.00	1.00	4.29	1.9:1
PDQ	0.07	0.12	0.61	0.40	0.82	—	—	—	5.02	1.6:1
DDC	0.07	0.11	0.61	0.40	0.81	—	—	—	5.00	1.6:1
RAC	0.06	0.10	0.49	0.31	0.62	—	—	—	4.05	1.8:1

TABLE 8.8
Error-free bit-plane coding results for Fig. 8.14(a): $H \approx 6.82$ bits/pixel

	WBS (1 × 8)	WBS (4 × 4)	RLC	PDQ	DDC	RAC
Code rate (bits/pixel)	0.48	0.39	0.32	0.23	0.22	0.23
Compression ratio	2.1:1	2.6:1	3.1:1	4.4:1	4.7:1	4.4:1

TABLE 8.9
Error-free binary image compression results for Fig. 8.14(b): $H \approx 0.55$ bits/pixel.



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a
b

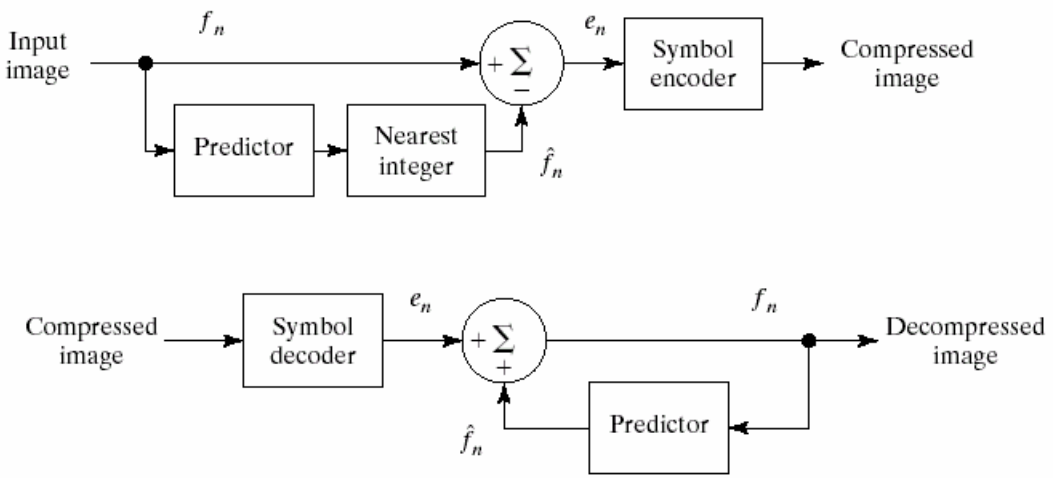
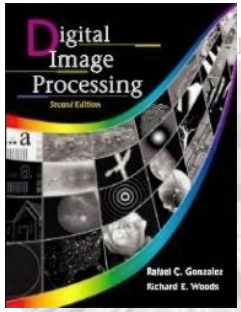


FIGURE 8.19 A lossless predictive coding model: (a) encoder; (b) decoder.



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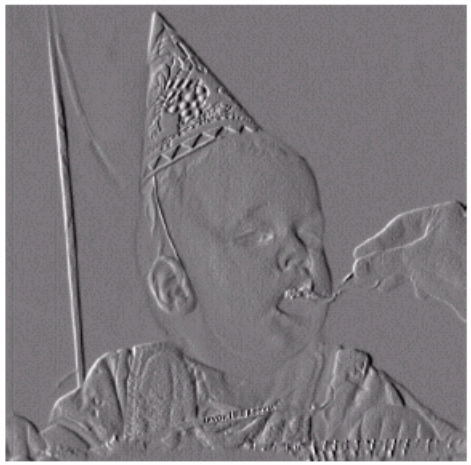
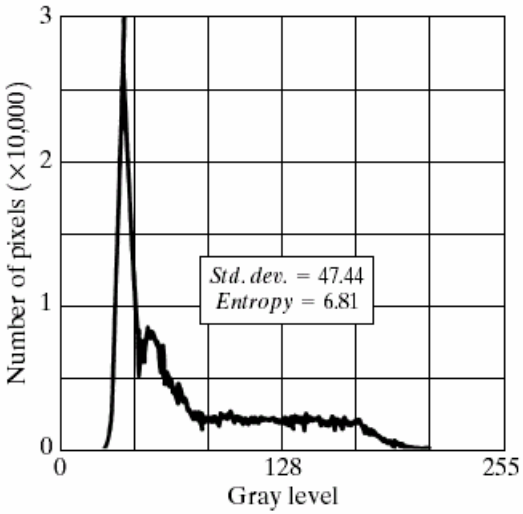
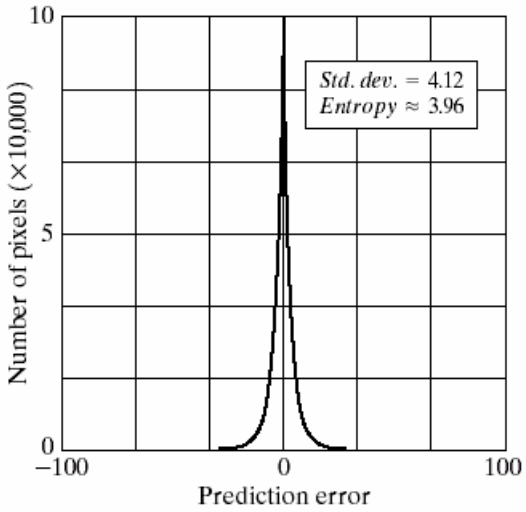
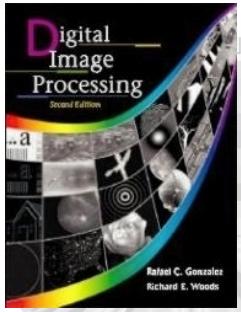
a
b c

FIGURE 8.20
(a) The prediction error image resulting from Eq. (8.4-9).
(b) Gray-level histogram of the original image.
(c) Histogram of the prediction error.









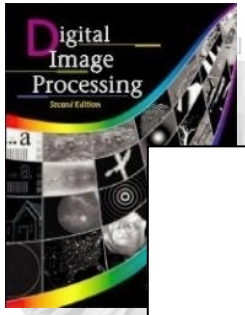
$$\dot{f}_n = \dot{e}_n + \hat{f}_n$$

· 表示有誤差

$$\hat{f}_n = \alpha * \dot{f}_{n-1} \quad \left(\hat{f}_n = \sum_{i=1}^{m} \alpha_i \dot{f}_{n-i} \right)$$

$$\dot{e}_n = \begin{cases} \\ \\ \end{cases} e_n$$

8.5 Lossy Compression



Compression Procedure

$$e_n = f_n - \hat{f}_n$$

where $\hat{f}_n = \alpha * f_{n-1}$ **(if 一階)**

where $\dot{f}_{n-1} = \dot{e}_{n-1} + \hat{f}_{n-1}$

where $\dot{e}_n = \begin{cases} +\mathbf{k} & \text{for } e_n > 0 \\ \mathbf{-k} & \textbf{otherwise} \end{cases}$

Ex: $\dot{e}_n = \begin{cases} +6.5 & \text{for } \eta\vartheta > 0 \\ -6.5 & \text{otherwise} \end{cases}$, $\alpha = 1$

$\dot{f}_0$: **initial data=14.0**

$$\hat{f}_1 = \alpha * \dot{f}_0 = 14.0$$

$$e_1 = f_1 - \hat{f}_1 = 15.0 - 14.0 = 1.0$$

$$\because e_1 > 0 \therefore \dot{e}_1 = +6.5$$

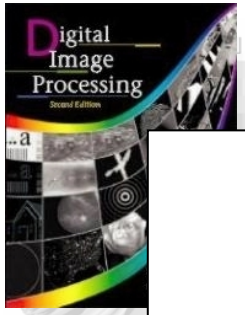
$$\dot{f}_1 = \dot{e}_1 + \hat{f}_1 = 6.5 + 14.0 = 20.5$$

$$\hat{f}_2 = \alpha * \dot{f}_1 = 20.5$$

$$e_2 = f_2 - \hat{f}_2 = -6.5$$

$$\because e_2 < 0 \therefore \dot{e}_2 = -6.5$$

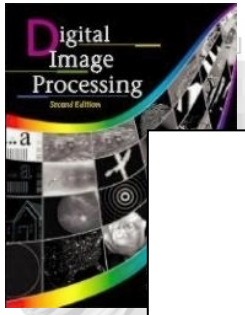
$$\dot{f}_2 = \dot{e}_2 + \hat{f}_2 = -6.5 + 20.5 = 14.0$$



求出 $\dot{e}$

n	f	$\hat{f}$	e	$\dot{e}$	$\dot{f}$
0	14.0	-	-	-	14.0
1	15.0	14.0	1.0	6.5	20.5
2	14.0	20.5	-6.5	-6.5	14.0

傳送 $\dot{f}_0$ 及 $\dot{e}$ 給接收端



接收端以 $\dot{f}_0$ 及 $\dot{e}$ 解壓縮計算 $\dot{f}$

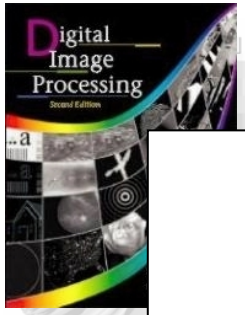
$$\dot{f}_0 = 14.0$$

$$\hat{f}_1 = \alpha * \dot{f}_0 = 14.0$$

$$\dot{f}_1 = \dot{e}_1 + \hat{f}_1 = 6.5 + 14.0 = 20.5$$

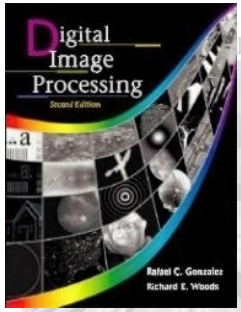
$$\hat{f}_2 = \alpha * \dot{f}_1 = 20.5$$

$$\dot{f}_2 = \dot{e}_2 + \hat{f}_2 = -6.5 + 20.5 = 14.0$$



解壓縮解出 $\dot{f}$

n	$\dot{e}$	$\dot{f}$	$\check{f}$	$\dot{f}$	$\lvert f - \dot{f} \rvert$
0	-	14.0	-	-	0.0
1	6.5	20.5	14.0	20.5	5.5
2	-6.5	14.0	20.5	14.0	0.0



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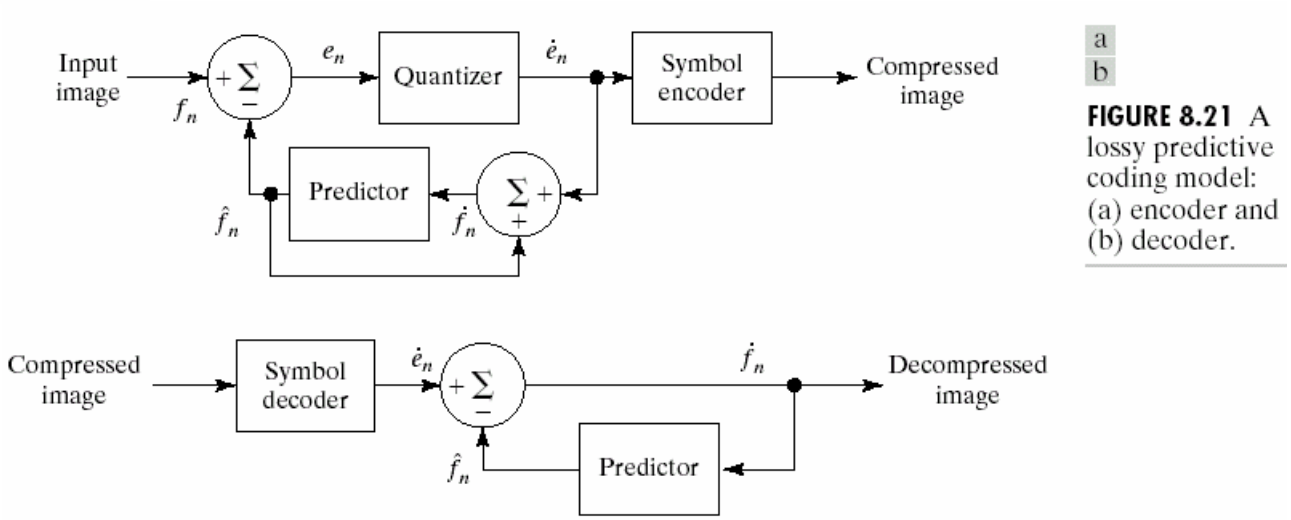

a
b

FIGURE 8.21 A lossy predictive coding model: (a) encoder and (b) decoder.

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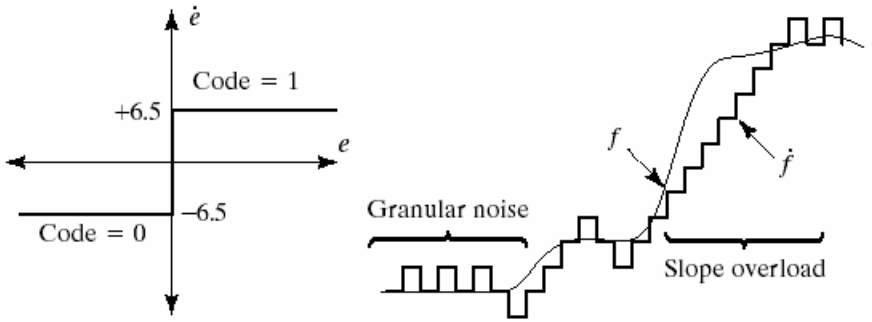

a b
c

FIGURE 8.22 An example of delta modulation.

Input		Encoder				Decoder		Error
n	f	$\hat{f}$	e	$\dot{e}$	$\dot{f}$	$\hat{f}$	$\dot{f}$	$[f-\dot{f}]$
0	14	—	—	—	14.0	—	14.0	0.0
1	15	14.0	1.0	6.5	20.5	14.0	20.5	–5.5
2	14	20.5	–6.5	–6.5	14.0	20.5	14.0	0.0
3	15	14.0	1.0	6.5	20.5	14.0	20.5	–5.5
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
14	29	20.5	8.5	6.5	27.0	20.5	27.0	2.0
15	37	27.0	10.0	6.5	33.5	27.0	33.5	3.5
16	47	33.5	13.5	6.5	40.0	33.5	40.0	7.0
17	62	40.0	22.0	6.5	46.5	40.0	46.5	15.5
18	75	46.5	28.5	6.5	53.0	46.5	53.0	22.0
19	77	53.0	24.0	6.5	59.6	53.0	59.6	17.5
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.

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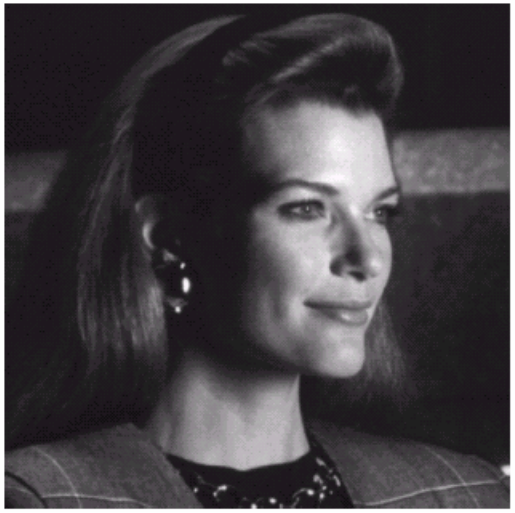
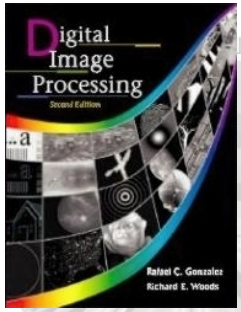


FIGURE 8.23 A 512 × 512 8-bit monochrome image.



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a b
c d

FIGURE 8.24 A comparison of four linear prediction techniques.

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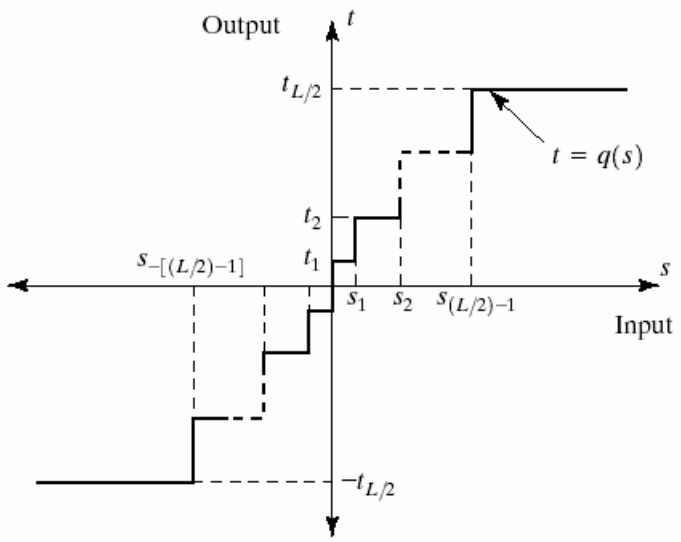

FIGURE 8.25 A typical quantization function.

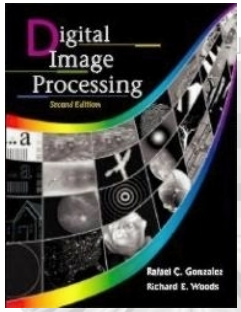

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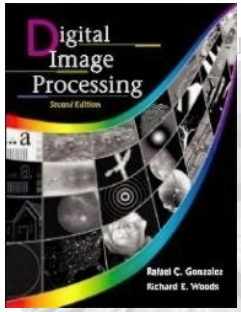
TABLE 8.10 Lloyd-Max quantizers for a Laplacian probability density function of unit variance.

Levels	2		4		8	
i	s_i	t_i	s_i	t_i	s_i	t_i
1	∞	0.707	1.102	0.395	0.504	0.222
2			∞	1.810	1.181	0.785
3					2.285	1.576
4					∞	2.994
θ	1.414		1.087		0.731	



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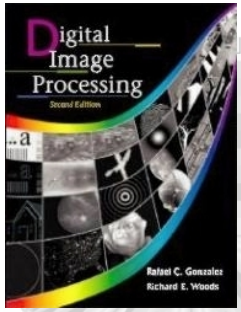
Predictor	Lloyd-Max Quantizer			Adaptive Quantizer		
	2-level	4-level	8-level	2-level	4-level	8-level
Eq. (8.5-16)	30.88	6.86	4.08	7.49	3.22	1.55
Eq. (8.5-17)	14.59	6.94	4.09	7.53	2.49	1.12
Eq. (8.5-18)	9.90	4.30	2.31	4.61	1.70	0.76
Eq. (8.5-19)	38.18	9.25	3.36	11.46	2.56	1.14
Compression	8.00:1	4.00:1	2.70:1	7.11:1	3.77:1	2.56:1

TABLE 8.11 Lossy DPCM root-mean-square error summary.



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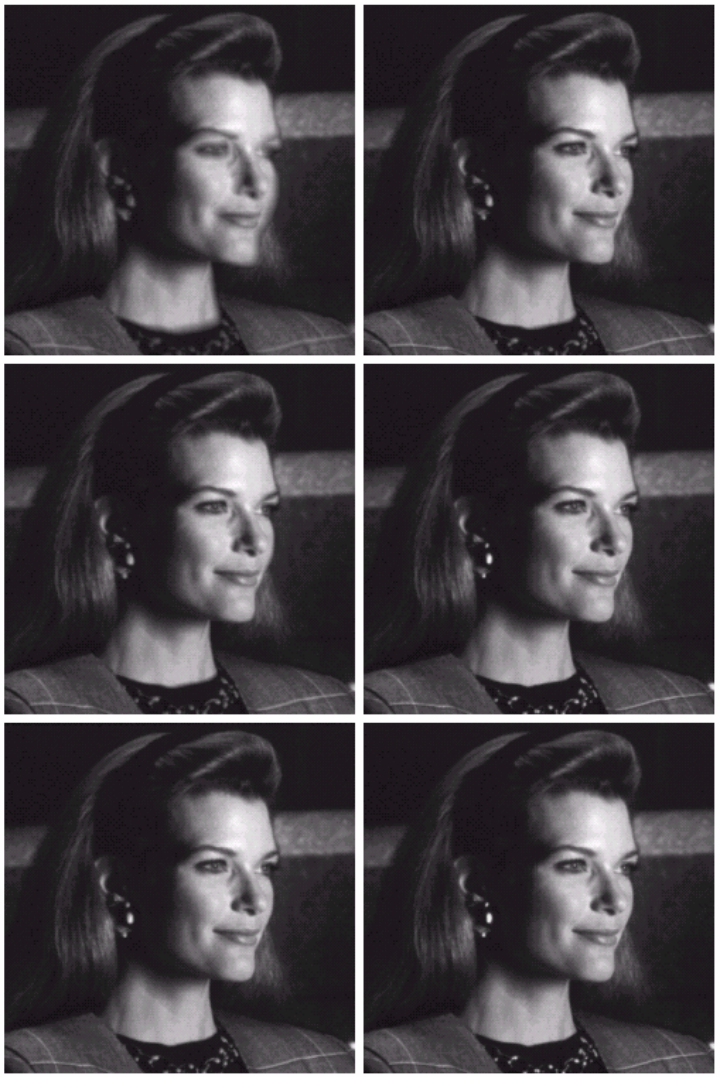

a b
c d
e f

FIGURE 8.26 DPCM result images: (a) 1.0; (b) 1.125; (c) 2.0; (d) 2.125; (e) 3.0; (f) 3.125 bits/pixel.

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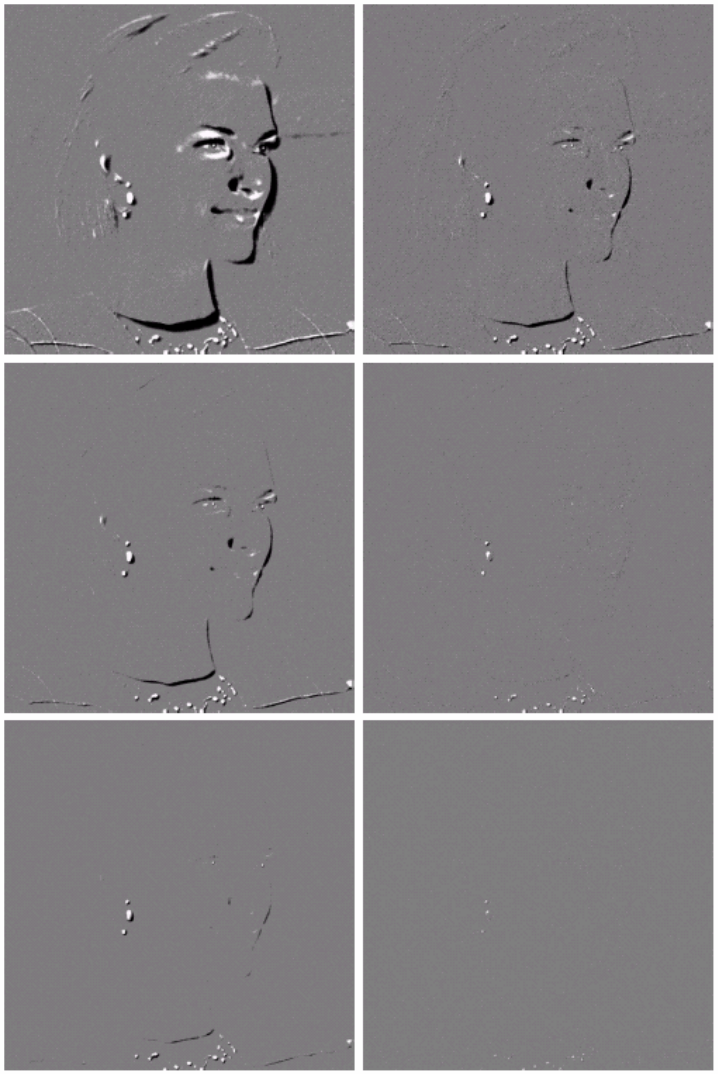
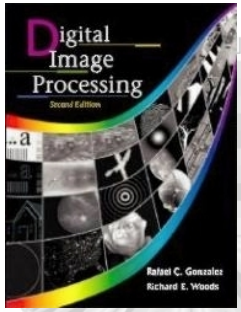


a b
c d
e f

FIGURE 8.27 The scaled (×8) DPCM error images that correspond to Figs. 8.26(a) through (f).



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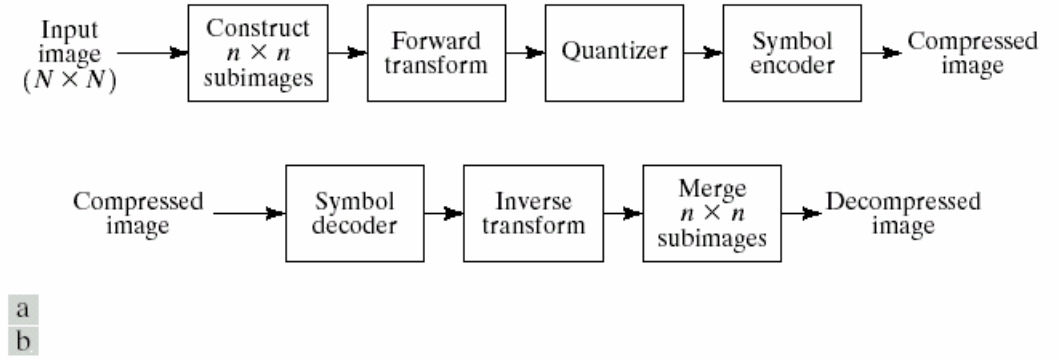

FIGURE 8.28 A transform coding system: (a) encoder; (b) decoder.

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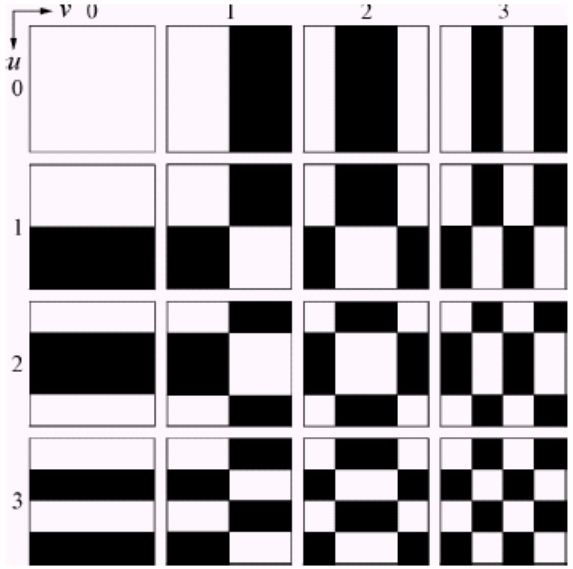
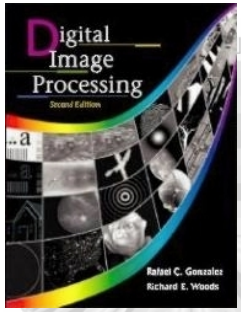


FIGURE 8.29 Walsh-Hadamard basis functions for $N = 4$. The origin of each block is at its top left.



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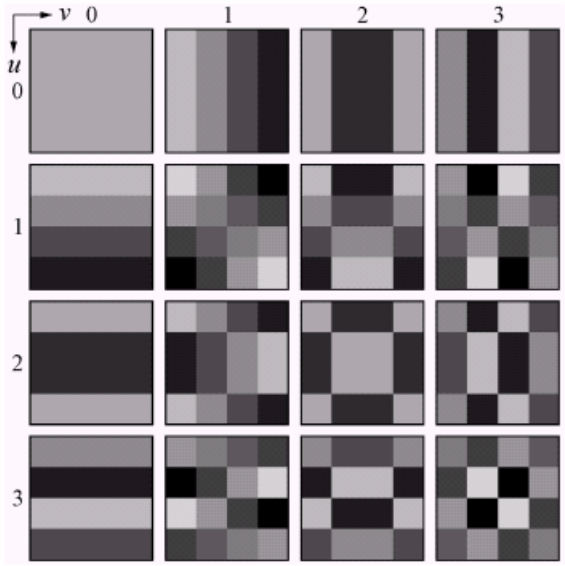
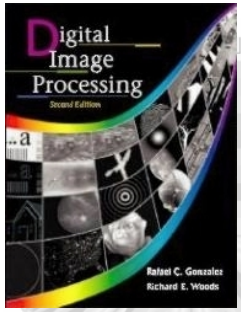


FIGURE 8.30 Discrete-cosine basis functions for $N = 4$. The origin of each block is at its top left.



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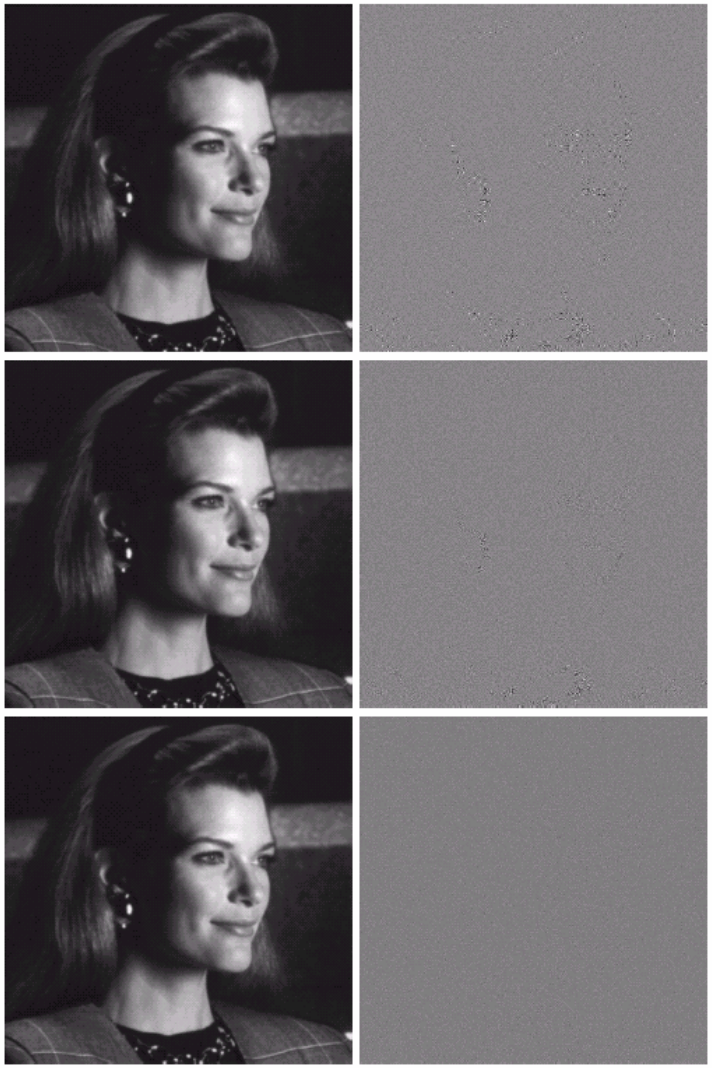

a b
c d
e f

FIGURE 8.31 Approximations of Fig. 8.23 using the (a) Fourier, (c) Hadamard, and (e) cosine transforms, together with the corresponding scaled error images.

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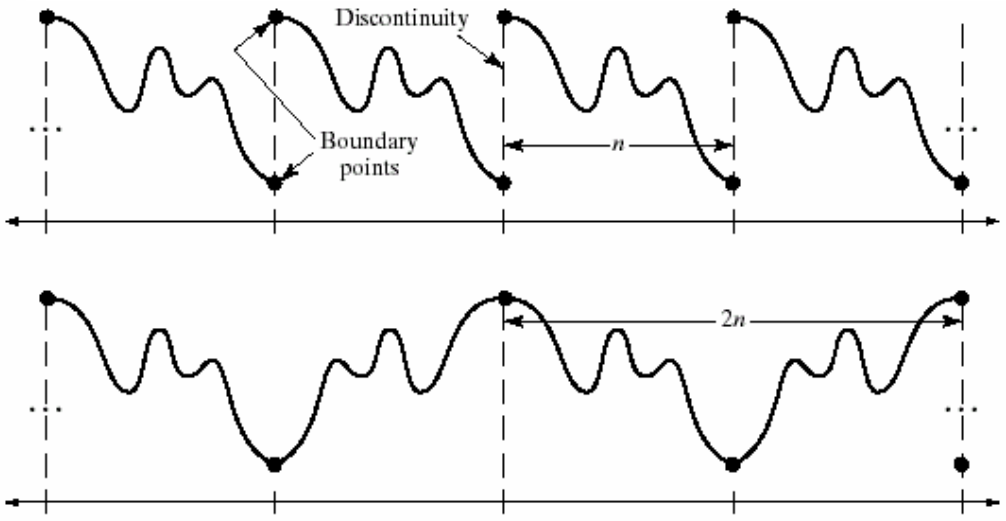
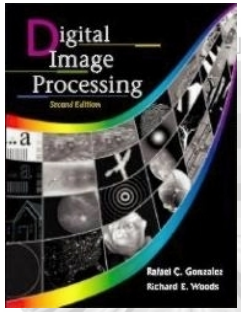


a
b

FIGURE 8.32 The periodicity implicit in the 1-D (a) DFT and (b) DCT.



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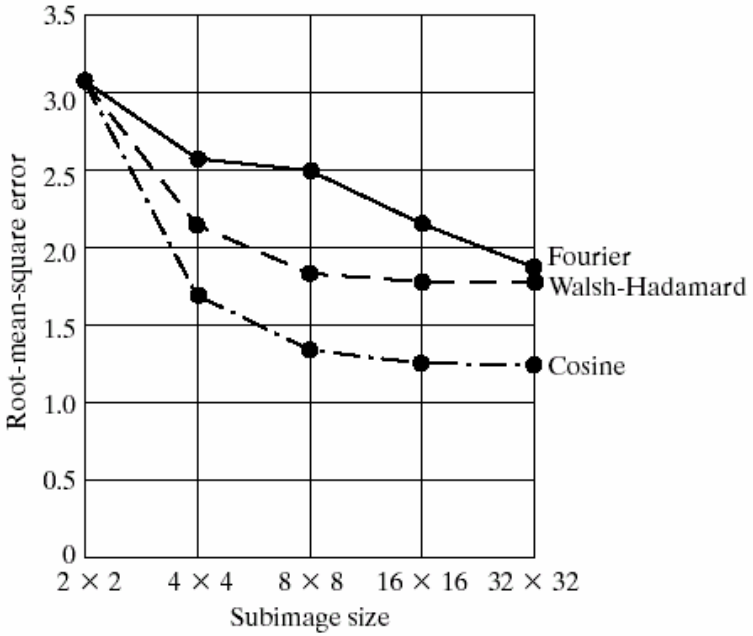
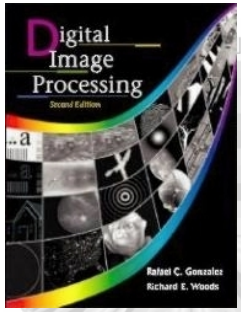


FIGURE 8.33 Reconstruction error versus subimage size.



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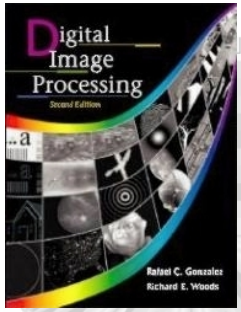
a b
c d
e f

FIGURE 8.34 Approximations of Fig. 8.23 using 25% of the DCT coefficients: (a) and (b) 8 × 8 subimage results; (c) zoomed original; (d) 2 × 2 result; (e) 4 × 4 result; and (f) 8 × 8 result.



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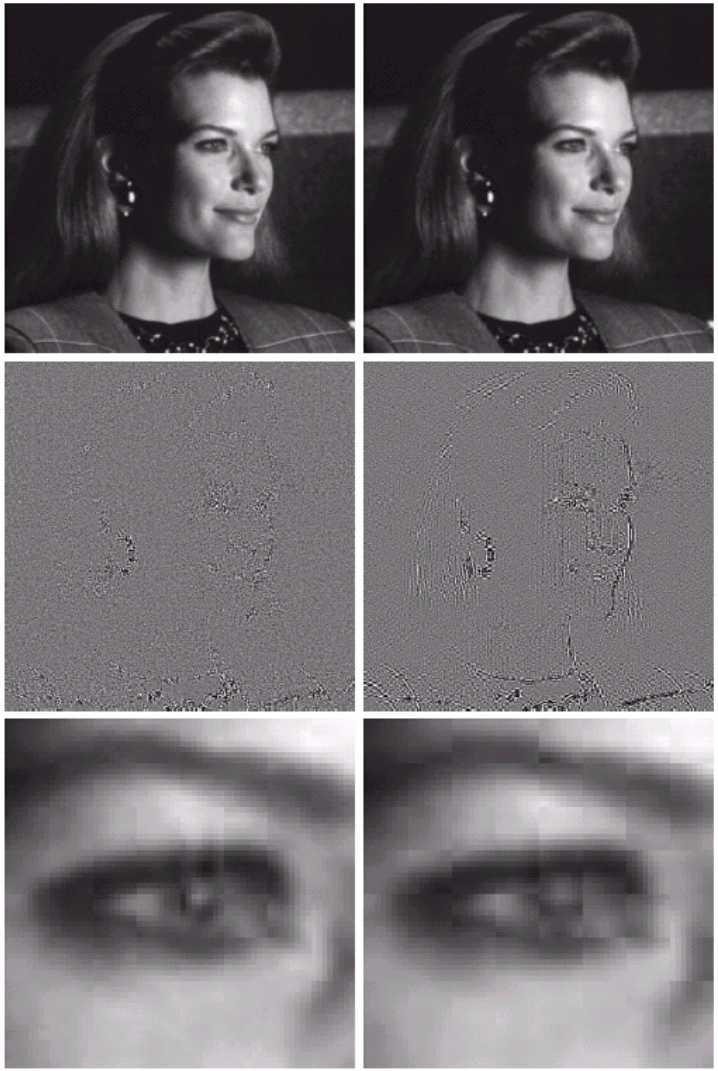
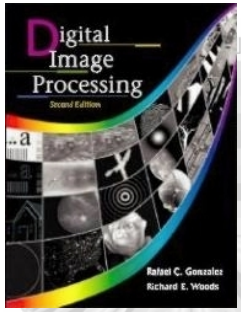


a b
c d
e f

FIGURE 8.35 Approximations of Fig. 8.23 using 12.5% of the 8×8 DCT coefficients: (a), (c), and (e) threshold coding results; (b), (d), and (f) zonal coding results.



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a b
c d

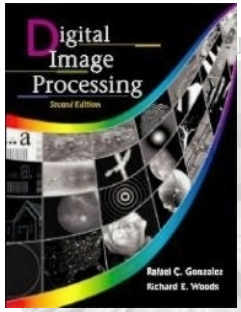
FIGURE 8.36 A typical (a) zonal mask, (b) zonal bit allocation, (c) threshold mask, and (d) thresholded coefficient ordering sequence. Shading highlights the coefficients that are retained.

1	1	1	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

8	7	6	4	3	2	1	0
7	6	5	4	3	2	1	0
6	5	4	3	3	1	1	0
4	4	3	3	2	1	0	0
3	3	3	2	1	1	0	0
2	2	1	1	1	0	0	0
1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0

1	1	0	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

0	1	5	6	14	15	27	28
2	4	7	13	16	26	29	42
3	8	12	17	25	30	41	43
9	11	18	24	31	40	44	53
10	19	23	32	39	45	52	54
20	22	33	38	46	51	55	60
21	34	37	47	50	56	59	61
35	36	48	49	57	58	62	63



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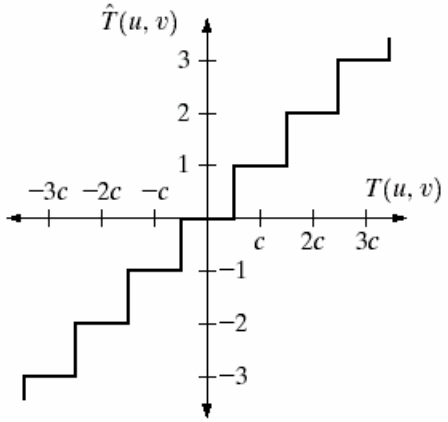
a b

FIGURE 8.37
(a) A threshold coding quantization curve [see Eq. (8.5-40)]. (b) A typical normalization matrix.



16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

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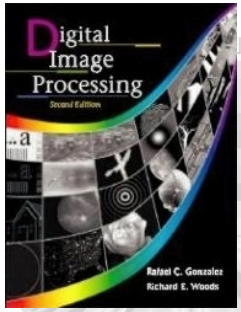
a b
c d
e f

FIGURE 8.38 Left column: Approximations of Fig. 8.23 using the DCT and normalization array of Fig. 8.37(b). Right column: Similar results for 4**Z**.



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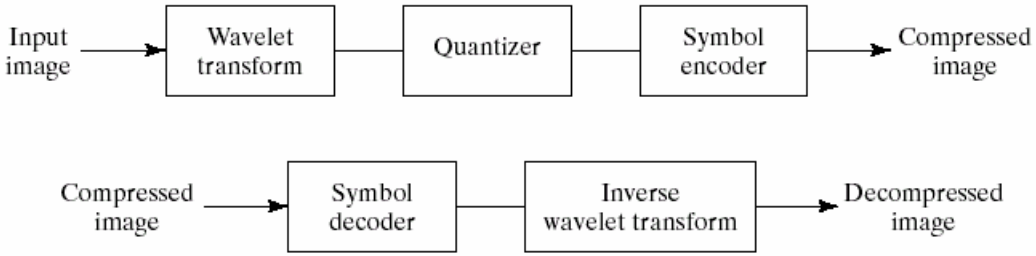
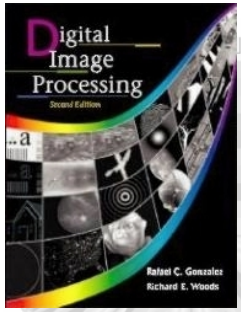
a
b

FIGURE 8.39 A wavelet coding system: (a) encoder; (b) decoder.





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a b
c d
e f

FIGURE 8.40 (a), (c), and (e) Wavelet coding results comparable to the transform-based results in Figs. 8.38(a), (c), and (e); (b), (d), and (f) similar results for Figs. 8.38(b), (d), and (f).

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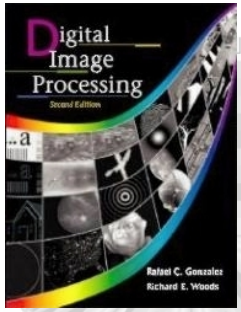
a b
c d
e f

FIGURE 8.41 (a), (c), and (e) Wavelet coding results with a compression ratio of 108 to 1; (b), (d), and (f) similar results for a compression of 167 to 1.



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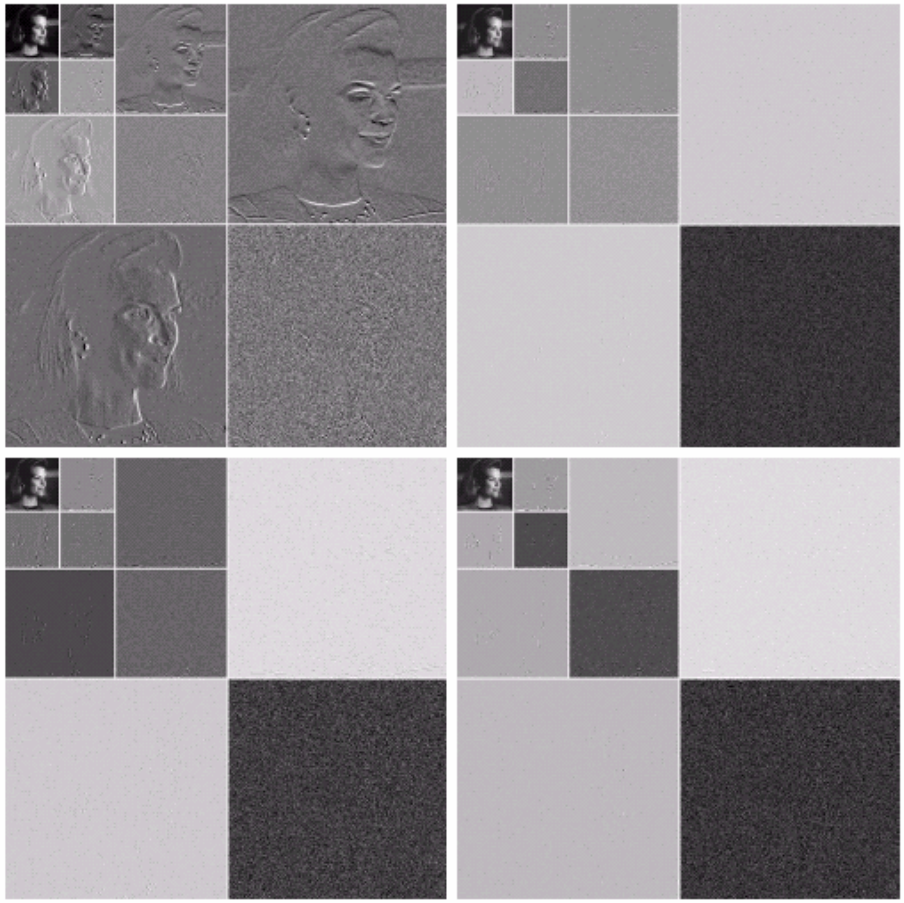
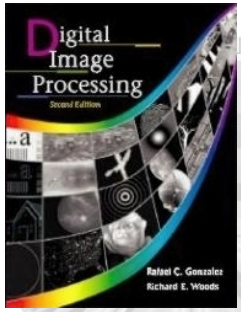


a b
c d

FIGURE 8.42 Wavelet transforms of Fig. 8.23 with respect to (a) Haar wavelets, (b) Daubechies wavelets, (c) symlets, and (d) Cohen-Daubechies-Feauveau biorthogonal wavelets.



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Wavelet	Filter Taps (Scaling + Wavelet)	Zeroed Coefficients
Haar (see Ex. 7.10)	2 + 2	46%
Daubechies (see Fig. 7.6)	8 + 8	51%
Symlet (see Fig. 7.24)	8 + 8	51%
Biorthogonal (see Fig. 7.37)	17 + 11	55%

TABLE 8.12 Wavelet transform filter taps and zeroed coefficients when truncating the transforms in Fig. 8.42 below 1.5.

Scales and Filter Bank Iterations	Approximation Coefficient Image	Truncated Coefficients (%)	Reconstruction Error (rms)
1	256 × 256	75%	1.93
2	128 × 128	93%	2.69
3	64 × 64	97%	3.12
4	32 × 32	98%	3.25
5	16 × 16	98%	3.27

TABLE 8.13 Decomposition level impact on wavelet coding the 512 × 512 image of Fig. 8.23.

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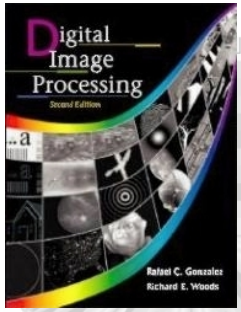

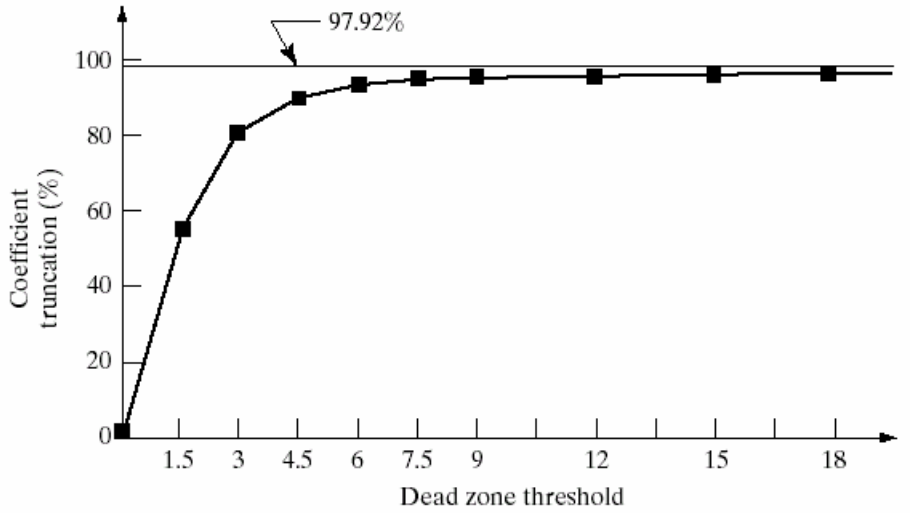

FIGURE 8.43 The impact of dead zone interval selection on wavelet coding.

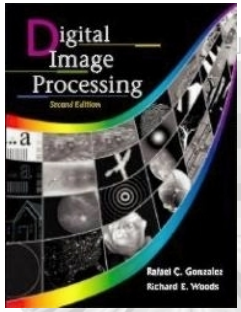

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TABLE 8.14
CCITT terminating codes.

Run Length	White Code Word	Black Code Word	Run Length	White Code Word	Black Code Word
0	00110101	0000110111	32	00011011	000001101010
1	000111	010	33	00010010	000001101011
2	0111	11	34	00010011	000011010010
3	1000	10	35	00010100	000011010011
4	1011	011	36	00010101	000011010100
5	1100	0011	37	00010110	000011010101
6	1110	0010	38	00010111	000011010110
7	1111	00011	39	00101000	000011010111
8	10011	000101	40	00101001	000001101100
9	10100	000100	41	00101010	000001101101
10	00111	0000100	42	00101011	000011011010
11	01000	0000101	43	00101100	000011011011
12	001000	0000111	44	00101101	000001010100
13	000011	00000100	45	00000100	000001010101
14	110100	00000111	46	00000101	000001010110
15	110101	000011000	47	00001010	000001010111

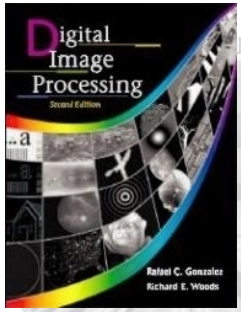

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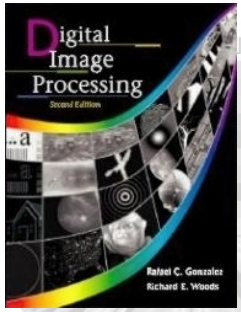
Table 8.14 (Cont')

16	101010	0000010111	48	00001011	000001100100
17	101011	0000011000	49	01010010	000001100101
18	0100111	0000001000	50	01010011	000001010010
19	0001100	00001100111	51	01010100	000001010011
20	0001000	00001101000	52	01010101	000000100100
21	0010111	00001101100	53	00100100	000000110111
22	0000011	00000110111	54	00100101	000000111000
23	0000100	00000101000	55	01011000	000000100111
24	0101000	00000010111	56	01011001	000000101000
25	0101011	00000011000	57	01011010	000001011000
26	0010011	000011001010	58	01011011	000001011001
27	0100100	000011001011	59	01001010	000000101011
28	0011000	000011001100	60	01001011	000000101100
29	00000010	000011001101	61	00110010	000001011010
30	00000011	000001101000	62	00110011	000001100110
31	00011010	000001101001	63	00110100	000001100111



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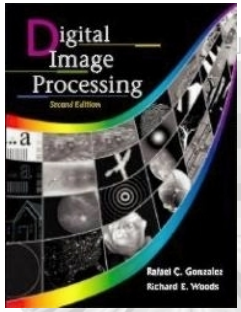
Run Length	White Code Word	Black Code Word	Run Length	White Code Word	Black Code Word
64	11011	0000001111	960	011010100	0000001110011
128	10010	000011001000	1024	011010101	0000001110100
192	010111	000011001001	1088	011010110	0000001110101
256	0110111	000001011011	1152	011010111	0000001110110
320	00110110	000000110011	1216	011011000	0000001110111
384	00110111	000000110100	1280	011011001	0000001010010
448	01100100	000000110101	1344	011011010	0000001010011
512	01100101	0000001101100	1408	011011011	0000001010100
576	01101000	0000001101101	1472	010011000	0000001010101
640	01100111	0000001001010	1536	010011001	0000001011010
704	011001100	0000001001011	1600	010011010	0000001011011
768	011001101	0000001001100	1664	011000	0000001100100
832	011010010	0000001001101	1728	010011011	0000001100101
896	011010011	0000001110010			

	Code Word		Code Word
1792	00000001000	2240	000000010110
1856	00000001100	2304	000000010111
1920	00000001101	2368	000000011100
1984	000000010010	2432	000000011101
2048	000000010011	2496	000000011110
2112	000000010100	2560	000000011111
2176	000000010101		

TABLE 8.15
CCITT makeup codes.



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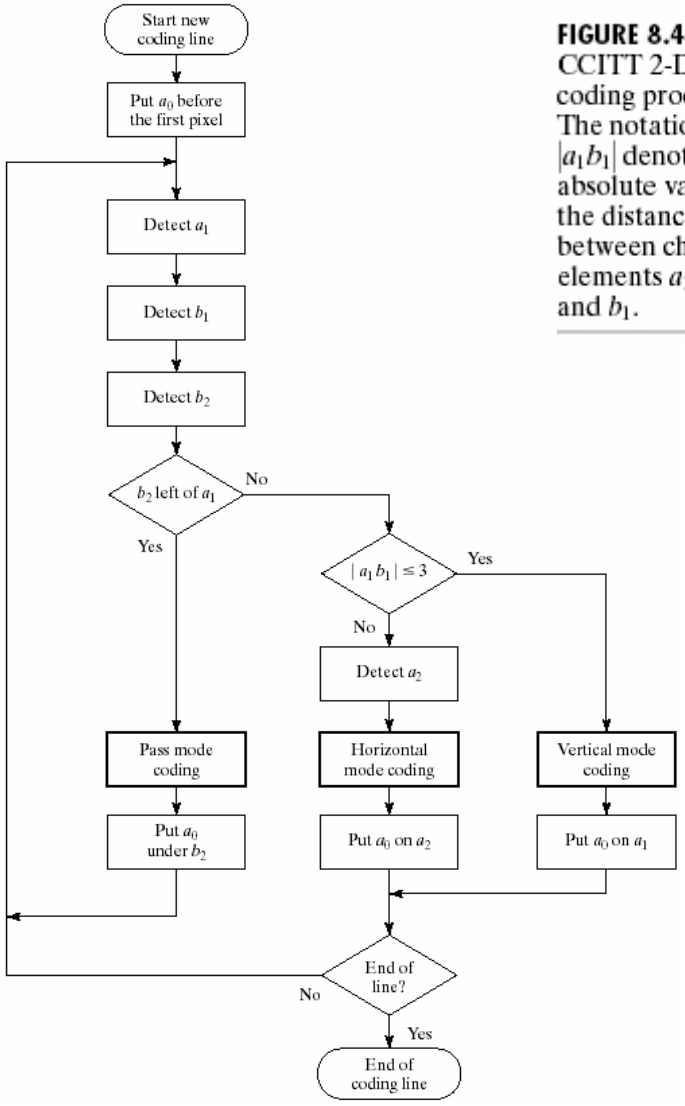

FIGURE 8.44 CCITT 2-D coding procedure. The notation $|a_1b_1|$ denotes the absolute value of the distance between changing elements a_1 and b_1.

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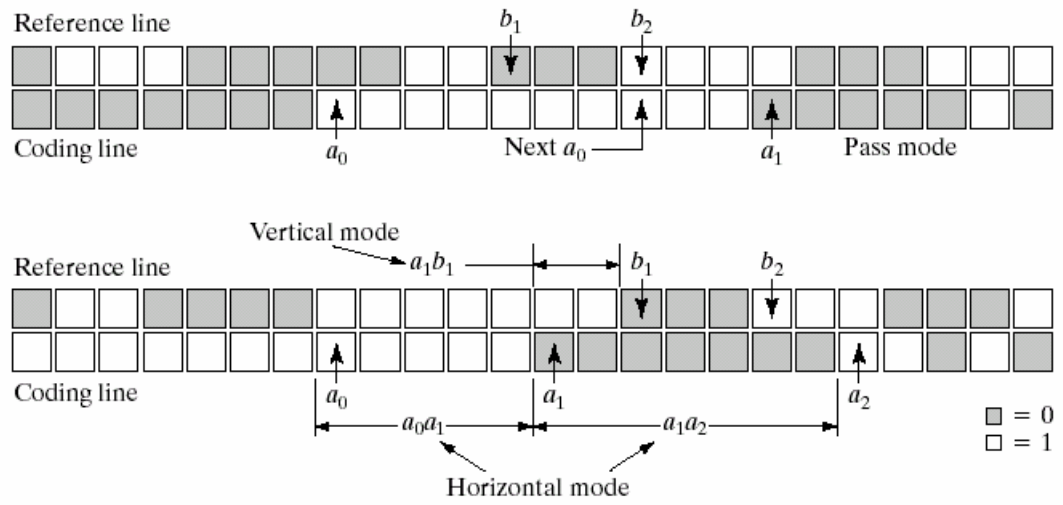

a
b

FIGURE 8.45 CCITT (a) pass mode and (b) horizontal and vertical mode coding parameters.

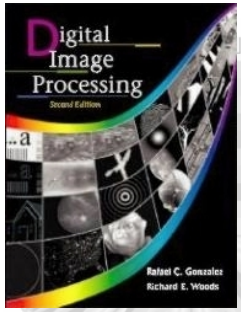

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TABLE 8.16
CCITT two-dimensional code table.

Mode	Code Word
Pass	0001
Horizontal	$001 + M(a_0a_1) + M(a_1a_2)$
Vertical	
a_1 below b_1	1
a_1 one to the right of b_1	011
a_1 two to the right of b_1	000011
a_1 three to the right of b_1	0000011
a_1 one to the left of b_1	010
a_1 two to the left of b_1	000010
a_1 three to the left of b_1	0000010
Extension	0000001×××

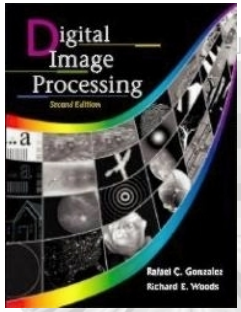

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TABLE 8.17
JPEG coefficient coding categories.

Range	DC Difference Category	AC Category
0	0	N/A
−1, 1	1	1
−3, −2, 2, 3	2	2
−7, … , −4, 4, … , 7	3	3
−15, … , −8, 8, … , 15	4	4
−31, … , −16, 16, … , 31	5	5
−63, … , −32, 32, … , 63	6	6
−127, … , −64, 64, … , 127	7	7
−255, … , −128, 128, … , 255	8	8
−511, … , −256, 256, … , 511	9	9
−1023, … , −512, 512, … , 1023	A	A
−2047, … , −1024, 1024, … , 2047	B	B
−4095, … , −2048, 2048, … , 4095	C	C
−8191, … , −4096, 4096, … , 8191	D	D
−16383, … , −8192, 8192, … , 16383	E	E
−32767, … , −16384, 16384, … , 32767	F	N/A

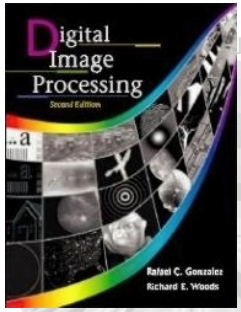

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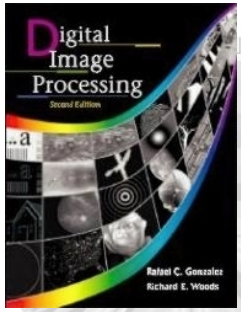
TABLE 8.18
JPEG default DC code (luminance).

Category	Base Code	Length	Category	Base Code	Length
0	010	3	6	1110	10
1	011	4	7	11110	12
2	100	5	8	111110	14
3	00	5	9	1111110	16
4	101	7	A	11111110	18
5	110	8	B	111111110	20



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Run/ Category	Base Code	Length	Run/ Category	Base Code	Length
0/0	**1010 (= EOB)**	**4**			
0/1	00	3	8/1	11111010	9
0/2	01	4	8/2	1111111111000000	17
0/3	100	6	8/3	1111111110110111	19
0/4	1011	8	8/4	1111111110111000	20
0/5	11010	10	8/5	1111111110111001	21
0/6	111000	12	8/6	1111111110111010	22
0/7	1111000	14	8/7	1111111110111011	23
0/8	1111110110	18	8/8	1111111110111100	24
0/9	1111111110000010	25	8/9	1111111110111101	25
0/A	1111111110000011	26	8/A	1111111110111110	26
1/1	1100	5	9/1	111111000	10
1/2	111001	8	9/2	1111111110111111	18
1/3	1111001	10	9/3	1111111111000000	19
1/4	111110110	13	9/4	1111111111000001	20
1/5	11111110110	16	9/5	1111111111000010	21
1/6	1111111110000100	22	9/6	1111111111000011	22
1/7	1111111110000101	23	9/7	1111111111000100	23
1/8	1111111110000110	24	9/8	1111111111000101	24
1/9	1111111110000111	25	9/9	1111111111000110	25
1/A	1111111110001000	26	9/A	1111111111000111	26
2/1	11011	6	A/1	111111001	10
2/2	11111000	10	A/2	1111111111001000	18
2/3	1111110111	13	A/3	1111111111001001	19
2/4	1111111110001001	20	A/4	1111111111001010	20
2/5	1111111110001010	21	A/5	1111111111001011	21
2/6	1111111110001011	22	A/6	1111111111001100	22
2/7	1111111110001100	23	A/7	1111111111001101	23

TABLE 8.19
JPEG default AC code (luminance) *(continues on next page)*.

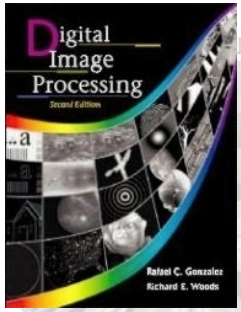

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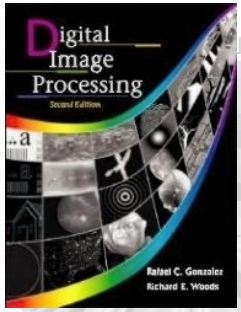
Table 8.19 (Con't)

2/8	1111111110001101	24	A/8	1111111111001110	24
2/9	1111111110001110	25	A/9	1111111111001111	25
2/A	1111111110001111	26	A/A	1111111111010000	26
3/1	111010	7	B/1	111111010	10
3/2	111110111	11	B/2	1111111111010001	18
3/3	11111110111	14	B/3	1111111111010010	19
3/4	1111111110010000	20	B/4	1111111111010011	20
3/5	1111111110010001	21	B/5	1111111111010100	21
3/6	1111111110010010	22	B/6	1111111111010101	22
3/7	1111111110010011	23	B/7	1111111111010110	23
3/8	1111111110010100	24	B/8	1111111111010111	24
3/9	1111111110010101	25	B/9	1111111111011000	25
3/A	1111111110010110	26	B/A	1111111111011001	26
4/1	111011	7	C/1	1111111010	11
4/2	1111111000	12	C/2	1111111111011010	18
4/3	1111111110010111	19	C/3	1111111111011011	19
4/4	1111111110011000	20	C/4	1111111111011100	20
4/5	1111111110011001	21	C/5	1111111111011101	21
4/6	1111111110011010	22	C/6	1111111111011110	22
4/7	1111111110011011	23	C/7	1111111111011111	23
4/8	1111111110011100	24	C/8	1111111111100000	24
4/9	1111111110011101	25	C/9	1111111111100001	25
4/A	1111111110011110	26	C/A	1111111111100010	26



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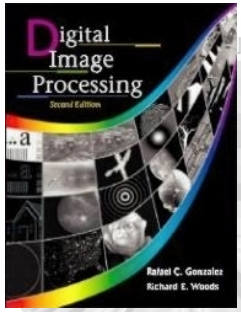
Filter Tap	Highpass Wavelet Coefficient	Lowpass Scaling Coefficient
0	−1.115087052456994	0.6029490182363579
±1	0.5912717631142470	0.2668641184428723
±2	0.05754352622849957	−0.07822326652898785
±3	−0.09127176311424948	−0.01686411844287495
±4	0	0.02674875741080976

TABLE 8.20 Impulse responses of the low and highpass analysis filters for an irreversible 9-7 wavelet transform.



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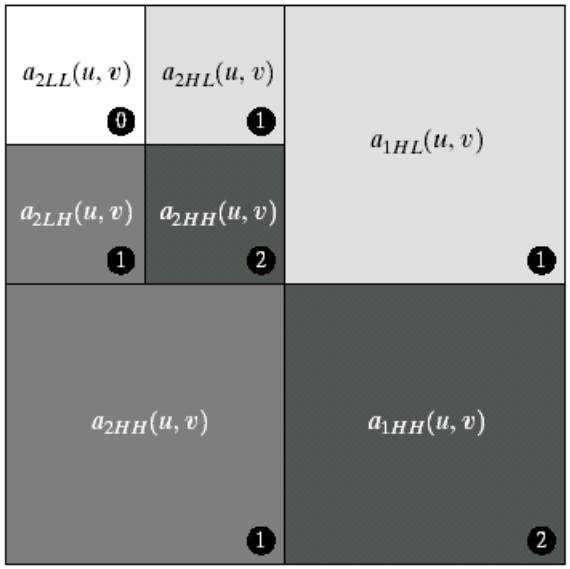
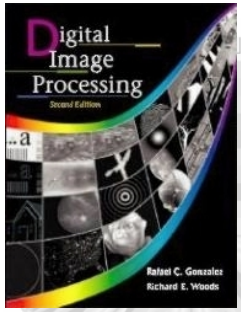


FIGURE 8.46 JPEG 2000 two-scale wavelet transform tile-component coefficient notation and analysis gain.



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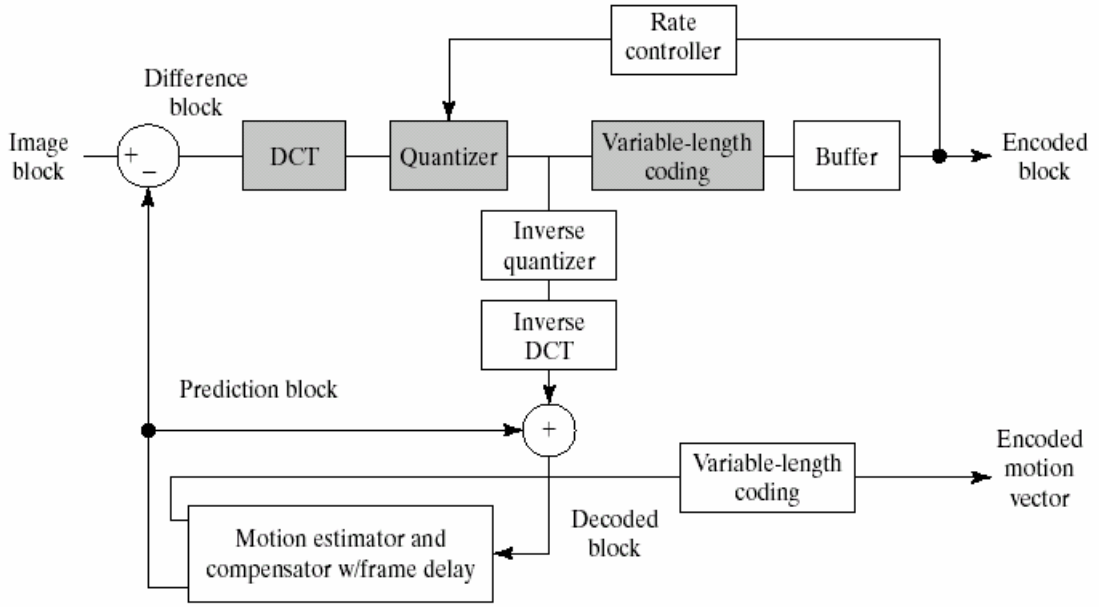

FIGURE 8.47 A basic DPCM/DCT encoder for motion compensated video compression.