Ch9 Morphological Image Processing

- 9.1 Preliminaries
- 9.1.1 Some Basic Concepts form Set Theory
- If every element of a set A is also an element of another set B, then A is said to be a subset of B, denoted as

$$A \subseteq B \qquad (9.1-3)$$

- The union of two sets A and B, denoted by

$$C = A \cup B \qquad (9.1-4)$$

- The intersection of two sets A and B, denote by

$$D = A \cap B \qquad (9.1.5)$$

- Disjoint or mutually exclusive

$$A \cap B = \varnothing \qquad (9.1.6)$$

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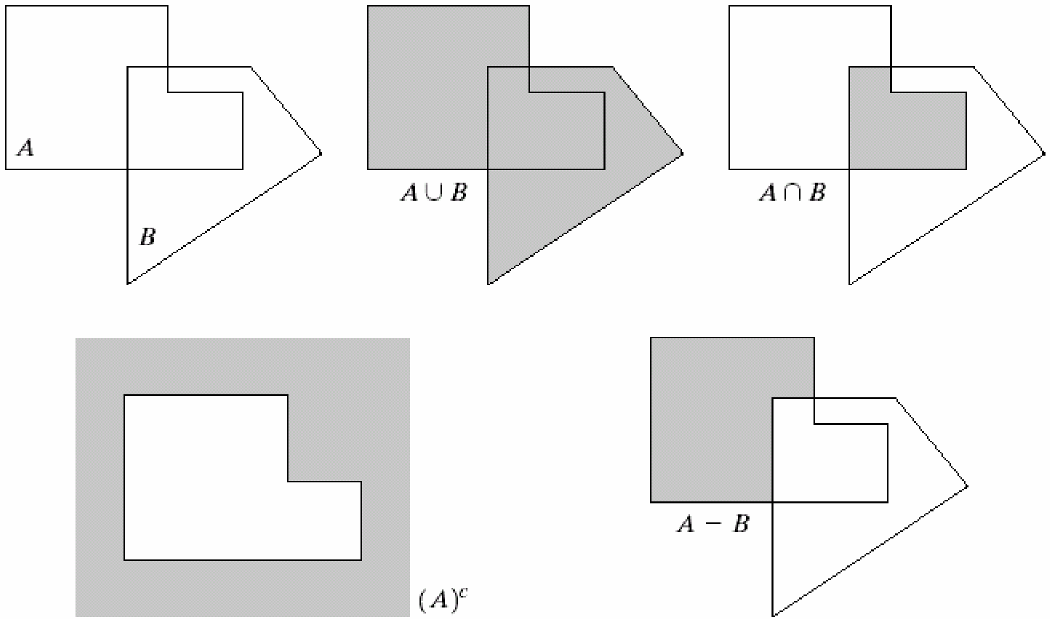

a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

- The complement of a set A is the set of elements not contained in A

$$A^c = \{\omega \mid \omega \notin A\} \qquad (9.1-7)$$

- The difference of two sets A and B, denoted A - B, is defined as

$$A - B = \{\omega \mid \omega \in A, \omega \notin B\} = A \cap B^c \qquad (9.1-8)$$

- The reflection of set B, denoted $\hat{B}$,is defined as

$$\hat{B} = \{\omega \mid \omega = -b, \; for \quad b \in B\} \qquad (9.1-9)$$

- The translation of set A by point z = (z_1, z_2), denoted $(A)_z$, ,is defined as

$$(A)_z = \{c \mid c = a + z, \; for \quad a \in A\} \qquad (9.1-10)$$

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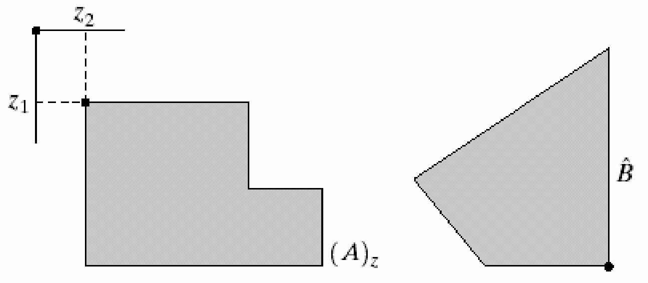

a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

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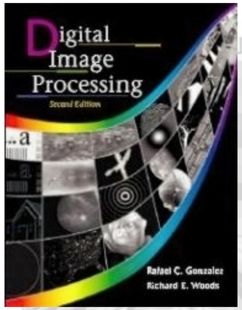
TABLE 9.1
The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0



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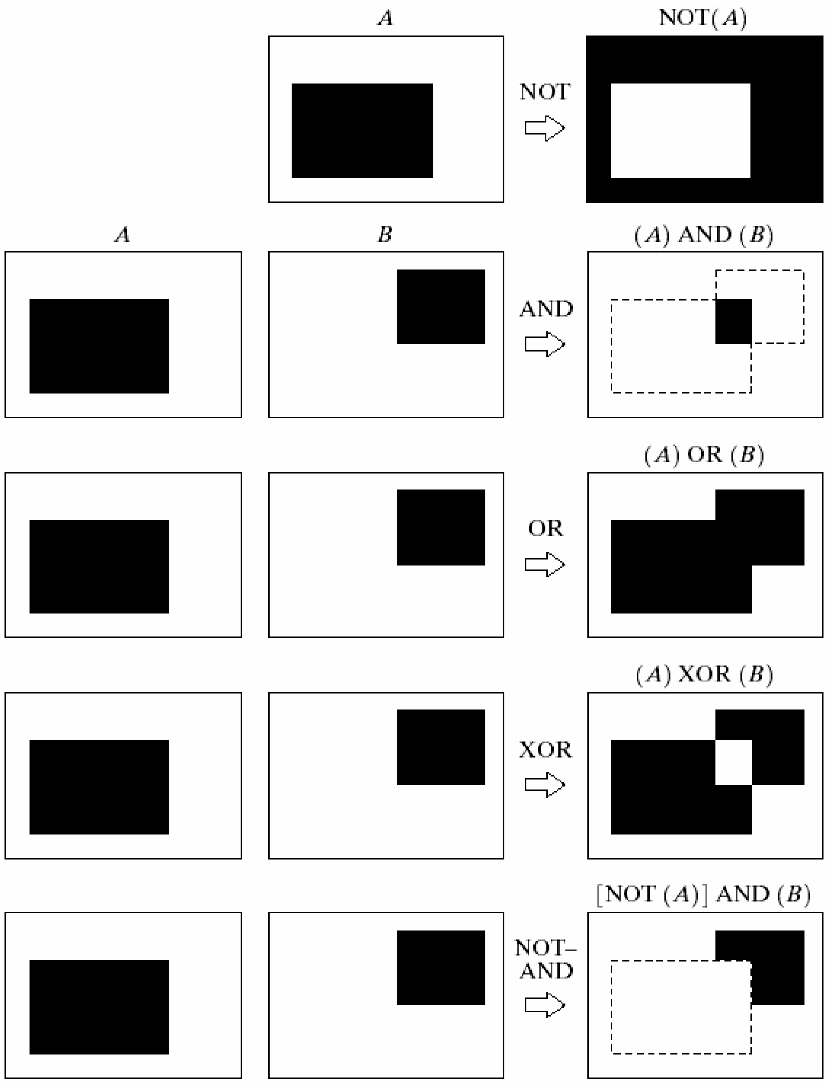

FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

9.2 Dilation and Erosion

- 9.2.1 Dilation
- The dilation of A by B, denoted $A \oplus B$, is defined as

$$A \oplus B = \left\{ z \,\middle|\, \left(\hat{B} \right)_z \cap A \neq \varnothing \right\} \qquad (9.2-1)$$

- The dilation of A by B then is the set of all displacements, z, such that $\hat{B}$ and A overlap by at least one element.

$$A \oplus B = \left\{ z \,\middle|\, \left[(\hat{B})_z \cap A \right] \subseteq A \right\} \qquad (9.2-2)$$

- Set B is commonly referred to as the structuring element.

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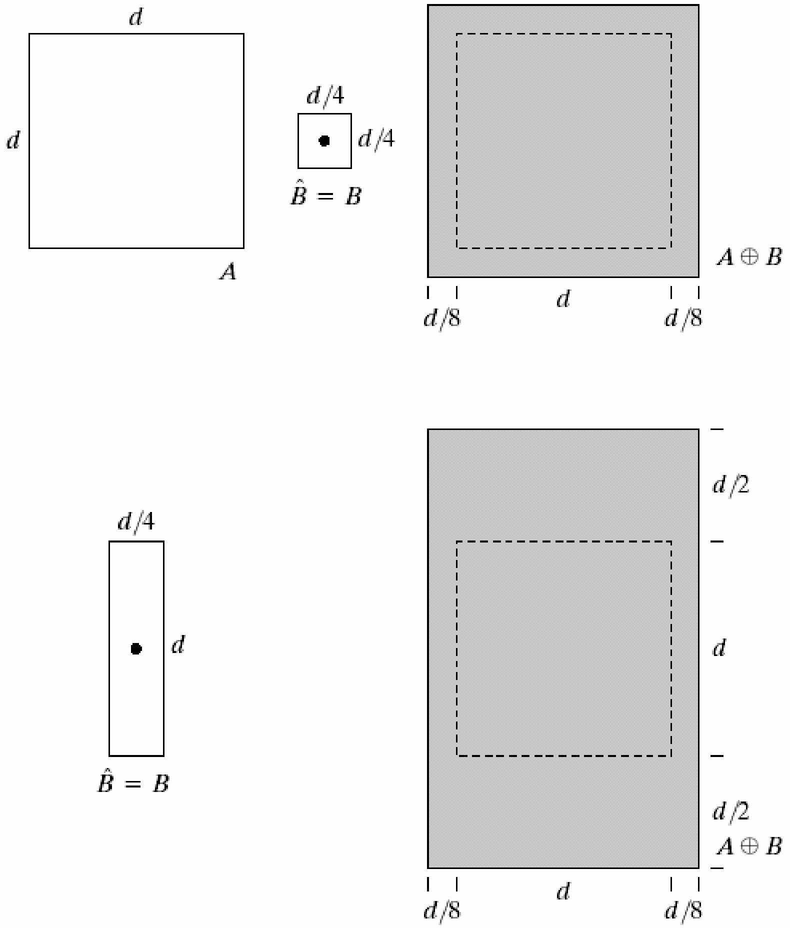

FIGURE 9.4 (a) Set A. (b) Square structuring element (dot is the center). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element.

- 9.2.2 Erosion
- The erosion of A by B, denoted $A \Theta B$,is defined as

$$A \Theta B = \left\{ z \middle| \left(B \right)_z \subseteq A \right\} \qquad (9.2-3)$$

- That the erosion of A by B is the set of all points z such that B, translated by z is contained in A.

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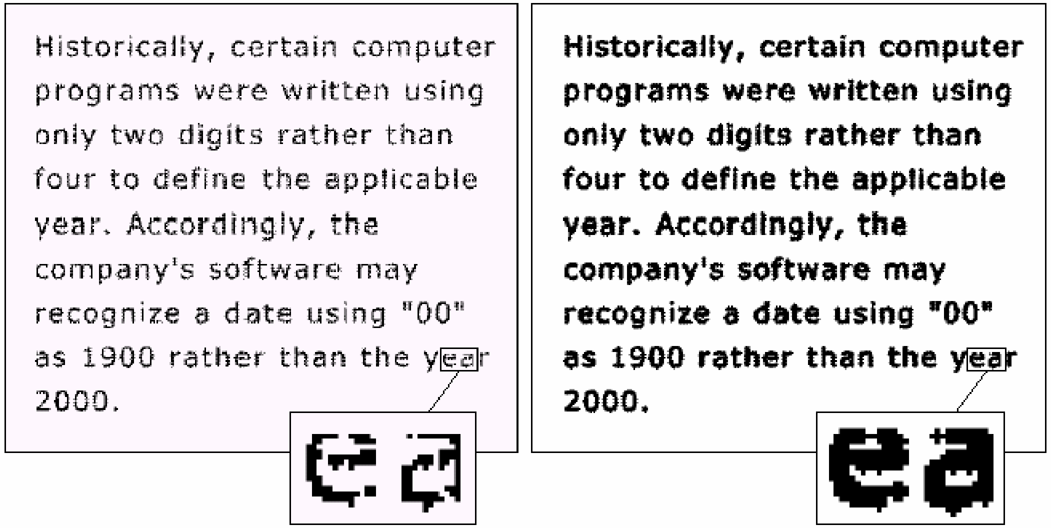

0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

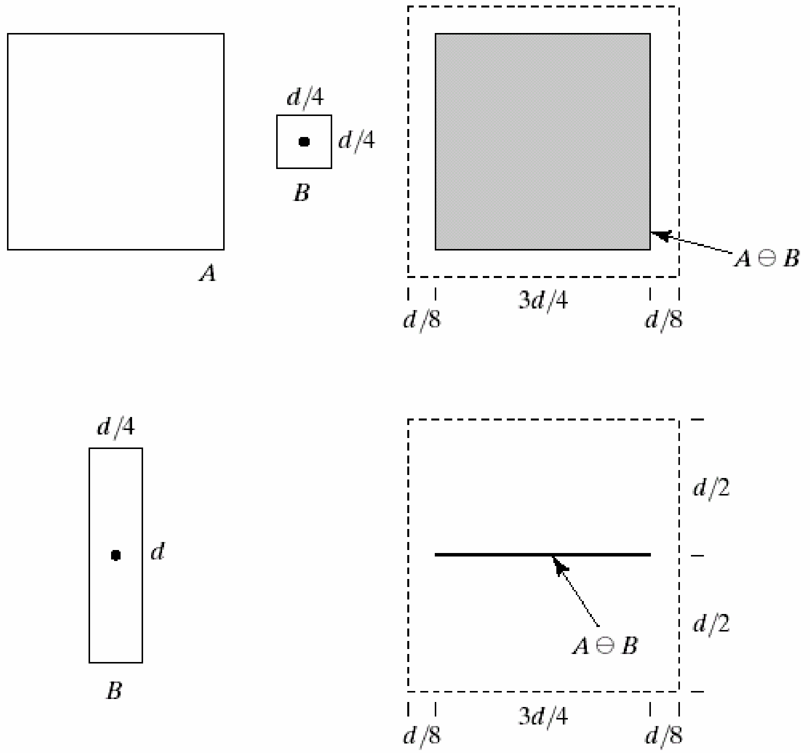

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

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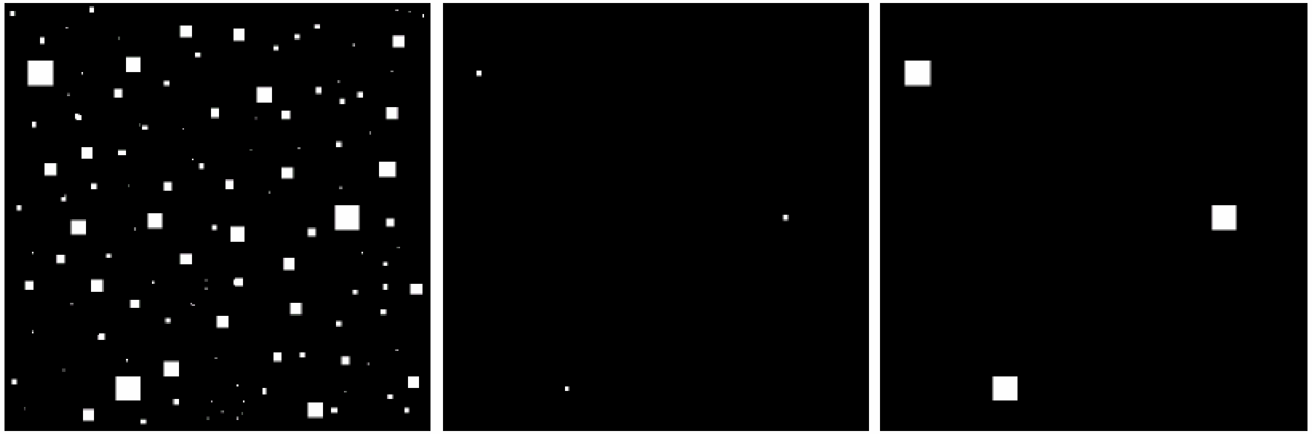

a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

9.3 Opening and Closing

- As we have seen, dilation expands an image and erosion shrinks it.
- Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions.
- Closing also tends to smooth sections of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.

- The opening of set A by structuring element B, denoted $A \circ B$

$$A \circ B = (A \Theta B) \oplus B \qquad (9.3-1)$$

- The closing of set A by structuring element B. denoted $A \bullet B$

$$A \bullet B = (A \oplus B) \Theta B \qquad (9.3-2)$$

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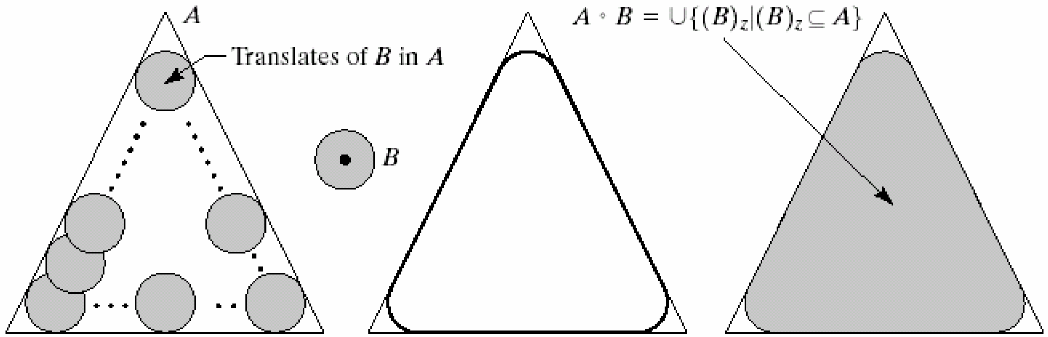

a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

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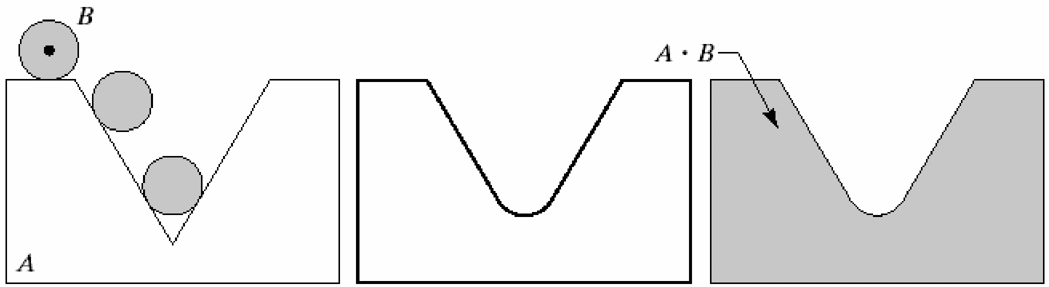
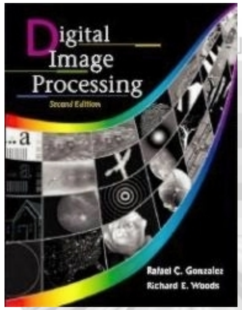


a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).



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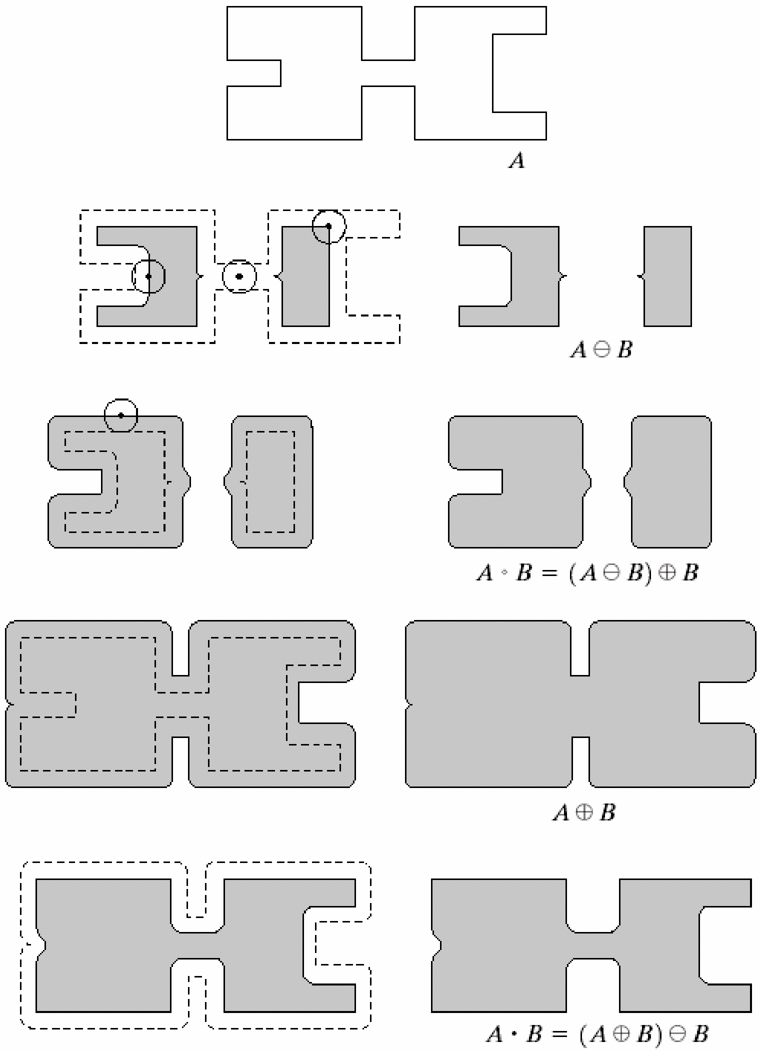
a	
b	c
d	e
f	g
h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



- The background noise was completely eliminated in the erosion stage of opening because in the case all noise components are physically smaller than the structuring element.
- We note in Fig 9.11(d) that the net effect of opening was to eliminate virtually all noise components in both the background and the fingerprint itself.
- However, new gaps between the fingerprint ridges were created .To counter this undesirable effect, we perform a dilation on the opening.

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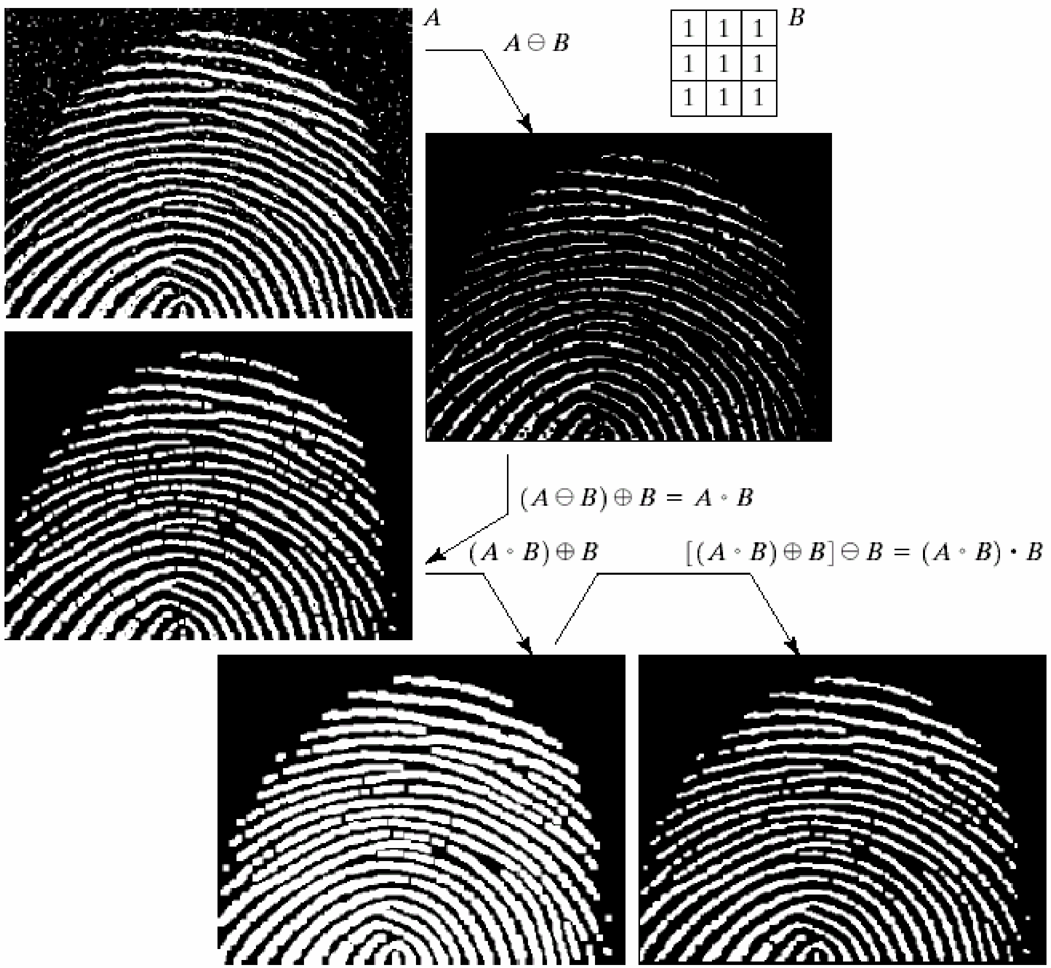

FIGURE 9.11 (a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

9.4 The Hit-or-Miss Transformation

- The morphological hit-or-miss transform is a basic tool for shape detection.
- The objective is to find the location of one of the shapes, say, X.
- $A \Theta X$ may be viewed geometrically as the set of all locations of the origin of X at which X found a match (hit) in A.

- B denotes the set composed of X and its background, the match (or set of matches) of B in A, denoted $A \circledast B$

$$A \circledast B = (A \ominus X) \cap \left[A^c \ominus (W - X) \right] \qquad (9.4-1)$$

$$A \circledast B = (A \ominus B_1) - \left(A \oplus \hat{B}_2 \right) \qquad (9.4-3)$$

- Morphological hit-or-miss transform.

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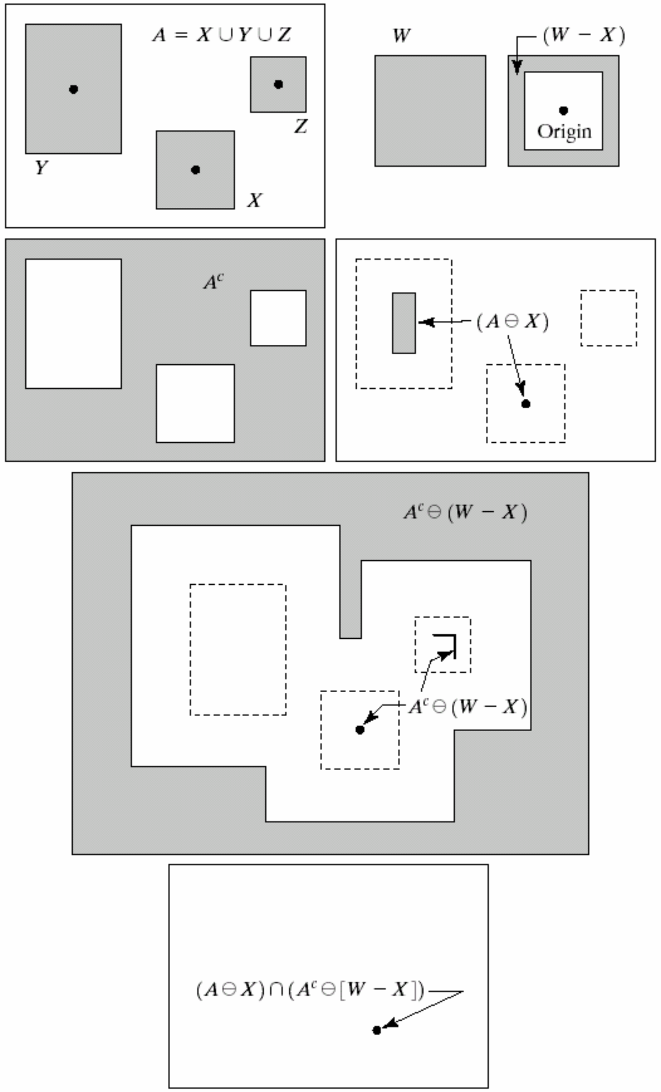

a b
c d
e
f

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$. (c) Complement of A. (d) Erosion of A by X. (e) Erosion of A^c by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

- The reason for using a structuring element B_1 associated with objects and an element B_2 associated with the background is based on an assumed definition that two or more objects are distinct only if they form disjoint (disconnected) sets.

9.5
Some Basic Morphological Algorithms

- We are now ready to consider some practical uses of morphology.
- In particular, we consider morphological algorithms for extracting boundaries, connected components, the convex hull, and the skeleton of a region.
- 9.5.1 Boundary Extraction

$$\beta(A) = A - (A \Theta B) \qquad (9.5-1)$$

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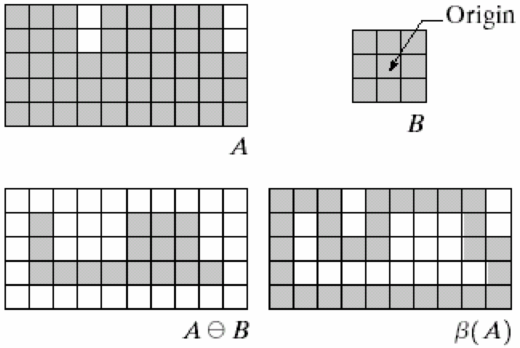

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

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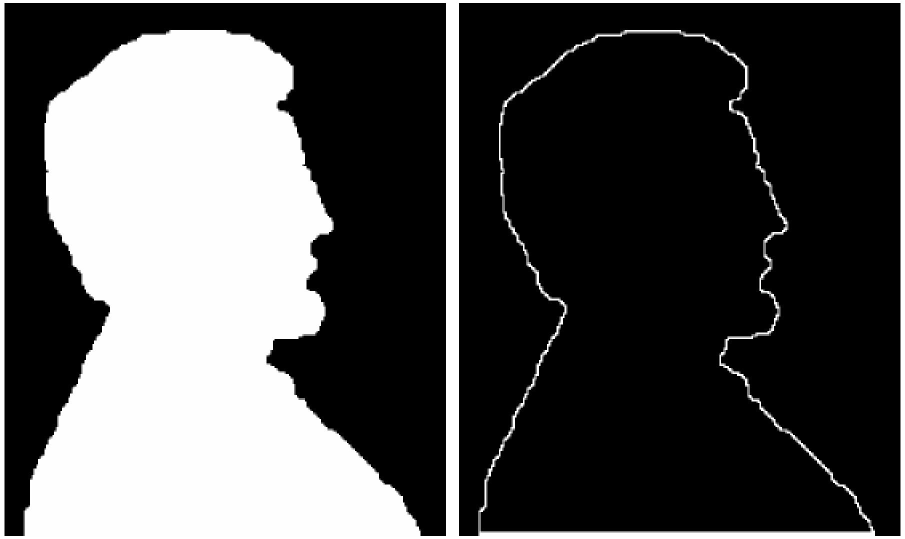

a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

- 9.5.2 Region Filling

$$X_K = (X_{K-1} \oplus B) \cap A^c \quad k = 1, 2, 3 \ldots . (9.5-2)$$

- Conditional dilation

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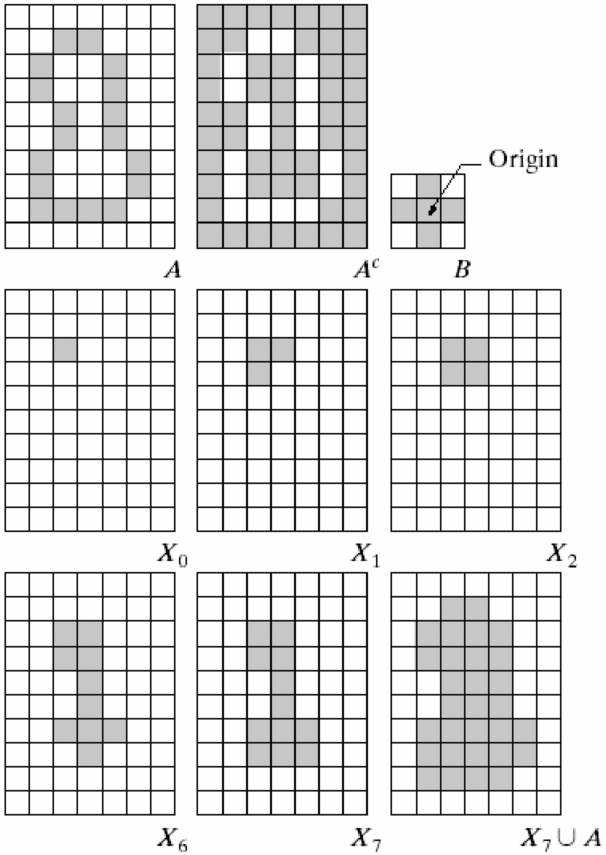

a b c
d e f
g h i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

- 9.5.3 Extraction of Connected Components

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3 \quad (9.5-3)$$

- We can make sure that only objects of "significant" size remain by eroding the thresholded image.

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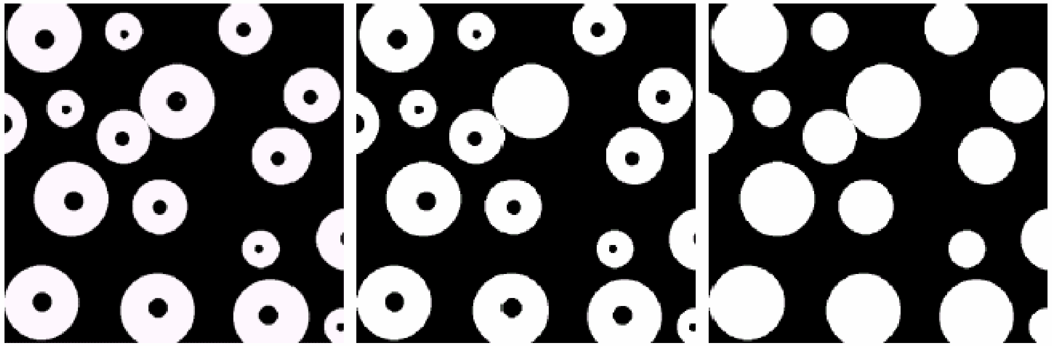

a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

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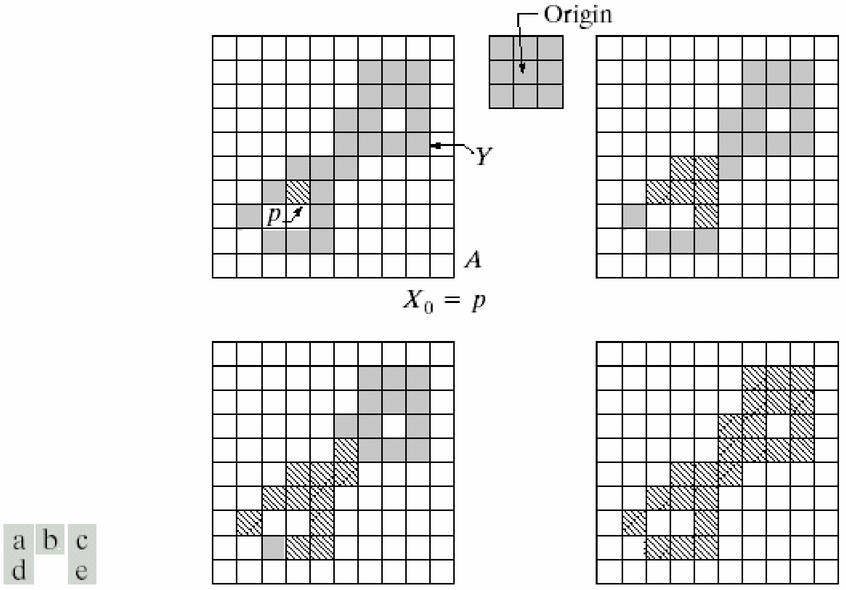

FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

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a
b
c d

FIGURE 9.18
(a) X-ray image of chicken filet with bone fragments.
(b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's.
(d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

- 9.5.4 Convex Hull
- A set A is said to be convex if the straight line segment joining any two points in A lies entirely within A.

$$X_k^i = (X_{k-1} \circledast B^i) \cup A \quad i = 1, 2, 3, 4 \; and \; k = 1, 2, 3 \ldots (9.5-4)$$

- With $X_0^i = A$. Now let $D^i = X_{conv}^i$, where the subscript "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. Then the convex hull of A is

$$C(A) = \bigcup_{i=1}^{4} D^i \qquad (9.5-5)$$

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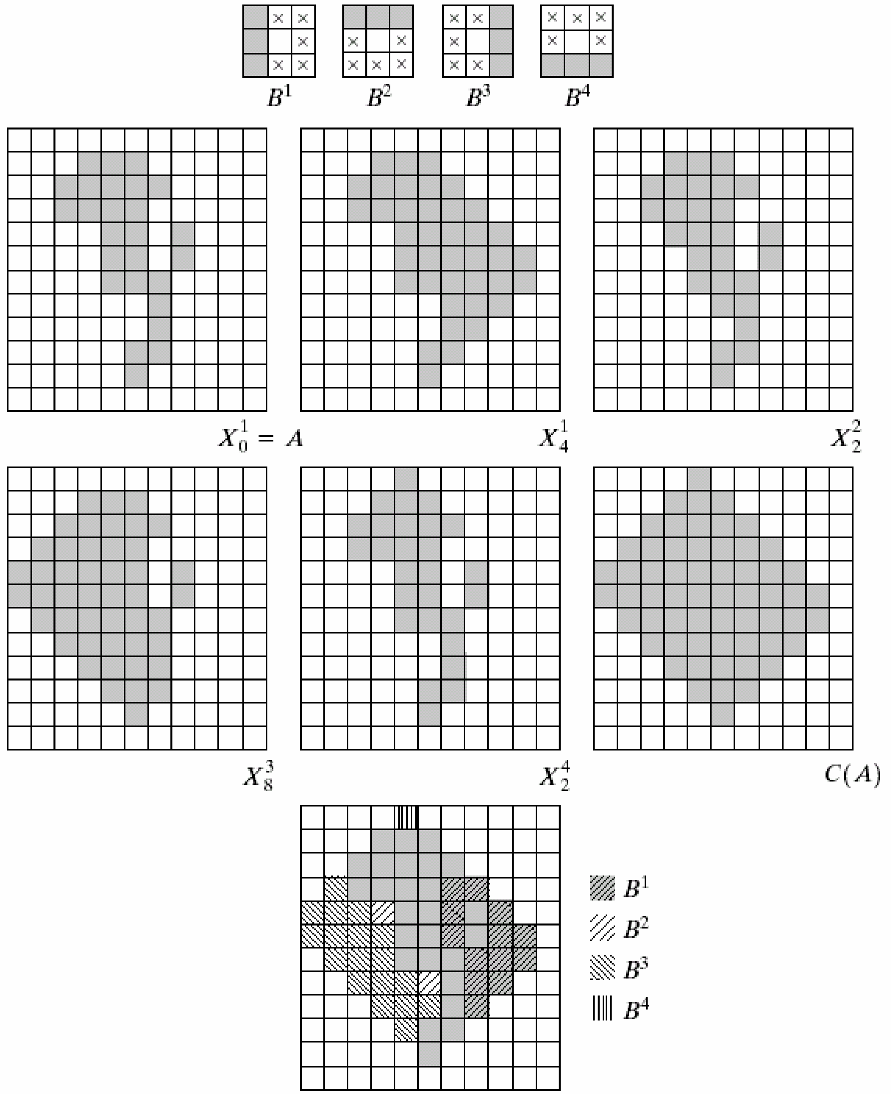

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

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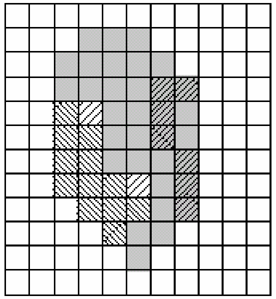


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

- 9.5.5 Thinning

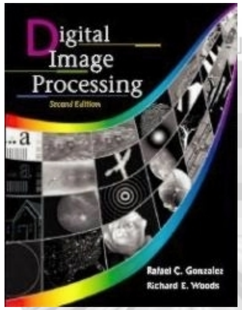
$$A \otimes B = A - (A \circledast B) \qquad (9.5-6)$$

- 9.5.6 Thickening
- Thinning and is defined by the expression

$$A \odot B = A \cup (A \circledast B) \qquad (9.5-9)$$

- As in thinning tickening can be defined as a sequential operation;

$$A \odot \{B\} = \left(\left(\left(\left(A \odot B^1\right) \odot B^2\right) \ldots\right) \odot B^n\right) \qquad (9.5-10)$$



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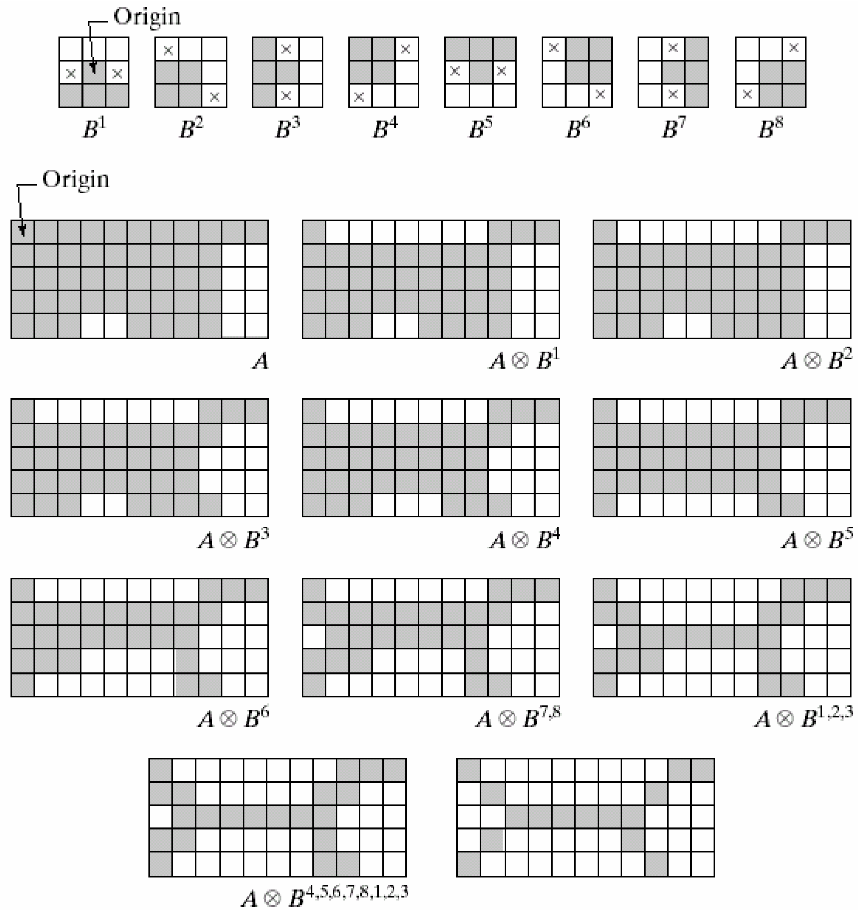

	a	
b	c	d
e	f	g
h	i	j
k	l	

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

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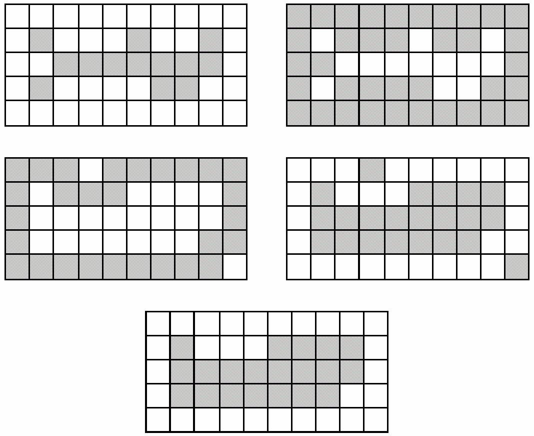

a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

- 9.5.7 Skeletons

$$S(A) = \bigcup_{k=0}^{k} S_k(A) \qquad (9.5-11)$$

$$S_k(A) = (A \Theta kB) - (A \Theta kB) \circ B \qquad (9.5-12)$$

$$(A \Theta kB) = (.....(A \Theta B) \Theta B) \Theta .....) \Theta B \qquad (9.5-13)$$

$$K = \max \left\{ k \,\middle|\, (A \Theta kB) \neq \varnothing \right\} \qquad (9.5-14)$$

- A can be reconstructed from these subsets by using the equation

$$A = \bigcup_{k=0}^{k} (S_k(A) \oplus kB) \qquad (9.5-15)$$

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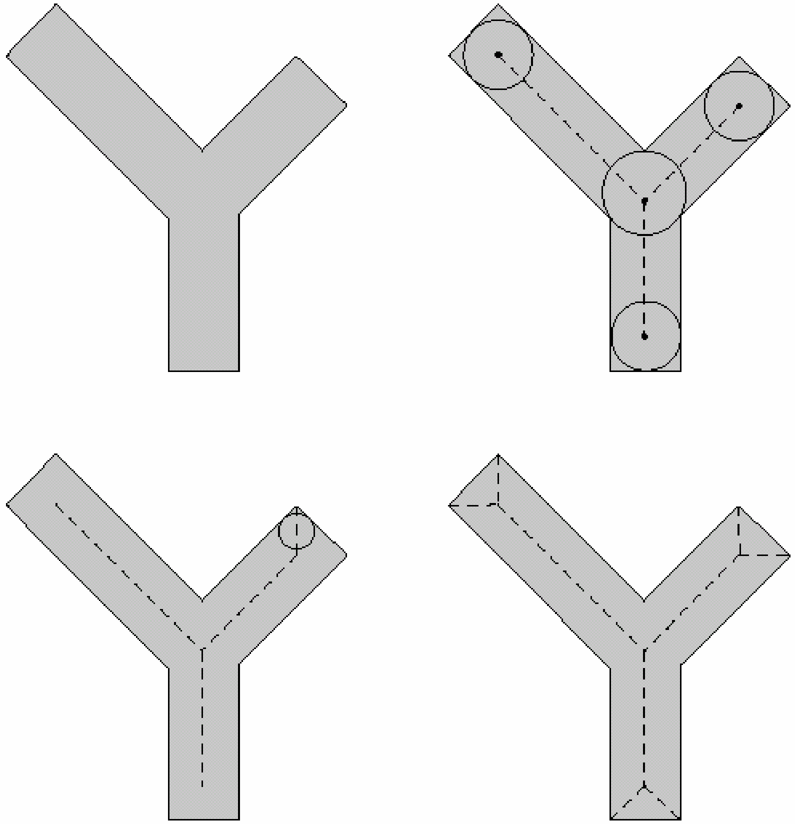

a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.

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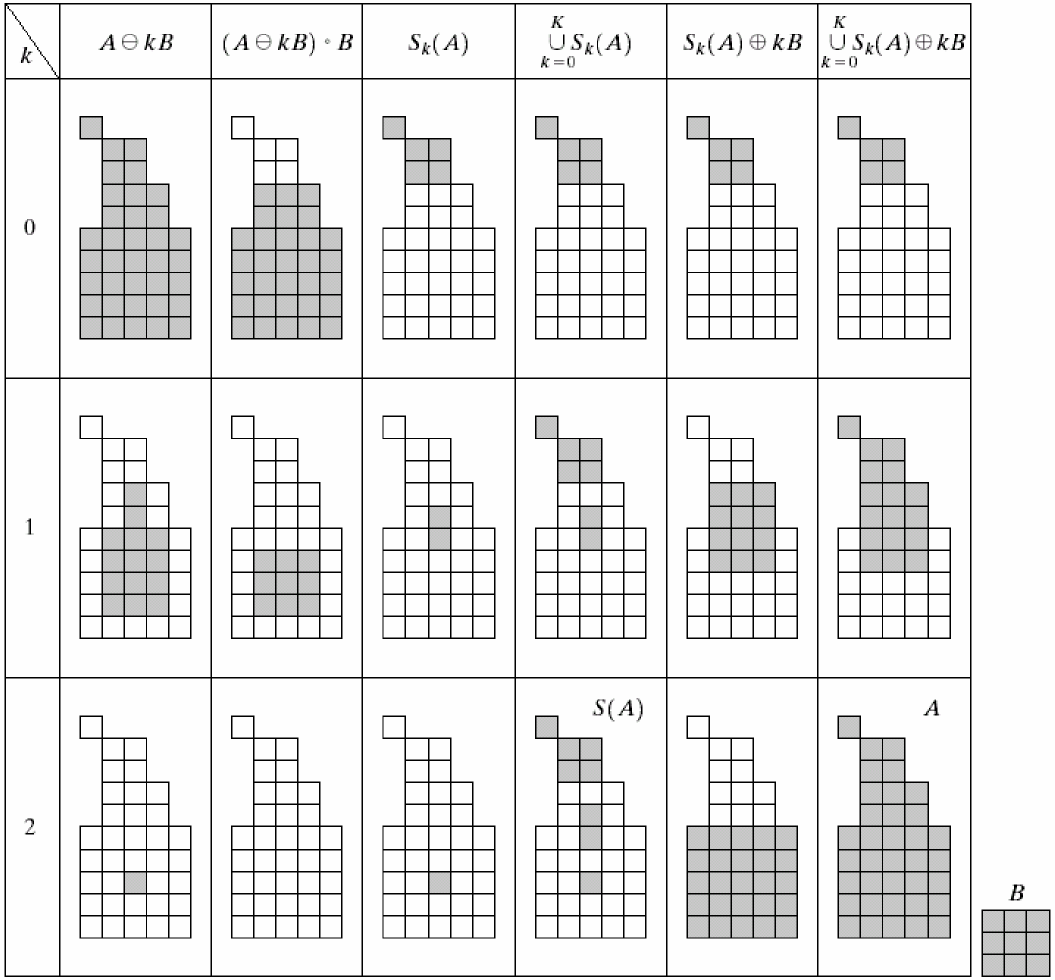

FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

- 9.5.8 Pruning
- Pruning methods are an essential complement to thinning and skeletonizing algorithms because these procedures tend to leave parasitic components that need to be “cleaned up”.

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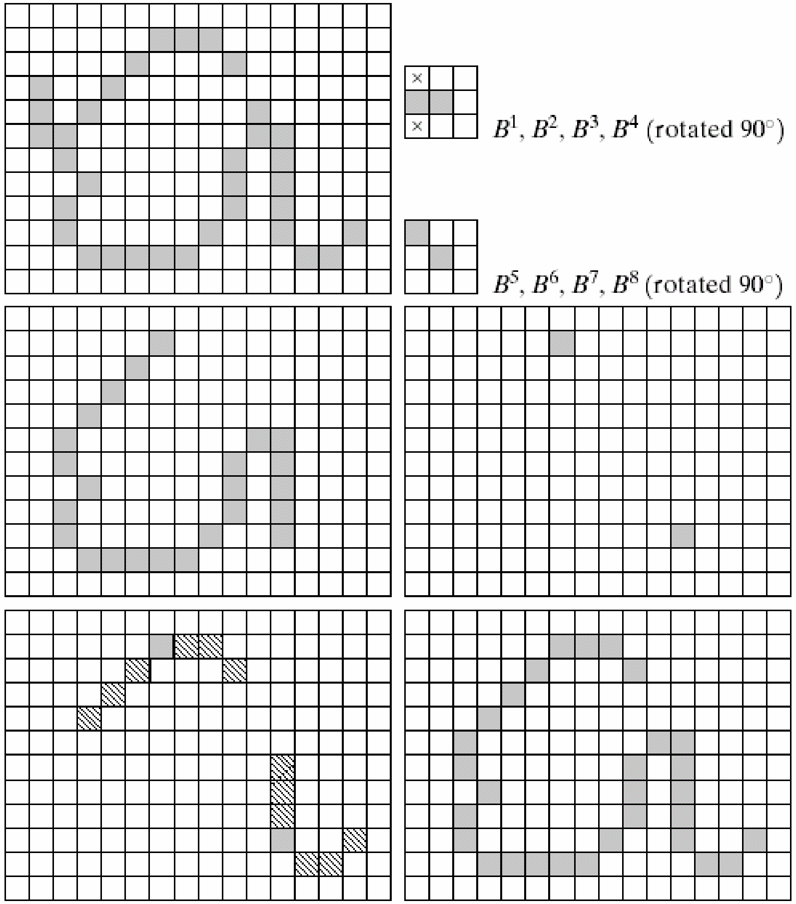

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

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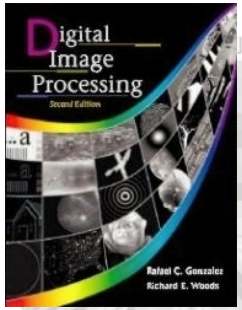
TABLE 9.2
Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

TABLE 9.2
Summary of morphological results and their properties.
(continued)



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Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

TABLE 9.2 Summary of morphological results and their properties. *(continued)*

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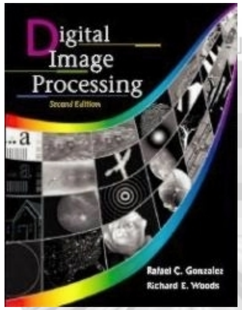
Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

TABLE 9.2
Summary of morphological results and their properties.
(continued)



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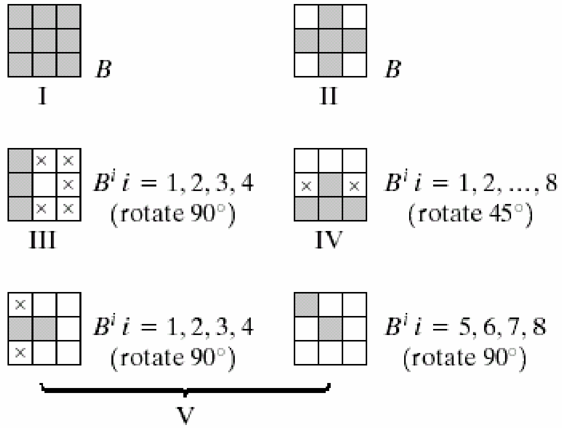

FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

9.6 Extensions to Gray-Scale Image

- 9.6.1 Dilation

$$(f \oplus b)(s,t) = \max\left\{ f(s-x,t-y) + b(x,y) \,\middle|\, \begin{array}{l} (s-x),(t-y) \in D_f; \\ (x,y) \in D_b \end{array} \right\} \quad (9.6-1)$$

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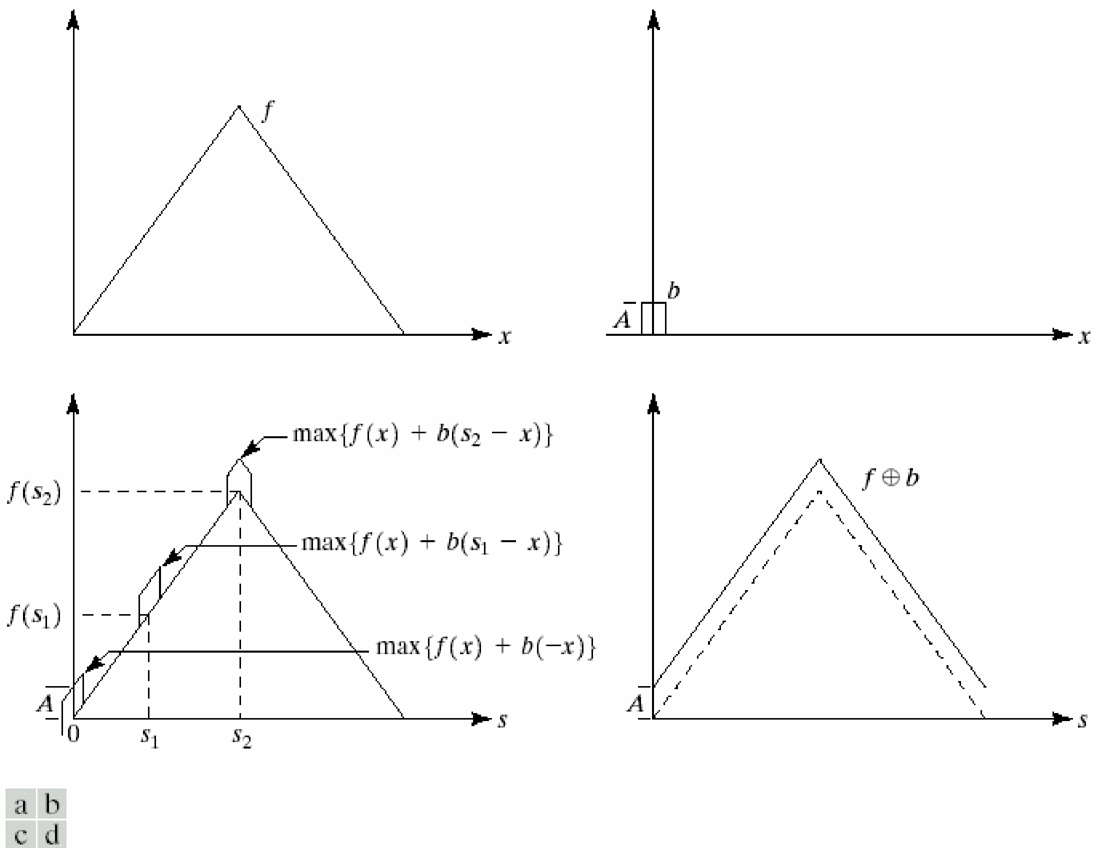

FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

- The general effect of performing dilation on a gray-scale image is twofold:

(1) If all the values of the structuring element are positive, the output image tends to be brighter than the input .

(2)Dark details either are reduced or eliminated, depending on how their values and shapes relate to the structuring element used for dilation.

- 9.6.2 Erosion

$$(f \ominus b)(s,t) = \min \left\{ f(s+x, t+y) - b(x,y) \;\middle|\; \begin{matrix} (s+x),(t+y) \in D_f; \\ (x,y) \in D_b \end{matrix} \right\} \quad (9.6-2)$$

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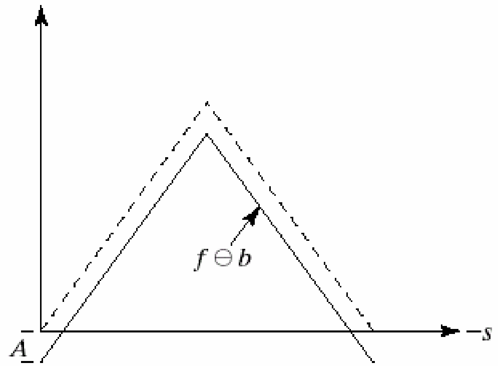

FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).

- The general effect of performing erosion on a gray-scale image is twofold:

(1) If all the elements of the structuring element are positive, the output image tends to be darker than the input image.

(2) The effect of bright details in the input image that are smaller in area than the structuring element is reduced.

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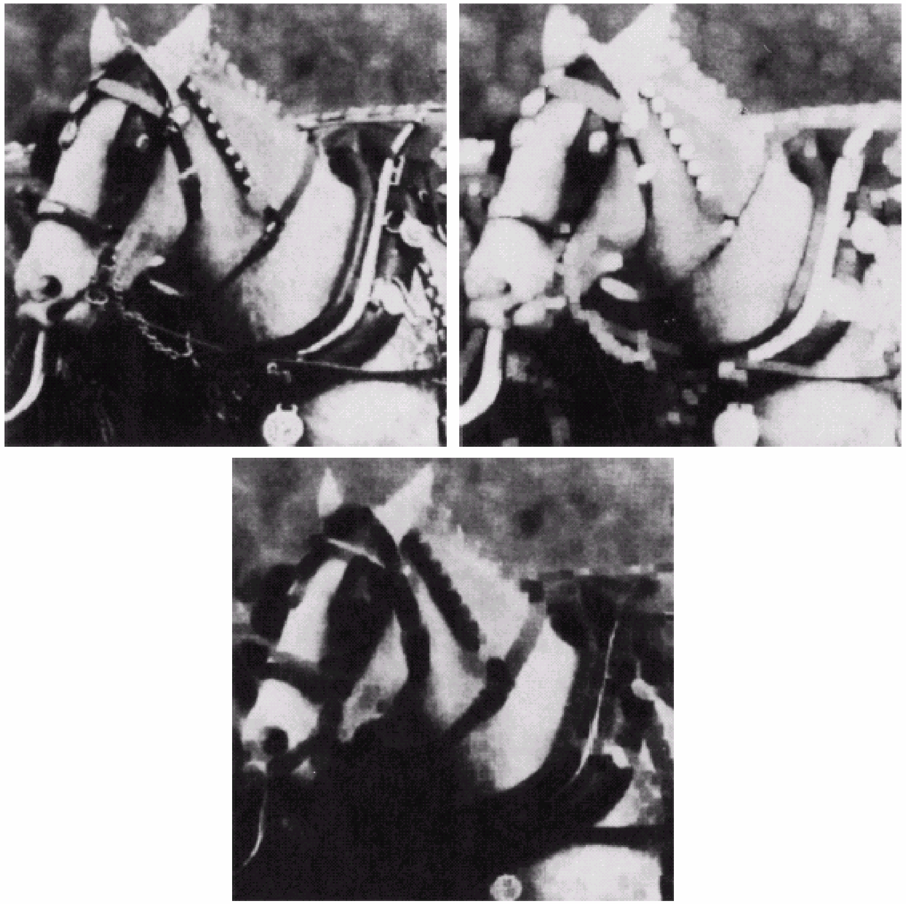

a b
c

FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

- 9.6.3 Opening and Closing

$$f \circ b = (f \ominus b) \oplus b \qquad (9.6-4)$$

$$f \bullet b = (f \oplus b) \ominus b \qquad (9.6-5)$$

$$(f \bullet b)^c = f^c \circ \hat{b} \qquad (9.6-6)$$

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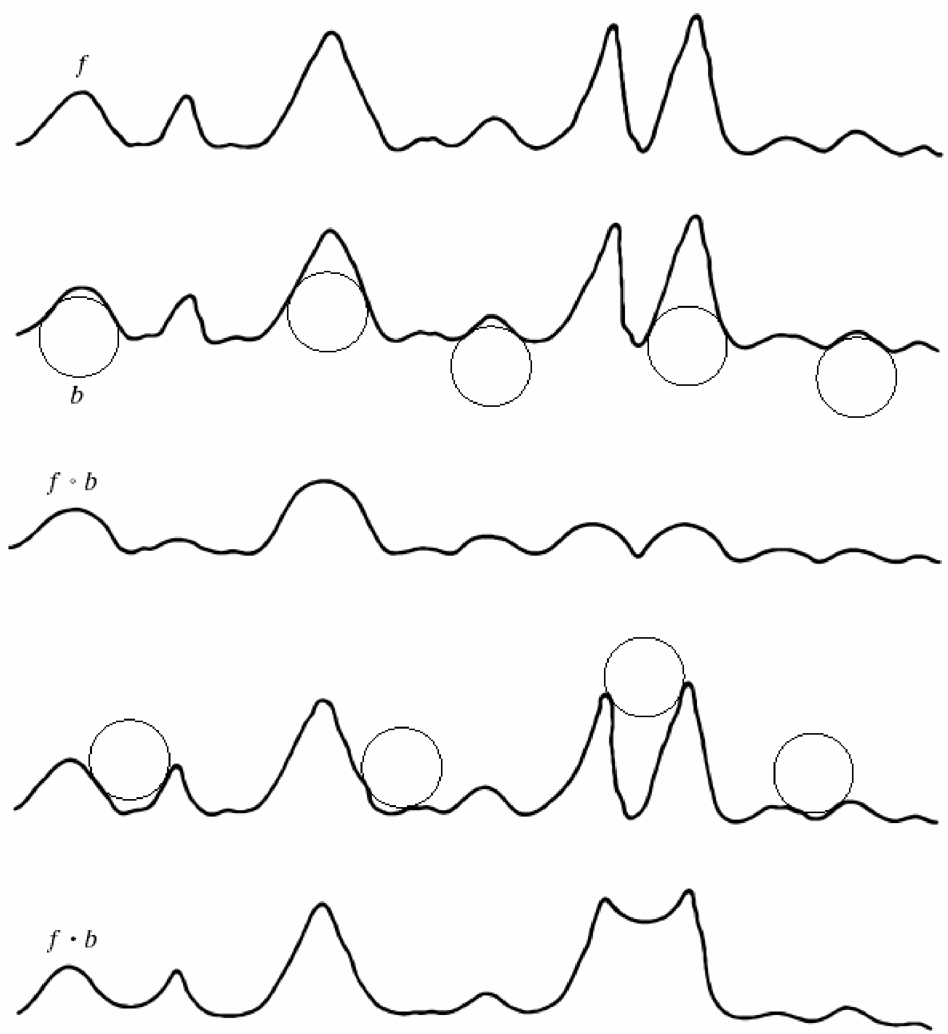

FIGURE 9.30 (a) A gray-scale scan line. (b) Positions of rolling ball for opening. (c) Result of opening. (d) Positions of rolling ball for closing. (e) Result of closing.

- The gray-scale opening operation satisfies the following properties:

$$(1)(f \circ b) \dashv f.$$

$$(2) If \ \ f_1 \dashv f_2, then (f_1 \circ b) \dashv (f_2 \circ b)$$

$$(3)(f \circ b) \circ b = f \circ b$$

- The closing operation satisfies the following properties:

$$(1)\ (f \bullet b) \sqsupseteq f.$$

$$(2)\ If\ \ f_1 \sqsubseteq f_2, then\ (f_1 \bullet b) \sqsubseteq (f_2 \bullet b)$$

$$(3)\ (f \bullet b) \bullet b = f \bullet b$$

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a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

- 9.6.4 Some Applications of Gray-Scale Morphology
- Morphological smoothing
- Morphological gradient
- Top-hat transformation
- Textural segmentation

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FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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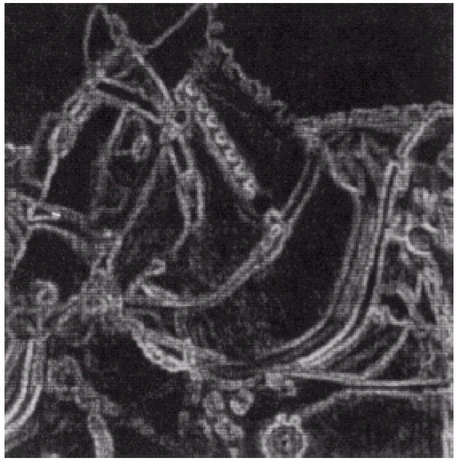

FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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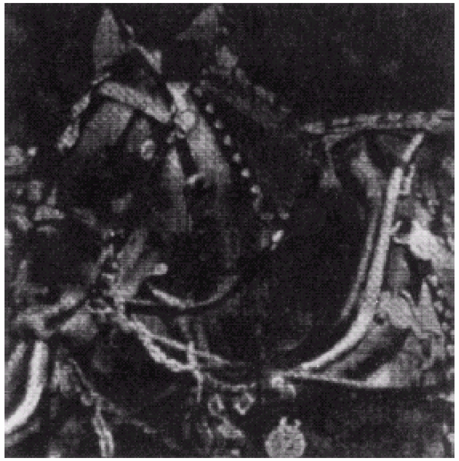

FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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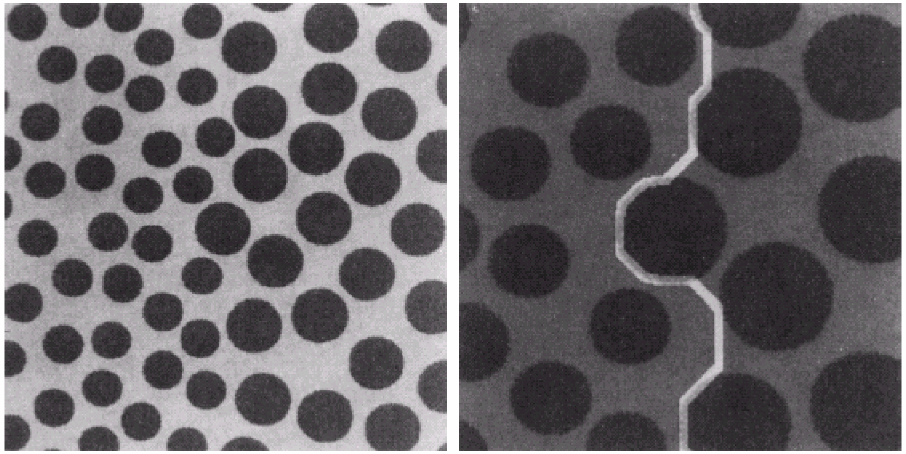

a b

FIGURE 9.35 (a) Original image. (b) Image showing boundary between regions of different texture. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

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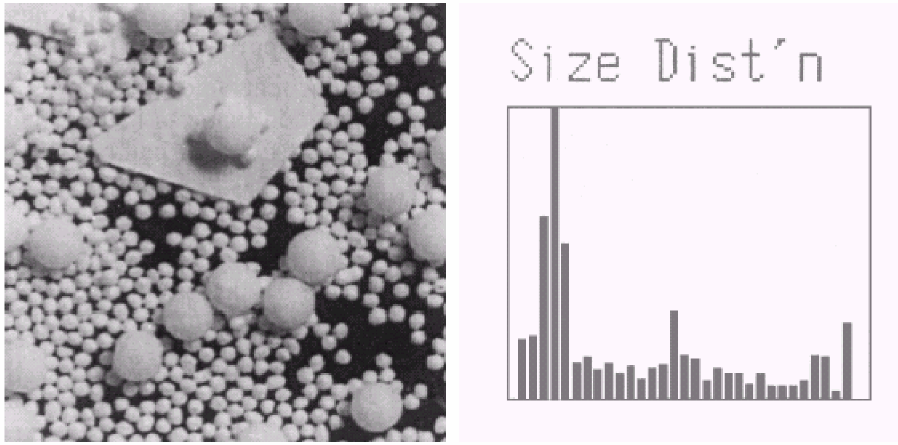

a b

FIGURE 9.36 (a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)