Appendix

Proof of Lemma 1: Denote L_1 and L_2 the Lagrange functions of operators 1 and 2 in problem (9) with

$$\begin{aligned} L_1 &= (1 - q_1 - bq_2)q_1 - (c_1 + r)q_1 - f + \lambda_1(q_1 - \delta) \text{ and} \\ L_2 &= (1 - q_2 - bq_1)q_2 - (c_2 + r)q_2 - f + \lambda_2(q_2 - \delta), \end{aligned}$$

where λ_1 and λ_2 are the respective Lagrange multipliers of operators 1 and 2. Then, the Kuhn-Tucker conditions for operator 1 are

$$\frac{\partial L_1}{\partial q_1} = 1 - 2q_1 - bq_2 - c_1 - r + \lambda_1 \leq 0,\ q_1\frac{\partial L_1}{\partial q_1} = 0 \text{ and} \tag{A1}$$

$$\frac{\partial L_1}{\partial \lambda_1} = q_1 - \delta \geq 0,\ \lambda_1\frac{\partial L_1}{\partial \lambda_1} = 0; \tag{A2}$$

and for operator 2 are

$$\frac{\partial L_2}{\partial q_2} = 1 - 2q_2 - bq_1 - c_2 - r + \lambda_2 \leq 0,\ q_2\frac{\partial L_2}{\partial q_2} = 0 \text{ and} \tag{A3}$$

$$\frac{\partial L_2}{\partial \lambda_2} = q_2 - \delta \geq 0,\ \lambda_2\frac{\partial L_2}{\partial \lambda_2} = 0. \tag{A4}$$

If $\frac{\partial L_1}{\partial q_1} < 0$, we have $q_1^* = 0$ by (A1). Then $q_1^* \geq \delta$ will not hold unless $\delta = 0$. Since this is not an interesting solution, we focus on solution q_1^*, which satisfies $\frac{\partial L_1}{\partial q_1} = 0$ in (A1). Similarly, we focus on solution q_2^*, which satisfies $\frac{\partial L_2}{\partial q_2} = 0$ in (A3). Based on the values of λ_1 and λ_2, there are four cases as follows.

Case 1: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then (A1) and (A3) suggest

$$\begin{aligned} &1 - 2q_1 - bq_2 - c_1 - r = 0 \text{ and} \\ &1 - 2q_2 - bq_1 - c_2 - r = 0. \end{aligned}$$

Solving these equations yields $q_1^* = \frac{1-r}{2+b} + \frac{bc_2-2c_1}{4-b^2}$ and $q_2^* = \frac{1-r}{2+b} + \frac{bc_1-2c_2}{4-b^2}$. The conditions in (12) imply non-negative q_i^* for $i = 1,\ 2$. To guarantee $q_1^* \geq \delta$ and $q_2^* \geq \delta$, condition

$0 \leq \delta \leq \delta_1 \equiv \frac{1-r}{2+b} + \frac{bc_1-2c_2}{4-b^2} = q_2^*$ should be met, because $c_1 < c_2$ implies $q_1^* \geq q_2^*$ and $q_2^* \geq \delta$ implies $q_1^* \geq \delta$. Substituting q_1^* and q_2^* into (1)-(2) yields $p_2^* = \frac{1+r(1+b)}{2+b} + \frac{[c_2(2-b^2)+c_1b]}{4-b^2} > p_1^* = \frac{1+r(1+b)}{2+b} + \frac{[c_1(2-b^2)+c_2b]}{4-b^2} > 0$, and into (4) yields $\pi_i^* = (q_i^*)^2 - f$ for $i = 1,\ 2$. These prove Lemma 1(i).

Case 2: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then, (A1), (A3), and (A4) suggest

$$(1-2q_1-bq_2-c_1-r)=0,\ (q_2-\delta)=0,\ \text{and}\ (1-2q_2-bq_1-c_2-r+\lambda_2)=0.$$

They in turn imply $q_1^* = \frac{1-r-c_1-b\delta}{2}$, $q_2^* = \delta$, and $\lambda_2^* = \frac{(4-b^2)(\delta-\delta_1)}{2}$. To guarantee $\lambda_2^* > 0$, conditions $\delta > \delta_1 \equiv \frac{1+c_1-2c_2-r}{3}$, $r \leq \bar{r}$, and $c_2 \leq \bar{c}_2$ are needed. On the other hand, to have $q_1^* \geq \delta$, conditions $\delta \leq \delta_2 \equiv \frac{1-c_1-r}{2+b}$ and $r \leq (1-c_1)$ are needed. Thus, the plausible range for δ is $\delta \in (\delta_1,\ \delta_2]$. Substituting q_1^* and q_2^* into (1)-(2) produces $p_2^* = \frac{1}{2}[(2-b)-(2-b^2)\delta + bc_1 + br] > p_1^* = \frac{1}{2}(1-b\delta+c_1+r) > 0$ if $\delta \leq \delta_2$, and into (4) gives $\pi_1^* = (q_1^*)^2 - f$ and $\pi_2^* = \frac{\delta}{2}[(2-b)(1-r)-(2-b^2)\delta + bc_1 - 2c_2] - f$. These prove Lemma 1(ii).

Case 3: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then (A1)-(A3) suggest

$$(q_1-\delta)=0,\ (1-2q_2-bq_1-c_2-r)=0,\ \text{and}\ (1-2q_1-bq_2-c_1-r+\lambda_1)=0.$$

Solving these equations yields

$$q_1^* = \delta,\ q_2^* = \frac{1-r-c_2-b\delta}{2},\ \text{and}\ \lambda_1^* = \frac{(4-b^2)}{2}[\delta - (\frac{1-r}{2+b} + \frac{bc_2-2c_1}{4-b^2})].$$

To guarantee $\lambda_1^* > 0$, conditions

$$\delta > \frac{1-r}{2+b} + \frac{bc_2-2c_1}{4-b^2}\ \text{and}\ r \leq 1 - \frac{2c_1-bc_2}{2-b} \qquad (A5)$$

are needed. On the other hand, $q_2^* \geq \delta$ is guaranteed by assuming

$$\delta \leq \frac{1-c_2-r}{2+b}. \qquad (A6)$$

However, (A5) and (A6) are incompatible with each other because $\frac{1-c_2-r}{2+b} - (\frac{1-r}{2+b} + \frac{bc_2-2c_1}{4-b^2}) = \frac{-2(c_2-c_1)}{4-b^2} < 0$. Thus, no solution exists in this case.

Case 4: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. Then, (A2) and (A4) suggest $q_1^* = \delta$ and $q_2^* = \delta$, and (A1) and (A3) imply $\lambda_1^* = -1 + (2+b)\delta + c_1 + r$, and $\lambda_2^* = -1 + (2+b)\delta + c_2 + r$. To have $\lambda_1^* > 0$ and $\lambda_2^* > 0$, conditions $\delta > \delta_2 \equiv \frac{1-c_1-r}{2+b}$ and $r \leq (1-c_1)$ are needed. Note that $r \leq (1-c_1)$ is inferred from $r \leq \bar{r} \equiv \frac{(2-b)+bc_1-2c_2}{2-b}$. Substituting $q_1^* = q_2^* = \delta$ into (1)-(2) produces $p_1^* = p_2^* = 1-(1+b)\delta > 0$ if $\delta < \frac{1}{1+b}$, and into (4) gives $\pi_i^* = \delta[1-(1+b)\delta - c_i - r] - f$ for $i = 1,\ 2$. These prove Lemma 1(iii). □

Proof of Lemma 2: The proofs are similar to those of Lemma 1, and thus omitted. □

Proof of Lemma 3: The proofs are similar to those of Lemma 1, and thus omitted. □

Proof of Lemma 4: Lemma 1 shows that operators' optimal choices depend on the values of δ. Thus, we have the following cases.

Case 1: Suppose $\delta \in [0,\ \delta_1]$. Then Lemma 1(i) implies $\pi_1^* > \pi_2^*$, and $f^* = \pi_2^* = \frac{1}{2}(q_2^*)^2 > 0$. The problem in (15) thus becomes

$$\max_{r,\ f,\ \delta}\ 2f + r(q_1^* + q_2^*)$$
$$\text{s.t.}\quad 0 \leq \delta \leq \delta_1 \text{ and } 0 < r \leq \bar{r}.$$

Its Lagrange function is

$$L = (q_2^*)^2 + r(q_1^* + q_2^*) + \lambda_1(\delta_1 - \delta) + \lambda_2(\bar{r} - r),$$

where λ_1 and λ_2 are the Lagrange multipliers associated with the inequality constraints. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = 2q_2^* \frac{\partial q_2^*}{\partial r} + r\left(\frac{\partial q_1^*}{\partial r} + \frac{\partial q_2^*}{\partial r}\right) + (q_1^* + q_2^*) + \lambda_1 \frac{\partial \delta_1}{\partial r} - \lambda_2 \leq 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \qquad (A7)$$

$$\frac{\partial L}{\partial \delta} = -\lambda_1 \leq 0,\ \delta \frac{\partial L}{\partial \delta} = 0, \qquad (A8)$$

$$\frac{\partial L}{\partial \lambda_1} = \delta_1 - \delta \geq 0,\ \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \text{ and} \qquad (A9)$$

$$\frac{\partial L}{\partial \lambda_2} = \bar{r} - r \geq 0,\ \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0. \qquad (A10)$$

Based on the values of λ_1 and λ_2, we have four sub-cases.

Case 1a: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then, (A7) becomes $\frac{1}{(2+b)^2}[(2+2b) - r(4b+6) - \frac{(4+2b-b^2)c_1}{(2-b)} + \frac{b^2c_2}{(2-b)}] = 0$, which implies $r^* = \frac{[(2+2b)+\frac{b^2c_2}{(2-b)}-\frac{(4+2b-b^2)c_1}{(2-b)}]}{(4b+6)} > 0$. It remains to check whether $r^* < \bar{r}$ holds. By some calculations, we have $r^* < \bar{r}$ iff $c_2 < \hat{c}_2 \equiv \frac{[2(2-b)+(2+3b)c_1]}{(b+6)}$. On the other hand, (A9) implies both $\delta^* \in [0,\ \delta_1]$ with $\delta_1 = \frac{1-r^*}{2+b} + \frac{bc_1-2c_2}{4-b^2} = \frac{2(2-b)+(2+3b)c_1-(6+b)c_2}{2(2-b)(3+2b)}$ and $f^* = \frac{1}{2}[\frac{2(2-b)+(2+3b)c_1-(6+b)c_2}{2(2-b)(3+2b)}]^2 > 0$. Thus, at the equilibrium, port authority's fee revenue equals

$$R^* = [\frac{2(2-b)+(2+3b)c_1-(6+b)c_2}{2(2-b)(3+2b)}]^2 + r^*[\frac{2(1-r^*)-(c_1+c_2)}{2+b}]. \qquad (A11)$$

Case 1b: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then, (A10) suggests $r^* = \bar{r} \equiv \frac{[(2-b)+bc_1-2c_2]}{(2-b)}$. However, at r^*, we have $f^* = 0$ due to $q_2^* = 0$, which contradicts the requirement of positive fixed fee. Thus, no solution exists in this case.

Case 1c: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then, (A9) suggests $\delta^* = \delta_1 > 0$. This in turn implies $\lambda_1^* = 0$ by (A8). It is a contradiction. Thus, no solution exists in this case.

Case 1d: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. As in Case 1b, we have $f^* = 0$. Again, no solution exists here.

Case 2: Suppose $\delta \in (\delta_1,\ \delta_2]$. Then Lemma 1(ii) implies $\pi_1^* > \pi_2^*$, and $f^* = \pi_2^* = \frac{\delta}{4}[(2-b)(1-r) - (2-b^2)\delta + bc_1 - 2c_2]$. We have $f^* > 0$ iff $\delta < \frac{(2-b)(1-r)+bc_1-2c_2}{(2-b^2)}$ and $r \leq \bar{r} \equiv \frac{(2-b)+bc_1-2c_2}{2-b}$. In addition, $\frac{(2-b)(1-r)+bc_1-2c_2}{(2-b^2)} > (\leq)\ \delta_2$ iff $r < (\geq)\ 1+(1+b)c_1-(2+b)c_2$. Thus, we have two sub-cases as follows.

Case 2a: Suppose $r \geq [1+(1+b)c_1-(2+b)c_2]$. Then, the problem in (15) becomes

$$\max_{r,\, f,\, \delta}\ 2f + r(q_1^* + q_2^*)$$

$$\text{s.t. } \delta_1 < \delta < \frac{(2-b)(1-r)+bc_1-2c_2}{(2-b^2)} \text{ and } [1+(1+b)c_1-(2+b)c_2] \leq r < \bar{r}. \quad (A12)$$

Its Lagrange function is

$$\begin{aligned} L &= \frac{\delta}{2}[(2-b)(1-r)-(2-b^2)\delta+bc_1-2c_2]+\frac{r}{2}[1+(2-b)\delta-c_1-r]+\lambda_1(\delta-\delta_1) \\ &\quad +\lambda_2[\frac{(2-b)(1-r)+bc_1-2c_2}{(2-b^2)}-\delta]+\lambda_3\{r-[1+(1+b)c_1-(2+b)c_2]\}+\lambda_4(\bar{r}-r). \end{aligned}$$

Then, the Kuhn-Tucker conditions are

$$\begin{aligned} \frac{\partial L}{\partial r} &= \frac{1}{2}(1-2r-c_1)+\frac{1}{2+b}\lambda_1-\frac{2-b}{(2-b^2)}\lambda_2+\lambda_3-\lambda_4 \leq 0,\ r\cdot\frac{\partial L}{\partial r}=0, \\ \frac{\partial L}{\partial \delta} &= \frac{1}{2}[(2-b)-2(2-b^2)\delta+bc_1-2c_2]+\lambda_1-\lambda_2 \leq 0,\ \delta\cdot\frac{\partial L}{\partial \delta}=0, \\ \frac{\partial L}{\partial \lambda_1} &= \delta-\delta_1 \geq 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1}=0, \\ \frac{\partial L}{\partial \lambda_2} &= \frac{(2-b)(1-r)+bc_1-2c_2}{(2-b^2)}-\delta \geq 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2}=0, \\ \frac{\partial L}{\partial \lambda_3} &= r-[1+(1+b)c_1-(2+b)c_2] \geq 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3}=0,\ \text{and} \\ \frac{\partial L}{\partial \lambda_4} &= \bar{r}-r \geq 0,\ \lambda_4\cdot\frac{\partial L}{\partial \lambda_4}=0, \end{aligned}$$

where $\lambda_1,\ \lambda_2,\ \lambda_3$, and λ_4 are the Lagrange multipliers for the four constraints in (A12). Since three of the constraints are strict inequalities, we must have $\lambda_1^*=\lambda_2^*=\lambda_4^*=0$. If $\lambda_3^*=0$, we have $\delta^*=\frac{(2-b)+bc_1-2c_2}{2(2-b^2)}$. By some calculations, we have $\frac{(2-b)(1-r^*)+bc_1-2c_2}{(2-b^2)}-\frac{(2-b)+bc_1-2c_2}{2(2-b^2)}=\frac{2(c_1-c_2)}{2(2-b^2)}<0$, which contradicts $\delta^*<\frac{(2-b)(1-r^*)+bc_1-2c_2}{(2-b^2)}$ required by problem (A12). Thus, no solution exists in this case. By contrast, if $\lambda_3^*>0$, we have $r^*=1+(1+b)c_1-(2+b)c_2,\ \delta^*=\frac{(2-b)+bc_1-2c_2}{2(2-b^2)}$, and $\lambda_3^*=\frac{1}{2}[1+(3+2b)c_1-2(2+b)c_2]$. Note that $\lambda_3^*>0$ iff $c_2<\frac{1+(3+2b)c_1}{2(2+b)}$. On the other hand, $\frac{\partial L}{\partial \lambda_2}\geq 0$ requires $\frac{(2-b)(1-r^*)+bc_1-2c_2}{(2-b^2)}-\delta^*\geq 0$. By some calculations, we have $\frac{(2-b)(1-r^*)+bc_1-2c_2}{(2-b^2)}-\delta^*=\frac{-(2-b)-(4+b-2b^2)c_1+2(3-b^2)c_2}{2(2-b^2)}\geq 0$ iff $c_2\geq\frac{(2-b)+(4+b-2b^2)c_1}{2(3-b^2)}$, which contradicts $c_2<\frac{1+(3+2b)c_1}{2(2+b)}$ required by $\lambda_3^*>0$. That is because $\frac{(2-b)+(4+b-2b^2)c_1}{2(3-b^2)}>\frac{1+(3+2b)c_1}{2(2+b)}$. Thus, no solution exists in this case.

<u>Case 2b</u>: Suppose $r<[1+(1+b)c_1-(2+b)c_2]$. Then the problem in (15) becomes

$$\max_{r,\,\delta}\ 2f^*+r(q_1^*+q_2^*)$$

$$\text{s.t. } \delta_1<\delta\leq\delta_2 \text{ and } 0<r<[1+(1+b)c_1-(2+b)c_2]. \qquad (A13)$$

Its Lagrange function is

$$\begin{aligned} L &= \frac{\delta}{2}[(2-b)(1-r)-(2-b^2)\delta+bc_1-2c_2]+\frac{r}{2}[1+(2-b)\delta-c_1-r] \\ &\quad +\lambda_1(\delta-\delta_1)+\lambda_2(\delta_2-\delta)+\lambda_3[1+(1+b)c_1-(2+b)c_2-r]. \end{aligned}$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r}=\frac{1}{2}(1-2r-c_1)+\frac{1}{2+b}\lambda_1-\frac{1}{2+b}\lambda_2-\lambda_3\leq 0,\ r\cdot\frac{\partial L}{\partial r}=0, \tag{A14}$$

$$\frac{\partial L}{\partial \delta}=\frac{1}{2}[(2-b)-2(2-b^2)\delta+bc_1-2c_2]+\lambda_1-\lambda_2\leq 0,\ \delta\cdot\frac{\partial L}{\partial \delta}=0, \tag{A15}$$

$$\frac{\partial L}{\partial \lambda_1}=\delta-\delta_1\geq 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1}=0, \tag{A16}$$

$$\frac{\partial L}{\partial \lambda_2}=\delta_2-\delta\geq 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2}=0,\ \text{and} \tag{A17}$$

$$\frac{\partial L}{\partial \lambda_3}=1+(1+b)c_1-(2+b)c_2-r\geq 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3}=0, \tag{A18}$$

where $\lambda_1,\ \lambda_2$, and λ_3 are the Lagrange multipliers for the three constraints in (A13). Constraints $\delta_1 < \delta$ and $r < [1+(1+b)c_1-(2+b)c_2]$ suggest $\lambda_1^* = \lambda_3^* = 0$ by (A16) and (A18). If $\lambda_2^* > 0$, (A14), (A15) and (A17) suggest $\frac{(1-2r-c_1)}{2} - \frac{\lambda_2}{2+b} = 0$, $\frac{[(2-b)-2(2-b^2)\delta+bc_1-2c_2]}{2} - \lambda_2 = 0$, and $(\delta_2-\delta) = 0$. Solving these equations yields $r^* = \frac{[2(1+b)-(4+3b)c_1+(2+b)c_2]}{2(3+2b)} > 0$, $\delta^* = \frac{(2-c_1-c_2)}{2(3+2b)}$, and $\lambda_2^* = \frac{(2+b)[1+(1+b)c_1-(2+b)c_2]}{2(3+2b)}$. By some calculations, we have $(\delta^*-\delta_1) = \frac{2(c_2-c_1)}{4-b^2} > 0$, and $\lambda_2^* > 0$ iff $c_2 < \frac{1+(1+b)c_1}{2+b}$. On the other hand, we have $[r^*-1+(1+b)c_1-(2+b)c_2] = \frac{(2+b)}{2(3+2b)}[-2-(5+4b)c_1+(7+4b)c_2]$. Thus , we get $r^* < 1+(1+b)c_1-(2+b)c_2$ iff $c_2 < \dot{c}_2 \equiv \frac{2+(5+4b)c_1}{(7+4b)}$.

By contrast, if $\lambda_2^* = 0$, we have $r^* = \frac{(1-c_1)}{2}$ and $\delta^* = \frac{(2-b)+bc_1-2c_2}{2(2-b^2)}$ by (A15) and (A16). Note that $\delta^* \leq \delta_2$ iff $c_2 > \frac{1+(1+b)c_1}{2+b}$. It in turn implies $[1+(1+b)c_1-(2+b)c_2] < 0 < r^*$, which contradicts $r^* < [1+(1+b)c_1-(2+b)c_2]$. Thus, no solution exists in this case.

In sum, if $c_2 \leq \dot{c}_2 \equiv \frac{2+(5+4b)c_1}{7+4b}$, an equilibrium with $r^* = \frac{[2(1+b)-(4+3b)c_1+(2+b)c_2]}{2(3+2b)} > 0$ and $\delta^* = \frac{(2-c_1-c_2)}{2(3+2b)}$ exists. At the equilibrium, port authority's fee revenues equals

$$R^* = \frac{(2-c_1-c_2)^2}{4(3+2b)}. \tag{A19}$$

Case 3: Suppose $\delta \in (\delta_2,\ \frac{1}{(1+b)})$. Then, Lemma 1(iii) implies $\pi_1^* > \pi_2^*$, and $f^* = \pi_2^* = \frac{1}{2}[1-(1+b)\delta - c_2 - r]\delta$ with $f^* > 0$ iff $\delta < \frac{1-c_2-r}{(1+b)}$ and $r < (1-c_2)$. Note that $r < (1-c_2)$ is implied by $r \leq \bar{r} = \frac{(2-b)+bc_1-2c_2}{2-b}$. Accordingly, the problem in (15) becomes

$$\max_{r,\ \delta}\ \delta[1-(1+b)\delta - c_2 - r] + 2r\delta$$

$$\text{s.t. } \delta_2 < \delta < \frac{1-c_2-r}{(1+b)} \text{ and } r < \bar{r}. \qquad (A20)$$

Its Lagrange function is

$$L = \delta[1-(1+b)\delta - c_2 - r] + 2r\delta + \lambda_1(\delta - \delta_2) + \lambda_2[\frac{1-c_2-r}{(1+b)} - \delta] + \lambda_3(\bar{r} - r).$$

Then, the Kuhn-Tucker conditions are

$$\begin{aligned}
\frac{\partial L}{\partial r} &= \delta + \frac{1}{2+b}\lambda_1 - \frac{1}{1+b}\lambda_2 - \lambda_3 \leq 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \\
\frac{\partial L}{\partial \delta} &= 1 - 2(1+b)\delta - c_2 + r + \lambda_1 - \lambda_2 \leq 0,\ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \\
\frac{\partial L}{\partial \lambda_1} &= \delta - \delta_2 \geq 0,\ \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \\
\frac{\partial L}{\partial \lambda_2} &= \frac{1-c_2-r}{(1+b)} - \delta \geq 0,\ \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0, \text{ and} \\
\frac{\partial L}{\partial \lambda_3} &= \bar{r} - r \geq 0,\ \lambda_3 \frac{\partial L}{\partial \lambda_3} = 0.
\end{aligned}$$

Since all constraints in problem (A20) are strict inequalities, we must have $\lambda_1^* = \lambda_2^* = \lambda_3^* = 0$. However, by some calculations, we discover that when $\lambda_1^* = \lambda_2^* = \lambda_3^* = 0$, $r^* = -1 + c_2 < 0$ contradicts $r^* \geq 0$. Thus, no solution exists in this case.

Based on the above, we can obtain the ensuing optimal two-part tariff contracts. Since $\dot{c}_2 \equiv \frac{2+(5+4b)c_1}{(7+4b)} < \hat{c}_2 \equiv \frac{[2(2-b)+(2+3b)c_1]}{(b+6)}$, we have two cases. First, if $c_2 \in (c_1,\ \dot{c}_2)$, one solution exists in Case 1a with $R^* = 2f^* + r^*[\frac{2(1-r^*)-(c_1+c_2)}{(2+b)}]$ in (A11), and one solution exists in Case 2b with $R^* = \frac{(2-c_1-c_2)^2}{4(3+2b)}$ in (A19). Because $\frac{(2-c_1-c_2)^2}{4(3+2b)} - 2f^* - r^*[\frac{2(1-r^*)-(c_1+c_2)}{(2+b)}] = \frac{(c_2-c_1)[(2-b)+bc_1-2c_2]}{(3+2b)(2-b)^2} > 0$ by $c_2 < \bar{c}_2$, port authority's optimal two-part tariff contract and the minimum throughput guarantee are those in Case 2b. These prove Lemma 4(i). Second, if $c_2 \in (\dot{c}_2,\ \hat{c}_2]$, there exists a unique solution in Case 1a, which is the optimal contract. Lemma 4(ii) is then proved. □

Proof of Lemma 5: Lemma 2 shows that q_1^u and q_2^u depend on the values of δ. Thus, we have three cases and their sub-cases as follows.

Case 1: Suppose $\delta \in [0,\ \delta_1]$. Lemma 2(i) implies $\pi_1^u > \pi_2^u \geq 0$ due to $r \leq \bar{r}$ and $c_2 > c_1$. Accordingly, the problem in (16) becomes

$$\max_{r,\, \delta}\ r(q_1^u + q_2^u)$$
$$\text{s.t.} \quad 0 \leq \delta \leq \delta_1 \text{ and } 0 < r \leq \bar{r}.$$

Its Lagrange function is

$$L = r[\frac{2(1-r) - c_1 - c_2}{2+b}] + \lambda_1(\delta_1 - \delta) + \lambda_2(\bar{r} - r).$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{2 - c_1 - c_2 - 4r}{(2+b)} - \frac{1}{(2+b)}\lambda_1 - \lambda_2 \leq 0,\ r\frac{\partial L}{\partial r} = 0, \tag{A21}$$

$$\frac{\partial L}{\partial \delta} = -\lambda_1 \leq 0,\ \delta\frac{\partial L}{\partial \delta} = 0, \tag{A22}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta_1 - \delta \geq 0,\ \lambda_1\frac{\partial L}{\partial \lambda_1} = 0, \text{ and} \tag{A23}$$

$$\frac{\partial L}{\partial \lambda_2} = \bar{r} - r \geq 0,\ \lambda_2\frac{\partial L}{\partial \lambda_2} = 0, \tag{A24}$$

where λ_1 and λ_2 are the Lagrange multipliers for the two inequality constraints. If $\lambda_1^* > 0$, then $\frac{\partial L}{\partial \delta} = -\lambda_1^* < 0$ and $\delta^* = 0$ by (A22). They in turn suggest $\frac{\partial L}{\partial \lambda_1} = \delta_1 > 0$ and $\lambda_1^* = 0$ by (A23). This is a contradiction. Thus, we must have $\lambda_1^* = 0$, and two sub-cases below.

Case 1a: Suppose $\lambda_2^* = 0$. Then, we have $r^u = \frac{2-c_1-c_2}{4}$ by (A21). In addition, $(\bar{r} - r^u) = \frac{[2(2-b)+(2+3b)c_1-(b+6)c_2]}{4(2-b)} \geq 0$ iff $c_2 \leq \hat{c}_2 \equiv \frac{[2(2-b)+(2+3b)c_1]}{(b+6)}$. Thus, port authority's equilibrium unit-fee revenue equals

$$R^u = \frac{(2-c_1-c_2)^2}{8(2+b)}, \tag{A25}$$

and the optimal minimum throughput guarantee is $\delta^u \in [0, \frac{2(2-b)+(2+3b)c_1-(b+6)c_2}{4(4-b^2)}]$ if $c_2 \le \hat{c}_2$.

Case 1b: Suppose $\lambda_2^* > 0$. Then (A24) suggests $r^u = \bar{r} \equiv \frac{(2-b)+bc_1-2c_2}{(2-b)}$, and (A21) suggests $\lambda_2^* = \frac{[-2(2-b)-(2+3b)c_1+(b+6)c_2]}{(4-b^2)}$. We have $r^u \ge 0$ iff $c_2 \le \bar{c}_2 \equiv \frac{(2-b)+bc_1}{2}$, $\lambda_2^* > 0$ iff $c_2 > \hat{c}_2$, and $\delta^u = 0$ due to $\delta_1 = \frac{1-r^u}{2+b} + \frac{bc_1-2c_2}{(4-b^2)} = 0$. At the equilibrium, port authority's fee revenue equals

$$R^u = \frac{(c_2-c_1)[(2-b)+bc_1-2c_2]}{(2-b)^2}. \tag{A26}$$

Case 2: Suppose $\delta \in (\delta_1,\ \delta_2]$. Then, Lemma 2(ii) implies $\pi_1^u > \pi_2^u = \frac{\delta}{2}[(2-b)(1-r) - (2-b^2)\delta + bc_1 - 2c_2]$ and $\pi_2^u \ge 0$ iff $\delta \le \frac{[(2-b)(1-r)+bc_1-2c_2]}{(2-b^2)}$ and $r \le \bar{r} \equiv \frac{(2-b)+bc_1-2c_2}{2-b}$. Moreover, because $\frac{[(2-b)(1-r)+bc_1-2c_2]}{(2-b^2)} \ge\ (<)\ \delta_2$ iff $r \le\ (>)\ [1+(1+b)c_1-(2+b)c_2]$, and $[1+(1+b)c_1-(2+b)c_2] < \bar{r}$ by $c_1 < c_2$, we have two sub-cases below.

Case 2a: Suppose $0 < r \le [1+(1+b)c_1-(2+b)c_2]$. Then the problem in (16) becomes

$$\max_{r,\,\delta}\ r(q_1^u + q_2^u)$$
$$\text{s.t.}\quad \delta_1 < \delta \le \delta_2 \text{ and } 0 < r \le [1+(1+b)c_1-(2+b)c_2].$$

Its Lagrange function is

$$L = \frac{r[1+(2-b)\delta - c_1 - r]}{2} + \lambda_1(\delta-\delta_1) + \lambda_2(\delta_2-\delta) + \lambda_3[1+(1+b)c_1-(2+b)c_2-r].$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{[1+(2-b)\delta-c_1-2r]}{2} + \frac{\lambda_1}{2+b} - \frac{\lambda_2}{2+b} - \lambda_3 \le 0,\ r\cdot\frac{\partial L}{\partial r} = 0, \tag{A27}$$

$$\frac{\partial L}{\partial \delta} = \frac{(2-b)r}{2} + \lambda_1 - \lambda_2 \le 0,\ \delta\frac{\partial L}{\partial \delta} = 0, \tag{A28}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_1 \ge 0,\ \lambda_1\frac{\partial L}{\partial \lambda_1} = 0, \tag{A29}$$

$$\frac{\partial L}{\partial \lambda_2} = \delta_2 - \delta \ge 0,\ \lambda_2\frac{\partial L}{\partial \lambda_2} = 0, \text{ and} \tag{A30}$$

$$\frac{\partial L}{\partial \lambda_3} = 1 + (1+b)c_1 - (2+b)c_2 - r \geq 0,\ \lambda_3 \frac{\partial L}{\partial \lambda_3} = 0. \tag{A31}$$

Since $\delta > \delta_1$, we have $\lambda_1^* = 0$ suggested by (A29). Thus, $\lambda_2^* > 0$ is inferred form (A28) and $r > 0$, and two sub-cases are as follows.

Case 2a-1: Suppose $\lambda_3^* > 0$. Then (A30) and (A31) suggest $r^u = 1 + (1+b)c_1 - (2+b)c_2$ and $\delta^u = \delta_2 = (c_2 - c_1) > 0$. Moreover, (A28) implies $\lambda_2^* = \frac{(2-b)r^u}{2} = \frac{(2-b)[1+(1+b)c_1-(2+b)c_2]}{2}$ and (A27) implies $\lambda_3^* = \frac{-2[1+(3+2b)c_1-2(2+b)c_2]}{(2+b)}$. By some calculations, we have $\delta^u = \delta_2 > \delta_1,\ r^u > 0,\ \lambda_2^* > 0$ iff $c_2 < c_2''' \equiv \frac{1+(1+b)c_1}{(2+b)}$, and $\lambda_3^* > 0$ iff $c_2 > \frac{1+(3+2b)c_1}{2(2+b)}$ with $\frac{1+(3+2b)c_1}{2(2+b)} < \frac{1+(1+b)c_1}{(2+b)}$. Thus, under $\frac{1+(3+2b)c_1}{2(2+b)} < c_2 < \frac{1+(1+b)c_1}{(2+b)}$, the equilibrium exists and port authority's fee revenue equals

$$R^u = 2(c_2 - c_1)[1 + (1+b)c_1 - (2+b)c_2]. \tag{A32}$$

Case 2a-2: Suppose $\lambda_3^* = 0$. Then (A27)-(A31) suggest $r^u = \frac{1-c_1}{2} > 0,\ \delta^u = \delta_2 = \frac{1-c_1}{2(2+b)} > 0$, and $\lambda_2^* = \frac{(2-b)(1-c_1)}{4} > 0$. By some calculations, we have $[1+(1+b)c_1-(2+b)c_2] - \frac{1-c_1}{2} = \frac{1}{2}[1+(3+2b)c_1-2(2+b)c_2] \geq 0$ iff $c_2 \leq \frac{1+(3+2b)c_1}{2(2+b)}$. Thus, under condition $c_2 \leq \breve{c}_2 \equiv \frac{1+(3+2b)c_1}{2(2+b)}$, we have $r^u \leq [1+(1+b)c_1-(2+b)c_2]$, and the equilibrium exists with port authority's fee revenue equal to

$$R^u = \frac{(1-c_1)^2}{2(2+b)}. \tag{A33}$$

Case 2b: Suppose $r > [1+(1+b)c_1-(2+b)c_2]$. Then, the problem in (16) becomes

$$\max_{r,\,\delta}\ r(q_1^u + q_2^u)$$
$$\text{s.t.}\quad \delta_1 < \delta \leq \frac{[(2-b)(1-r) + bc_1 - 2c_2]}{(2-b^2)} \text{ and } [1 + (1+b)c_1 - (2+b)c_2] < r \leq \bar{r}.$$

Its Lagrange function is

$$\begin{aligned} L \;=\; & \frac{r}{2}[1 + (2-b)\delta - c_1 - r] + \lambda_1(\delta - \delta_1) + \lambda_2\{\frac{[(2-b)(1-r) + bc_1 - 2c_2]}{(2-b^2)} - \delta\} \\ & + \lambda_3\{r - [1 + (1+b)c_1 - (2+b)c_2]\} + \lambda_4(\bar{r} - r). \end{aligned}$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{[1+(2-b)\delta - c_1 - 2r]}{2} + \frac{\lambda_1}{2+b} - \frac{(2-b)\lambda_2}{(2-b^2)} + \lambda_3 - \lambda_4 \leq 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \quad (A34)$$

$$\frac{\partial L}{\partial \delta} = \frac{r(2-b)}{2} + \lambda_1 - \lambda_2 \leq 0,\ \delta \frac{\partial L}{\partial \delta} = 0, \quad (A35)$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_1 \geq 0,\ \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \quad (A36)$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{(2-b)(1-r) + bc_1 - 2c_2}{(2-b^2)} - \delta \geq 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad (A37)$$

$$\frac{\partial L}{\partial \lambda_3} = r - [1+(1+b)c_1 - (2+b)c_2] \geq 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \text{ and} \quad (A38)$$

$$\frac{\partial L}{\partial \lambda_4} = \bar{r} - r \geq 0,\ \lambda_4 \frac{\partial L}{\partial \lambda_4} = 0. \quad (A39)$$

Since $\delta > \delta_1$ and $r > [1+(1+b)c_1 - (2+b)c_2]$, $\lambda_1^* = \lambda_3^* = 0$ are implied by (A36) and (A38). Thus, (A35) suggests $\lambda_2^* > 0$, and two sub-cases are as follows.

Case 2b-1: Suppose $\lambda_4^* > 0$. Then, $r^u = \bar{r}$ is implied by (A39), and $\delta^u = 0$ by (A37). However, since $\delta_1 = \frac{1-r^u}{2+b} + \frac{bc_1 - 2c_2}{4-b^2} = 0$, we have $\delta^u = \delta_1 = 0$, which contradicts $\delta > \delta_1$. Thus, no solution exists in this case.

Case 2b-2: Suppose $\lambda_4^* = 0$. Then (A34)-(A35), (A37), and $\lambda_2^* > 0$ suggest $r^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]}{2(3-2b)}$, $\delta^u = \frac{(2-b)(1-r^u)+bc_1-2c_2}{(2-b^2)}$, and $\lambda_2^* = \frac{(2-b)r^u}{2}$. By some calculations, we have $r^u - [1+(1+b)c_1 - (2+b)c_2] = \frac{[-(3-2b)-(7+b-4b^2)c_1+(10-b-4b^2)c_2]}{2(3-2b)}$ and $(\bar{r} - r^u) = \frac{[(2-b)(3-2b)+(2+3b-3b^2)c_1-(8-4b-b^2)c_2]}{2(2-b)(3-2b)}$. Thus, $r^u > [1+(1+b)c_1 - (2+b)c_2]$ iff $c_2 > c_2' \equiv \frac{(3-2b)+(7+b-4b^2)c_1}{(10-b-4b^2)}$, $r^u \leq \bar{r}$ iff $c_2 \leq \tilde{c}_2 \equiv \frac{(2-b)(3-2b)+(2+3b-3b^2)c_1}{(8-4b-b^2)}$, and $r^u > 0$ iff $c_2 < \frac{(3-2b)-(1-b)c_1}{(2-b)}$ with $\frac{(3-2b)+(7+b-4b^2)c_1}{(10-b-4b^2)} < \frac{(2-b)(3-2b)+(2+3b-3b^2)c_1}{(8-4b-b^2)} < \frac{(3-2b)-(1-b)c_1}{(2-b)}$. Moreover, we have $\lambda_2^* > 0$ and $\delta^u > \delta_1$ implied by $r^u > 0$ and $r^u \leq \bar{r}$, respectively. Thus, under condition $\frac{(3-2b)+(7+b-4b^2)c_1}{(10-b-4b^2)} < c_2 \leq \frac{(2-b)(3-2b)+(2+3b-3b^2)c_1}{(8-4b-b^2)}$, the equilibrium exists and port authority's fee revenue equals

$$R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}. \quad (A40)$$

Case 3: Suppose $\delta \in (\delta_2, \frac{1}{(1+b)})$. Then, Lemma 2(iii) suggests $\pi_1^u > \pi_2^u = \delta[1-(1+b)\delta - c_2 - r] \geq 0$ iff $\delta \leq \frac{1-c_2-r}{(1+b)}$ and $r < (1-c_2)$ with $(1-c_2) > \bar{r}$. Thus, the problem in (16) becomes

$$\max_{r,\,\delta} \; 2r\delta$$
$$\text{s.t.} \quad \delta_2 < \delta \leq \frac{(1-c_2-r)}{(1+b)} \text{ and } 0 < r \leq \bar{r}.$$

Its Lagrange function is

$$L = 2r\delta + \lambda_1[\delta - \delta_2] + \lambda_2[\frac{(1-c_2-r)}{(1+b)} - \delta] + \lambda_3(\bar{r} - r).$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = 2\delta + \frac{\lambda_1}{(2+b)} - \frac{\lambda_2}{(1+b)} - \lambda_3 \leq 0,\; r\frac{\partial L}{\partial r} = 0, \tag{A41}$$

$$\frac{\partial L}{\partial \delta} = 2r + \lambda_1 - \lambda_2 \leq 0,\; \delta\frac{\partial L}{\partial \delta} = 0, \tag{A42}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_2 \geq 0,\; \lambda_1\frac{\partial L}{\partial \lambda_1} = 0, \tag{A43}$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{1-c_2-r}{(1+b)} - \delta \geq 0,\; \lambda_2\frac{\partial L}{\partial \lambda_2} = 0, \text{ and} \tag{A44}$$

$$\frac{\partial L}{\partial \lambda_3} = \bar{r} - r \geq 0,\; \lambda_3\frac{\partial L}{\partial \lambda_3} = 0. \tag{A45}$$

Since $\delta > \delta_2$, we have $\lambda_1^* = 0$ by (A43), and $\lambda_2^* > 0$ by $r > 0$ and (A42). Thus, there are two sub-cases as follows.

Case 3a: Suppose $\lambda_3^* = 0$. Then, (A41), (A42), and (A44) suggest $r^u = \frac{1-c_2}{2} > 0$, $\delta^u = \frac{1-c_2}{2(1+b)} > 0$, and $\lambda_2^* = 2r^u > 0$. Under the circumstance, we have $(\bar{r} - r^u) = \frac{(2-b)+bc_1-2c_2}{(2-b)} - \frac{1-c_2}{2} = \frac{(2-b)+2bc_1-(2+b)c_2}{2(2-b)}$ and $(\delta^u - \delta_2) = \frac{1-c_2}{2(1+b)} - \frac{(1-c_1-r^u)}{(2+b)} = \frac{[1+2(1+b)c_1-(3+2b)c_2]}{2(1+b)(2+b)}$. Thus, $\bar{r} \geq r^u$ iff $c_2 \leq \frac{(2-b)+2bc_1}{(2+b)}$, and $\delta^u > \delta_2$ iff $c_2 < c_2'' \equiv \frac{1+2(1+b)c_1}{(3+2b)}$ with $\frac{(2-b)+2bc_1}{(2+b)} > \frac{1+2(1+b)c_1}{(3+2b)}$. For $c_2 < \frac{1+2(1+b)c_1}{(3+2b)}$, the equilibrium exists with $r^u = \frac{1-c_2}{2}$ and $\delta^u = \frac{1-c_2}{2(1+b)}$. At the equilibrium, port authority's fee revenue equals

$$R^u = \frac{(1-c_2)^2}{2(1+b)}. \tag{A46}$$

Case 3b: Suppose $\lambda_3^* > 0$. Then we have $r^u = \bar{r}$ by (A45) and $\delta^u = \frac{b(c_2-c_1)}{(1+b)(2-b)}$ by (A41). However, condition $(\delta^u - \delta_2) = \frac{b(c_2-c_1)}{(1+b)(2-b)} - \frac{1-c_1-r^u}{(2+b)} = \frac{-(2-b^2)(c_2-c_1)}{(1+b)(4-b^2)} < 0$ contradicts $\delta > \delta_2$. Thus, no solution exists in this case.

Based on the above, we can derive optimal unit-fee schemes as follows. We first compare the values of $\hat{c}_2 = \frac{[2(2-b)+(2+3b)c_1]}{(b+6)}$ in Case 1a, $c_2''' = \frac{1+(1+b)c_1}{(2+b)}$ in Case 2a-1 and $\breve{c}_2 = \frac{1+(3+2b)c_1}{2(2+b)}$ in Case 2a-2, $c_2' = \frac{(3-2b)+(7+b-4b^2)c_1}{(10-b-4b^2)}$ and $\tilde{c}_2 = \frac{(2-b)(3-2b)+(2+3b-3b^2)c_1}{(8-4b-b^2)}$ in Case 2b-2, and $c'' = \frac{1+2(1+b)c_1}{(3+2b)}$ in Case 3a. Some calculations show $\breve{c}_2 < c_2' < c_2'' < \min\{c_2''',\ \hat{c}_2\} < \max\{c_2''', \hat{c}_2\} < \tilde{c}_2 < \bar{c}_2$.[1] Thus, we have the ensuing six cases.

First, for $c_2 \in (c_1, \breve{c}_2]$, equilibria of $R^u = \frac{(2-c_1-c_2)^2}{8(2+b)}$ in (A25) of Case 1a, $R^u = \frac{(1-c_1)^2}{2(2+b)}$ in (A33) of Case 2a-2, and $R^u = \frac{(1-c_2)^2}{2(1+b)}$ in (A46) of Case 3a exist. Defining $M_1 = \frac{(1-c_2)^2}{2(1+b)} - \frac{(2-c_1-c_2)^2}{8(2+b)}$ and $M_2 = \frac{(1-c_2)^2}{2(1+b)} - \frac{(1-c_1)^2}{2(2+b)}$. Since $\frac{\partial M_1}{\partial c_2} = \frac{-(1-c_2)}{(1+b)} + \frac{(2-c_1-c_2)}{4(2+b)} < \frac{-(1-\breve{c}_2)}{(1+b)} + \frac{(2-c_1-\breve{c}_2)}{4(2+b)} = \frac{-(1-c_1)(17+17b+4b^2)}{8(1+b)(2+b)^2} < 0$ and $\frac{\partial^2 M_1}{\partial c_2^2} = \frac{(7+3b)}{4(1+b)(2+b)} > 0$, we have $M_1 = \frac{(1-c_2)^2}{2(1+b)} - \frac{(2-c_1-c_2)^2}{8(2+b)} > \frac{(1-\breve{c}_2)^2}{2(1+b)} - \frac{(2-c_1-\breve{c}_2)^2}{8(2+b)} = \frac{(1-c_1)^2(23+27b+8b^2)}{32(1+b)(2+b)^3} > 0$, and thus $\frac{(1-c_2)^2}{2(1+b)} > \frac{(2-c_1-c_2)^2}{8(2+b)}$. Similarly, since $\frac{\partial M_2}{\partial c_2} = \frac{-(1-c_2)}{(1+b)} < 0$ and $\frac{\partial^2 M_2}{\partial c_2^2} = \frac{1}{1+b} > 0$, we have $M_2 = \frac{(1-c_2)^2}{2(1+b)} - \frac{(1-c_1)^2}{2(2+b)} > \frac{(1-\breve{c}_2)^2}{2(1+b)} - \frac{(1-c_1)^2}{2(2+b)} = \frac{(1-c_1)^2}{8(1+b)(2+b)^2} > 0$, and thus $\frac{(1-c_2)^2}{2(1+b)} > \frac{(1-c_1)^2}{2(2+b)}$. Hence, for $c_2 \in (c_1, \breve{c}_2]$, the port authority will choose the unit-fee scheme in Case 3a with $r^u = \frac{(1-c_2)}{2}$ and $\delta^u = \frac{(1-c_2)}{2(1+b)}$, and obtain the equilibrium fee revenue $R^u = \frac{(1-c_2)^2}{2(1+b)}$ in (A46).

Second, for $c_2 \in (\breve{c}_2, c_2']$, equilibria of $R^u = \frac{(2-c_1-c_2)^2}{8(2+b)}$ in (A25) of Case 1a, $R^u = 2(c_2 - c_1)[1 + (1 + b)c_1 - (2 + b)c_2]$ in (A32) of Case 2a-1, and $R^u = \frac{(1-c_2)^2}{2(1+b)}$ in (A46) of Case 3a exist. In this parameter range, we have $\frac{\partial M_1}{\partial c_2} = \frac{-(1-c_2)}{(1+b)} + \frac{(2-c_1-c_2)}{4(2+b)} < \frac{-(1-c_2')}{(1+b)} + \frac{(2-c_1-c_2')}{4(2+b} < \frac{-(1-c_2'')}{(1+b)} + \frac{(2-c_1-c'')}{4(2+b)} = \frac{-(1-c_1)(11+4b)}{4(2+b)(3+2b)} < 0$ and $\frac{\partial^2 M_1}{\partial c_2^2} = \frac{(7+3b)}{4(1+b)(2+b)} > 0$. Accordingly, $M_1 = \frac{(1-c_2)^2}{2(1+b)} - \frac{(2-c_1-c_2)^2}{8(2+b)} > \frac{(1-c_2')^2}{2(1+b)} - \frac{(2-c_1-c_2')^2}{8(2+b)} > \frac{(1-c_2'')^2}{2(1+b)} - \frac{(2-c_1-c_2'')^2}{8(2+b)} =$

[1] That is because $(c_2' - \breve{c}_2) = \frac{(2-b)(1-c_1)}{2(2+b)(10-b-4b^2)} > 0$, $(c'' - c_2') = \frac{(1-b)(1-c_1)}{(3+2b)(10-b-4b^2)} > 0$, $(c_2''' - c'') = \frac{(1+b)(1-c_1)}{(2+b)(3+2b)} > 0$, $(\hat{c}_2 - c'') = \frac{(6+b-4b^2)(1-c_1)}{(b+6)(3+2b)} > 0$, $(\hat{c}_2 - c''') = \frac{(2-b-2b^2)(1-c_1)}{(b+6)(2+b)} >\ (<)\ 0$ iff $(2 - b - 2b^2) >\ (<)\ 0$, $(\tilde{c}_2 - \hat{c}_2) = \frac{(2-b)^2(1-c_1)}{(b+6)(8-4b-b^2)} > 0$, $(\tilde{c}_2 - c''') = \frac{2(1-b)(2-b^2)(1-c_1)}{(2+b)(8-4b-b^2)} > 0$, and $(\bar{c}_2 - \tilde{c}_2) = \frac{(2-b)(2-b^2)(1-c_1)}{2(8-4b-b^2)} > 0$.

$\frac{(1-c_1)^2(7+8b)}{8(2+b)(3+2b)^2} > 0$, and thus $\frac{(1-c_2)^2}{2(1+b)} > \frac{(2-c_1-c_2)^2}{8(2+b)}$. On the other hand, we have $\frac{(1-c_2)^2}{2(1+b)} - 2(c_2-c_1)[1+(1+b)c_1-(2+b)c_2] = \frac{[1+2(1+b)c_1-(3+2b)c_2]^2}{2(1+b)} > 0$. Accordingly, for $c_2 \in (c_1,\ \breve{c}_2]$, the port authority will choose the unit-fee scheme in Case 3a with $r^u = \frac{1-c_2}{2}$ and $\delta^u = \frac{1-c_2}{2(1+b)}$, and obtain the equilibrium fee revenue $R^u = \frac{(1-c_2)^2}{2(1+b)}$ in (A46).

Third, for $c_2 \in (c_2',\ c'')$, equilibria of $R^u = \frac{(2-c_1-c_2)^2}{8(2+b)}$ in (A25) of Case 1a, $R^u = 2(c_2-c_1)[1+(1+b)c_1-(2+b)c_2]$ in (A32) of Case 2a-1, $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40) of Case 2b-2, and $R^u = \frac{(1-c_2)^2}{2(1+b)}$ in (A46) of Case 3a exist. Similarly, we can show $\frac{(1-c_2)^2}{2(1+b)} > 2(c_2-c_1)[1+(1+b)c_1-(2+b)c_2]$ and $\frac{(1-c_2)^2}{2(1+b)} > \frac{(2-c_1-c_2)^2}{8(2+b)}$. Thus, it remains to compare $\frac{(1-c_2)^2}{2(1+b)}$ and $\frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$. Define $M_3 = \frac{(1-c_2)^2}{2(1+b)} - \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$. Since $\frac{\partial M_3}{\partial c_2} = \frac{-(1-c_2)}{(1+b)} + \frac{(2-b)[(3-2b)-(1-b)c_1-(2-b)c_2]}{2(3-2b)(2-b^2)}$ and $\frac{\partial^2 M_3}{\partial c_2^2} = \frac{(1-b)(8-3b^2)}{2(1+b)(3-2b)(2-b^2)} > 0$, we have $\frac{\partial M_3}{\partial c_2} < \frac{-(1-c_2'')}{(1+b)} + \frac{(2-b)[(3-2b)-(1-b)c_1-(2-b)c_2'']}{2(3-2b)(2-b^2)} = \frac{-(1-b)(1-c_1)(10-b-4b^2)}{2(2-b^2)(9-4b^2)} < 0$, $M_3 < \frac{(1-c_2')^2}{2(1+b)} - \frac{[(3-2b)-(1-b)c_1-(2-b)c_2']^2}{4(3-2b)(2-b^2)} = \frac{(1-b)^2(1-c_1)^2}{2(1+b)(10-b-4b^2)^2} > 0$, and $M_3 > \frac{(1-c_2'')^2}{2(1+b)} - \frac{[(3-2b)-(1-b)c_1-(2-b)c_2'']^2}{4(3-2b)(2-b^2)} = \frac{-(1-b)^2(1-c_1)^2}{4(3-2b)(2-b^2)(3+2b)^2} < 0$. These imply that M_3 can be positive or negative for $c_2 \in (c_2', c'')$, and there must exist some $\ddot{c}_2$, $c_2' < \ddot{c}_2 < c_2''$, at which $M_3 = 0$. Solving $M_3 = 0$ yields $\ddot{c}_2 = \frac{1}{(8-3b^2)}\{[(2+b)(3-2b)+(1+b)(2-b)c_1] - \sqrt{2(1+b)(3-2b)(2-b^2)(1-c_1)^2}\}$. Accordingly, we have $\frac{(1-c_2)^2}{2(1+b)} \geq\ (<)\ \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ for $c_2' < c_2 \leq \ddot{c}_2$ ($\ddot{c}_2 < c_2 < c_2''$). For $c_2 \in (c_2',\ c'')$, there are two sub-cases. If $c_2 \in (c_2', \ddot{c}_2]$, the port authority will choose the unit-fee scheme in Case 3a with $r^u = \frac{1-c_2}{2}$ and $\delta^u = \frac{1-c_2}{2(1+b)}$, and obtain the equilibrium fee revenue $R^u = \frac{(1-c_2)^2}{2(1+b)}$ in (A46). If $c_2 \in (\ddot{c}_2,\ c_2'')$, the port authority will choose the unit-fee scheme in Case 2b-2 with $r^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]}{2(3-2b)}$ and $\delta^u = \frac{(2-b)(1-r^u)+bc_1-2c_2}{(2-b^2)}$, and obtain the equilibrium fee revenue $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40).

Fourth, we have either $c_2''' \equiv \frac{1+(1+b)c_1}{(2+b)} < \hat{c}_2 \equiv \frac{1}{(b+6)}[2(2-b)+(2+3b)c_1]$ or $\hat{c}_2 < c_2'''$. If $c_2''' < \hat{c}_2$, then $(2-b-2b^2) > 0$. For $c_2 \in [c_2'', c_2''')$, equilibria of $R^u = \frac{(2-c_1-c_2)^2}{8(2+b)}$ in (A25) of Case 1a, $R^u = 2(c_2-c_1)[1+(1+b)c_1-(2+b)c_2]$ in (A32) of Case 2a-1, and $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40) of Case 2b-2 exist. Define $M_4 = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} - \frac{(2-c_1-c_2)^2}{8(2+b)}$. Since $\frac{\partial M_4}{\partial c_2} = \frac{-(2-b)[(3-2b)-(1-b)c_1-(2-b)c_2]}{2(3-2b)(2-b^2)} + \frac{(2-c_1-c_2)}{4(2+b)}$ and

$\frac{\partial^2 M_4}{\partial c_2^2} = \frac{10-4b-b^2}{4(2+b)(3-2b)(2-b^2)} > 0$, we have $\frac{\partial M_4}{\partial c_2} < \frac{-(2-b)[(3-2b)-(1-b)c_1-(2-b)c_2''']}{2(3-2b)(2-b^2)} + \frac{(2-c_1-c_2''')}{4(2+b)} = \frac{-7(1-c_1)}{4(3-2b)(2+b)^2} < 0$ and $M_4 > \frac{[(3-2b)-(1-b)c_1-(2-b)c_2''']^2}{4(3-2b)(2-b^2)} - \frac{(2-c_1-c_2''')^2}{8(2+b)} = \frac{(5-2b-4b^2)(1-c_1)^2}{8(3-2b)(2+b)^3} > 0$ by $(5-2b-4b^2) > (2-b-2b^2) > 0$. Then, we have $\frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > \frac{(2-c_1-c_2)^2}{8(2+b)}$. On the other hand, we have $\frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} - 2(c_2-c_1)[1+(1+b)c_1-(2+b)c_2] = \frac{[-(3-2b)-(7+b-4b^2)c_1+(10-b-4b^2)c_2]^2}{4(3-2b)(2-b^2)} > 0$. Thus, $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40) is optimal for the port authority if $c_2 \in [c_2'', c_2''')$ with $c_2''' < \hat{c}_2$.

For $c_2 \in [c_2''',\ \hat{c}_2)$, equilibria of $R^u = \frac{(2-c_1-c_2)^2}{8(2+b)}$ in (A25) of Case 1a and $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40) of Case 2b-2 exist. Since $\frac{\partial^2 M_4}{\partial c_2^2} = \frac{10-4b-b^2}{4(2+b)(3-2b)(2-b^2)} > 0$ and $\frac{\partial M_4}{\partial c_2} < \frac{-(2-b)[(3-2b)-(1-b)c_1-(2-b)\hat{c}_2]}{2(3-2b)(2-b^2)} + \frac{(2-c_1-\hat{c}_2)}{4(2+b)} = \frac{-(8-4b-b^2)(1-c_1)}{2(b+6)(3-2b)(2-b^2)} < 0$, we have $M_4 > \frac{[(3-2b)-(1-b)c_1-(2-b)\hat{c}_2]^2}{4(3-2b)(2-b^2)} - \frac{(2-c_1-\hat{c}_2)^2}{8(2+b)} = \frac{(2-b)^2(1-c_1)^2}{4(b+6)^2(3-2b)(2-b^2)} > 0$. Thus, $\frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > \frac{(2-c_1-c_2)^2}{8(2+b)}$. It implies that for $c_2 \in [c_2'',\ \hat{c}_2)$ with $c_2''' < \hat{c}_2$, the port authority will choose the unit-fee scheme in Case 2b-2 with $r^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]}{2(3-2b)}$ and $\delta^u = \frac{(2-b)(1-r^u)+bc_1-2c_2}{(2-b^2)}$, and get the equilibrium fee revenue $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40).

By contrast, if $c_2''' > \hat{c}_2$, then $(2-b-2b^2) < 0$. For $c_2 \in [c_2'', \hat{c}_2)$, equilibria of $R^u = \frac{(2-c_1-c_2)^2}{8(2+b)}$ in (A25) of Case 1a, $R^u = 2(c_2-c_1)[1+(1+b)c_1-(2+b)c_2]$ in (A32) of Case 2a-1, and $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40) of Case 2b-2 exist. Similarly, we can show $\frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > \frac{(2-c_1-c_2)^2}{8(2+b)}$ and $\frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > 2(c_2-c_1)[1+(1+b)c_1-(2+b)c_2]$. For $c_2 \in (\hat{c}_2,\ c_2''')$, equilibria of $R^u = 2(c_2-c_1)[1+(1+b)c_1-(2+b)c_2]$ in (A32) of Case 2a-1 and $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40) of Case 2b-2 exist. We can show $\frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > 2(c_2-c_1)[1+(1+b)c_1-(2+b)c_2]$. These imply that for $c_2 \in [c_2'',\ c_2''')$ with $\hat{c}_2 < c_2'''$, the port authority will choose the unit-fee scheme in Case 2b-2 with $r^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]}{2(3-2b)}$ and $\delta^u = \frac{(2-b)(1-r^u)+bc_1-2c_2}{(2-b^2)}$, and get the equilibrium fee revenue $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40).

Fifth, we have either $c_2 \in [\hat{c}_2,\ \tilde{c}_2)$ with $c_2''' < \hat{c}_2$ or $c_2 \in [c_2''',\ \tilde{c}_2)$ with $c_2''' > \hat{c}_2$. In both cases, equilibria of $R^u = \frac{(c_2-c_1)[(2-b)+bc_1-2c_2]}{(2-b)^2}$ in (A26) of Case 1b and $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40) of Case 2b-2 always exist. By some calculations, we can get $\frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} - \frac{(c_2-c_1)[(2-b)+bc_1-2c_2]}{(2-b)^2} = \frac{[(-6+7b-2b^2)-(2+3b-3b^2)c_1+(8-4b-b^2)c_2]^2}{4(3-2b)(2-b^2)(2-b)^2} > 0$. Thus, the port authority will choose the unit-fee scheme in Case 2b-2 with $r^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]}{2(3-2b)}$ and $\delta^u = \frac{(2-b)(1-r^u)+bc_1-2c_2}{(2-b^2)}$, and obtain the equilibrium fee revenue $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40).

Sixth, for $c_2 \in [\tilde{c}_2,\ \bar{c}_2)$, there is a unique equilibrium in Case 1b. Thus, the port authority will choose the unit-fee scheme with $r^u = \bar{r} \equiv \frac{(2-b)+bc_1-2c_2}{2-b}$ and $\delta^u = 0$, and obtain the equilibrium fee revenue $R^u = \frac{(c_2-c_1)[(2-b)+bc_1-2c_2]}{(2-b)^2}$ in (A26).

In sum, the first, second and third cases yield Lemma 5(i), the third, fourth and fifth cases provide Lemma 5(ii), and the sixth case gives Lemma 5(iii). □

Proof of Lemma 6: Lemma 3 shows that q_1^f and q_2^f depend on the values of δ. Thus, we have three cases below.

Case 1: Suppose $\delta \in (0,\ \delta_3]$ with $\delta_3 = \frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}$. Then Lemma 3(i) suggests $\pi_1^f > \pi_2^f = (q_2^f)^2 - f$, and $f^f = \frac{1}{2}[\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}]^2$. The problem in (17) thus becomes

$$\max_{\delta}\ [\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}]^2$$

$$\text{s.t.}\ \ \delta \leq \delta_3 = \frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}. \qquad (A47)$$

Since the constraint in (A47) is independent of the objective function of this problem, equilibrium δ^f can be any value within $[0,\ \delta_3]$, and port authority's equilibrium fee revenue equals

$$R^f = [\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}]^2. \qquad (A48)$$

Case 2: Suppose $\delta \in (\delta_3,\ \delta_4]$ with $\delta_4 = \frac{1-c_1}{2+b}$. Then Lemma 3(ii) suggests $\pi_1^f > \pi_2^f = \frac{\delta}{2}[(2-b)-(2-b^2)\delta + bc_1 - 2c_2] - f$, and $f^f = \pi_2^f = \frac{\delta}{4}[(2-b)-(2-b^2)\delta + bc_1 - 2c_2]$. We will have $f^f > 0$ iff $\delta < \frac{(2-b)+bc_1-2c_2}{(2-b^2)}$ and $c_2 < \bar{c}_2 \equiv \frac{(2-b)+bc_1}{2}$. On the other hand, we have $\frac{(2-b)+bc_1-2c_2}{(2-b^2)} > \ (\leq)\ \delta_4$ iff $c_2 < \ (\geq)\ \frac{1+(1+b)c_1}{(2+b)}$ with $\frac{1+(1+b)c_1}{(2+b)} < \bar{c}_2$. Thus, there are two sub-cases as follows.

Case 2a: Suppose $c_2 \leq \frac{1+(1+b)c_1}{(2+b)}$. Then the problem in (17) becomes

$$\begin{aligned} \max_\delta \quad & \frac{\delta}{2}[(2-b)-(2-b^2)\delta+bc_1-2c_2] \\ \text{s.t.} \quad & \delta_3 < \delta \leq \delta_4 = \frac{1-c_1}{2+b}. \end{aligned}$$

Its Lagrange function is

$$L = \frac{\delta}{2}[(2-b)-(2-b^2)\delta+bc_1-2c_2] + \lambda_1(\delta-\delta_3) + \lambda_2(\delta_4-\delta).$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial \delta} = \frac{1}{2}[(2-b)-2(2-b^2)\delta+bc_1-2c_2]+\lambda_1-\lambda_2 \leq 0,\ \delta\frac{\partial L}{\partial \delta} = 0, \tag{A49}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta-\delta_3 \geq 0,\ \lambda_1\frac{\partial L}{\partial \lambda_1} = 0,\ \text{and} \tag{A50}$$

$$\frac{\partial L}{\partial \lambda_2} = \delta_4-\delta \geq 0,\ \lambda_2\frac{\partial L}{\partial \lambda_2} = 0. \tag{A51}$$

Because $\delta > \delta_3$, we have $\lambda_1^* = 0$ by (A50), and the following two sub-cases.

Case 2a-1: Suppose $\lambda_2^* > 0$. Then, (A51) implies $\delta^f = \delta_4 = \frac{1-c_1}{2+b}$ and (A49) suggests $\lambda_2^* = \frac{[b^2+(4+2b-b^2)c_1-2(2+b)c_2]}{2(2+b)}$. To have $\lambda_2^* > 0$, we need $c_2 < \frac{b^2+(4+2b-b^2)c_1}{2(2+b)}$. In addition, we have $(\delta^f - \delta_3) = \frac{2(c_2-c_1)}{4-b^2} > 0$. With condition $c_2 < \frac{b^2+(4+2b-b^2)c_1}{2(2+b)}$, port authority's equilibrium fee revenue equals

$$R^f = \frac{(1-c_1)[1+(1+b)c_1-(2+b)c_2]}{(2+b)^2}. \tag{A52}$$

Case 2a-2: Suppose $\lambda_2^* = 0$. Then we have $\delta^f = \frac{(2-b)+bc_1-2c_2}{2(2-b^2)}$ by (A49). By some calculations, we have $(\delta_4 - \delta^f) = \frac{[-b^2-(4+2b-b^2)c_1+2(2+b)c_2]}{2(2+b)(2-b^2)} \geq 0$ iff $c_2 \geq \frac{b^2+(4+2b-b^2)c_1}{2(2+b)}$, and $\delta^f > \delta_3$ iff $c_2 < \bar{c}_2$. Combining this condition with $c_2 < \frac{1+(1+b)c_1}{(2+b)}$ and $\frac{b^2+(4+2b-b^2)c_1}{2(2+b)} < \frac{1+(1+b)c_1}{(2+b)}$, we assume $\frac{b^2+(4+2b-b^2)c_1}{2(2+b)} \leq c_2 < \frac{1+(1+b)c_1}{(2+b)}$. Under the circumstance, port authority's equilibrium fee revenue equals

$$R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}. \tag{A53}$$

Case 2b: Suppose $\frac{1+(1+b)c_1}{(2+b)} \le c_2 < \bar{c}_2 \equiv \frac{(2-b)+bc_1}{2}$. Then, the problem in (17) becomes

$$\begin{aligned} \max_\delta \quad & \frac{\delta}{2}[(2-b)-(2-b^2)\delta+bc_1-2c_2] \\ \text{s.t.} \quad & \delta_3 < \delta < \frac{(2-b)+bc_1-2c_2}{(2-b^2)}. \end{aligned}$$

Its Lagrange function is

$$L = \frac{\delta}{2}[(2-b)-(2-b^2)\delta+bc_1-2c_2]+\lambda_1(\delta-\delta_3)+\lambda_2[\frac{(2-b)+bc_1-2c_2}{(2-b^2)}-\delta].$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial \delta} = \frac{1}{2}[(2-b)-2(2-b^2)\delta+bc_1-2c_2]+\lambda_1-\lambda_2 \le 0,\ \delta\frac{\partial L}{\partial \delta}=0, \tag{A54}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta-\delta_3 \ge 0,\ \lambda_1\frac{\partial L}{\partial \lambda_1}=0,\ \text{and} \tag{A55}$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{(2-b)+bc_1-2c_2}{(2-b^2)}-\delta \ge 0,\ \lambda_2\frac{\partial L}{\partial \lambda_2}=0. \tag{A56}$$

Since $\delta_3 < \delta < \frac{(2-b)+bc_1-2c_2}{(2-b^2)}$, we have $\lambda_1^* = \lambda_2^* = 0$ by (A55)-(A56) and $\delta^f = \frac{(2-b)+bc_1-2c_2}{2(2-b^2)}$ by (A54). Obviously, $\delta^f < \frac{(2-b)+bc_1-2c_2}{(2-b^2)}$ and $\delta^f > \delta_3$ iff $c_2 < \bar{c}_2 = \frac{(2-b)+bc_1}{2}$. For $\frac{1+(1+b)c_1}{(2+b)} \le c_2 < \bar{c}_2$, port authority's equilibrium fee revenue equals

$$R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}. \tag{A57}$$

Case 3: Suppose $\delta \in (\delta_4,\ \frac{1}{1+b})$. Then, Lemma 3(iii) suggests $\pi_i^f = [1-(1+b)\delta-c_i]\delta-f$ for $i=1,\ 2$, and thus $f^f = \frac{\delta}{2}[1-(1+b)\delta-c_2]$. To have $f^f > 0$, condition $\delta < \frac{(1-c_2)}{(1+b)}$ is needed. The problem in (17) thus becomes

$$\begin{aligned} \max_\delta \quad & [1-(1+b)\delta-c_2]\delta \\ \text{s.t.} \quad & \frac{1-c_1}{2+b} < \delta < \frac{1-c_2}{1+b}. \end{aligned}$$

Its Lagrange function is

$$L = [1-(1+b)\delta-c_2]\delta+\lambda_1(\delta-\frac{1-c_1}{2+b})+\lambda_2(\frac{1-c_2}{1+b}-\delta).$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial \delta} = 1 - c_2 - 2(1+b)\delta + \lambda_1 - \lambda_2 \leq 0,\ \delta\frac{\partial L}{\partial \delta} = 0, \quad (A58)$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \frac{1-c_1}{(2+b)} \geq 0,\ \lambda_1\frac{\partial L}{\partial \lambda_1} = 0,\ \text{and} \quad (A59)$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{1-c_2}{(1+b)} - \delta \geq 0,\ \lambda_2\frac{\partial L}{\partial \lambda_2} = 0. \quad (A60)$$

Because $\frac{1-c_1}{(2+b)} < \delta < \frac{1-c_2}{(1+b)}$, we have $\lambda_1^* = \lambda_2^* = 0$ by (A59)-(A60). We can also obtain $\delta^f = \frac{1-c_2}{2(1+b)} > 0$ by (A58). However, since $(\delta^f - \delta_4) = \frac{-b(1-c_1)-(2+b)(c_2-c_1)}{2(1+b)(2+b)} < 0$ due to $0 < c_1 < c_2 < 1$, no solution exists in this case.

Based on the above, we can derive optimal fixed-fee schemes as follows. Since $\dddot{c}_2 \equiv \frac{b^2+(4+2b-b^2)c_1}{2(2+b)} < \frac{1+(1+b)c_1}{(2+b)} < \bar{c}_2 \equiv \frac{(2-b)+bc_1}{2}$, we have three cases below.

First, for $c_2 \in (c_1,\ \dddot{c}_2)$, equilibria of $R^f = [\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}]^2$ in (A48) of Case 1, and $R^f = \frac{(1-c_1)[1+(1+b)c_1-(2+b)c_2]}{(2+b)^2}$ in (A52) of Case 2a-1 exist. Define $M_5 = [\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}]^2 - \frac{(1-c_1)[1+(1+b)c_1-(2+b)c_2]}{(2+b)^2}$. Then, we have $M_5 = 0$ when $c_2 = c_1$, and $M_5 = \frac{-b^4(1-c_1)^2(2-b)^2}{2(2-b)^2(2+b)^4}$ when $c_2 = \dddot{c}_2$. Since $\frac{\partial M_5}{\partial c_2} = \frac{8(c_2-c_1)-b^2(2-b)(1-c_1)}{(2-b)^2(2+b)^2}$ and $\frac{\partial^2 M_5}{\partial c_2^2} = \frac{8}{(2-b)^2(2+b)^2} > 0$, we have $M_5 < \max\{M_5|_{c_2=c_1},\ M_5|_{c_2=\dddot{c}_2}\} = \{0,\ \frac{-b^4(1-c_1)^2(2-b)^2}{2(2-b)^2(2+b)^4}\} = 0$, and thus $[\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}]^2 < \frac{(1-c_1)[1+(1+b)c_1-(2+b)c_2]}{(2+b)^2}$ if $c_2 \in (c_1,\ \dddot{c}_2)$. The port authority will choose the fixed-fee scheme in Case 2a-1 with $f^f = \frac{(1-c_1)[1+(1+b)c_1-(2+b)c_2]}{2(2+b)^2}$ and $\delta^f = \delta_4 = \frac{1-c_1}{2+b}$, and get the equilibrium fee revenue $R^f = \frac{(1-c_1)[1+(1+b)c_1-(2+b)c_2]}{(2+b)^2}$. These prove Lemma 6(i).

Second, for $c_2 \in [\dddot{c}_2,\ \frac{1+(1+b)c_1}{(2+b)})$, an equilibrium exists in Case 1 with $R^f = [\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}]^2$ in (A48), and an equilibrium exists in Case 2a-2 with $R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}$ in (A53). Since $\frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)} - [\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}]^2 = \frac{b^4[(2-b)+bc_1-2c_2]^2}{8(2-b^2)(2-b)^2(2+b)^2} > 0$, the port authority will choose the fixed-fee scheme in Case 2a-2 with $f^f = \frac{[(2-b)+bc_1-2c_2]^2}{16(2-b^2)}$ and $\delta^f = \frac{(2-b)+bc_1-2c_2}{2(2-b^2)}$, and obtain the equilibrium fee revenue $R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}$. These prove Lemma 6(ii).

Third, for $c_2 \in [\frac{1+(1+b)c_1}{(2+b)},\ \bar{c}_2)$, an equilibrium exists in Case 1 with $R^f = [\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}]^2$ in (A48), and an equilibrium exists in Case 2b with $R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}$ in

(A57). Since $\frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)} > [\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}]^2$, the port authority will choose the fixed-fee scheme in Case 2b with $f^f = \frac{[(2-b)+bc_1-2c_2]^2}{16(2-b^2)}$ and $\delta^f = \frac{(2-b)+bc_1-2c_2}{2(2-b^2)}$, and obtain the equilibrium fee revenue $R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}$. These prove Lemma 6(ii). □

Proof of Proposition 1: We can obtain optimal concession contracts by comparing port authority's equilibrium fee revenues derived in Lemmas 4-6. Since $(\dot{c}_2 - \dddot{c}_2) = \frac{(8+4b-7b^2-4b^3)(1-c_1)}{2(2+b)(7+4b)} > 0$, $(c_2' - \dot{c}_2) = \frac{(1-c_1)}{(7+4b)(10-b-4b^2)} > 0$, and $c_2' < \ddot{c}_2 < \hat{c}_2 < \tilde{c}_2 < \bar{c}_2$ as shown in the proof of Lemma 5, we have $\dddot{c}_2 < \dot{c}_2 < \ddot{c}_2 < \hat{c}_2 < \tilde{c}_2 < \bar{c}_2$. Thus, we have the following six cases.

Case 1: Suppose $c_2 \in [\tilde{c}_2,\ \bar{c}_2)$. Then Lemma 5(iii) shows $R^u = \frac{(c_2-c_1)[(2-b)+bc_1-2c_2]}{(2-b)^2} > 0$ in (A26), and Lemma 6(ii) displays $R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}$ in (A57). Some calculations yield $(R^u - R^f) = \frac{[(2-b)+bc_1-2c_2][-(2-b)^3-(16+4b-12b^2+b^3)c_1+(24-8b-6b^2)c_2]}{8(2-b^2)(2-b)^2} > 0$ due to $[-(2-b)^3 - (16+4b-12b^2+b^3)c_1 + (24-8b-6b^2)c_2] > 0$ by $c_2 \geq \tilde{c}_2$ and $\tilde{c}_2 - \frac{(2-b)^3+(16+4b-12b^2+b^3)c_1}{(24-8b-6b^2)} = \frac{(2-b)(2-b^2)(20-12b-b^2)(1-c_1)}{(8-4b-b^2)(24-8b-6b^2}} > 0$, and $[(2-b)+bc_1-2c_2] > 0$ by $c_2 < \bar{c}_2$. Thus, $R^u > R^f$. Then, port authority's best choice is the unit-fee contract with $r^u = \bar{r} \equiv \frac{(2-b)+bc_1-2c_2}{2-b}$, $\delta^u = 0$, and $R^u = \frac{(c_2-c_1)[(2-b)+bc_1-2c_2]}{(2-b)^2}$.

Case 2: Suppose $c_2 \in [\hat{c}_2,\ \tilde{c}_2)$. Then Lemma 5(ii) shows $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40), and Lemma 6(ii) displays $R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}$ in (A57). Define $M_6 = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} - \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)} = \frac{[(3-2b)-2(3-2b)c_1+4c_1c_2+(1-2b)c_1^2-2c_2^2]}{8(3-2b)}$. Since $\frac{\partial M_6}{\partial c_2} = \frac{(c_1-c_2)}{2(3-2b)} < 0$, we have $M_6 > \frac{[(3-2b)-(1-b)c_1-(2-b)\tilde{c}_2]^2}{4(3-2b)(2-b^2)} - \frac{[(2-b)+bc_1-2\tilde{c}_2]^2}{8(2-b^2)} = \frac{(2-b^2)(20-12b-b^2)(1-c_1)^2}{8(8-4b-b^2)^2}$ > 0. Thus, $R^u > R^f$, which implies that port authority's best choice is the unit-fee contract with $r^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]}{2(3-2b)}$, $\delta^u = \frac{(2-b)(1-r^u)+bc_1-2c_2}{(2-b^2)}$, and $R^u = \frac{1}{4(3-2b)(2-b^2)}[(3-2b) - (1-b)c_1 - (2-b)c_2]^2$.

Case 3: Suppose $c_2 \in (\ddot{c}_2,\ \hat{c}_2)$. Then Lemma 4(ii) demonstrates $R^* = [\frac{2(2-b)+(2+3b)c_1-(6+b)c_2}{2(2-b)(3+2b)}]^2$ $+r^*[\frac{2(1-r^*)-(c_1+c_2)}{2+b}]$ in (A11), Lemma 5(ii) shows $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ in (A40), and Lemma 6(ii) displays $R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}$ in (A57). Define $M_7 = \frac{1}{4(3-2b)(2-b^2)}[(3-2b)-(1-b)c_1-(2-b)c_2]^2 - [\frac{2(2-b)+(2+3b)c_1-(6+b)c_2}{2(2-b)(3+2b)}]^2 - r^*[\frac{2(1-r^*)-(c_1+c_2)}{2+b}] = \frac{1}{-4(2-b^2)(2-b)^2(9-4b^2)}$

$[(-12+20b+2b^3-11b^2)+(24-40b+10b^2+10b^3-4b^4)c_1+(12b^2-14b^3+4b^4)c_2+(-48+16b+32b^2-2b^3-8b^4)c_1c_2+(12+12b-21b^2-4b^3+6b^4)c_1^2+(24-8b-22b^2+8b^3+2b^4)c_2^2]$ with $\frac{\partial M_7}{\partial c_2} = \frac{b^2(12-14b+4b^2)(1-c_1)+4(12-4b-11b^2+4b^3+b^4)(c_2-c_1)}{-4(2-b^2)(2-b)^2(9-4b^2)}$. Since $(12-14b+4b^2) > 0$ by $\frac{\partial(12-14b+4b^2)}{\partial b} = -14+8b < 0$, and $(12-4b-11b^2+4b^3+b^4) > 2 > 0$ by $\frac{\partial(12-4b-11b^2+4b^3+b^4)}{\partial b} = -4-22b+12b^2+4b^3 < 0$, we have $\frac{\partial M_7}{\partial c_2} < 0$ and $M_7 > \frac{[(3-2b)-(1-b)c_1-(2-b)\hat{c}_2]^2}{4(3-2b)(2-b^2)} - [\frac{2(2-b)+(2+3b)c_1-(6+b)\hat{c}_2}{2(2-b)(3+2b)}]^2 - \bar{r}[\frac{2(1-\bar{r})-(c_1+\hat{c}_2)}{2+b}] = \frac{(2-b)^2(1-c_1)^2}{4(3-2b)(2-b^2)(b+6)^2} > 0$. Thus, we have $R^u > R^*$ and $R^u > R^f$ by $c_2 < \hat{c}_2 < \tilde{c}_2$ as in Case 2. That is, port authority's best choice is the unit-fee contract with $r^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]}{2(3-2b)}$, $\delta^u = \frac{(2-b)(1-r^u)+bc_1-2c_2}{(2-b^2)}$, and $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$.

Case 4: Suppose $c_2 \in [\dot{c}_2,\ \ddot{c}_2]$. Then Lemma 4(ii) demonstrates $R^* = [\frac{2(2-b)+(2+3b)c_1-(6+b)c_2}{2(2-b)(3+2b)}]^2 + r^*[\frac{2(1-r^*)-(c_1+c_2)}{2+b}]$ in (A11), Lemma 5(i) shows $R^u = \frac{(1-c_2)^2}{2(1+b)}$ in (A46), and Lemma 6(ii) displays $R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}$ in (A57). We can show $R^u > R^f$ by $c_2 < \ddot{c}_2 < \tilde{c}_2$ as in Case 2, $R^u > R^*$ due to $\frac{(1-c_2)^2}{2(1+b)} \geq \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ by $c_2 \leq \ddot{c}_2 < \hat{c}_2$ as in the proof of Case 3 of Lemma 5, and $\frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > R^*$ by $c_2 < \hat{c}_2$ as shown in Case 3. Thus, $R^u > R^*$ and $R^u > R^f$. Then, port authority's best choice is the unit-fee contract with $r^u = \frac{1-c_2}{2}$, $\delta^u = \frac{1-c_2}{2(1+b)}$, and $R^u = \frac{(1-c_2)^2}{2(1+b)}$.

Case 5: Suppose $c_2 \in [\dddot{c}_2,\ \dot{c}_2)$. Then Lemma 4(i) demonstrates $R^* = \frac{(2-c_1-c_2)^2}{4(3+2b)}$ in (A11), Lemma 5(i) shows $R^u = \frac{(1-c_2)^2}{2(1+b)}$ in (A46), and Lemma 6(ii) displays $R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}$ in (A57). As in Case 4, we have $R^u > R^f$ by $c_2 < \dot{c}_2 < \ddot{c}_2 < \hat{c}_2$. It remains to compare R^* and R^u. Define $M_8 = \frac{(1-c_2)^2}{2(1+b)} - \frac{(2-c_1-c_2)^2}{4(3+2b)}$. Since $\frac{\partial M_8}{\partial c_2} = \frac{[-2(2+b)-(1+b)c_1+(5+3b)c_2]}{2(1+b)(3+2b)}$ and $\frac{\partial^2 M_8}{\partial c_2^2} = \frac{(5+3b)}{2(1+b)(3+2b)} > 0$, we have $\frac{\partial M_8}{\partial c_2} < \frac{[-2(2+b)-(1+b)c_1+(5+3b)\dot{c}_2]}{2(1+b)(3+2b)} = \frac{-(3+2b)(1-c_1)}{(1+b)(7+4b)} < 0$ and $M_8 > \frac{(1-\dot{c}_2)^2}{2(1+b)} - \frac{(2-c_1-\dot{c}_2)^2}{4(3+2b)} = \frac{(1-c_1)^2}{2(1+b)(7+4b)^2} > 0$. These imply $R^u > R^*$. Thus, the port authority will choose the unit-fee scheme with $r^u = \frac{1-c_2}{2}$, $\delta^u = \frac{1-c_2}{2(1+b)}$, and $R^u = \frac{(1-c_2)^2}{2(1+b)}$.

Case 6: Suppose $c_2 \in (c_1,\ \dddot{c}_2)$. Then Lemma 4(i) demonstrates $R^* = \frac{(2-c_1-c_2)^2}{4(3+2b)}$ in (A19), Lemma 5(i) shows $R^u = \frac{(1-c_2)^2}{2(1+b)}$ in (A46), and Lemma 6(i) displays $R^f = \frac{(1-c_1)[1+(1+b)c_1-(2+b)c_2]}{(2+b)^2}$ in (A52). As in Case 5, we have $R^u > R^*$ by $c_2 < \dddot{c}_2 < \dot{c}_2$.

It remains to compare R^u and R^f. Define $M_9 = \frac{(1-c_2)^2}{2(1+b)} - \frac{(1-c_1)[1+(1+b)c_1-(2+b)c_2]}{(2+b)^2}$. Since $\frac{\partial M_9}{\partial c_2} = \frac{[-1-(1+b)c_1+(2+b)c_2]}{(1+b)(2+b)}$ and $\frac{\partial^2 M_9}{\partial c_2^2} = \frac{1}{(1+b)} > 0$, we have $\frac{\partial M_9}{\partial c_2} < \frac{[-1-(1+b)c_1+(2+b)\ddddot{c}_2]}{(1+b)(2+b)} = \frac{-(2-b)(1-c_1)}{(4+6b+2b^2)} < 0$ and $M_9 > \frac{(1-\ddddot{c}_2)^2}{2(1+b)} - \frac{(1-c_1)[1+(1+b)c_1-(2+b)\ddddot{c}_2]}{(2+b)^2} = \frac{(8+8b+b^4)(1-c_1)^2}{8(1+b)(2+b)^2} > 0$. These imply $R^u > R^f$. Thus, the port authority will choose the unit-fee scheme with $r^u = \frac{1-c_2}{2}$, $\delta^u = \frac{1-c_2}{2(1+b)}$, and $R^u = \frac{(1-c_2)^2}{2(1+b)}$.

In sum, Cases 4-6, Cases 2-3, and Case 1 show Proposition 1(i), Proposition 1(ii), and Proposition 1(iii), respectively. □

Proof of Proposition 2: Denote $R_1^u \equiv \frac{(1-c_2)^2}{2(1+b)}$, $R_2^u \equiv \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$, and $R_3^u \equiv \frac{(c_2-c_1)[(2-b)+bc_1-2c_2]}{(2-b)^2}$ port authority's equilibrium fee revenues in Lemma 5(i), Lemma 5(ii), and Lemma 5(iii), respectively. In addition, denote $R_1^* \equiv \frac{(2-c_1-c_2)^2}{4(3+2b)}$ and $R_2^* \equiv [\frac{2(2-b)+(2+3b)c_1-(6+b)c_2}{2(2-b)(3+2b)}]^2 + r^*[\frac{2(1-r^*)-(c_1+c_2}{2+b}]$ port authority's equilibrium fee revenues in Lemma 4(i) and Lemma 4(ii), respectively. Note that $R^{c*} = R_2^*$ by (24) and $R^{cu} = R_3^u$ by (25). Recall that $c_1 < \ddddot{c}_2 < \dot{c}_2 < \ddot{c}_2 < \hat{c}_2 < \tilde{c}_2 < \bar{c}_2$. First, for $c_2 \in (c_1,\ \dot{c}_2)$, Lemma 4(i) and Proposition 1(i) show $R_1^* > R_2^* = R^{c*}$ and $R_1^u > R_1^*$, and thus $R_1^u > R^{c*}$. Second, for $c_2 \in [\dot{c}_2,\ \ddot{c}_2)$, Lemma 4(ii) and Proposition 1(i) show $R_1^u > R_2^* = R^{c*}$. Third, for $c_2 \in [\ddot{c}_2,\ \hat{c}_2)$, Lemma 4(ii) and Proposition 1(ii) show $R_2^u > R_2^* = R^{c*}$. Fourth, for $c_2 \in [\hat{c}_2,\ \tilde{c}_2)$, no optimal two-part tariff contract exists by Lemma 4, and Proposition 1(ii) shows $R_3^u > R_2^u$. Finally, for $c_2 \in [\tilde{c}_2,\ \bar{c}_2]$, no optimal two-part tariff contract exists by Lemma 4, and $R_3^u = R^{cu}$ is the optimal concession contract by Proposition 1(iii). In sum, for $c_2 \in (c_1,\ \hat{c}_2)$, the port authority will be better off by imposing the minimum throughput requirements on operators, while it will have the same equilibrium fee revenues in both scenarios if $c_2 \in [\hat{c}_2,\ \bar{c}_2]$. These prove Proposition 2. □

Proof of Lemma 7: The proofs are straightforward, and thus omitted. □

Lemma 8. *Given two-part tariff scheme* $(r,\ f)$ *and minimum throughput guarantee* δ, *optimal behaviors of the two operators are as follows.*

(i) *For* $\delta \in [0,\ \delta_{p1}]$ *with* $\delta_{p1} = \frac{(1-r)}{(1+b)(2-b)} + \frac{[bc_1-(2-b^2)c_2]}{(1-b^2)(4-b^2)}$, *both operators' equilibrium service prices are* $p^*_{p1} = \frac{1-b+r}{2-b} + \frac{2c_1+bc_2}{4-b^2} > 0$ *and* $p^*_{p2} = \frac{1-b+r}{2-b} + \frac{bc_1+2c_2}{4-b^2} > 0$, *and the equilibrium cargo-handling amounts are* $q^*_{p1} = \frac{1-r}{(1+b)(2-b)} + \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)} > \delta_{p1}$ *and* $q^*_{p2} = \delta_{p1}$. *The equilibrium profit of operator* i *is* $\pi^*_{pi} = (1-b^2)(q^*_{pi})^2 - f$ *for* $i = 1, 2$.

(ii) *For* $\delta \in (\delta_{p1},\ \delta_{p2}]$ *with* $\delta_{p2} \equiv \frac{1-c_1-r}{(1+b)(2-b)}$, *both operators' equilibrium service prices are* $p^*_{p1} = \frac{(1-b^2)(1-b\delta)+c_1+r}{2-b^2}$ *and* $p^*_{p2} = \frac{[(1-b)(2+b)-2(1-b^2)\delta+bc_1+br]}{(2-b^2)} > 0$, *their equilibrium cargo-handling amounts are* $q^*_{p1} = \frac{[1-b\delta-c_1-r]}{2-b^2}$ *and* $q^*_{p2} = \delta$, *and their equilibrium profits are* $\pi^*_{p1} = (1-b^2)(q^*_{p1})^2 - f$ *and* $\pi^*_{p2} = \frac{\delta}{(2-b^2)}[(1-b)(2+b)(1-r) - 2(1-b^2)\delta + bc_1 - (2-b^2)c_2] - f$.

(iii) *For* $\delta \in (\delta_{p2},\ \frac{1}{1+b})$, *operators' equilibrium service prices are* $p^*_{p1} = p^*_{p2} = 1-(1+b)\delta > 0$, *their equilibrium cargo-handling amounts are* $q^*_{p1} = q^*_{p1} = \delta$, *and operator i's equilibrium profit is* $\pi^*_{pi} = \delta[1-(1+b)\delta - r - c_i] - f$ *for* $i = 1, 2$.

Proof of Lemma 8: Denote L_1 and L_2 the respective Lagrange functions of operators 1 and 2 in problem (30) with

$$\begin{aligned} L_1 &= (p_1-c_1-r)[\frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2}] - f + \lambda_1[\frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} - \delta] \text{ and} \\ L_2 &= (p_2-c_2-r)[\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2}] - f + \lambda_2[\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2} - \delta], \end{aligned}$$

where λ_1 and λ_2 are the Lagrange multipliers associated with the operators. Then, the Kuhn-Tucker conditions for operator 1 are

$$\frac{\partial L_1}{\partial p_1} = [\frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2}] - \frac{(p_1-c_1-r)}{1-b^2} - \frac{\lambda_1}{1-b^2} \leq 0,\ p_1 \cdot \frac{\partial L_1}{\partial p_1} = 0, \quad (A61)$$

$$\frac{\partial L_1}{\partial \lambda_1} = \frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} - \delta \geq 0,\ \lambda_1 \cdot \frac{\partial L_1}{\partial \lambda_1} = 0, \quad (A62)$$

and for operator 2 are

$$\frac{\partial L_2}{\partial p_2} = [\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2}] - \frac{(p_2-c_2-r)}{1-b^2} - \frac{\lambda_2}{1-b^2} \leq 0,\ p_2 \cdot \frac{\partial L_2}{\partial p_2} = 0, \quad (A63)$$

$$\frac{\partial L_2}{\partial \lambda_2} = \frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2} - \delta \geq 0,\ \lambda_2 \cdot \frac{\partial L_2}{\partial \lambda_2} = 0. \quad (A64)$$

According to the values of λ_1 and λ_2, there are four cases as follows.

Case 1: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then (A61) and (A63) become

$$[\tfrac{1}{1+b} - \tfrac{p_1}{1-b^2} + \tfrac{bp_2}{1-b^2}] - \tfrac{(p_1-c_1-r)}{1-b^2} = 0 \text{ and}$$
$$[\tfrac{1}{1+b} - \tfrac{p_2}{1-b^2} + \tfrac{bp_1}{1-b^2}] - \tfrac{(p_2-c_2-r)}{1-b^2} = 0.$$

Solving these equations yields $p_{p1}^* = \frac{1-b+r}{2-b} + \frac{2c_1+bc_2}{4-b^2} > 0$ and $p_{p2}^* = \frac{1-b+r}{2-b} + \frac{bc_1+2c_2}{4-b^2} > 0$. Substituting p_{p1}^* and p_{p2}^* into (27)-(28) yields $q_{p1}^* = \frac{1-r}{(1+b)(2-b)} + \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)}$ and $q_{p2}^* = \frac{1-r}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)}$. To guarantee $q_{p1}^* \geq \delta$ and $q_{p2}^* \geq \delta$, condition $0 \leq \delta \leq \delta_{p1} \equiv \frac{1-r}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)} = q_{p2}^*$ is needed. Substituting p_{p1}^* and p_{p2}^* into (29) yields $\pi_{pi}^* = (1-b^2)(q_{pi}^*)^2 - f$ for $i = 1, 2$. These prove Lemma 8(i).

Case 2: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then (A61), (A63), and (A64) suggest

$$[\tfrac{1}{1+b} - \tfrac{p_1}{1-b^2} + \tfrac{bp_2}{1-b^2}] - \tfrac{(p_1-c_1-r)}{1-b^2} = 0,$$
$$[\tfrac{1}{1+b} - \tfrac{p_2}{1-b^2} + \tfrac{bp_1}{1-b^2}] - \tfrac{(p_2-c_2-r)}{1-b^2} - \tfrac{\lambda_2}{1-b^2} = 0, \text{ and}$$
$$\tfrac{1}{1+b} - \tfrac{p_2}{1-b^2} + \tfrac{bp_1}{1-b^2} - \delta = 0.$$

Solving these equations yields $\lambda_2^* = \frac{(1-b^2)(4-b^2)(\delta-\delta_1)}{2-b^2}$, $p_{p1}^* = \frac{(1-b^2)(1-b\delta)+c_1+r}{(2-b^2)}$, and $p_{p2}^* = \frac{[(1-b)(2+b)-2(1-b^2)\delta+bc_1+br]}{2-b^2}$. Substituting p_{p1}^* and p_{p2}^* into (27)-(28) yields $q_{p1}^* = \frac{(1-b\delta-c_1-r)}{(2-b^2)}$ and $q_{p2}^* = \delta$. To guarantee $\lambda_2^* > 0$, conditions $\delta > \delta_{p1}$, $r \leq \bar{r}_p$, and $c_2 < \bar{c}_{p2}$ are needed; and condition $\delta \leq \delta_{p2} \equiv \frac{1-c_1-r}{(1+b)(2-b)}$ is needed to guarantee $q_{p1}^* \geq \delta$. Thus, the plausible range for δ is $\delta \in (\delta_{p1},\ \delta_{p2}]$. Under the circumstance, $p_{p1}^* = \frac{(1-b^2)(1-b\delta)+c_1+r}{(2-b^2)} > p_{p2}^* = \frac{[(1-b)(2+b)-2(1-b^2)\delta+bc_1+br]}{2-b^2} > 0$ if $\delta \leq \delta_{p2}$. Substituting p_{p1}^* and p_{p2}^* into (29) gives $\pi_{p1}^* = (1-b^2)(q_{p1}^*)^2 - f$ and $\pi_{p2}^* = \frac{\delta}{(2-b^2)}[(1-b)(2+b)(1-r)-2(1-b^2)\delta+bc_1-(2-b^2)c_2]-f$. These prove Lemma 8(ii).

Case 3: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then (A61)-(A63) suggest

$$\tfrac{1}{1+b} - \tfrac{p_1}{1-b^2} + \tfrac{bp_2}{1-b^2} - \delta = 0,$$
$$[\tfrac{1}{1+b} - \tfrac{p_1}{1-b^2} + \tfrac{bp_2}{1-b^2}] - \tfrac{(p_1-c_1-r)}{1-b^2} - \tfrac{\lambda_1}{1-b^2} = 0, \text{ and}$$
$$[\tfrac{1}{1+b} - \tfrac{p_2}{1-b^2} + \tfrac{bp_1}{1-b^2}] - \tfrac{(p_2-c_2-r)}{1-b^2} = 0.$$

Solving these equations yields $p_{p1}^* = \frac{[(1-b)(2+b)-2(1-b^2)\delta+bc_1+br]}{(2-b^2)}$, $p_{p2}^* = \frac{(1-b^2)(1-b\delta)+c_2+r}{2-b^2}$, and $\lambda_1^* = \frac{(1-b^2)(4-b^2)}{2-b^2}[\delta - \frac{1-r}{(1+b)(2-b)} - \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)}]$. Substituting p_{p1}^* and p_{p2}^* into (27)-(28) yields $q_{p1}^* = \delta$ and $q_{p2}^* = \frac{(1-b\delta-c_2-r)}{2-b^2}$. To guarantee $\lambda_1^* > 0$, condition $\delta > \frac{1-r}{(1+b)(2+b)} + \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)}$ is needed. On the other hand, $q_{p2}^* \geq \delta$ is guaranteed if $\delta \leq \frac{1-c_2-r}{(1+b)(2-b)}$. However, these two conditions are incompatible with each other because $\frac{1-c_2-r}{(1+b)(2-b)} - \frac{1-r}{(1+b)(2-b)} - \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)} = \frac{-(2-b^2)(c_2-c_1)}{(1-b^2)(4-b^2)} < 0$. Thus, no solution exists in this case.

Case 4: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. Then (A61)-(A64) suggest

$$\frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} - \delta = 0,$$

$$\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2} - \delta = 0,$$

$$[\frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2}] - \frac{(p_1-c_1-r)}{1-b^2} - \frac{\lambda_1}{1-b^2} = 0, \text{ and}$$

$$[\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2}] - \frac{(p_2-c_2-r)}{1-b^2} - \frac{\lambda_2}{1-b^2} = 0.$$

Solving these equations yields $p_{p1}^* = p_{p2}^* = 1-(1+b)\delta$, $\lambda_1^* = (1+b)(2-b)\delta-(1-c_1-r)$, and $\lambda_2^* = (1+b)(2-b)\delta-(1-c_2-r)$. To guarantee $p_{p1}^* = p_{p2}^* > 0$, condition $\delta < \frac{1}{1+b}$ is needed. To have $\lambda_1^* > 0$ and $\lambda_2^* > 0$, conditions $\delta > \delta_{p2} \equiv \frac{1-c_1-r}{(1+b)(2-b)}$ and $r < (1-c_1)$ are needed. Note that $r < (1-c_1)$ is implied by $r \leq \bar{r}_p \equiv \frac{(1-b)(2+b)+bc_1-(2-b^2)c_2}{(1-b)(2+b)}$. Substituting $p_{p1}^* = p_{p2}^* = 1-(1+b)\delta$ into (27)-(28) generates $q_{p1}^* = q_{p2}^* = \delta$, and into (29) gives $\pi_{pi}^* = \delta[1-(1+b)\delta - c_i - r] - f$ for $i = 1, 2$. These prove Lemma 8(iii). □

Lemma 9. *Suppose conditions in (31) hold. Given* $(q_{p1}^*,\ q_{p2}^*,\ \pi_{p1}^*,\ \pi_{p2}^*)$ *in Lemma 8, port authority's optimal two-part tariff scheme and minimum throughout requirement* $(r_p^*,\ f_p^*,\ \delta_p^*)$ *are as follows.*

(i) *If* $c_2 \in (c_1,\ c_{p2}')$ *with* $c_{p2}' = \frac{2(1-b)+(5-b)c_1}{(7-3b)}$, *then we have* $r_p^* = \frac{[2-(4-b)c_1+(2-b)c_2]}{2(3-b)}$, $f_p^* = \frac{\{(2-c_1-c_2)[2(1-b)+(5-b)c_1-(7-3b)c_2]\}}{8(1+b)(3-b)^2}$, *and* $\delta_p^* = \frac{(2-c_1-c_2)}{2(1+b)(3-b)} > 0$. *At the equilibrium, operator i's cargo-handling amount is* $q_{pi}^* = \delta_p^* > 0$ *for* $i = 1,\ 2$, *as in Lemma 8(ii), and port authority's fee revenue equals* $R_p^* = \frac{(2-c_1-c_2)^2}{4(1+b)(3-b)} > 0$.

(ii) *If* $c_2 \in (c_{p2}',\ \hat{c}_{p2}]$ *with* $\hat{c}_{p2} = \frac{[2(1-b)(2+b)+(2+3b-b^2)c_1]}{(6+b-3b^2)} > c_{p2}'$, *then we have* $r_p^* = \frac{[2(2+b)-(4+2b-b^2)c_1-b^2c_2]}{2(2+b)(3-b)}$, $f_p^* = \frac{(1-b^2)}{2}[\frac{2(1-b)(2+b)+(2+3b-b^2)c_1-(6+b-3b^2)c_2}{2(2+b)(3-b)(1-b^2)}]^2$, *and* $\delta_p^* \in [0,$

$\frac{2(1-b)(2+b)+(2+3b-b^2)c_1-(6+b-3b^2)c_2}{2(2+b)(3-b)(1-b^2)}]$. *At the equilibrium, operators' cargo-handling amounts are* $q_{p1}^* = \frac{1-r_p^*}{(1+b)(2-b)} + \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)}$ *and* $q_{p2}^* = \frac{1-r_p^*}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)}$ *as in Lemma 8(i), and port authority's fee revenue equals* $R_p^* = (1-b^2)[\frac{2(1-b)(2+b)+(2+3b-b^2)c_1-(6+b-3b^2)c_2}{2(2+b)(3-b)(1-b^2)}]^2 + r_p^*[\frac{2(1-r_p^*)-(c_1+c_2)}{(1+b)(2-b)}]$.

Lemma 10. *Suppose conditions in (31) hold. Given* $(q_{p1}^u, q_{p2}^u, \pi_{p1}^u, \pi_{p2}^u)$ *derived in problem (32), port authority's optimal unit-fee scheme and minimum throughout requirement* $(r_p^u,\ \delta_p^u)$ *are as follows.*
(i) *If* $c_2 \in (0,\ \dot{c}_{p2}]$ *with* $\dot{c}_{p2} = \frac{[(6-b-b^2)+(2+b)c_1]-2\sqrt{(3+b)(1-2c_1+c_1^2)}}{(8-b^2)}$, *then we have* $r_p^u = \frac{(1-c_2)}{2} > 0$ *and* $\delta_p^u = \frac{(1-c_2)}{2(1+b)} > 0$. *At the equilibrium, operator i's cargo-handling amount is* $q_{pi}^u = \delta_p^u > 0$ *for* $i = 1,\ 2$, *and port authority's fee revenue equals* $R_p^u = \frac{(1-c_2)^2}{2(1+b)}$.
(ii) *If* $c_2 \in (\dot{c}_{p2},\ \tilde{c}_{p2}]$ *with* $\tilde{c}_{p2} = \frac{(1-b)(2+b)(3+b)+(2+5b+b^2)c_1}{(8+4b-3b^2-b^3)}$ *and* $\dot{c}_{p2} < \tilde{c}_{p2}$, *then we have* $r_p^u = \frac{[(3+b)-c_1-(2+b)c_2]}{2(3+b)}$ *and* $\delta_p^u = \frac{(1-b)(2+b)(1-r_p^u)+bc_1-(2-b^2)c_2}{2(1-b^2)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{p1}^u = \frac{(1-b\delta_p^u-c_1-r_p^u)}{(2-b^2)}$ *and* $q_{p2}^u = \delta_p^u$, *and port authority's fee revenue equals* $R_p^u = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$.
(iii) *If* $c_2 \in (\tilde{c}_{p2},\ \bar{c}_{p2}]$ *with* $\bar{c}_{p2} = \frac{(1-b)(2+b)+bc_1}{(2-b^2)}$, *then we have* $r_p^u = \bar{r}_p = \frac{(1-b)(2+b)+bc_1-(2-b^2)c_2}{(1-b)(2+b)} > 0$ *and* $\delta_p^u = 0$. *At the equilibrium, operators' cargo-handling amounts are* $q_{p1}^u = \frac{(c_2-c_1)}{(1-b)(2+b)}$ *and* $q_{p2}^u = 0$, *and port authority's fee revenue equals* $R_p^u = \frac{(c_2-c_1)}{(1-b)^2(2+b)^2}[(1-b)(2+b)+bc_1-(2-b^2)c_2]$.

Lemma 11. *Suppose conditions in (31) hold. Then, given* $(q_{p1}^f, q_{p2}^f, \pi_{p1}^f, \pi_{p2}^f)$ *derived in problem (33) and* $c_2 \in (c_1,\ \bar{c}_{p2}]$ *with* $\bar{c}_{p2} = \frac{(1-b)(2+b)+bc_1}{(2-b^2)}$, *we have optimal fixed fee* $f_p^f = \frac{(1-b^2)}{2}[\frac{1}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)}]^2$ *and optimal minimum throughput guarantee* $\delta_p^f \in [0,\ \frac{1}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)}]$. *At the equilibrium, operators' cargo-handling amounts are* $q_{p1}^f = \frac{1}{(1+b)(2-b)} + \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)}$ *and* $q_{p2}^f = \frac{1}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)}$, *and port authority's fee revenue equals* $R_p^f = (1-b^2)[\frac{1}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)}]^2$.

Proof of Lemmas 9-11 and Proposition 3: Denote L the following problem

$$\max_{r,\,f,\,\delta}\ 2f + r(q_{p1}^* + q_{p2}^*)$$

$$\text{s.t. } 0 \leq \delta < \frac{1}{1+b},\ 0 \leq r \leq \bar{r}_p,\ \pi^*_{p1} \geq 0,\ \pi^*_{p2} \geq 0,\ \text{and } 0 \leq f \leq \min\{\pi^*_{p1},\ \pi^*_{p2}\}. \quad (A65)$$

Moreover, denote $L_1,\ L_2$, and L_3 the problems given in (34), (35), and (36), respectively. Let S_L be the set of $(r,\ f,\ \delta)$ satisfying all constraints in problem L, and S_{L_1} the set of $(r,\ f,\ \delta)$ satisfying all constraints in problem L_1. Similarly, S_{L_2} is the set of $(r,\ \delta)$ satisfying all constraints in problem L_2, and S_{L_3} is the set of $(f,\ \delta)$ satisfying all constraints in problem L_3.

First, since $q^*_{pi} = q^u_{pi}$ for all $(r,\ f,\ \delta)$, we have $\pi^*_{pi} \leq \pi^u_{pi}$ with equality held at $f = 0$, and $\pi^*_{pi} \geq 0$ will hold if operators handle nonzero cargo amounts for $i = 1,\ 2$. Thus, we have $S_{L_1} \subset S_L$, and the maximum fee revenue in problem L is not less than that in problem L_1. Second, we have $\pi^*_{pi} = (\pi^u_{pi} - f)$ for all $(r,\ f,\ \delta)$ because $q^*_{pi} = q^u_{pi}$ for $i = 1,\ 2$. Thus, for any $(r,\ \delta)$ with $\pi^u_{pi} \geq 0$, we have $\pi^*_{pi} = (\pi^u_{pi} - f) \geq \pi^u_{pi} - \min\{\pi^*_{p1},\ \pi^*_{p2}\} = \pi^u_{pi} - \min\{(\pi^u_{p1} - f),\ (\pi^u_{p2} - f)\} \geq f \geq 0$ by $0 < f \leq \min\{\pi^*_{p1},\ \pi^*_{p2}\}$. These imply $S_{L_2} \subset S_L$. On the other hand, we have $2f + r(q^*_{p1} + q^*_{p2}) \geq r(q^u_{p1} + q^u_{p2})$ for any given $(r,\ f,\ \delta)$. Thus, the maximum fee revenue in problem L is not less than that in problem L_2. They will be equal if the solutions in problem L have $f^* = 0$. Third, for any solution $(r^*,\ f^*,\ \delta^*)$ of problem L with $r^* = 0$, it must also be the solution of problem L_2. However, if $r^* > 0$, then the solution of problem L will have higher value than that of problem L_3. Thus, the maximum fee revenue in problem L is not less than that in problem L_3 as well. In sum, deriving optimal concession contracts is equivalent to solving problem L in (A65). If the associated solution $(r^*,\ f^*,\ \delta^*)$ has nonzero $(r^*,\ f^*)$, then the two-part tariff scheme is port authority's best choice, the unit-fee scheme is the best if $f^* = 0$, and the fixed-fee scheme is the best if $r^* = 0$. Thus, all the solutions of problem L are derived below.

Case 1: Suppose $\delta \in [0,\ \delta_{p1}]$. Then, Lemma 8(i) implies $\pi^*_{p1} > \pi^*_{p2}$ and $f^*_p = \pi^*_{p2} = (1-b^2)\frac{1}{2}(q^*_{p2})^2 > 0$. Thus, the problem in (A65) becomes

$$\begin{aligned} \max_{r,\ f,\ \delta} \quad & 2f + r[q^*_{p1} + q^*_{p2}] \\ \text{s.t.} \quad & 0 \leq \delta \leq \delta_{p1} \text{ and } 0 < r \leq \bar{r}_p. \end{aligned}$$

Its Lagrange function is

$$L = (1-b^2)(q_{p2}^*)^2 + r[q_{p1}^* + q_{p2}^*] + \lambda_1(\delta_{p1} - \delta) + \lambda_2(\bar{r}_p - r),$$

where λ_1 and λ_2 are the Lagrange multipliers associated with the inequality constraints of this problem. The Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = 2(1-b^2)q_{p2}^*\frac{\partial q_{p2}^*}{\partial r} + r[\frac{\partial q_{p1}^*}{\partial r} + \frac{\partial q_{p2}^*}{\partial r}] + (q_{p1}^* + q_2^*) + \lambda_1\frac{\partial \delta_{p1}}{\partial r} - \lambda_2 \leq 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \quad (A66)$$

$$\frac{\partial L}{\partial \delta} = -\lambda_1 \leq 0,\ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad (A67)$$

$$\frac{\partial L}{\partial \lambda_1} = \delta_{p1} - \delta \geq 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \text{ and} \quad (A68)$$

$$\frac{\partial L}{\partial \lambda_2} = \bar{r}_p - r \geq 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0. \quad (A69)$$

Based on the values of λ_1 and λ_2, there are four cases as follows.

Case 1a: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then (A66) becomes

$$\frac{[2(2+b) - 2(2+b)(3-b)r - (4+2b-b^2)c_1 - b^2c_2]}{(1+b)(2-b)(4-b^2)} = 0.$$

Solving this equation yields $r_p^* = \frac{1}{3-b} - \frac{(4+2b-b^2)c_1 + b^2c_2}{2(2+b)(3-b)} > 0$. It remains to check whether $r_p^* < \bar{r}_p$ holds. By some calculations, we have $r_p^* < \bar{r}_p$ iff $c_2 < \hat{c}_{p2} \equiv \frac{[2(1-b)(2+b)+(2+3b-b^2)c_1]}{(6+b-3b^2)}$. In addition, (A68) implies $\delta^* \in [0,\ \delta_{p1}]$ with $\delta_{p1} = \frac{1}{2(2+b)(3-b)(1-b^2)}$ $[2(1-b)(2+b) + (2+3b-b^2)c_1 - (6+b-3b^2)c_2]$ and $f^* = \frac{1}{2}(1-b^2)\cdot$ $[\frac{2(1-b)(2+b)+(2+3b-b^2)c_1-(6+b-3b^2)c_2}{2(2+b)(3-b)(1-b^2)}]^2 > 0$. Accordingly, port authority's equilibrium fee revenue equals

$$R_p^* = 2f_p^* + r_p^*[\frac{2(1-r_p^*) - (c_1+c_2)}{(1+b)(2-b)}]. \quad (A70)$$

Case 1b: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then we have $r_p^* = \bar{r}_p \equiv \frac{(1-b)(2+b)+bc_1-(2-b^2)c_2}{(1-b)(2+b)}$ by (A69) and $\lambda_2^* = \frac{[-2(1-b)(2+b)-(2+3b-b^2)c_1+(6+b-3b^2)c_2]}{(1-b^2)(4-b^2)}$ by (A66). Note that $r_p^* > 0$ iff $c_2 < \bar{c}_{p2} \equiv \frac{(1-b)(2+b)+bc_1}{2-b^2}$, $\lambda_2^* > 0$ iff $c_2 > \hat{c}_{p2}$, $f_p^* = 0$, and $\delta_p^* = 0$ due to $\delta_{p1} = \frac{1-r_p^*}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)} = 0$. At the equilibrium, port authority's fee revenue equals

$$R_p^* = \bar{r}_p q_{p1}^* = \frac{(c_2-c_1)[(1-b)(2+b) + bc_1 - (2-b^2)c_2]}{(1-b)^2(2+b)^2}. \quad (A71)$$

Case 1c: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then, (A68) suggests $\delta_p^* = \delta_{p1} > 0$. This in turn implies $\lambda_1^* = 0$ by (A67). It is a contradiction. Thus, no solution exists in this case.

Case 1d: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. As in Case 1b, we have $f_p^* = 0,\ r_p^* = \bar{r}_p,\ \delta_p^* = 0$, and $R_p^* = \bar{r}_p q_{p1}^* = \frac{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)^2(2+b)^2}$.

Case 2: Suppose $\delta \in (\delta_{p1},\ \delta_{p2}]$. Then, Lemma 8(ii) implies $\pi_{p1}^* > \pi_{p2}^*$, and $f_p^* = \frac{\delta[(1-b)(2+b)(1-r)-2(1-b^2)\delta+bc_1-(2-b^2)c_2]}{2(2-b^2)}$ with $f_p^* \geq 0$ iff $\delta \leq \frac{(1-b)(2+b)(1-r)+bc_1-(2-b^2)c_2}{2(1-b^2)}$ and $r \leq \frac{(1-b)(2+b)+bc_1-(2-b^2)c_2}{(1-b)(2+b)}$. In addition, $\frac{(1-b)(2+b)(1-r)+bc_1-(2-b^2)c_2}{2(1-b^2)} \geq (<)\ \delta_{p2}$ iff $r \leq (>)\frac{[(1-b)+c_1-(2-b)c_2]}{1-b}$. Thus, we have two sub-cases.

Case 2a: Suppose $r \leq \frac{[(1-b)+c_1-(2-b)c_2]}{1-b}$. Then the problem in (A65) becomes

$$\max_{r,\ f,\ \delta}\ 2f + r[q_{p1}^* + q_{p2}^*]$$

$$\text{s.t. } \delta_{p1} < \delta \leq \delta_{p2} \text{ and } 0 \leq r \leq \frac{[(1-b)+c_1-(2-b)c_2]}{1-b}. \qquad (A72)$$

Its Lagrange function is

$$\begin{aligned} L \ = \ & \frac{\delta}{(2-b^2)}[(1-b)(2+b)(1-r) - 2(1-b^2)\delta + bc_1 - (2-b^2)c_2] \\ & + \frac{r[1+(1-b)(2+b)-c_1-r]}{2-b^2} + \lambda_1(\delta - \delta_{p1}) + \lambda_2(\delta_{p2} - \delta) \\ & + \lambda_3\{\frac{1}{1-b}[(1-b)+c_1-(2-b)c_2] - r\}. \end{aligned}$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{(1-2r-c_1)}{2-b^2} + \frac{\lambda_1}{(1+b)(2-b)} - \frac{\lambda_2}{(1+b)(2-b)} - \lambda_3 \leq 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \quad (A73)$$

$$\frac{\partial L}{\partial \delta} = \frac{1}{2-b^2}[(1-b)(2+b) - 4(1-b^2)\delta + bc_1 - (2-b^2)c_2] + \lambda_1 - \lambda_2 \leq 0,\ \delta\frac{\partial L}{\partial \delta} = 0, \ (A74)$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_{p1} \geq 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \qquad (A75)$$

$$\frac{\partial L}{\partial \lambda_2} = \delta_{p2} - \delta \geq 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \text{ and} \qquad (A76)$$

$$\frac{\partial L}{\partial \lambda_3} = \frac{1}{1-b}[(1-b)+c_1-(2-b)c_2] - r \geq 0, \ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \tag{A77}$$

where λ_1, λ_2 and λ_3 are the Lagrange multipliers associated with the inequality constraints in (A72). Constraint $\delta_{p1} < \delta$ suggests $\lambda_1^* = 0$ by (A75). If $\lambda_2^* = 0$, we have $\delta_p^* = \frac{(1-b)(2+b)+bc_1-(2-b^2)c_2}{4(1-b^2)}$ by (A74). Note that $(\delta_{p2} - \delta_p^*) = \frac{(2-b^2)[-(1-b)-c_1+(2-b)c_2]}{4(1-b^2)(2-b)} \geq 0$ iff $c_2 \geq \frac{(1-b)+c_1}{(2-b)}$. However, since $r \leq \frac{[(1-b)+c_1-(2-b)c_2]}{1-b} \leq 0$ if $c_2 \geq \frac{(1-b)+c_1}{(2-b)}$, it is a contradiction. Thus, we have $\lambda_2^* > 0$. Based on the values of λ_3, there are two sub-cases.

<u>Case 2a-1</u>: Suppose $\lambda_3^* = 0$. Then (A73), (A74), and (A76) suggest $\frac{(1-2r-c_1)}{2-b^2} - \frac{\lambda_2}{(1+b)(2-b)} = 0$, $\frac{[(1-b)(2+b)-4(1-b^2)\delta+bc_1-(2-b^2)c_2]}{2-b^2} - \lambda_2 = 0$, and $(\delta_{p2} - \delta) = 0$. Solving these equations yields $r_p^* = \frac{[2-(4-b)c_1+(2-b)c_2]}{2(3-b)} > 0$, $\delta_p^* = \delta_{p2} = \frac{(2-c_1-c_2)}{2(1+b)(3-b)} > 0$, and $\lambda_2^* = \frac{(1+b)(2-b)[(1-b)+c_1-(2-b)c_2]}{(3-b)(2-b^2)}$. Since $\frac{[(1-b)+c_1-(2-b)c_2]}{1-b} - r_p^* = \frac{(2-b)[2(1-b)+(5-b)c_1-(7-3b)c_2]}{2(1-b)(3-b)} \geq 0$ iff $c_2 \leq \frac{2(1-b)+(5-b)c_1}{7-3b}$, we have $r_p^* \leq \frac{[(1-b)+c_1-(2-b)c_2]}{1-b}$ if $c_2 \leq \frac{2(1-b)+(5-b)c_1}{7-3b}$. On the other hand, $\lambda_2^* > 0$ iff $c_2 < \frac{(1-b)+c_1}{2-b}$. Since $\frac{(1-b)+c_1}{2-b} - \frac{2(1-b)+(5-b)c_1}{7-3b} = \frac{(1-b)(3-b)(1-c_1)}{(2-b)(7-3b)} > 0$, the equilibrium exists with fee revenue

$$R_p^* = \frac{(2-c_1-c_2)^2}{4(1+b)(3-b)} \tag{A78}$$

if $c_2 \leq \frac{2(1-b)+(5-b)c_1}{7-3b}$.

<u>Case 2a-2</u>: Suppose $\lambda_3^* > 0$. Then (A76) and (A77) suggest $r_p^* = \frac{[(1-b)+c_1-(2-b)c_2]}{1-b}$ and $\delta_p^* = \delta_{p2} = \frac{(c_2-c_1)}{(1-b^2)} > 0$. Moreover, (A74) implies $\lambda_2^* = \frac{[(1-b)(2+b)+(4+b)c_1-(6-b^2)c_2]}{2-b^2}$, and (A73) implies $\lambda_3^* = \frac{[-2(1-b)-(5-b)c_1+(7-3b)c_2]}{(2-b)(1-b^2)}$. By some calculations, we get $\delta_p^* = \delta_{p2} > \delta_{p1}$, $r^* \geq 0$ iff $c_2 \leq \frac{(1-b)+c_1}{2-b}$, $\lambda_2^* > 0$ iff $c_2 < \frac{(1-b)(2+b)+(4+b)c_1}{(6-b^2)}$, and $\lambda_3^* > 0$ iff $c_2 > \frac{2(1-b)+(5-b)c_1}{7-3b}$ with $\frac{2(1-b)+(5-b)c_1}{7-3b} < \frac{(1-b)(2+b)+(4+b)c_1}{(6-b^2)} < \frac{(1-b)+c_1}{2-b}$. Thus, under condition $\frac{2(1-b)+(5-b)c_1}{7-3b} < c_2 < \frac{(1-b)(2+b)+(4+b)c_1}{(6-b^2)}$, the equilibrium exists with fee revenue

$$R_p^* = \frac{2(c_2-c_1)[(1-b)+c_1-(2-b)c_2]}{(1+b)(1-b)^2}. \tag{A79}$$

<u>Case 2b</u>: Suppose $r > \frac{[(1-b)+c_1-(2-b)c_2]}{1-b}$. Then the problem in (A65) becomes

$$\max_{r,\, f,\, \delta} \ 2f + r[q_{p1}^* + q_{p2}^*]$$

$$\text{s.t. } \delta_{p1} < \delta \le \frac{(1-b)(2+b)(1-r)+bc_1-(2-b^2)c_2}{2(1-b^2)} \text{ and } \frac{[(1-b)+c_1-(2-b)c_2]}{1-b} < r < \bar{r}_p. \tag{A80}$$

Its Lagrange function is

$$\begin{aligned} L &= \frac{\delta[(1-b)(2+b)(1-r)-2(1-b^2)\delta+bc_1-(2-b^2)c_2]}{(2-b^2)} \\ &+\frac{r[1+(1-b)(2+b)\delta-c_1-r]}{(2-b^2)}+\lambda_1(\delta-\delta_{p1}) \\ &+\lambda_2[\frac{(1-b)(2+b)(1-r)+bc_1-(2-b^2)c_2}{2(1-b^2)}-\delta] \\ &+\lambda_3[r-\frac{(1-b)+c_1-(2-b)c_2}{(1-b)}]+\lambda_4(\bar{r}_p-r). \end{aligned}$$

Then, the associated Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{1-2r-c_1}{2-b^2}+\frac{\lambda_1}{(1+b)(2-b)}-\frac{(1-b)(2+b)\lambda_2}{2(1-b^2)}+\lambda_3-\lambda_4 \le 0,\ r\cdot\frac{\partial L}{\partial r}=0, \tag{A81}$$

$$\frac{\partial L}{\partial \delta} = \frac{1}{2-b^2}[(1-b)(2+b)-4(1-b^2)\delta+bc_1-(2-b^2)c_2]+\lambda_1-\lambda_2 \le 0,\ \delta\frac{\partial L}{\partial \delta}=0, \tag{A82}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta-\delta_{p1} \ge 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1}=0, \tag{A83}$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{(1-b)(2+b)(1-r)+bc_1-(2-b^2)c_2}{2(1-b^2)}-\delta \ge 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2}=0, \tag{A84}$$

$$\frac{\partial L}{\partial \lambda_3} = r-\frac{[(1-b)+c_1-(2-b)c_2]}{1-b} \ge 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3}=0, \text{ and} \tag{A85}$$

$$\frac{\partial L}{\partial \lambda_4} = \bar{r}_p-r \ge 0,\ \lambda_4\cdot\frac{\partial L}{\partial \lambda_4}=0. \tag{A86}$$

We have $\lambda_1^* = \lambda_3^* = 0$ due to the strict inequalities in (A80), (A83), and (A85). Thus, there are four sub-cases as follows.

<u>Case 2b-1</u>: Suppose $\lambda_2^* > 0$ and $\lambda_4^* = 0$. Then, (A81), (A82), and (A84) suggest

$$\begin{gathered} \tfrac{1-2r-c_1}{2-b^2}-\tfrac{(1-b)(2+b)}{2(1-b^2)}\lambda_2=0, \\ \tfrac{1}{2-b^2}[(1-b)(2+b)-4(1-b^2)\delta+bc_1-(2-b^2)c_2]-\lambda_2=0, \text{ and} \\ \tfrac{1}{2(1-b^2)}[(1-b)(2+b)(1-r)+bc_1-(2-b^2)c_2]-\delta=0. \end{gathered}$$

Solving these equations yields $r_p^* = \frac{[(3+b)-c_1-(2+b)c_2]}{2(3+b)}$, $\delta_p^* = \frac{(1-b)(2+b)(1-r_p^*)+bc_1-(2-b^2)c_2}{2(1-b^2)}$, and $\lambda_2^* = \frac{2(1+b)(c_2-c_1)}{(3+b)(2-b^2)} > 0$. Since $[r_p^* - \frac{[(1-b)+c_1-(2-b)c_2]}{1-b}] = \frac{-(3-2b-b^2)-(7+b)c_1+(10-b-b^2)c_2}{2(3-2b-b^2)}$ and $(\bar{r}_p - r_p^*) = \frac{[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)(2+b)} - \frac{[(3+b)-c_1-(2+b)c_2]}{2(3+b)} = \frac{1}{2(1-b)(2+b)(3+b)}[(1-b)(2+b)(3+b)+(2+5b+b^2)c_1-(8+4b-3b^2-b^3)c_2]$, we have $r_p^* > \frac{[(1-b)+c_1-(2-b)c_2]}{1-b}$ iff $c_2 > \frac{(3-2b-b^2)+(7+b)c_1}{(10-b-b^2)}$, $r_p^* \leq \bar{r}_p$ iff $c_2 \leq \frac{(1-b)(2+b)(3+b)+(2+5b+b^2)c_1}{(8+4b-3b^2-b^3)}$, and $r_p^* \geq 0$ iff $c_2 \leq \frac{(3+b)-c_1}{(2+b)}$ with $\frac{(3-2b-b^2)+(7+b)c_1}{(10-b-b^2)} < \frac{(1-b)(2+b)(3+b)+(2+5b+b^2)c_1}{(8+4b-3b^2-b^3)} < \frac{(3+b)-c_1}{(2+b)}$. Moreover, we have $\delta_p^* > \delta_{p1}$ by $r_p^* \leq \bar{r}_p$. Thus, under condition $\frac{(3-2b-b^2)+(7+b)c_1}{(10-b-b^2)} < c_2 \leq \frac{(1-b)(2+b)(3+b)+(2+5b+b^2)c_1}{(8+4b-3b^2-b^3)}$, the equilibrium exists with fee revenue

$$R_p^* = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}. \qquad (A87)$$

Case 2b-2: Suppose $\lambda_2^* > 0$ and $\lambda_4^* > 0$. Then, $r_p^* = \bar{r}_p$ is implied by (A86), and $\delta_p^* = \frac{(1-b)(2+b)(1-r_p^*)+bc_1-(2-b^2)c_2}{2(1-b^2)} = 0$ by (A84). However, since $\delta_{p1} = 0$ at $r_p^* = \bar{r}_p$, we have $\delta_p^* = \delta_{p1} = 0$, which contradicts $\delta > \delta_{p1}$. Thus, no solution exists in this case.

Case 2b-3: Suppose $\lambda_2^* = 0$ and $\lambda_4^* = 0$. Then, (A81) and (A82) suggest $r_p^* = \frac{1-c_1}{2} > 0$ and $\delta_p^* = \frac{(1-b)(2+b)+bc_1-(2-b^2)c_2}{4(1-b^2)}$. Note that $\frac{(1-b)(2+b)(1-r_p^*)+bc_1-(2-b^2)c_2}{2(1-b^2)} - \frac{(1-b)(2+b)+bc_1-(2-b^2)c_2}{4(1-b^2)} = \frac{-(2-b^2)(c_2-c_1)}{4(1-b^2)} < 0$, which contradicts $\delta \leq \frac{(1-b)(2+b)(1-r)+bc_1-(2-b^2)c_2}{2(1-b^2)}$ required by (A80). Thus, no solution exists in this case.

Case 2b-4: Suppose $\lambda_2^* = 0$ and $\lambda_4^* > 0$. Then, $r_p^* = \bar{r}_p$ is implied by (A86), and $\delta_p^* = \frac{(1-b)(2+b)+bc_1-(2-b^2)c_2}{4(1-b^2)}$ by (A82). Since $\frac{(1-b)(2+b)(1-r_p^*)+bc_1-(2-b^2)c_2}{2(1-b^2)} - \delta_p^* = \frac{-(1-b)(2+b)-bc_1+(2-b^2)c_2}{4(1-b^2)} < 0$ due to $c_2 < \bar{c}_{p2} \equiv \frac{(1-b)(2+b)+bc_1}{2-b^2}$, which contradicts $\delta \leq \frac{1}{2(1-b^2)}[(1-b)(2+b)(1-r)+bc_1-(2-b^2)c_2]$. Thus, no solution exists in this case.

Case 3: Suppose $\delta \in (\delta_{p2}, \frac{1}{1+b}]$. Then, Lemma 8(iii) implies $\pi_{p1}^* > \pi_{p2}^*$, and $f_p^* = \pi_{p2}^* = \frac{[1-(1+b)\delta-c_2-r]\delta}{2}$ with $f_p^* \geq 0$ iff $\delta \leq \frac{1-c_2-r}{(1+b)}$ and $r < (1-c_2)$. Note that $r < (1-c_2)$ is implied by $r \leq \bar{r}_p \equiv \frac{[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)(2+b)}$. Accordingly, the problem in (A65) becomes

$$\max_{r,\,\delta} \; \delta[1-(1+b)\delta-c_2-r]+2r\delta$$

$$\text{s.t. } \delta_{p2} < \delta \leq \frac{(1-c_2-r)}{(1+b)} \text{ and } 0 \leq r \leq \bar{r}_p. \qquad (A88)$$

Its Lagrange function associated with the problem in (A88) is

$$L = \delta[1-(1+b)\delta - c_2 - r] + 2r\delta + \lambda_1(\delta - \delta_{p2}) + \lambda_2[\frac{(1-c_2-r)}{(1+b)} - \delta] + \lambda_3(\bar{r}_p - r).$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \delta + \frac{\lambda_1}{(1+b)(2-b)} - \frac{\lambda_2}{1+b} - \lambda_3 \leq 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \tag{A89}$$

$$\frac{\partial L}{\partial \delta} = 1 - 2(1+b)\delta - c_2 + r + \lambda_1 - \lambda_2 \leq 0,\ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \tag{A90}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_{p2} \geq 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \tag{A91}$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{1-c_2-r}{(1+b)} - \delta \geq 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0,\ \text{and} \tag{A92}$$

$$\frac{\partial L}{\partial \lambda_3} = \bar{r}_p - r \geq 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0. \tag{A93}$$

Since $\delta > \delta_{p2}$, we have $\lambda_1^* = 0$ by (A91). According to the values of λ_2^* and λ_3^*, there are four sub-cases.

Case 3a: Suppose $\lambda_2^* > 0$ and $\lambda_3^* = 0$. Then, (A89), (A90) and (A92) suggest $\delta - \frac{\lambda_2}{1+b} = 0$, $1 - 2(1+b)\delta - c_2 + r - \lambda_2 = 0$, and $\frac{(1-c_2-r)}{(1+b)} - \delta = 0$. Solving these equations yields $r_p^* = \frac{(1-c_2)}{2} > 0$, $\delta_p^* = \frac{1-c_2}{2(1+b)} > 0$, and $\lambda_2^* = (1+b)\delta_p^* > 0$. Some calculations yield $(\bar{r}_p - r_p^*) = \frac{[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)(2+b)} - \frac{(1-c_2)}{2} = \frac{[(1-b)(2+b)+2bc_1-(1+b)(2-b)c_2]}{2(1-b)(2+b)}$ and $(\delta_p^* - \delta_{p2}) = \frac{1-c_2}{2(1+b)} - \frac{1-c_1-r_p^*}{(1+b)(2-b)} = \frac{(1-b)+2c_1-(3-b)c_2}{2(1+b)(2-b)}$. Thus, we have $r_p^* \leq \bar{r}_p$ iff $c_2 \leq \frac{(1-b)(2+b)+2bc_1}{(1+b)(2-b)}$, and $\delta_{p2} < \delta_p^*$ iff $c_2 < \frac{(1-b)+2c_1}{(3-b)}$ with $\frac{(1-b)(2+b)+2bc_1}{(1+b)(2-b)} > \frac{(1-b)+2c_1}{(3-b)}$. Under condition $c_2 < \frac{(1-b)+2c_1}{(3-b)}$, the equilibrium exists with fee revenue

$$R_p^* = \frac{(1-c_2)^2}{2(1+b)}. \tag{A94}$$

Case 3b: Suppose $\lambda_2^* > 0$ and $\lambda_3^* > 0$. Then we have $r_p^* = \bar{r}_p$ by (A93), and $\delta_p^* = \frac{b(c_2-c_1)}{(1-b^2)(2+b)} > 0$ by $r_p^* = \bar{r}_p$ and (A92). However, we have $(\delta_p^* - \delta_{p2}) = \frac{-2(c_2-c_1)}{(1+b)(4-b^2)} < 0$, which contradicts $\delta > \delta_{p2}$. Thus, no solution exists in this case.

Case 3c: Suppose $\lambda_2^* = 0$ and $\lambda_3^* = 0$. Then we have $\delta_p^* \leq 0$ by (A89), which contradicts $\delta > \delta_{p2} > 0$. Thus, no solution exists in this case.

Case 3d: Suppose $\lambda_2^* = 0$ and $\lambda_3^* > 0$. Then (A90) and (A93) suggest $r_p^* = \bar{r}_p$ and $\delta_p^* = \frac{1+\bar{r}_p}{2(1+b)}$. Some calculations yield $(\delta_p^* - \delta_{p2}) = \frac{2(1-b)(4-b^2)+(4+2b-3b^2)c_1-(2-b^2)(4-b)c_2}{2(1-b^2)(4-b^2)} > 0$ iff $c_2 < \frac{[2(1-b)(4-b^2)+(4+2b-3b^2)c_1]}{(2-b^2)(4-b)}$, and $[\frac{(1-c_2-r_p^*)}{(1+b)} - \delta_p^*] = \frac{1-c_2-\bar{r}_p}{(1+b)} - \frac{1+\bar{r}_p}{2(1+b)} = \frac{1}{2(1-b^2)(2+b)}[-2(2-b-b^2) - 3bc_1 + (2+2b-b^2)c_2] \geq 0$ iff $c_2 \geq \frac{2(2-b-b^2)+3bc_1}{(2+2b-b^2)}$, which contradicts $c_2 < \frac{2(1-b)(4-b^2)+(4+2b-3b^2)c_1}{(2-b^2)(4-b)}$ derived above. Thus, no solution exists in this case.

Finally, by comparing port authority's equilibrium fee revenues in Cases 1-3, we can derive the best concession contracts. Before doing this, we need to know relative sizes of the critical points in Cases 1-3, including $\hat{c}_{p2} \equiv \frac{2(1-b)(2+b)+(2+3b-b^2)c_1}{(6+b-3b^2)}$ and $\bar{c}_{p2} \equiv \frac{(1-b)(2+b)+bc_1}{2-b^2}$ in Case 1, $\check{c}_{p2} \equiv \frac{2(1-b)+(5-b)c_1}{7-3b}$ and $c_{p2}''' \equiv \frac{(1-b)(2+b)+(4+b)c_1}{(6-b^2)}$ in Case 2a, $c_{p2}' \equiv \frac{(3-2b-b^2)+(7+b)c_1}{(10-b-b^2)}$ and $\tilde{c}_{p2} \equiv \frac{(1-b)(2+b)(3+b)+(2+5b+b^2)c_1}{(8+4b-3b^2-b^3)}$ in Case 2b-1, and $c_{p2}'' \equiv \frac{(1-b)+2c_1}{(3-b)}$ in Case 3a. Since $(c_{p2}' - \check{c}_{p2}) = \frac{(1+b)(1-b)^2(1-c_1)}{(7-3b)(10-b-b^2)} > 0$, $(c_{p2}'' - c_{p2}') = \frac{(1-b)^2(1-c_1)}{(3-b)(10-b-b^2)} > 0$, $(c_{p2}''' - c_{p2}'') = \frac{(1-b)(1-c_1)}{(3-b)(6-b^2)} > 0$, $(\hat{c}_{p2} - c_{p2}''') = \frac{(1-b)(2+b)(6-b+b^2)(1-c_1)}{(6-b^2)(6+b-3b^2)} > 0$, $(\tilde{c}_{p2} - \hat{c}_{p2}) = \frac{(1+b)(1-b)^2(2+b)^2(1-c_1)}{(8+4b-3b^2-b^3)} > 0$, and $(\bar{c}_{p2} - \tilde{c}_{p2}) = \frac{2(1-b^2)(2+b)(1-c_1)}{(2-b^2)(8+4b-3b^2-b^3)} > 0$, we have $\check{c}_{p2} < c_{p2}' < c_{p2}'' < c_{p2}''' < \hat{c}_{p2} < \tilde{c}_{p2} < \bar{c}_{p2}$, and seven cases as follows.

Case A: For $c_2 \in (c_1, \ \check{c}_{p2}]$, an equilibrium exists in Case 1a with $R_p^* = 2f_p^* + r_p^*[\frac{2(1-r^*)-(c_1+c_2)}{(1+b)(2-b)}]$ in (A70), where $r_p^* = \frac{1}{3-b} - \frac{(4+2b-b^2)c_1+b^2c_2}{2(2+b)(3-b)}$ and $f_p^* = \frac{1}{2}(1-b^2)\cdot$ $[\frac{2(1-b)(2+b)+(2+3b-b^2)c_1-(6+b-3b^2)c_2}{2(2+b)(3-b)(1-b^2)}]^2$, an equilibrium in Case 2a-1 with $R_p^* = \frac{(2-c_1-c_2)^2}{4(1+b)(3-b)}$ in (A78), and an equilibrium in Case 3a with $R_p^* = \frac{(1-c_2)^2}{2(1+b)}$ in (A94). Define $M_1 = \frac{(1-c_2)^2}{2(1+b)} - 2f_p^* - r_p^*[\frac{2(1-r_p^*)-(c_1+c_2)}{(1+b)(2-b)}] = \frac{1}{4(1-b^2)(3-b)(2+b)^2}[(8-8b-6b^2+4b^3+2b^4)+(8-4b-4b^2)c_1 + (-24+20b+16b^2-8b^3-4b^4)c_2+(12b+6b^2-2b^3)c_1c_2-(4+4b+b^2-b^3)c_1^2+(12-16b-11b^2+5b^3+2b^4)c_2^2]$ with $\frac{\partial M_1}{\partial c_2} = \frac{(-12+10b+8b^2-4b^3-2b^4)+(6b+3b^2-b^3)c_1+(12-16b-11b^2+5b^3+2b^4)c_2}{2(1-b^2)(3-b)(2+b)^2}$. By some calculations, we get $\frac{\partial M_1}{\partial c_2}|_{c_2=c_1} = \frac{-(3-b-b^2)(1-c_1)}{(1+b)(2+b)(3-b)} < 0$, $\frac{\partial M_1}{\partial c_2}|_{c_2=\check{c}_{p2}} = \frac{1}{(3-b)(1+b)(2+b)^2(7-3b)}$ $[-(30+5b-13b^2-3b^3+b^4)(1-c_1)] < 0$, $M_1|_{c_2=c_1} = \frac{(1-b)(1-c_1)^2}{2(1+b)(3-b)} > 0$, and $M_1|_{c_2=\check{c}_{p2}} = \frac{(1-b)(52+40b-9b^2-4b^3+b^4)(1-c_1)^2}{2(3-b)(1+b)(2+b)^2(7-3b)^2} > 0$. Thus, we have $M_1 \geq \max\{M_1|_{c_2=c_1}, \ M_1|_{c_2=\check{c}_{p2}}\} > 0$, which implies $\frac{(1-c_2)^2}{2(1+b)} > 2f_p^* + r_p^*[\frac{2(1-r_p^*)-(c_1+c_2)}{(1+b)(2-b)}]$.

Next, define $M_2 = \frac{(1-c_2)^2}{2(1+b)} - \frac{(2-c_1-c_2)^2}{4(1+b)(3-b)} = \frac{2(1-b)+4c_1-4(2-b)c_2-2c_1c_2-c_1^2+(5-2b)c_2^2}{4(1+b)(3-b)}$. Since $\frac{\partial M_2}{\partial c_2} = \frac{-2(2-b)-c_1+(5-2b)c_2}{2(1+b)(3-b)}$ and $\frac{\partial^2 M_2}{\partial c_2^2} = \frac{(5-2b)}{2(1+b)(3-b)} > 0$, we have $\frac{\partial M_2}{\partial c_2} < \frac{-2(2-b)-c_1+(5-2b)\breve{c}_{p2}}{2(1+b)(3-b)} = \frac{-(3-b)(1-c_1)}{(1+b)(7-3b)} < 0$. Moreover, since $M_2 > \frac{(1-\breve{c}_{p2})^2}{2(1+b)} - \frac{(2-c_1-\breve{c}_{p2})^2}{4(1+b)(3-b)} = \frac{(1-b)^2(1-c_1)^2}{2(1+b)(7-3b)^2} > 0$ implied by $\frac{\partial M_2}{\partial c_2} < 0$ and $c_2 \leq \breve{c}_{p2}$, we have $\frac{(1-c_2)^2}{2(1+b)} > \frac{(2-c_1-c_2)^2}{4(1+b)(3-b)}$. Thus, the port authority will choose the optimal unit-fee contract in Case 3a with $r_p^* = \frac{(1-c_2)}{2}$, $\delta_p^* = \frac{1-c_2}{2(1+b)}$, $f_p^* = 0$, and $R_p^* = \frac{(1-c_2)^2}{2(1+b)}$.

Case B: For $c_2 \in (\breve{c}_{p2},\ c'_{p2}]$, an equilibrium exists in Case 1a with $R_p^* = 2f_p^* + r_p^*[\frac{2(1-r_p^*)-(c_1+c_2)}{(1+b)(2-b)}]$ in (A70), where $r_p^* = \frac{1}{3-b} - \frac{(4+2b-b^2)c_1+b^2c_2}{2(2+b)(3-b)}$ and $f_p^* = \frac{1}{2}(1-b^2)\cdot [\frac{2(1-b)(2+b)+(2+3b-b^2)c_1-(6+b-3b^2)c_2}{2(2+b)(3-b)(1-b^2)}]^2$, an equilibrium in Case 2a-2 with $R_p^* = \frac{1}{(1+b)(1-b)^2}$ $\{2(c_2-c_1)[(1-b)+c_1-(2-b)c_2]\}$ in (A79), and an equilibrium in Case 3a with $R_p^* = \frac{(1-c_2)^2}{2(1+b)}$ in (A94). As in Case A, some calculations show $\frac{\partial M_1}{\partial c_2}|_{c_2=c'_{p2}} = \frac{-(84+44b-25b^2-20b^3-3b^4)(1-c_1)}{2(1+b)(3-b)(2+b)^2(10-b-b^2)}$ < 0 and $M_1|_{c_2=c'_{p2}} = \frac{(1-b)(188+280b+133b^2+31b^3+7b^4+b^5)(1-c_1)^2}{4(1+b)(3-b)(2+b)^2(10-b-b^2)^2} > 0$. Thus, we have $M_1 \geq \max\{M_1|_{c_2=\breve{c}_{p2}},\ M_1|_{c_2=c'_{p2}}\} > 0$, and hence $\frac{(1-c_2)^2}{2(1+b)} > 2f_p^* + r_p^*[\frac{2(1-r_p^*)-(c_1+c_2)}{(1+b)(2-b)}]$. Moreover, we have $\frac{(1-c_2)^2}{2(1+b)} - \frac{2(c_2-c_1)[(1-b)+c_1-(2-b)c_2]}{(1+b)(1-b)^2} = \frac{[(1-b)+2c_1-(3-b)c_2]^2}{2(1+b)(1-b)^2} > 0$. Accordingly, the port authority will choose the optimal unit-fee contract in Case 3a with $r_p^* = \frac{1-c_2}{2}$, $\delta_p^* = \frac{(1-c_2)}{2(1+b)}$, $f_p^* = 0$, and $R_p^* = \frac{(1-c_2)^2}{2(1+b)}$.

Case C: For $c_2 \in (c'_{p2},\ c''_{p2})$, an equilibrium exists in Case 1a with $R_p^* = 2f_p^* + r_p^*[\frac{2(1-r_p^*)-(c_1+c_2)}{(1+b)(2-b)}]$ in (A70), where $r_p^* = \frac{1}{(3-b)} - \frac{(4+2b-b^2)c_1+b^2c_2}{2(2+b)(3-b)}$ and $f_p^* = \frac{1}{2}(1-b^2)\cdot [\frac{2(1-b)(2+b)+(2+3b-b^2)c_1-(6+b-3b^2)c_2}{2(2+b)(3-b)(1-b^2)}]^2$, an equilibrium in Case 2a-2 with $R_p^* = \frac{1}{(1+b)(1-b)^2}$ $\{2(c_2-c_1)[(1-b)+c_1-(2-b)c_2]\}$ in (A79), an equilibrium in Case 2b-1 with $R_p^* = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$ in (A87), and an equilibrium in Case 3a with $R_p^* = \frac{(1-c_2)^2}{2(1+b)}$ in (A94). As in Case A, some calculations yield $\frac{\partial M_1}{\partial c_2}|_{c_2=c''_{p2}} = \frac{-(24+10b-9b^2-5b^3)(1-c_1)}{2(1+b)(3-b)^2(2+b)^2} < 0$ and $M_1|_{c_2=c''_{p2}} = \frac{(1-b)(12+20b+7b^2+b^3)(1-c_1)^2}{4(1+b)(2+b)^2(3-b)^3} > 0$. Thus, we have $M_1 \geq \max\{M_1|_{c_2=c'_{p2}},\ M_1|_{c_2=c''_{p2}}\}$ > 0, and hence $\frac{(1-c_2)^2}{2(1+b)} > 2f_p^* + r_p^*[\frac{2(1-r_p^*)-(c_1+c_2)}{(1+b)(2-b)}]$. Moreover, we have $\frac{(1-c_2)^2}{2(1+b)} > \frac{1}{(1+b)(1-b)^2}$ $\{2(c_2-c_1)[(1-b)+c_1-(2-b)c_2]\}$ from Case B. Next, define $M_3 = \frac{(1-c_2)^2}{2(1+b)} - \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$ with $\frac{\partial M_3}{\partial c_2} = \frac{-(6-b-b^2)-(2+b)c_1+(8-b^2)c_2}{4(1+b)(3+b)}$ and $\frac{\partial^2 M_3}{\partial c_2^2} = \frac{(8-b^2)}{4(1+b)(3+b)} > 0$. Accordingly, $\frac{\partial M_3}{\partial c_2} < \frac{-(6-b-b^2)-(2+b)c_1+(8-b^2)c''_{p2}}{4(1+b)(3+b)} = \frac{-(1-c_1)(10-b-b^2)}{4(1+b)(9-b^2)} < 0$, $M_3 < \frac{(1-c'_{p2})^2}{2(1+b)} -$

$\frac{[(3+b)-c_1-(2+b)c'_{p2}]^2}{8(1+b)(3+b)} = \frac{(1-b)^2(1-c_1)^2}{2(1+b)(10-b-b^2)^2} > 0$, and $M_3 > \frac{(1-c''_{p2})^2}{2(1+b)} - \frac{[(3+b)-c_1-(2+b)c''_{p2}]^2}{8(1+b)(3+b)} = \frac{-(1-b)^2(1-c_1)^2}{8(1+b)(3+b)(3-b)^2} < 0$. Since M_3 can be positive or negative for $c_2 \in (c'_{p2},\ c''_{p2})$, there must exist some $\dot{c}_{p2}$ with $c'_{p2} < \dot{c}_{p2} < c''_{p2}$ so that $M_3 = 0$ at $c_2 = \dot{c}_{p2} = \frac{1}{(8-b^2)}\{[(6-b-b^2)+(2+b)c_1] - 2\sqrt{(3+b)(1-2c_1+c_1^2)}\}$. Accordingly, we have $\frac{(1-c_2)^2}{2(1+b)} \geq \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$ for $c_1 \in (c'_{p2},\ \dot{c}_{p2}]$, and $\frac{(1-c_2)^2}{2(1+b)} < \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$ for $c_2 \in (\dot{c}_{p2},\ < c''_{p2})$.

Thus, if $c_2 \in (c'_{p2},\ c''_{p2})$, there are two sub-cases. For $c_2 \in (c'_{p2},\ \dot{c}_{p2}]$, the port authority will choose the unit-fee contract in Case 3a with $r_p^* = \frac{(1-c_2)}{2}$, $f_p^* = 0$, $\delta_p^* = \frac{(1-c_2)}{2(1+b)}$, and $R_p^* = \frac{(1-c_2)^2}{2(1+b)}$ in (A94). For $c_2 \in (\dot{c}_{p2},\ c''_{p2})$, the port authority will choose the unit-fee contract in Case 2b-1 with $r_p^* = \frac{[(3+b)-c_1-(2+b)c_2]}{2(3+b)}$, $\delta_p^* = \frac{(1-b)(2+b)(1-r_p^*)+bc_1-(2-b^2)c_2}{2(1-b^2)}$, $f_p^* = 0$, and $R_p^* = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$ in (A87).

<u>Case D</u>: For $c_2 \in [c''_{p2},\ c'''_{p2})$, an equilibrium exists in Case 1a with $R_p^* = 2f_p^* + r_p^*[\frac{2(1-r_p^*)-(c_1+c_2)}{(1+b)(2-b)}]$ in (A70), where $r_p^* = \frac{1}{3-b} - \frac{(4+2b-b^2)c_1+b^2c_2}{2(2+b)(3-b)}$ and $f_p^* = \frac{1}{2}(1-b^2)\cdot[\frac{2(1-b)(2+b)+(2+3b-b^2)c_1-(6+b-3b^2)c_2}{2(2+b)(3-b)(1-b^2)}]^2$, an equilibrium in Case 2a-2 with $R_p^* = \frac{1}{(1+b)(1-b)^2}\cdot\{2(c_2-c_1)[(1-b)+c_1-(2-b)c_2]\}$ in (A79), and an equilibrium in Case 2b-1 with $R_p^* = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$ in (A87). Define $M_4 = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)} - 2f_p^* - r_p^*[\frac{2(1-r_p^*)-(c_1+c_2)}{(1+b)(2-b)}] = \frac{1}{8(1-b)(2+b)^2(9-b^2)}[(12-8b-13b^2+3b^3+5b^4+b^5)+(-24+16b+14b^2-4b^3-2b^4)c_1+(12b^2-2b^3-8b^4-2b^5)c_2+(48+32b-16b^2-2b^3+2b^4)c_1c_2+(-12-24b+b^2+3b^3)c_1^2+(-24-16b+2b^2+2b^3+3b^4+b^5)c_2^2]$. Since $\frac{\partial M_4}{\partial c_2} = \frac{1}{4(1-b)(2+b)^2(9-b^2)}[(6b^2-b^3-4b^4-b^5)+(24+16b-8b^2-b^3+b^4)c_1+(-24-16b+2b^2+2b^3+3b^4+b^5)c_2]$ and $\frac{\partial^2 M_4}{\partial c_2^2} = \frac{-(24+16b-2b^2-2b^3-3b^4-b^5)c_2}{4(1-b)(2+b)^2(9-b^2)} < 0$, we have $\frac{\partial M_4}{\partial c_2} < \frac{1}{4(1-b)(2+b)^2(9-b^2)}[(6b^2-b^3-4b^4-b^5)+(24+16b-8b^2-b^3+b^4)c_1+(-24-16b+2b^2+2b^3+3b^4+b^5)c''_{p2}] = \frac{-(24+16b-20b^2-11b^3-b^4)(1-c_1)}{4(3+b)(2+b)^2(3-b)^2} < 0$, and $M_4 > \frac{1}{8(1-b)(2+b)^2(9-b^2)}[(12-8b-13b^2+3b^3+5b^4+b^5)+(-24+16b+14b^2-4b^3-2b^4)c_1+(12b^2-2b^3-8b^4-2b^5)c_2+(48+32b-16b^2-2b^3+2b^4)c_1c_2+(-12-24b+b^2+3b^3)c_1^2+(-24-16b+2b^2+2b^3+3b^4+b^5)c_2^2] = \frac{(1-b)(42+10b+b^2+b^3)(1-c_1)^2}{4(9-b^2)(6-b^2)^2} > 0$. These imply $\frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)} > 2f_p^* + r_p^*[\frac{2(1-r_p^*)-(c_1+c_2)}{(1+b)(2-b)}]$. Moreover, $\frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)} - \frac{2(c_2-c_1)[(1-b)+c_1-(2-b)c_2]}{(1+b)(1-b)^2} = \frac{[(3-2b-b^2)+(7+b)c_1-(10-b-b^2)c_2]^2}{8(1+b)(3+b)(1-b)^2} > 0$. Thus, the port authority will choose the unit-fee contract in Case 2b-1 with $r_p^* = \frac{[(3+b)-c_1-(2+bc_2]}{2(3+b)}$, $f_p^* = 0$, $\delta_p^* = \frac{(1-b)(2+b)(1-r_p^*)+bc_1-(2-b^2)c_2}{2(1-b^2)}$, and $R_p^* = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$ in

(A87).

Case E: For $c_2 \in [c'''_{p2},\ \hat{c}_{p2})$, an equilibrium exists in Case 1a with $R_p^* = 2f_p^* + r_p^*[\frac{2(1-r_p^*)-(c_1+c_2)}{(1+b)(2-b)}]$ in (A70), where $r^* = \frac{1}{(3-b)} - \frac{(4+2b-b^2)c_1+b^2c_2}{2(2+b)(3-b)}$ and $f_p^* = \frac{1}{2}(1-b^2)\cdot [\frac{2(1-b)(2+b)+(2+3b-b^2)c_1-(6+b-3b^2)c_2}{2(2+b)(3-b)(1-b^2)}]^2$, and an equilibrium in Case 2b-1 with $R_p^* = \frac{1}{8(1+b)(3+b)}[(3+b)-c_1-(2+b)c_2]^2$ in (A87). Since $\frac{\partial M_4}{\partial c_2} < 0$ by the results of Case D, we have $M_4 > \frac{1}{8(1-b)(2+b)^2(9-b^2)}[(12-8b-13b^2+3b^3+5b^4+b^5)+(-24+16b+14b^2-4b^3-2b^4)c_1+(12b^2-2b^3-8b^4-2b^5)\hat{c}_{p2}+(48+32b-16b^2-2b^3+2b^4)c_1\hat{c}_{p2}+(-12-24b+b^2+3b^3)c_1^2+(-24-16b+2b^2+2b^3+3b^4+b^5)\hat{c}_{p2}^2] = \frac{(1+b)(1-b)^2(2+b)^2(1-c_1)^2}{8(3+b)(6+b-3b^2)^2} > 0$. Thus, the port authority will choose the unit-fee contract in Case 2b-1 with $r_p^* = \frac{[(3+b)-c_1-(2+b)c_2]}{2(3+b)}$, $f_p^* = 0$, $\delta_p^* = \frac{(1-b)(2+b)(1-r_p^*)+bc_1-(2-b^2)c_2}{2(1-b^2)}$, and $R_p^* = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$ in (A87).

Case F: For $c_2 \in [\hat{c}_{p2},\ \tilde{c}_{p2}]$, an equilibrium exists in Case 1b with $R_p^* = \bar{r}_p q_{p1}^* = \frac{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)^2(2+b)^2}$ in (A71), and an equilibrium in Case 2b-1 with $R_p^* = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$ in (A87). By some calculations, we have $\frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)} - \frac{1}{(1-b)^2(2+b)^2}\{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]\} = \frac{[(6-b-4b^2-b^3)+(2+5b+b^2)c_1-(8+4b-3b^2-b^3)c_2]^2}{8(1+b)(3+b)(1-b)^2(2+b)^2} > 0$. Thus, the best choice for the port authority is the unit-fee contract with $r_p^* = \frac{[(3+b)-c_1-(2+b)c_2]}{2(3+b)}$, $\delta_p^* = \frac{(1-b)(2+b)(1-r_p^*)+bc_1-(2-b^2)c_2}{2(1-b^2)}$, $f_p^* = 0$, and $R_p^* = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$ in (A87).

Case G: For $c_2 \in (\tilde{c}_{p2},\ \bar{c}_{p2})$, only the equilibrium in Case 1b exists with $R_p^* = \bar{r}_p q_{p1}^* = \frac{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)^2(2+b)^2}$ in (A71). Thus, the best choice for the port authority is the unit-fee contract with $r_p^* = \bar{r}_p \equiv \frac{(1-b)(2+b)+bc_1-(2-b^2)c_2}{(1-b)(2+b)}$, $\delta_p^* = 0$, $f_p^* = 0$, and $R_p^* = \bar{r}_p q_{p1}^* = \frac{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)^2(2+b)^2}$ in (A71). □

Proof of Corollary 4: By some calculations, we have $\dot{c}_{p2} < \ddot{c}_2$, $\tilde{c}_{p2} > (\leq)\ \ddot{c}_2$ if $b < (\geq)\ 0.830504$, $\tilde{c}_{p2} < \tilde{c}_2$, $\bar{c}_{p2} < \bar{c}_2$, $\tilde{c}_2 < (\geq)\ \bar{c}_{p2}$ iff $b < (\geq)\ 0.807374$, and $\bar{c}_{p2} > (\leq)\ \ddot{c}_2$ if $b < (\geq)\ 0.830504$. According to relative sizes of c_1, $\dot{c}_{p2}$, $\ddot{c}_2$, $\tilde{c}_{p2}$, $\tilde{c}_2$, $\bar{c}_{p2}$, and $\bar{c}_2$, we can prove Corollary 4 by the ensuing Lemmas A, B, C and D.

Lemma A. *Suppose* $b < 0.807374$. *Then we have the following.*

(i) *For* $c_2 \in (c_1,\ \dot{c}_{p2}]$, *terminal operators' best choices and port authority's optimal contracts under both competition modes are the same.*

(ii) *For* $c_2 \in (\dot{c}_{p2},\ \ddot{c}_2]$, *operators will rent terminals under both competition modes, but operator 2's equilibrium profit equals zero. Moreover, we have* $R_p^u > R^u$, $\pi_{p2}^u = \pi_2^u = 0$, $\pi_{p1}^u \leq \pi_1^u$ *for* $c_2 \in (\dot{c}_{p2},\ v]$, $\pi_{p1}^u > \pi_1^u$ *for* $c_2 \in (v,\ \ddot{c}_2]$ *if* $b > 0.226985$, *and* $\pi_{p1}^u < \pi_1^u$ *for* $c_2 \in (\dot{c}_{p2},\ \ddot{c}_2]$ *if* $b < 0.226985$, *where* v *is defined in the proofs below.*

(iii) *For* $c_2 \in (\ddot{c}_2,\ \tilde{c}_{p2}]$, *we have* $R_p^u > R^u$, $\pi_{p2}^u = \pi_2^u = 0$, $\pi_{p1}^u \geq \pi_1^u$ *for* $c_2 \in (\ddot{c}_2,\ t]$ *and* $\pi_{p1}^u < \pi_1^u$ *for* $c_2 \in (t,\ \tilde{c}_{p2}]$ *if* $0.391041 < b < 0.45193$, $\pi_{p1}^u > \pi_1^u$ *for* $c_2 \in (\ddot{c}_2,\ \tilde{c}_{p2}]$ *if* $b > 0.45193$ *and* $\pi_{p1}^u < \pi_1^u$ *for* $c_2 \in (\ddot{c}_2,\ \tilde{c}_{p2}]$ *if* $b < 0.391041$, *where* t *is defined in the proofs below.*

(iv) *For* $c_2 \in (\tilde{c}_{p2},\ \tilde{c}_2)$, *operator 2 will not rent terminals under price competition, but operator 1 will always rent them. Accordingly, we have* $R_p^u > R^u$ *for* $c_2 \in (\tilde{c}_{p2},\ \tilde{c}_2)$ *if* $b < 0.643333$, $R_p^u \geq R^u$ *for* $c_2 \in (\tilde{c}_{p2},\ y]$ *and* $R_p^u < R^u$ *for* $c_2 \in (y,\ \tilde{c}_2)$ *if* $b > 0.643333$, $\pi_{p2}^u = \pi_2^u = 0$, $\pi_{p1}^u > \pi_1^u$ *for* $c_2 \in (\tilde{c}_{p2},\ \tilde{c}_2)$ *if* $b > 0.45193$, $\pi_{p1}^u \leq \pi_1^u$ *for* $c_2 \in (\tilde{c}_{p2},\ k]$ *and* $\pi_{p1}^u > \pi_1^u$ *for* $c_2 \in (k,\ \tilde{c}_2)$ *if* $b < 0.45193$,*where* y *and* k *are defined in the proofs below.*

(v) *For* $c_2 \in [\tilde{c}_2,\ \bar{c}_{p2})$, *operator 2 will not rent terminals under both competition modes, but operator 1 will always rent them. Accordingly, we have* $R_p^u < R^u$ *for* $c_2 \in [\tilde{c}_2,\ \bar{c}_{p2})$ *if* $b > 0.643333$, $R_p^u > R^u$ *for* $c_2 \in [\tilde{c}_2,\ w)$ *and* $R_p^u \leq R^u$ *for* $c_2 \in [w,\ \bar{c}_{p2})$ *if* $b < 0.643333$, $\pi_{p2}^u = \pi_2^u = 0$, *and* $\pi_{p1}^u > \pi_1^u$, *where* w *is defined in the proofs below.*

Proof. For $b < 0.807374$, we have $c_1 < \dot{c}_{p2} < \ddot{c}_2 < \tilde{c}_{p2} < \tilde{c}_2 < \bar{c}_{p2} < \bar{c}_2$. Then, there are five sub-cases.

(i) For $c_2 \in (c_1,\ \dot{c}_{p2}]$, Lemma 2(iii), Lemma 5(i), Lemma 8(iii), and Lemma 10(i) imply $r_p^u = r^u = \frac{1-c_2}{2}$, $\delta_p^u = \delta^u = \frac{1-c_2}{2(1+b)}$, $R_p^u = R^u = \frac{(1-c_2)^2}{2(1+b)}$, $q_{pi}^u = q_i^u = \frac{1-c_2}{2(1+b)}$, $p_{pi}^u = p_i^u = \frac{1+c_2}{2}$, $\pi_{p1}^u = \pi_1^u = \frac{(1-c_2)(c_2-c_1)}{2(1+b)}$, and $\pi_{p2}^u = \pi_2^u = 0$ for $i = 1,\ 2$.

(ii) For $c_2 \in (\dot{c}_{p2},\ \ddot{c}_2]$, by Lemma 2(iii) and Lemma 5(i), we have $R^u = \frac{(1-c_2)^2}{2(1+b)}$, $\pi_1^u = \frac{(1-c_2)(c_2-c_1)}{2(1+b)}$, and $\pi_2^u = 0$. Moreover, Lemma 8(ii) and Lemma 10(ii) suggest $R_p^u = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$, $\pi_{p1}^u = \frac{[-(3-2b-b^2)+(5+3b)c_1-(2+5b+b^2)c_2]}{16(1-b^2)(3+b)^2}$, and $\pi_{p2}^u = 0$. Thus, we can get $\frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)} > \frac{(1-c_2)^2}{2(1+b)}$ if $c_2 > \dot{c}_{p2}$. This means $R_p^u > R^u$. On the other hand, $(\pi_{p1}^u - \pi_1^u) = \frac{[-(3-2b-b^2)+(5+3b)c_1-(2+5b+b^2)c_2]}{16(1-b^2)(3+b)^2} - \frac{(1-c_2)(c_2-c_1)}{2(1+b)}$ is a convex function of c_2 because $\frac{\partial^2(\pi_{p1}^u-\pi_1^u)}{\partial c_2^2} = \frac{76-4b-11b^2+2b^3+b^4}{8(1-b^2)(3+b)^2} > 0$. By some calculations, we have $\pi_{p1}^u = \pi_1^u$ at $c_2 = u$ and $c_2 = v$, where

$$u = \frac{[(30-23b-11b^2+3b^3+b^4)+(46+19b-b^3)c_1] - 2(1-c_1)\sqrt{2(3+b)^3(1-b)^3}}{(76-4b-11b^2+2b^3+b^4)}$$

and

$$v = \frac{[(30-23b-11b^2+3b^3+b^4)+(46+19b-b^3)c_1] + 2(1-c_1)\sqrt{2(3+b)^3(1-b)^3}}{(76-4b-11b^2+2b^3+b^4)}$$

with $v > u$. Then, we can know relative sizes of π_{p1}^u and π_1^u by comparing $\dot{c}_{p2},\ \ddot{c}_2, u$, and v. Since $\frac{\partial^2(\pi_{p1}^u-\pi_1^u)}{\partial c_2^2} > 0$, $\frac{\partial(\pi_{p1}^u-\pi_1^u)}{\partial c_2} = \frac{-(1-b)(2+b)(7+b)(1-c_1)}{8(1+b)(3-b)(3+b)^2} < 0$ at $c_2 = c_2''$, and $(\pi_{p1}^u - \pi_1^u) = \frac{-(3+b)(1-b)^2(1-c_1)^2}{2(1+b)(10-b-b^2)^2} < 0$ at $c_2 = c_2'$ with $c_2' < \dot{c}_{p2} < c_2''$, where $c_2' = \frac{(3-2b-b^2)+(7+b)c_1}{(10-b-b^2)}$ and $c'' = \frac{(1-b)+2c_1}{(3-b)}$; we have $(\pi_{p1}^u - \pi_1^u) < 0$ at $c_2 = \dot{c}_{p2}$ and hence $u < \dot{c}_{p2} < v$. Then, we can show $(v - \ddot{c}_2) = \frac{(1-c_1)}{(8-3b^2)(76-4b-11b^2+2b^3+b^4)}[-(216+84b-36b^2-62b^3-15b^4+4b^5+b^6) + 2(8-3b^2)\sqrt{2(3+b)^3(1-b)^3} + (76-4b-11b^2+2b^3+b^4)\sqrt{2(6+2b-7b^2-b^3+2b^4)} \geq (\leq)\ 0$ iff $b \leq\ (\geq)\ 0.226985$. Accordingly, $u < \dot{c}_{p2} < \ddot{c}_2 < v$ if $b < 0.226985$, and $u < \dot{c}_{p2} < v < \ddot{c}_2$ if $b > 0.226985$. These suggest $\pi_{p1}^u < \pi_1^u$ if $b < 0.226985$, and $\pi_{p1}^u \leq \pi_1^u$ for $c_2 \in (\dot{c}_{p2},\ v]$ and $\pi_{p1}^u > \pi_1^u$ for $c_2 \in (v,\ \ddot{c}_2]$ if $b > 0.226985$.

(iii) For $c_2 \in (\ddot{c}_2,\ \tilde{c}_{p2}]$, Lemma 2(ii) and Lemma 5(ii) imply $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$, $\pi_1^u = \frac{[(3-5b+2b^2)-(5-2b-b^2)c_1+(2+3b-3b^2)c_2]^2}{4(3-2b)^2(2-b^2)^2}$, and $\pi_2^u = 0$. Moreover, Lemma 8(ii) and Lemma 10(ii) suggest $R_p^u = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$, $\pi_{p1}^u = \frac{[-(3-2b-b^2)+(5+3b)c_1-(2+5b+b^2)c_2]}{16(1-b^2)(3+b)^2}$, and $\pi_{p2}^u = 0$. By some calculations, we have $(R_p^u - R^u) = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)} - \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ with $\frac{\partial^2(R_p^u-R^u)}{\partial c_2^2} = \frac{-b^2(4+8b-3b^2-2b^3)}{4(1+b)(3+b)(3-2b)(2-b^2)} < 0$. That is, $(R_p^u - R^u)$ is a concave function of c_2. In addition, we can get $(R_p^u - R^u) = \frac{b^2(1-b)(72+48b-44b^2-16b^3+3b^4+b^5)(1-c_1)^2}{4(2-b^2)(3-2b)(8+4b-3b^2-b^3)^2} > 0$ at $c_2 = \tilde{c}_{p2}$, and $R_p^u > R^u$ at $c_2 = \ddot{c}_2$ because $\frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)} > \frac{(1-c_2)^2}{2(1+b)}$ for $c_2 > \dot{c}_{p2}$

and $\frac{(1-c_2)^2}{2(1+b)} = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ at $c_2 = \ddot{c}_2$. Thus, $R_p^u > R^u$ for $c_2 \in (\ddot{c}_2,\ \tilde{c}_{p2}]$ by $\frac{\partial^2(R_p^u - R^u)}{\partial c_2^2} < 0$, and $R_p^u > R^u$ at $c_2 = \tilde{c}_{p2}$ and $c_2 = \ddot{c}_2$.

On the other hand, we have $(\pi_{p1}^u - \pi_1^u) = \frac{b^2 H(b, c_1,\ c_2)}{16(1-b^2)(3+b)^2(3-2b)^2(2-b^2)^2}$, where $H(b,\ c_1,\ c_2)$ $= (81b^2-216b^3+162b^4+12b^5-47b^6+4b^7+4b^8)+(72-168b-238b^2+554b^3-66b^4-210b^5+32b^6+24b^7)c_1+(-72+168b+76b^2-122b^3-258b^4+186b^5+62b^6-32b^7-8b^8)c_2+(168-88b-524b^2-158b^3+504b^4+106b^5-112b^6-24b^7)c_1c_2+(-120+128b+381b^2-198b^3-219b^4+52b^5+40b^6)c_1^2+(-48-40b+224b^2+140b^3-123b^4-146b^5+25b^6+28b^7+4b^8)c_2^2$ with $\frac{\partial^2 H(b,\ c_1,\ c_2)}{\partial c_2^2} = 2b^2(-48-40b+224b^2+140b^3-123b^4-146b^5+25b^6+28b^7+4b^8) \leq\ (\geq)\ 0$ iff $b \leq\ (\geq)\ 0.53173$. This implies that $H(b,\ c_1,\ c_2)$ is a concave (convex) function of c_2 if $b <\ (>)\ 0.53173$. In addition, $H(b,\ c_1,\ c_2) = 0$ at $c_2 = s$ and $c_2 = t$, where

$$\begin{aligned} s \;=\; & \frac{1}{(-48-40b+224b^2+140b^3-123b^4-146b^5+25b^6+28b^7+4b^8)} \times [(36-84b \\ & -38b^2+61b^3+129b^4-93b^5-31b^6+16b^7+4b^8)+(-84+44b+262b^2+79b^3-252b^4 \\ & -53b^5+56b^6+12b^7)c_1+2(1-c_1)\sqrt{(1+b)(1-b)^5(18-6b-13b^2+3b^3+2b^4)^2}], \end{aligned}$$

and

$$\begin{aligned} t \;=\; & \frac{1}{(-48-40b+224b^2+140b^3-123b^4-146b^5+25b^6+28b^7+4b^8)} \times [(36-84b \\ & -38b^2+61b^3+129b^4-93b^5-31b^6+16b^7+4b^8)+(-84+44b+262b^2+79b^3-252b^4 \\ & -53b^5+56b^6+12b^7)c_1+2(1-c_1)\sqrt{(1+b)(1-b)^5(18-6b-13b^2+3b^3+2b^4)^2}], \end{aligned}$$

and $s \leq\ (\geq)\ t$ iff $b \leq\ (\geq)\ 0.53173$. Under the circumstance, $(s - \ddot{c}_2) = (1-c_1)(s_1 + s_2 + s_3) \times \frac{1}{(8-3b^2)(-48-40b+224b^2+140b^3-123b^4-146b^5+25b^6+28b^7+4b^8)}$, where $s_1 = -(3-2b)(2-b^2)(2b^7-3b^6-90b^5-133b^4+218b^3+278b^2+16b-96) \leq\ (\geq)\ 0$ iff $b \geq\ (\leq)\ 0.506603$, $s_2 = 2(8-3b^2)\sqrt{(1+b)(1-b)^5(18-6b-13b^2+3b^3+2b^4)^2} > 0$, and $s_3 = (-48-40b+224b^2+140b^3-123b^4-146b^5+25b^6+28b^7+4b^8)\sqrt{2(6+2b-7b^2-b^3+2b^4)} \leq\ (\geq)\ 0$ iff $b \leq\ (\geq)\ 0.53173$. By some calculations, we can show $(s_1+s_2+s_3) > 0$ if $b < 0.53173$, and $(s_1+s_2+s_3) < 0$ if $b > 0.53173$. Thus, $\ddot{c}_2 > s$. On the other hand, we have $(\tilde{c}_{p2} - t) = \frac{t_1+t_2}{(8+4b-3b^2-b^3)(-48-40b+224b^2+140b^3-123b^4-146b^5+25b^6+28b^7+4b^8)}$, where $t_1 = 2(8+4b-3b^2-b^3)\sqrt{(1+b)(1-b)^5(18-6b-13b^2+3b^3+2b^4)^2} > 0$, and $t_2 =$

$4(1-b^2)(3+b)^2(4b^6+11b^5-38b^4-10b^3+37b^2+20b-16) \leq\ (\geq)\ 0$ iff $b \leq\ (\geq)\ 0.4991$. Some calculations can show $(t_1+t_2) < 0$ if $b < 0.45193$ and $(t_1+t_2) > 0$ if $b > 0.45193$. That is, $\tilde{c}_{p2} > t$ if $b < 0.45193$, $\tilde{c}_{p2} < t$ if $0.45193 < b < 0.53173$, and $\tilde{c}_{p2} > t$ if $b > 0.53173$. Accordingly, if $b < 0.45193$, then $s < t$, $s < \ddot{c}_2$, $t < \tilde{c}_{p2}$, and $H(b,\ c_1,\ c_2)$ is a concave function of c_2. If $0.45193 < b < 0.53173$, we have $s < \ddot{c}_2 < \tilde{c}_{p2} < t$; and we have $t < s < \ddot{c}_2 < \tilde{c}_{p2}$ and $H(b,\ c_1,\ c_2)$ is a convex function of c_2 if $b > 0.53173$.

Next, we compare $\ddot{c}_2$ and t as $b < 0.45193$. By some calculations, we have $(t-\ddot{c}_2) = \frac{(1-c_1)(s_1-s_2+s_3)}{(8-3b^2)(-48-40b+224b^2+140b^3-123b^4-146b^5+25b^6+28b^7+4b^8)}$ with $s_1 > 0$, $s_2 > 0$, and $s_3 < 0$ if $b < 0.45193$. We can show $t < \ddot{c}_2$ if $b < 0.391041$ and $t > \ddot{c}_2$ if $0.391041 < b < 0.45193$. Thus, if $b < 0.53173$, then $H(b,\ c_1,\ c_2)$ is a convex function of c_2. Moreover, $s < t < \ddot{c}_2 < \tilde{c}_{p2}$, $\pi^u_{p1} < \pi^u_1$ for $c_2 \in (\ddot{c}_2,\ \tilde{c}_{p2}]$ if $b < 0.391041$; $s < \ddot{c}_2 < t < \tilde{c}_{p2}$, $\pi^u_{p1} \geq \pi^u_1$ for $c_2 \in (\ddot{c}_2,\ t]$ and $\pi^u_{p1} < \pi^u_1$ for $c_2 \in (t,\ \tilde{c}_{p2}]$ if $0.391041 < b < 0.45193$; $s < \ddot{c}_2 < \tilde{c}_{p2} < t$, $\pi^u_{p1} > \pi^u_1$ for $c_2 \in (\ddot{c}_2,\ \tilde{c}_{p2}]$ if $0.45193 < b < 0.53173$. As $b > 0.53173$, $H(b,\ c_1,\ c_2)$ is a convex function of c_2, $t < s < \ddot{c}_2 < \tilde{c}_{p2}$, and $\pi^u_{p1} > \pi^u_1$ for $c_2 \in (\ddot{c}_2,\ \tilde{c}_{p2}]$.

(iv) For $c_2 \in (\tilde{c}_{p2},\ \tilde{c}_2)$, Lemma 2(ii) and Lemma 5(ii) suggest $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$, $\pi^u_1 = \frac{[(3-5b+2b^2)-(5-2b-b^2)c_1+(2+3b-3b^2)c_2]^2}{4(3-2b)^2(2-b^2)^2}$, and $\pi^u_2 = 0$. On the other hand, Lemma 8(i) and Lemma 10(iii) imply $R^u_p = \frac{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)^2(2+b)^2}$, $\pi^u_{p1} = \frac{(1+b)(c_2-c_1)^2}{(1-b)(2+b)^2}$, and $\pi^u_{p2} = 0$. Thus, we have $(R^u_p - R^u) = \frac{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)^2(2+b)^2} - \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ with $\frac{\partial^2(R^u_p-R^u)}{\partial c_2^2} = \frac{-(64-64b-40b^2+48b^3+5b^4-10b^5+b^6)}{2(2-b^2)(3-2b)(1-b)^2(2+b)^2} < 0$, and $(R^u_p - R^u)$ is a strictly concave function of c_2. In addition, $R^u_p = R^u$ at $c_2 = x$ and $c_2 = y$, where

$$\begin{aligned} x \;=\; & \frac{1}{(64-64b-40b^2+48b^3+5b^4-10b^5+b^6)} \times [(48-80b+2b^2+47b^3-12b^4-7b^5 \\ & +2b^6) + (16+16b-42b^2+b^3+17b^4-3b^5-b^6)c_1 - 2b(1-b)(6-b-2b^2) \\ & \sqrt{(1-b)(2-b^2)(1-c_1)^2}], \end{aligned}$$

and

$$y = \frac{1}{(64-64b-40b^2+48b^3+5b^4-10b^5+b^6)}[(48-80b+2b^2+47b^3-12b^4-7b^5 +2b^6)+(16+16b-42b^2+b^3+17b^4-3b^5-b^6c_1+2b(1-b)(6-b-2b^2) \sqrt{(1-b)(2-b^2)(1-c_1)^2}]$$

with $(y-x) = \frac{4b(1-b)(6-b-2b^2)\sqrt{(1-b)(2-b^2)(1-c_1)^2}}{(64-64b-40b^2+48b^3+5b^4-10b^5+b^6)} > 0$. Thus, $R_p^u > R^u$ if $x < c_2 < y$, $R_p^u < R^u$ if $c_2 < x$, and $R_p^u < R^u$ if $c_2 > y$. By some calculations, we can obtain $(R_p^u - R^u) = \frac{b^2(1-b)(72+48b-44b^2-16b^3+3b^4+b^5)(1-c_1)^2}{4(2-b^2)(3-2b)(8+4b-3b^2-b^3)^2} > 0$ at $c_2 = \tilde{c}_{p2}$, $\frac{\partial(R_p^u-R^u)}{\partial c_2} = \frac{-b^2(4+2b-5b^2+b^3)(1-c_1)}{(1-b)^2(2+b)^2(8-4b-b^2)} < 0$, and $(R_p^u - R^u) = \frac{b^2(3-2b)(4-8b+3b^2-b^3+b^4)(1-c_1)^2}{(1-b)^2(2+b)^2(8-4b-b^2)^2} \leq (\geq)\ 0$ iff $b \geq (\leq)\ 0.643333$ at $c_2 = \tilde{c}_2$. These suggest $x < \tilde{c}_{p2} < y$, $\tilde{c}_2 < y$ if $b < 0.643333$, and $\tilde{c}_2 > y$ if $b > 0.643333$. Accordingly, $R_p^u > R^u$ for $c_2 \in (\tilde{c}_{p2},\ \tilde{c}_2)$ if $b < 0.643333$, $R_p^u \geq R^u$ for $c_2 \in (\tilde{c}_{p2},\ y]$, and $R_p^u < R^u$ for $c_2 \in (y,\ \tilde{c}_2)$ if $b > 0.643333$. On the other hand, we have $(\pi_{p1}^u - \pi_1^u) = \frac{(1+b)(c_2-c_1)^2}{(1-b)(2+b)^2} - \frac{[(3-5b+2b^2)-(5-2b-b^2)c_1+(2+3b-3b^2)c_2]^2}{4(3-2b)^2(2-b^2)^2}$ with $\frac{\partial^2(\pi_{p1}^u-\pi_1^u)}{\partial c_2^2} = \frac{128-96b-248b^2+224b^3+131b^4-133b^5-23b^6+25b^7}{2(1-b)(2+b)^2(3-2b)^2(2-b^2)^2} > 0$. That is, $(\pi_{p1}^u - \pi_1^u)$ is a strictly convex function of c_2. Moreover, $\pi_{p1}^u = \pi_1^u$ at $c_2 = j$ and $c_2 = k$, where

$$j = \frac{1}{(128-96b-248b^2+224b^3+131b^4-133b^5-23b^6+25b^7)} \times [(24-4b-98b^2 +81b^3+37b^4-43b^5-3b^6+6b^7)+(104-92b-150b^2+143b^3+94b^4-90b^5 -20b^6+19b^7)c_1 - 2(1-b)(2+b)(2-b^2)(3-2b)^2\sqrt{(1-b^2)(1-c_1)^2}]$$

and

$$k = \frac{1}{(128-96b-248b^2+224b^3+131b^4-133b^5-23b^6+25b^7)} \times [(24-4b-98b^2 +81b^3+37b^4-43b^5-3b^6+6b^7)+(104-92b-150b^2+143b^3+94b^4-90b^5 -20b^6+19b^7)c_1 + 2(1-b)(2+b)(2-b^2)(3-2b)^2\sqrt{(1-b^2)(1-c_1)^2}]$$

with $k > j$. Then, $\pi_{p1}^u < \pi_1^u$ if $j < c_2 < k$, and $\pi_{p1}^u > \pi_1^u$ if $c_2 < j$ or $c_2 > k$. Since $\frac{\partial(\pi_{p1}^u-\pi_1^u)}{\partial c_2} = \frac{(288-240b-492b^2+436b^3+338b^4-231b^5-129b^6+43b^7+19b^8)}{2(2+b)(3-2b)^2(2-b^2)^2(8+4b-3b^2-b^3)} > 0$, $(\pi_{p1}^u - \pi_1^u) = \frac{b^2(1-b)(-144+120b+544b^2-44b^3-421b^4-25b^5+81b^6+17b^7)(1-c_1)^2}{4(3-2b)^2(2-b^2)^2(8+4b-3b^2-b^3)^2} \leq (\geq)\ 0$ iff $b \leq (\geq)\ 0.45193$ at $c_2 = \tilde{c}_{p2}$, and $(\pi_{p1}^u - \pi_1^u) = \frac{2b^3(3-2b)^2(1-c_1)^2}{(1-b)(2+b)^2(8-4b-b^2)^2} > 0$ at $c_2 = \tilde{c}_2$; we can infer $\tilde{c}_2 > k$, $\tilde{c}_{p2} < k$

if $b < 0.45193$, and $\tilde{c}_{p2} > k$ if $b > 0.45193$. Thus, $\pi^u_{p1} > \pi^u_1$ for $c_2 \in (\tilde{c}_{p2},\ \tilde{c}_2)$ if $b > 0.45193$, $\pi^u_{p1} \leq \pi^u_1$ for $c_2 \in (\tilde{c}_{p2},\ k]$, and $\pi^u_{p1} > \pi^u_1$ for $c_2 \in (k,\ \tilde{c}_2)$ if $b < 0.45193$.

(v) For $c_2 \in [\tilde{c}_2,\ \bar{c}_{p2})$, Lemma 2(i) and Lemma 5(iii) suggest $R^u = \frac{(c_2-c_1)[(2-b)+bc_1-2c_2]}{(2-b)^2}$, $\pi^u_1 = (\frac{c_2-c_1}{2-b})^2$, and $\pi^u_2 = 0$. On the other hand, Lemma 8(i) and Lemma 10(iii) imply $R^u_p = \frac{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)^2(2+b)^2}$, $\pi^u_{p1} = \frac{(1+b)(c_2-c_1)^2}{(1-b)(2+b)^2}$, and $\pi^u_{p2} = 0$. Thus, we have $(R^u_p - R^u) = \frac{b^2[(4-4b-b^2+b^3)+b(4-2b-b^2)c_1-(4-3b^2)c_2](c_2-c_1)}{(1-b)^2(4-b^2)}$ with $\frac{\partial^2(R^u_p-R^u)}{\partial c_2^2} = \frac{-2b^2(4-3b^2)}{(1-b)^2(4-b^2)} < 0$. These mean that $(R^u_p - R^u)$ is a strictly concave function of c_2. In addition, $R^u_p = R^u$ at $c_2 = c_1$ and

$$c_2 = \frac{(4-4b-b^2+b^3)+b(4-2b-b^2)c_1}{(4-3b^2)} \equiv w,$$

with $(w - c_1) = \frac{(4-4b-b^2+b^3)(1-c_1)}{(4-3b^2)} > 0$. Then, we can infer $c_1 < \tilde{c}_2 < w < \bar{c}_{p2}$ if $b < 0.643333$, and $w < \tilde{c}_2 < \bar{c}_{p2}$ if $b > 0.643333$. Thus, $R^u_p < R^u$ for $c_2 \in [\tilde{c}_2,\ \bar{c}_{p2})$ if $b > 0.643333$, $R^u_p > R^u$ for $c_2 \in [\tilde{c}_2,\ w)$, and $R^u_p \leq R^u$ for $c_2 \in [w,\ \bar{c}_{p2})$ if $b < 0.643333$. We also have $(\pi^u_{p1} - \pi^u_1) = \frac{2b^3(c_2-c_1)^2}{(1-b)(4-b^2)^2} > 0$. □

Lemma B. *Suppose* $0.807374 < b < 0.830504$. *Then we have the following.*

(i) *For* $c_2 \in (c_1,\ \dot{c}_{p2}]$, *the results are the same as those in Lemma A(i).*

(ii) *For* $c_2 \in (\dot{c}_{p2},\ \ddot{c}_2]$, *we have* $R^u_p > R^u$, $\pi^u_{p2} = \pi^u_2 = 0$, $\pi^u_{p1} \leq \pi^u_1$ *for* $c_2 \in (\dot{c}_{p2},\ v]$, *and* $\pi^u_{p1} > \pi^u_1$ *for* $c_2 \in (v,\ \ddot{c}_2]$.

(iii) *For* $c_2 \in (\ddot{c}_2,\ \tilde{c}_{p2}]$, *we have* $R^u_p > R^u$, $\pi^u_{p2} = \pi^u_2 = 0$, *and* $\pi^u_{p1} > \pi^u_1$ *for* $c_2 \in (\ddot{c}_2,\ \tilde{c}_{p2}]$.

(iv) *For* $c_2 \in (\tilde{c}_{p2},\ \bar{c}_{p2})$, *operator 2 will not rent terminals under both competition modes, but operator 1 will always rent them. Accordingly, we have* $R^u_p \geq R^u$ *for* $c_2 \in (\tilde{c}_{p2},\ y]$ *and* $R^u_p < R^u$ *for* $c_2 \in (y,\ \tilde{c}_2)$, $\pi^u_{p2} = \pi^u_2 = 0$, *and* $\pi^u_{p1} > \pi^u_1$ *for* $c_2 \in (\tilde{c}_{p2},\ \bar{c}_{p2})$.

Proof. For $0.807374 < b < 0.830504$, we have $c_1 < \dot{c}_{p2} < \ddot{c}_2 < \tilde{c}_{p2} < \bar{c}_{p2} < \tilde{c}_2 < \bar{c}_2$.

Then, there are four sub-cases.

(i) For $c_2 \in (c_1,\ \dot{c}_{p2}]$, we have $r_p^u = r^u = \frac{1-c_2}{2}$, $\delta_p^u = \delta^u = \frac{1-c_2}{2(1+b)}$, $R_p^u = R^u = \frac{(1-c_2)^2}{2(1+b)}$, $\pi_{p1}^u = \pi_1^u = \frac{(1-c_2)(c_2-c_1)}{2(1+b)}$, and $\pi_{p2}^u = \pi_2^u = 0$.

(ii) For $c_2 \in (\dot{c}_{p2},\ \ddot{c}_2]$, the proofs are the same as those in Lemma A(ii).

(iii) For $c_2 \in (\ddot{c}_2,\ \tilde{c}_{p2}]$, the proofs are the same as those in Lemma A(iii).

(iv) For $c_2 \in (\tilde{c}_{p2},\ \bar{c}_{p2})$, Lemma 2(ii) and Lemma 5(ii) imply $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$, $\pi_1^u = \frac{[(3-5b+2b^2)-(5-2b-b^2)c_1+(2+3b-3b^2)c_2]^2}{4(3-2b)^2(2-b^2)^2}$, and $\pi_2^u = 0$. On the other hand, Lemma 8(i) and Lemma 10(iii) suggest $R_p^u = \frac{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)^2(2+b)^2}$, $\pi_{p1}^u = \frac{(1+b)(c_2-c_1)^2}{(1-b)(2+b)^2}$, and $\pi_{p2}^u = 0$. We have $R_p^u \geq R^u$ for $c_2 \in (\tilde{c}_{p2},\ y]$ and $R_p^u < R^u$ for $c_2 \in (y,\ \tilde{c}_2)$, because $\frac{\partial^2(R_p^u-R^u)}{\partial c_2^2} = \frac{-(64-64b-40b^2+48b^3+5b^4-10b^5+b^6)}{2(2-b^2)(3-2b)(1-b)^2(2+b)^2} < 0$, $(R_p^u - R^u) = \frac{1}{4(2-b^2)(3-2b)(8+4b-3b^2-b^3)^2}[b^2(1-b)(72+48b-44b^2-16b^3+3b^4+b^5)(1-c_1)^2] > 0$ at $c_2 = \tilde{c}_{p2}$, and $(R_p^u - R^u) = \frac{-(2-2b^2+b^3)^2(1-c_1)^2}{4(3-2b)(2-b^2)^3} < 0$ at $c_2 = \bar{c}_{p2}$. As shown in the proof of Lemma A(iv), we have $\pi_{p1}^u > \pi_1^u$ by $\frac{\partial^2(\pi_{p1}^u-\pi_1^u)}{\partial c_2^2} > 0$, $\frac{\partial(\pi_{p1}^u-\pi_1^u)}{\partial c_2} > 0$, and $(\pi_{p1}^u - \pi_1^u) = \frac{1}{4(3-2b)^2(2-b^2)^2(8+4b-3b^2-b^3)^2}[b^2(1-b)(-144+120b+544b^2-44b^3-421b^4-25b^5+81b^6+17b^7)(1-c_1)^2] > 0$ at $c_2 = \tilde{c}_{p2}$ when $0.807374 < b < 0.830504$. □

Lemma C. *Suppose* $0.830504 < b < 0.918708$. *Then we have the following.*

(i) *For* $c_2 \in (c_1,\ \dot{c}_{p2}]$, *the results are the same as those in Lemma A(i).*

(ii) *For* $c_2 \in (\dot{c}_{p2},\ \tilde{c}_{p2}]$, *we have* $R_p^u > R^u$, $\pi_{p2}^u = \pi_2^u = 0$, $\pi_{p1}^u \leq \pi_1^u$ *for* $c_2 \in (\dot{c}_{p2},\ v]$, *and* $\pi_{p1}^u > \pi_1^u$ *for* $c_2 \in (v,\ \tilde{c}_{p2}]$.

(iii) *For* $c_2 \in (\tilde{c}_{p2},\ \ddot{c}_2]$, *operator 2 will not rent terminals under both competition modes, but operator 1 will always rent them. Accordingly, we have* $R_p^u > R^u$ *for* $c_2 \in (\tilde{c}_{p2},\ \ddot{c}_2]$ *if* $b < 0.872981$, $R_p^u \geq R^u$ *for* $c_2 \in (\tilde{c}_{p2},\ g]$ *and* $R_p^u < R^u$ *for* $c_2 \in (g,\ \ddot{c}_2]$ *if* $b > 0.872981$, $\pi_{p1}^u > \pi_1^u$, *and* $\pi_{p2}^u = \pi_2^u = 0$, *where* g *is defined in the proofs below.*

(iv) *For* $c_2 \in (\ddot{c}_2,\ \bar{c}_{p2})$, *we have* $R_p^u < R^u$ *for* $c_2 \in (\ddot{c}_2,\ \bar{c}_{p2})$ *if* $b > 0.872981$, $R_p^u \geq R^u$

for $c_2 \in (\ddot{c}_2,\ y]$ *and* $R^u_p < R^u$ *for* $c_2 \in (y,\ \bar{c}_{p2})$ *if* $b < 0.872981$, $\pi^u_{p1} > \pi^u_1$ *for* $c_2 \in (\ddot{c}_2,\ \bar{c}_{p2})$ *and* $\pi^u_{p2} = \pi^u_2 = 0$.

Proof. For $0.830504 < b < 0.918708$, we have $c_1 < \dot{c}_{p2} < \tilde{c}_{p2} < \ddot{c}_2 < \bar{c}_{p2} < \tilde{c}_2 < \bar{c}_2$. Then, there are four sub-cases.

(i) For $c_2 \in (c_1,\ \dot{c}_{p2}]$, the proofs are the same as those in Lemma A(i).

(ii) For $c_2 \in (\dot{c}_{p2},\ \tilde{c}_{p2}]$, Lemma 2(iii) and Lemma 5(i) suggest $R^u = \frac{(1-c_2)^2}{2(1+b)}$, $\pi^u_1 = \frac{(1-c_2)(c_2-c_1)}{2(1+b)}$, and $\pi^u_2 = 0$. On the other hand, Lemma 8(ii) and Lemma 10(ii) imply $R^u_p = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$, $\pi^u_{p1} = \frac{[-(3-2b-b^2)+(5+3b)c_1-(2+5b+b^2)c_2]}{16(1-b^2)(3+b)^2}$, and $\pi^u_{p2} = 0$. Thus, we can get $(R^u_p - R^u) = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)} - \frac{(1-c_2)^2}{2(1+b)} > 0$ from Lemma B(i). On the other hand, $(\pi^u_{p1} - \pi^u_1) = \frac{[-(3-2b-b^2)+(5+3b)c_1-(2+5b+b^2)c_2]}{16(1-b^2)(3+b)^2} - \frac{(1-c_2)(c_2-c_1)}{2(1+b)}$ is a convex function of c_2 because $\frac{\partial^2(\pi^u_{p1}-\pi^u_1)}{\partial c_2^2} = \frac{76-4b-11b^2+2b^3+b^4}{8(1-b^2)(3+b)^2} > 0$. By some calculations, we have $\pi^u_{p1} = \pi^u_1$ at $c_2 = u$ and $c_2 = v$, where

$$u = \frac{[(30-23b-11b^2+3b^3+b^4)+(46+19b-b^3)c_1]-2(1-c_1)\sqrt{2(3+b)^3(1-b)^3}}{(76-4b-11b^2+2b^3+b^4)}$$

and

$$v = \frac{[(30-23b-11b^2+3b^3+b^4)+(46+19b-b^3)c_1]+2(1-c_1)\sqrt{2(3+b)^3(1-b)^3}}{(76-4b-11b^2+2b^3+b^4)}$$

with $v > u$. Then, it remains to compare $\dot{c}_2$, $\tilde{c}_{p2}$, u, and v, which will decide relative sizes of π^u_{p1} and π^u_1. Since $\frac{\partial^2(\pi^u_{p1}-\pi^u_1)}{\partial c_2^2} > 0$, $\frac{\partial(\pi^u_{p1}-\pi^u_1)}{\partial c_2} = \frac{-(1-b)(2+b)(7+b)(1-c_1)}{8(1+b)(3-b)(3+b)^2} < 0$ at $c_2 = c''_2$, and $(\pi^u_{p1} - \pi^u_1) = \frac{-(3+b)(1-b)^2(1-c_1)^2}{2(1+b)(10-b-b^2)^2} < 0$ at $c_2 = c'_2$ with $c'_2 < \dot{c}_2 < c''_2$, where $c'_2 = \frac{(3-2b-b^2)+(7+b)c_1}{(10-b-b^2)}$ and $c'' = \frac{(1-b)+2c_1}{(3-b)}$; we have $(\pi^u_{p1} - \pi^u_1) < 0$ at $c_2 = \dot{c}_{p2}$. It means $u < \dot{c}_{p2} < v$. Thus, we have $\frac{\partial(\pi^u_{p1}-\pi^u_1)}{\partial c_2} = \frac{(6+b+b^2)(1-c_1)}{2(1+b)(8+4b-3b^2-b^3)} > 0$ and $(\pi^u_{p1} - \pi^u_1) = \frac{(1-b)(3+b)(2+2b+3b^2+b^3)(1-c_1)^2}{2(1+b)(8+4b-3b^2-b^3)^2} > 0$ at $c_2 = \tilde{c}_{p2}$. These suggest $\tilde{c}_{p2} > v$, and we can obtain $u < \dot{c}_{p2} < v < \tilde{c}_{p2}$. Accordingly, $\pi^u_{p1} \le \pi^u_1$ for $c_2 \in (\dot{c}_{p2},\ v]$ and $\pi^u_{p1} > \pi^u_1$ for $c_2 \in (v,\ \tilde{c}_{p2}]$.

(iii) For $c_2 \in (\tilde{c}_{p2},\ \ddot{c}_2]$, Lemma 2(iii) and Lemma 5(i) suggest $R^u = \frac{(1-c_2)^2}{2(1+b)}$, $\pi^u_1 = \frac{(1-c_2)(c_2-c_1)}{2(1+b)}$, and $\pi^u_2 = 0$, while Lemma 8(i) and Lemma 10(iii) imply $R^u_p = \frac{1}{(1-b)^2(2+b)^2}[(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]]$, $\pi^u_{p1} = \frac{(1+b)(c_2-c_1)^2}{(1-b)(2+b)^2}$, and $\pi^u_{p2} = 0$. Thus, $(R^u_p -$

$R^u) = \frac{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)^2(2+b)^2} - \frac{(1-c_2)(c_2-c_1)}{2(1+b)}$ is a strictly concave function of c_2 by $\frac{\partial^2(R_p^u-R^u)}{\partial c_2^2} = \frac{-(8-5b^2+b^4)}{(1+b)(1-b)^2(2+b)^2} < 0$. In addition, $R_p^u = R^u$ at $c_2 = e$ and $c_2 = g$, where

$$e = \frac{[(6-3b-5b^2+b^3+b^4)+(2+3b-b^3)c_1 - \sqrt{(1+b)(2+b)^2(1-b)^3}(1-c_1)]}{(8-5b^2+b^4)},$$

and

$$g = \frac{[(6-3b-5b^2+b^3+b^4)+(2+3b-b^3)c_1 + \sqrt{(1+b)(2+b)^2(1-b)^3}(1-c_1)]}{(8-5b^2+b^4)}$$

with $g > e$. By some calculations, we have $e < \tilde{c}_{p2} < g$ by $\frac{\partial^2(R_p^u-R^u)}{\partial c_2^2} < 0$ and $(R_p^u - R^u) = \frac{(1-b)(8+10b+7b^2+b^3)(1-c_1)^2}{2(1+b)(8+4b-3b^2-b^3)} > 0$ at $c_2 = \tilde{c}_{p2}$, and $(g - \ddot{c}_2) = \frac{1}{(8-3b^2)(8-5b^2+b^4)}[(8-3b^2)\sqrt{(1+b)(2+b)^2(1-b)^3} - b(1+b)(4-b^2)(4-b-b^2) + (8-5b^2+b^4)\times \sqrt{2(6+2b-7b^2-b^3+2b^4)}] \geq (\leq) 0$ iff $b \leq (\geq) 0.872981$. Then, we can get $e < \tilde{c}_{p2} < \ddot{c}_2 < g$ if $b < 0.872981$ and $e < \tilde{c}_{p2} < g < \ddot{c}_2$ if $b > 0.872981$. Thus, $R_p^u > R^u$ for $c_2 \in (\tilde{c}_{p2}, \ \ddot{c}_2]$ if $b < 0.872981$, $R_p^u \geq R^u$ for $c_2 \in (\tilde{c}_{p2}, \ g]$, and $R_p^u < R^u$ for $c_2 \in (g, \ \ddot{c}_2]$ if $b > 0.872981$. On the other hand, we have $(\pi_{p1}^u - \pi_1^u) = \frac{-(4-3b^2-b^3)-(2+4b+2b^2)c_1+(6+4b-b^2-b^3)c_2}{2(1-b^2)(2+b)^2} > \frac{-(4-3b^2-b^3)-(2+4b+2b^2)c_1+(6+4b-b^2-b^3)\tilde{c}_{p2}}{2(1-b^2)(2+b)^2} = \frac{(1-b)(3+b)(2+2b+3b^2+b^3)(1-c_1)^2}{2(1+b)(8+4b-3b^2-b^3)} > 0$.

(iv) For $c_2 \in (\ddot{c}_2, \ \bar{c}_{p2})$, Lemma 2(ii) and Lemma 5(ii) imply $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$, $\pi_1^u = \frac{[(3-5b+2b^2)-(5-2b-b^2)c_1+(2+3b-3b^2)c_2]^2}{4(3-2b)^2(2-b^2)^2}$, and $\pi_2^u = 0$, while Lemma 8(i) and Lemma 10(iii) suggest $R_p^u = \frac{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)^2(2+b)^2}$, $\pi_{p1}^u = \frac{(1+b)(c_2-c_1)^2}{(1-b)(2+b)^2}$, and $\pi_{p2}^u = 0$. Thus, $(R_p^u - R^u) = \frac{(c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2]}{(1-b)^2(2+b)^2} - \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)}$ is a strictly concave function of c_2 by $\frac{\partial^2(R_p^u-R^u)}{\partial c_2^2} = \frac{-(64-64b-40b^2+48b^3+5b^4-10b^5+b^6)}{2(2-b^2)(3-2b)(1-b)^2(2+b)^2} < 0$. In addition, $R_p^u = R^u$ at $c_2 = x$ and $c_2 = y$. Since $(R_p^u - R^u) = \frac{b^2(1-b)(72+48b-44b^2-16b^3+3b^4+b^5)(1-c_1)^2}{4(2-b^2)(3-2b)(8+4b-3b^2-b^3)^2} > 0$ at $c_2 = \tilde{c}_{p2}$, and $(R_p^u - R^u) = \frac{-(2-2b^2+b^3)^2(1-c_1)^2}{4(3-2b)(2-b^2)^3} < 0$ at $c_2 = \bar{c}_{p2}$; we have $x < \tilde{c}_{p2} < \ddot{c}_2$ and $\bar{c}_{p2} > y$. Moreover, we can obtain $(y - \ddot{c}_2) = \frac{(1-c_1)(y_2+y_3-y_1)}{(8-3b^2)(64-64b-40b^2+48b^3+5b^4-10b^5+b^6)} \geq (\leq) 0$ iff $b \leq (\geq) 0.872981$, where $y_1 = 2b(3-2b)(2-b^2)(4-b^2)(4-b-b^2) > 0$, $y_2 = 2b(1-b)(8-3b^2)(6-b-2b^2)\sqrt{(1-b)(2-b^2)} > 0$, and $y_3 = (64-64b-40b^2+48b^3+5b^4-10b^5+b^6)\sqrt{2(6+2b-7b^2-b^3+2b^4)}$. Some calculations can show $(y_2+y_3)^2 - (y_1)^2 \geq (\leq) 0$ iff $b \leq (\geq) 0.872981$. Thus, $x < \ddot{c}_2 < y < \bar{c}_{p2}$ if $b < 0.872981$ and $x < y < \ddot{c}_2 < \bar{c}_{p2}$

if $b > 0.872981$. We have $R_p^u < R^u$ for $c_2 \in (\ddot{c}_2,\ \bar{c}_{p2})$ if $b > 0.872981$, $R_p^u \geq R^u$ for $c_2 \in (\ddot{c}_2,\ y]$, and $R_p^u < R^u$ for $c_2 \in (y,\ \bar{c}_{p2})$ if $b < 0.872981$. As in the proofs of Lemma A(iv) and Lemma B(iv), we also have $\pi_{p1}^u > \pi_1^u$ by $\ddot{c}_2 > \tilde{c}_{p2}$ for $c_2 \in (\ddot{c}_2,\ \bar{c}_{p2})$. □

Lemma D. *Suppose* $b > 0.918708$. *Then we have the following.*

(i) *For* $c_2 \in (c_1,\ \dot{c}_{p2}]$, *the results are the same as those in Lemma A(i).*

(ii) *For* $c_2 \in (\dot{c}_{p2},\ \tilde{c}_{p2}]$, *the results are the same as those in Lemma C(ii).*

(iii) *For* $c_2 \in (\tilde{c}_{p2},\ \bar{c}_{p2})$, *the results are the same as those in Lemma B(iv).*

Proof. For $b > 0.918708$, we have $c_1 < \dot{c}_{p2} < \tilde{c}_{p2} < \bar{c}_{p2} < \ddot{c}_2 < \tilde{c}_2 < \bar{c}_2$. Then, there are three sub-cases.

(i) For $c_2 \in (c_1,\ \dot{c}_{p2}]$, the proofs are the same as those in Lemma A(i).

(ii) For $c_2 \in (\dot{c}_{p2},\ \tilde{c}_{p2}]$, the proofs are the same as those in Lemma C(ii).

(iii) For $c_2 \in (\tilde{c}_{p2},\ \bar{c}_{p2})$, the proofs are the same as those in Lemma B(iv). □

In sum, Lemmas A-D imply that operator 2 will always earn zero profit. However, we can have either $R_p^u \geq R^u$ or $R_p^u < R^u$, and similarly either $\pi_{p1}^u \geq \pi_1^u$ or $\pi_{p1}^u < \pi_1^u$. □

Lemma 12. *Given two-part tariff scheme* $(r,\ f)$ *and minimum throughput guarantee* δ, *terminal operators' optimal behaviors are given below. Define* $\alpha_1 \equiv (1 + \frac{d_1}{K_1})$ *and* $\alpha_2 \equiv (1 + \frac{d_2}{K_2})$.

(i) *Suppose* $\alpha_1 \leq \alpha_2$.
(ia) *For* $\delta \in [0,\ \delta_{c1}]$ *with* $\delta_{c1} = \frac{(2\alpha_1 - b)(1-r) + bc_1 - 2\alpha_1 c_2}{4\alpha_1\alpha_2 - b^2}$, *both operators' equilibrium cargo-handling amounts are*

$$q_{c1}^* = \frac{(2\alpha_2 - b)(1-r) - 2\alpha_2 c_1 + bc_2}{(4\alpha_1\alpha_2 - b^2)} > \delta_{c1} \text{ and } q_{c2}^* = \frac{(2\alpha_1 - b)(1-r) + bc_1 - 2\alpha_1 c_2}{(4\alpha_1\alpha_2 - b^2)} = \delta_{c1},$$

the equilibrium service prices are $p_{ci}^* = c_i + r + (1 + \frac{2d_i}{K_i})q_{ci}^* > 0$, *and their equilibrium profits are* $\pi_{ci}^* = \alpha_i (q_{ci}^*)^2 - f$ *for* $i = 1,\ 2$.

(ib) *For* $\delta \in (\delta_{c1},\ \delta_{c2}]$ *with* $\delta_{c2} = \frac{1-c_1-r}{2\alpha_1+b}$, *both operators' equilibrium cargo-handling amounts are*

$$q^*_{c1} = \frac{(1-b\delta-c_1-r)}{2\alpha_1} \text{ and } q^*_{c2} = \delta,$$

the equilibrium prices are $p^*_{c1} = \frac{[(2\alpha_1-1)(1-b\delta)+c_1+r]}{2\alpha_1} > 0$ *and* $p^*_{c2} = \frac{[(2\alpha_1-b)-(2\alpha_1-b^2)\delta+bc_1+br]}{2\alpha_1} > 0$, *and their equilibrium profits are* $\pi^*_{c1} = \alpha_1(q^*_{c1})^2 - f$ *and* $\pi^*_{c2} = \frac{\delta}{2\alpha_1}[(2\alpha_1-b)(1-r)-(2\alpha_1\alpha_2-b^2)\delta+bc_1-2\alpha_1c_2]-f$.

(ic) *For* $\delta \in (\delta_{c2},\ \frac{1}{1+b})$, *both operators' equilibrium cargo-handling amounts are*

$$q^*_{c1} = \delta \text{ and } q^*_{c2} = \delta,$$

the equilibrium service prices are $p^*_{c1} = p^*_{c2} = 1-(1+b)\delta > 0$, *and their equilibrium profits are* $\pi^*_{ci} = \delta[1-(\alpha_i+b)\delta-r-c_i]-f$ *for* $i = 1, 2$.

(ii) *Suppose* $\alpha_1 > \alpha_2$ *and* $r \geq r_{12} \equiv \frac{2(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1-(2\alpha_1+b)c_2}{2(\alpha_1-\alpha_2)}$. *Then operators' optimal behaviors are the same as those in part (i).*

(iii) *Suppose* $\alpha_1 > \alpha_2$ *and* $r < r_{12}$.

(iiia) *For* $\delta \in [0,\ \delta'_{c1}]$ *with* $\delta'_{c1} = \frac{(2\alpha_2-b)(1-r)+bc_2-2\alpha_2c_1}{4\alpha_1\alpha_2-b^2}$, *both operators' equilibrium cargo-handling amounts are*

$$q^*_{c1} = \frac{(2\alpha_2-b)(1-r)-2\alpha_2c_1+bc_2}{(4\alpha_1\alpha_2-b^2)} = \delta'_{c1} \text{ and } q^*_{c2} = \frac{(2\alpha_1-b)(1-r)+bc_1-2\alpha_1c_2}{(4\alpha_1\alpha_2-b^2)} > \delta'_{c1},$$

the equilibrium service prices are $p^*_{ci} = c_i + r + (1+\frac{2d_i}{K_i})q^*_{ci} > 0$, *and their equilibrium profits are* $\pi^*_{ci} = \alpha_i(q^*_{ci})^2 - f$ *for* $i = 1,\ 2$.

(iiib) *For* $\delta \in (\delta'_{c1},\ \delta'_{c2}]$ *with* $\delta'_{c2} = \frac{1-c_2-r}{2\alpha_2+b}$, *both operators' equilibrium cargo-handling amounts are*

$$q^*_{c1} = \delta \text{ and } q^*_{c2} = \frac{(1-b\delta-c_2-r)}{2\alpha_2},$$

the equilibrium prices are $p^*_{c1} = \frac{[(2\alpha_2-b)-(2\alpha_2-b^2)\delta+bc_2+br]}{2\alpha_2} > 0$ *and* $p^*_{c2} = \frac{[(2\alpha_2-1)(1-b\delta)+c_2+r]}{2\alpha_2} > 0$, *and their equilibrium profits are* $\pi^*_{c1} = \frac{1}{2\alpha_2}\{\delta[(2\alpha_2-b)(1-r)-(2\alpha_1\alpha_2-b^2)\delta+bc_2-2\alpha_2c_1]\}-f$ *and* $\pi^*_{c2} = \alpha_2(q^*_{c2})^2 - f$.

(iiic) *For* $\delta \in (\delta_2', \frac{1}{1+b})$, *both operators' equilibrium cargo-handling amounts are*

$$q_{c1}^* = \delta \text{ and } q_{c2}^* = \delta,$$

the equilibrium service prices are $p_{c1}^* = p_{c2}^* = 1 - (1+b)\delta > 0$, *and their equilibrium profits are* $\pi_{ci}^* = \delta[1 - (\alpha_i + b)\delta - r - c_i] - f$ *for* $i = 1, 2$.

<u>*Proof of Lemma 12*</u>: Denote L_1 and L_2 the Lagrange functions of operators 1 and 2, respectively, in problem (39) with

$$\begin{aligned} L_1 &= [1 - q_1 - bq_2 - c_1 - r - \frac{d_1}{K_1} q_1] q_1 - f + \lambda_1 (q_1 - \delta) \text{and} \\ L_2 &= [1 - q_2 - bq_1 - c_2 - r - \frac{d_2}{K_2} q_2] q_2 - f + \lambda_2 (q_2 - \delta), \end{aligned}$$

where λ_1 and λ_2 are their associated Lagrange multipliers. Then, the Kuhn-Tucker conditions for operator 1 are

$$\frac{\partial L_1}{\partial q_1} = 1 - 2(1 + \frac{d_1}{K_1}) q_1 - bq_2 - c_1 - r + \lambda_1 \leq 0, \; q_1 \cdot \frac{\partial L_1}{\partial q_1} = 0 \text{ and} \tag{A95}$$

$$\frac{\partial L_1}{\partial \lambda_1} = q_1 - \delta \geq 0, \; \lambda_1 \cdot \frac{\partial L_1}{\partial \lambda_1} = 0, \tag{A96}$$

and for operator 2 are

$$\frac{\partial L_2}{\partial q_2} = 1 - 2(1 + \frac{d_2}{K_2}) q_2 - bq_1 - c_2 - r + \lambda_2 \leq 0, \; q_2 \cdot \frac{\partial L_2}{\partial q_2} = 0 \text{ and} \tag{A97}$$

$$\frac{\partial L_2}{\partial \lambda_2} = q_2 - \delta \geq 0, \; \lambda_2 \cdot \frac{\partial L_2}{\partial \lambda_2} = 0. \tag{A98}$$

Based on the values of λ_1 and λ_2, there are four cases as follows.

<u>Case 1</u>: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then (A95) and (A97) become

$$\begin{aligned} 1 - 2\alpha_1 q_1 - bq_2 - c_1 - r &= 0 \text{ and} \\ 1 - 2\alpha_2 q_2 - bq_1 - c_2 - r &= 0 \end{aligned}$$

Solving these equations yields $q_{c1}^* = \frac{(2\alpha_2 - b)(1-r) - 2\alpha_2 c_1 + bc_2}{4\alpha_1\alpha_2 - b^2}$ and $q_{c2}^* = \frac{(2\alpha_1 - b)(1-r) + bc_1 - 2\alpha_1 c_2}{4\alpha_1\alpha_2 - b^2}$. Then we have $(q_{c1}^* - q_{c2}^*) = \frac{[2(\alpha_2 - \alpha_1)(1-r) - (2\alpha_2 + b)c_1 + (2\alpha_1 + b)c_2]}{(4\alpha_1\alpha_2 - b^2)} \geq 0$ iff $\alpha_1 \leq \alpha_2$, or $\alpha_1 > \alpha_2$

and $r \geq \frac{2(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1-(2\alpha_1+b)c_2}{2(\alpha_1-\alpha_2)} \equiv r_{12}$, and $(q^*_{c1} - q^*_{c2}) < 0$ iff $\alpha_1 > \alpha_2$ and $r < r_{12}$. Based on relative sizes of q^*_{c1} and q^*_{c2}, there are three sub-cases as follows.

Case 1a: Suppose $\alpha_1 \leq \alpha_2$. To guarantee $q^*_{c1} \geq \delta$ and $q^*_{c2} \geq \delta$, condition $0 \leq \delta \leq \delta_{c1} \equiv \frac{(2\alpha_1-b)(1-r)+bc_1-2\alpha_1 c_2}{4\alpha_1\alpha_2-b^2} = q^*_{c2}$ is needed because $c_1 < c_2$ implies $q^*_{c1} > q^*_{c2}$, and $q^*_{c2} \geq \delta$ implies $q^*_{c1} \geq \delta$. Substituting q^*_{c1} and q^*_{c2} into (1)-(2) yields $p^*_{ci} = c_i + r + (1 + \frac{2d_i}{K_i})q^*_{ci} > 0$, and into (38) yields $\pi^*_{ci} = \alpha_i(q^*_{ci})^2 - f$ for $i = 1, 2$. To guarantee $q^*_{c1} \geq 0$ and $q^*_{c2} \geq 0$, conditions $r < \bar{r}_c \equiv \frac{[(2\alpha_1-b)+bc_1-2\alpha_1 c_2]}{2\alpha_1-b}$ and $c_2 < \bar{c}_{c2} \equiv \frac{(2\alpha_1-b)+bc_1}{2\alpha_1}$ are needed. That is because $r < \bar{r}_c$ and $c_2 < \bar{c}_{c2}$ imply $q^*_{c2} \geq 0$, and $r < \frac{[(2\alpha_2-b)+bc_2-2\alpha_2 c_1]}{2\alpha_2-b}$ implies $q^*_{c1} \geq 0$ with $\frac{[(2\alpha_2-b)+bc_2-2\alpha_2 c_1]}{2\alpha_2-b} - \bar{r}_c = \frac{(4\alpha_1\alpha_2-b^2)(c_2-c_1)}{(2\alpha_1-b)(2\alpha_2-b)} > 0$. These prove Lemma 12(ia).

Case 1b: Suppose $\alpha_1 > \alpha_2$ and $r \geq r_{12}$. Then, $q^*_{c1} > q^*_{c2}$, and the analyses are the same as those in Case 1a. To guarantee $q^*_{c1} \geq \delta$ and $q^*_{c2} \geq \delta$, condition $0 \leq \delta \leq \delta_{c1}$ is needed. Under the circumstance, we have $p^*_{ci} = c_i + r + (1 + \frac{2d_i}{K_i})q^*_{ci} > 0$ and $\pi^*_{ci} = \alpha_i(q^*_{ci})^2 - f$ for $i = 1, 2$. These prove Lemma 12(iia).

Case 1c: Suppose $\alpha_1 > \alpha_2$ and $r < r_{12}$. Then, $q^*_{c2} > q^*_{c1}$. To guarantee $q^*_{c1} \geq \delta$ and $q^*_{c2} \geq \delta$, condition $0 \leq \delta \leq \delta'_{c1} \equiv \frac{(2\alpha_2-b)(1-r)-2\alpha_2 c_1+bc_2}{4\alpha_1\alpha_2-b^2} = q^*_{c1}$ is needed. That is because $\alpha_1 > \alpha_2$ and $r < r_{12}$ imply $q^*_{c2} > q^*_{c1}$, and $q^*_{c1} \geq \delta$ implies $q^*_{c2} \geq \delta$. Substituting q^*_{c1} and q^*_{c2} into (1)-(2) yields $p^*_{ci} = c_i + r + (1 + \frac{2d_i}{K_i})q^*_{ci} > 0$, and into (38) yields $\pi^*_{ci} = \alpha_i(q^*_{ci})^2 - f$ for $i = 1, 2$. These prove Lemma 12(iiia).

Case 2: Suppose $\lambda^*_1 = 0$ and $\lambda^*_2 > 0$. Then (A95), (A97) and (A98) suggest

$$1 - 2\alpha_1 q_1 - bq_2 - c_1 - r = 0, \; 1 - 2\alpha_2 q_2 - bq_1 - c_2 - r + \lambda_2 = 0, \text{ and } q_2 - \delta = 0.$$

Solving these equations yields $q^*_{c1} = \frac{(1-b\delta-c_1-r)}{2\alpha_1}$, $q^*_{c2} = \delta$, and $\lambda^*_2 = \frac{(4\alpha_1\alpha_2-b^2)(\delta-\delta_1)}{2\alpha_1}$. To guarantee $\lambda^*_2 > 0$ and $\delta_{c1} \geq 0$, conditions $\delta > \delta_{c1}$, $r \leq \bar{r}_c$, and $c_2 \leq \bar{c}_{c2}$ are needed. On the other hand, to have $q^*_{c1} \geq \delta$, condition $\delta \leq \delta_{c2} \equiv \frac{1-c_1-r}{2\alpha_1+b}$ should be imposed, where $(\delta_{c2} - \delta_{c1}) = \frac{2\alpha_1[2(\alpha_2-\alpha_1)(1-r)-(2\alpha_2+b)c_1+(2\alpha_1+b)c_2]}{(2\alpha_1+b)(4\alpha_1\alpha_2-b^2)} > 0$ iff $\alpha_{c1} \leq \alpha_{c2}$, or $\alpha_{c1} > \alpha_{c2}$ and $r > r_{12}$. In sum, if $\alpha_1 \leq \alpha_2$, or $\alpha_1 > \alpha_2$ and $r > r_{12}$, the plausible range for δ is $(\delta_{c1},\ \delta_{c2}]$. Accordingly, substituting q^*_{c1} and q^*_{c2} into (1)-(2) gives $p^*_{c1} = \frac{[(2\alpha_1-1)(1-b\delta)+c_1+r]}{2\alpha_1} > 0$ and

$p^*_{c2} = \frac{[(2\alpha_1 - b) - (2\alpha_1 - b^2)\delta + bc_1 + br]}{2\alpha_1} > 0$ if $\delta \le \delta_{c2}$, and into (41) yields $\pi^*_{c1} = \alpha_1 (q^*_{c1})^2 - f$ and $\pi^*_{c2} = \frac{\delta[(2\alpha_1 - b)(1-r) - (2\alpha_1\alpha_2 - b^2)\delta + bc_1 - 2\alpha_1 c_2]}{2\alpha_1} - f$. These prove Lemma 12(ib) and Lemma 12(ii).

<u>Case 3</u>: Suppose $\lambda^*_1 > 0$ and $\lambda^*_2 = 0$. Then (A95)-(A97) suggest

$$q_1 - \delta = 0,\ (1 - 2\alpha_1 q_1 - bq_2 - c_1 - r + \lambda_1) = 0,\ \text{and}\ (1 - 2\alpha_2 q_2 - bq_1 - c_2 - r) = 0.$$

Solving these equations yields $q^*_{c1} = \delta$, $q^*_{c2} = \frac{(1 - b\delta - c_2 - r)}{2\alpha_2}$, and $\lambda^*_1 = \frac{(4\alpha_1\alpha_2 - b^2)(\delta - \delta'_{c1})}{2\alpha_2}$. To guarantee $\lambda^*_1 > 0$, condition $\delta > \delta'_{c1}$ is needed. On the other hand, $q^*_{c2} \ge \delta$ is implied by assuming $\delta \le \delta'_{c2} \equiv \frac{1 - c_2 - r}{2\alpha_2 + b}$. However, we have $(\delta'_{c2} - \delta'_{c1}) = \frac{2\alpha_2[2(\alpha_1 - \alpha_2)(1-r) - (2\alpha_1 + b)c_2 + (2\alpha_2 + b)c_1]}{(2\alpha_2 + b)(4\alpha_1\alpha_2 - b^2)}$ > 0 iff $\alpha_1 > \alpha_2$ and $r < r_{12}$. Thus, if $\alpha_1 > \alpha_2$ and $r < r_{12}$, the plausible range for δ is $(\delta'_{c1},\ \delta'_{c2}]$. Substituting q^*_{c1} and q^*_{c2} into (1)-(2) yields $p^*_{c2} = \frac{[(2\alpha_2 - 1)(1 - b\delta) + c_2 + r]}{2\alpha_2} > 0$ and $p^*_{c1} = \frac{[(2\alpha_2 - b) - (2\alpha_2 - b^2)\delta + bc_2 + br]}{2\alpha_2} > 0$ if $\delta \le \delta'_{c2}$, and into (38) gives $\pi^*_{c1} = \frac{1}{2\alpha_2}\delta[(2\alpha_2 - b)(1-r) - (2\alpha_1\alpha_2 - b^2)\delta + bc_2 - 2\alpha_2 c_1] - f$ and $\pi^*_{c2} = \alpha_2 (q^*_{c2})^2 - f$. These prove Lemma 12(iiib).

<u>Case 4</u>: Suppose $\lambda^*_1 > 0$ and $\lambda^*_2 > 0$. Then (A95)-(A98) suggest

$$q^*_{c1} = q^*_{c2} = \delta,\ \lambda^*_1 = -1 + (2\alpha_1 + b)\delta + c_1 + r,\ \text{and}\ \lambda^*_2 = -1 + (2\alpha_2 + b)\delta + c_2 + r.$$

To have $\lambda^*_1 > 0$ and $\lambda^*_2 > 0$, conditions $\delta > \delta_{c2} \equiv \frac{1 - c_1 - r}{2\alpha_1 + b}$, $\delta > \delta'_{c2} \equiv \frac{1 - c_2 - r}{2\alpha_2 + b}$, and $r < (1 - c_2)$ are needed. Note that $r < (1 - c_2)$ is implied by $r < \bar{r}_c$. In addition, we have $(\delta_{c2} - \delta'_{c2}) = \frac{[2(\alpha_2 - \alpha_1)(1-r) - (2\alpha_2 + b)c_1 + (2\alpha_1 + b)c_2]}{(2\alpha_1 + b)(2\alpha_2 + b)} \ge 0$ iff $\alpha_1 \le \alpha_2$, or $\alpha_1 > \alpha_2$ and $r \ge r_{12}$, and $(\delta_{c2} - \delta'_{c2}) < 0$ iff $\alpha_1 > \alpha_2$ and $r < r_{12}$. Thus, if $\alpha_1 \le \alpha_2$, or $\alpha_1 > \alpha_2$ and $r \ge r_{12}$, the conditions needed are $\delta > \delta_{c2} \equiv \frac{1 - c_1 - r}{2\alpha_1 + b}$ and $r < \bar{r}_c$. Then, substituting $q^*_{c1} = q^*_{c2} = \delta$ into (1)-(2) gives $p^*_{c1} = p^*_{c2} = 1 - (1+b)\delta > 0$ if $\delta < \frac{1}{1+b}$, and into (38) gives $\pi^*_{ci} = \delta[1 - (\alpha_i + b)\delta - r - c_i] - f$ for $i = 1, 2$. These prove Lemma 12(ic) and 12(ii). By contrast, if $\alpha_1 > \alpha_2$ and $r < r_{12}$, condition $\delta > \delta'_{c2}$ is needed. Substituting $q^*_{c1} = q^*_{c2} = \delta$ into (1)-(2) generates $p^*_{c1} = p^*_{c2} = 1 - (1+b)\delta > 0$ if $\delta < \frac{1}{1+b}$, and into (38) gives $\pi^*_{ci} = \delta[1 - (\alpha_i + b)\delta - r - c_i] - f$ for $i = 1, 2$. These prove Lemma 12(iiic).
□

Lemma 13. *Suppose the conditions in (40) hold. Then we have the following.*

(i) *Suppose* $\alpha_1 \leq \frac{\alpha_2}{2}$. *Then, for* $c_2 \in (c_1,\ \hat{c}_{c2})$ *with* $\hat{c}_{c2} = \frac{1}{(4\alpha_1^2+8\alpha_1\alpha_2-4b\alpha_1-b^2)}[(4\alpha_1^2 + 4\alpha_1\alpha_2 - 6b\alpha_1 - 2b\alpha_2 + 2b^2) + (4\alpha_1\alpha_2 + 2b\alpha_1 + 2b\alpha_2 - 3b^2)c_1]$, *port authority's optimal two-part tariff contract and minimum throughput requirement are* $(r_c^*,\ f_c^*,\ \delta_c^*)$ *with* $r_c^* = \frac{[(2\alpha_2-b)(4\alpha_1\alpha_2+2b\alpha_1-2b^2)-(8\alpha_1\alpha_2^2-4b^2\alpha_2+b^3)c_1+b^2(2\alpha_1-b)c_2]}{2(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4b\alpha_1\alpha_2-3b^2\alpha_2-2b^2\alpha_1+2b^3)}$, $f_c^* = \frac{\alpha_2}{2}[\frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1c_2}{4\alpha_1\alpha_2-b^2}]^2$, *and* $\delta_c^* \in [0,\ \frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1c_2}{4\alpha_1\alpha_2-b^2}]$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^* = \frac{(2\alpha_2-b)(1-r_c^*)-2\alpha_2c_1+bc_2}{4\alpha_1\alpha_2-b^2}$ *and* $q_{c2}^* = \frac{1}{(4\alpha_1\alpha_2-b^2)}[(2\alpha_1 - b)(1 - r_c^*) + bc_1 - 2\alpha_1c_2]$ *as in Lemma 12(ia), and port authority's equilibrium fee revenue equals* $R_c^* = 2f_c^* + r_c^*[\frac{2(\alpha_1+\alpha_2-b)(1-r_c^*)-(2\alpha_2-b)c_1-(2\alpha_1-b)c_2}{4\alpha_1\alpha_2-b^2}]$.

(ii) *Suppose* $\frac{\alpha_2}{2} < \alpha_1 \leq \alpha_2$.

(iia) *If* $c_2 \in (c_1,\ \ddot{c}_{c2})$ *with* $\ddot{c}_{c2} = \frac{2(2\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1}{(6\alpha_1+\alpha_2+4b)}$, *then port authority's optimal two-part tariff contract and minimum throughput requirement are* $(r_c^*,\ f_c^*,\ \delta_c^*)$ *with* $r_c^* = \frac{2(\alpha_2+b)-(2\alpha_1+2\alpha_2+3b)c_1+(2\alpha_1+b)c_2}{2(2\alpha_1+\alpha_2+2b)}$, $f_c^* = \frac{1}{8(2\alpha_1+\alpha_2+2b)^2}\{(2 - c_1 - c_2)[2(2\alpha_1 - \alpha_2) + (2\alpha_1 + 3\alpha_2 + 4b)c_1 - (6\alpha_1 + \alpha_2 + 4b)c_2]\}$, *and* $\delta_c^* = \frac{2-c_1-c_2}{2(2\alpha_1+\alpha_2+2b)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{ci}^* = \delta_c^*$ *for* $i = 1,\ 2$, *as in Lemma 12(ia), and port authority's fee revenue equals* $R_c^* = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$.

(iib) *If* $c_2 \in [\ddot{c}_{c2},\ \hat{c}_{c2})$, *then port authority's optimal two-part tariff contract and minimum throughput requirement are the same as those in Lemma 13(i).*

(iii) *Suppose* $\alpha_2 < \alpha_1 \leq 2\alpha_2$.

(iiia) *If* $c_2 \in (c_1,\ \dot{c}_{c2})$ *with* $\dot{c}_{c2} \equiv \frac{2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1}{(2\alpha_1+\alpha_2+2b)}$, *then port authority's optimal two-part tariff contract and minimum throughput requirement are* $(r_c^*,\ f_c^*,\ \delta_c^*)$ *with* $r_c^* = \frac{[2(\alpha_1+b)+(2\alpha_2+b)c_1-(2\alpha_1+2\alpha_2+3b)c_2]}{2(\alpha_1+2\alpha_2+2b)}$, $f_c^* = \frac{(2-c_1-c_2)[2(2\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1-(6\alpha_1+\alpha_2+4b)c_2]}{8(2\alpha_1+\alpha_2+2b)^2}$, *and* $\delta_c^* = \frac{2-c_1-c_2}{2(\alpha_1+2\alpha_2+2b)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{ci}^* = \delta_c^*$ *for* $i = 1,\ 2$, *as in Lemma 12(iiib), and port authority's fee revenue equals* $R_c^* = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$.

(iiib) *If* $c_2 \in [\dot{c}_{c2},\ \ddot{c}_{c2})$, *then port authority's optimal two-part tariff contract and minimum throughput requirement are the same as those in Lemma 13(iia).*

(iiic) *If* $c_2 \in [\ddot{c}_{c2},\ \hat{c}_{c2})$, *then port authority's optimal two-part tariff contract and minimum throughput requirement are the same as those in Lemma 13(i).*

(iv) *Suppose* $\alpha_1 > 2\alpha_2$.

(iva) *If* $c_2 \in (c_1,\ c''_{c2}]$ *with* $c''_{c2} = \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{\alpha_1+b}$, *then port authority's optimal two-part tariff contract and minimum throughput requirement are* $(r_c^*,\ f_c^*,\ \delta_c^*)$ *with* $r_c^* = \frac{1-c_2}{2}$, $f_c^* = \frac{(2\alpha_2-b-2\alpha_2c_1+bc_2)(c_2-c_1)}{8(2\alpha_1\alpha_2-b^2)}$, *and* $\delta_c^* = \frac{(2\alpha_2-b)+bc_2-2\alpha_2c_1}{2(2\alpha_1\alpha_2-b^2)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^* = \delta_c^*$ *and* $q_{c2}^* = \frac{1-b\delta_c^*-c_2-r_c^*}{2\alpha_2}$ *as in Lemma 12(iiib), and port authority's fee revenue equals* $R_c^* = \frac{1}{4(2\alpha_1\alpha_2-b^2)}[(\alpha_1+2\alpha_2-2b) - 2(2\alpha_2-b)c_1 - 2(\alpha_1-b)c_2 - 2bc_1c_2 + 2\alpha_2c_1^2 + \alpha_1c_2^2]$.

(ivb) *If* $c_2 \in (c''_{c2},\ \dot{c}_{c2})$, *then port authority's optimal two-part tariff contract and minimum throughput requirement are the same as those in Lemma 13(iiia).*

(ivc) *Suppose* $\dot{c}_{c2} < \ddot{c}_{c2}$ *and* $c_2 \in [\dot{c}_{c2},\ \ddot{c}_{c2})$. *Then port authority's optimal two-part tariff contract and minimum throughput requirement are the same as those in Lemma 13(iia).*

(ivd) *If* $\max\{\dot{c}_{c2},\ \ddot{c}_{c2}\} \le c_2 < \hat{c}_{c2}$, *then port authority's optimal two-part tariff contract and minimum throughput requirement are the same as those in Lemma 13(i).*

Proof of Lemma 13: Based on the values of α_1 and α_2, we have three cases as follows.

Case 1: Suppose $\alpha_1 \le \alpha_2$. Then, there are two sub-cases.

Case 1(1): Suppose $\delta \in [0,\ \delta_{c1}]$. Lemma 12(ia) implies $\pi_{c1}^* > \pi_{c2}^*$ and $f_c^* = \pi_{c2}^* = \frac{1}{2}\alpha_2(q_{c2}^*)^2 > 0$. Thus, the problem in (43) becomes

$$\begin{aligned} \max_{r,\ f,\ \delta} \quad & 2f + r(q_{c1}^* + q_{c2}^*) \\ \text{s.t.} \quad & 0 \le \delta \le \delta_{c1} \text{ and } 0 < r < \bar{r}_c. \end{aligned}$$

Denote L its Lagrange function with $L = \alpha_2(q^*_{c2})^2 + r(q^*_{c1} + q^*_{c2}) + \lambda_1(\delta_{c1} - \delta) + \lambda_2(\bar{r}_c - r)$, where λ_1 and λ_2 are the Lagrange multipliers associated with the constraints. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = 2\alpha_2 q^*_{c2}\frac{\partial q^*_{c2}}{\partial r} + r(\frac{\partial q^*_{c1}}{\partial r} + \frac{\partial q^*_{c2}}{\partial r}) + (q^*_{c1} + q^*_{c2}) + \lambda_1\frac{\partial \delta_1}{\partial r} - \lambda_2 \leq 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \quad (A99)$$

$$\frac{\partial L}{\partial \delta} = -\lambda_1 \leq 0,\ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad (A100)$$

$$\frac{\partial L}{\partial \lambda_1} = \delta_{c1} - \delta \geq 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0,\ \text{and} \quad (A101)$$

$$\frac{\partial L}{\partial \lambda_2} = \bar{r}_c - r \geq 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0. \quad (A102)$$

Constraint $r < \bar{r}_c$ suggests $\lambda^*_2 = 0$ by (A102). Based on the values of λ_1, we have two situations.

Case 1(1)a: Suppose $\lambda^*_1 = 0$. Then (A99) becomes $\frac{1}{(4\alpha_1\alpha_2-b^2)^2}[(2\alpha_2 - b)(4\alpha_1\alpha_2 + 2b\alpha_1 - 2b^2) - (8\alpha_1\alpha_2^2 - 4b^2\alpha_2 + b^3)c_1 + b^2(2\alpha_1 - b)c_2 - [16\alpha_1\alpha_2^2 + (8\alpha_1^2 - 8b\alpha_1 - 6b^2)\alpha_2 - 4b^2(\alpha_1 - b)]r] = 0$. Solving this equation yields $r^*_c = \frac{1}{2(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4b\alpha_1\alpha_2-3b^2\alpha_2-2b^2\alpha_1+2b^3)}$ $[(2\alpha_2 - b)(4\alpha_1\alpha_2 + 2b\alpha_1 - 2b^2) - (8\alpha_1\alpha_2^2 - 4b^2\alpha_2 + b^3)c_1 + b^2(2\alpha_1 - b)c_2]$. It remains to check whether $r^*_c < \bar{r}_c$ holds. By some calculations, we have $r^*_c < \bar{r}_c$ iff $c_2 < \hat{c}_{c2} \equiv \frac{(4\alpha_1^2+4\alpha_1\alpha_2-6b\alpha_1-2b\alpha_2+2b^2)+(4\alpha_1\alpha_2+2b\alpha_1+2b\alpha_2-3b^2)c_1}{(4\alpha_1^2+8\alpha_1\alpha_2-4b\alpha_1-b^2)}$. In addition, (A101) implies both $\delta^*_c \in [0,\ \delta_{c1}]$ with $\delta_{c1} = \frac{(2\alpha_1-b)(1-r^*_c)+bc_1-2\alpha_1c_2}{4\alpha_1\alpha_2-b^2}$ and $f^*_c = \frac{\alpha_2}{2}[\frac{(2\alpha_1-b)(1-r^*_c)+bc_1-2\alpha_1c_2}{4\alpha_1\alpha_2-b^2}]^2 > 0$. Thus, at the equilibrium, port authority's fee revenue equals

$$R^*_c = \alpha_2[\frac{(2\alpha_1 - b)(1 - r^*_c) + bc_1 - 2\alpha_1c_2}{4\alpha_1\alpha_2 - b^2}]^2 + r^*_c[\frac{2(\alpha_1 + \alpha_2 - b)(1 - r^*_c) - (2\alpha_2 - b)c_1 - (2\alpha_1 - b)c_2}{4\alpha_1\alpha_2 - b^2}] = R^*_1. \quad (A103)$$

Case 1(1)b: Suppose $\lambda^*_1 > 0$. Then, (A101) suggests $\delta^*_c = \delta_{c1} > 0$. This in turn implies $\lambda^*_1 = 0$ by (A100). It is a contradiction. Thus, no solution exists in this case.

Case 1(2): Suppose $\delta \in (\delta_{c1},\ \delta_{c2}]$. Then, Lemma 12(ib) implies $\pi^*_{c1} > \pi^*_{c2}$ and $f^*_c = \pi^*_{c2} = \frac{\delta[(2\alpha_1-b)(1-r)-(2\alpha_1\alpha_2-b^2)\delta+bc_1-2\alpha_1c_2]}{4\alpha_1}$. We have $f^*_c > 0$ iff $\delta < \frac{(2\alpha_1-b)(1-r)+bc_1-2\alpha_1c_2}{(2\alpha_1\alpha_2-b^2)} \equiv \tilde{\delta}_c$ and $r \leq \bar{r}_c \equiv \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]}{(2\alpha_1-b)}$. Moreover, if $(2\alpha_1 - \alpha_2) > 0$, we have $\tilde{\delta}_c >\ (\leq)\ \delta_{c2}$ iff $r <\ (\geq)\ \frac{[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{(2\alpha_1-\alpha_2)} \equiv \tilde{r}_c$, and if $(2\alpha_1 - \alpha_2) \leq 0$, we have $\tilde{\delta}_c < \delta_{c2}$.

Also, $(\bar{r}_c - \tilde{r}_c) = \frac{(2\alpha_1\alpha_2-b^2)(c_2-c_1)}{(2\alpha_1-b)(2\alpha_1-\alpha_2)} > \ (\leq)\ 0$ iff $(2\alpha_1 - \alpha_2) > \ (\leq)\ 0$. Thus, we have two sub-cases below.

Case 1(2)-1: Suppose $(2\alpha_1 - \alpha_2) > 0$. Then, there are two situations.

Case 1(2)-1a: Suppose $r < \tilde{r}_c$. Then the problem in (43) becomes

$$\max_{r,\, f,\, \delta} \ 2f + r(q_{c1}^* + q_{c2}^*)$$
$$\text{s.t. } \delta_1 < \delta \leq \delta_{c2} \text{ and } 0 < r < \tilde{r}_c. \qquad (A104)$$

Denote L its Lagrange function with $L = \frac{\delta}{2\alpha_1}[(2\alpha_1 - b)(1-r) - (2\alpha_1\alpha_2 - b^2)\delta + bc_1 - 2\alpha_1 c_2] + \frac{r}{2\alpha_1}[1 + (2\alpha_1 - b)\delta - c_1 - r] + \lambda_1(\delta - \delta_{c1}) + \lambda_2(\delta_{c2} - \delta) + \lambda_3[\tilde{r}_c - r]$. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{(1 - 2r - c_1)}{2\alpha_1} + \frac{(2\alpha_1 - b)\lambda_1}{4\alpha_1\alpha_2 - b^2} - \frac{\lambda_2}{2\alpha_1 + b} - \lambda_3 \leq 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \qquad (A105)$$

$$\frac{\partial L}{\partial \delta} = \frac{1}{2\alpha_1}[(2\alpha_1 - b) - 2(2\alpha_1\alpha_2 - b^2)\delta + bc_1 - 2\alpha_1 c_2] + \lambda_1 - \lambda_2 \leq 0,\ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad (A106)$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_{c1} \geq 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \qquad (A107)$$

$$\frac{\partial L}{\partial \lambda_2} = \delta_{c2} - \delta \geq 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \text{ and} \qquad (A108)$$

$$\frac{\partial L}{\partial \lambda_3} = \tilde{r}_c - r \geq 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \qquad (A109)$$

where $\lambda_1,\ \lambda_2$, and λ_3 are the Lagrange multipliers associated with the constraints in (A104). Constraints $\delta_{c1} < \delta$ and $r < \tilde{r}_c$ suggest $\lambda_1^* = \lambda_3^* = 0$ by (A107) and (A109). If $\lambda_2^* = 0$, we have $r_c^* = \frac{1-c_1}{2}$ and $\delta_c^* = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)}$ by (A106) and (A107). We have $(\delta_{c2} - \delta_c^*) = \frac{\alpha_1[-(2\alpha_1-\alpha_2)-(\alpha_2+b)c_1+(2\alpha_1+b)c_2]}{(2\alpha_1+b)(2\alpha_1\alpha_2-b^2)} \geq 0$ iff $c_2 \geq \frac{(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1}{2\alpha_1+b}$, and $\tilde{r}_c \leq 0 < r_c^*$ iff $c_2 \geq \frac{(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1}{2\alpha_1+b}$. This is a contradiction. Thus, no solution exists in this case.

If $\lambda_2^* > 0$, (A105), (A106), and (A108) suggest

$$\frac{(1 - 2r - c_1)}{2\alpha_1} - \frac{1}{2\alpha_1 + b}\lambda_2 = 0,\ \frac{[(2\alpha_1 - b) - 2(2\alpha_1\alpha_2 - b^2)\delta + bc_1 - 2\alpha_1 c_2]}{2\alpha_1} - \lambda_2 = 0,$$

and $(\delta_{c2} - \delta) = 0$.

Solving these equations yields $r_c^* = \frac{[2(\alpha_2+b)-(2\alpha_1+2\alpha_2+3b)c_1+(2\alpha_1+b)c_2]}{2(2\alpha_1+\alpha_2+2b)} > 0,\ \delta_c^* = \frac{2-c_1-c_2}{2(2\alpha_1+\alpha_2+2b)}$, and $\lambda_2^* = \frac{(2\alpha_1+b)[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{2\alpha_1(2\alpha_1+\alpha_2+2b)}$. By some calculations, we have $(\delta_c^* - \delta_{c1}) = \frac{2\alpha_1[2(\alpha_2-\alpha_1)-(\alpha_1+2\alpha_2+2b)c_1+(3\alpha_1+2b)c_2]}{(2\alpha_1+\alpha_2+2b)(4\alpha_1\alpha_2-b^2)} > 0$, and $\lambda_2^* > 0$ iff $c_2 < \frac{(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1}{2\alpha_1+b}$. Since $(r_c^* - \tilde{r}_c) = \frac{(2\alpha_1+b)[-2(2\alpha_1-\alpha_2)-(2\alpha_1+3\alpha_2+4b)c_1+(6\alpha_1+\alpha_2+4b)c_2]}{2(2\alpha_1-\alpha_2)(2\alpha_1+\alpha_2+2b)}$, we have $r_c^* < \tilde{r}_c$ iff $c_2 < \frac{2(2\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1}{(6\alpha_1+\alpha_2+4b)} \equiv \ddot{c}_{c2}$. In addition, we have $\ddot{c}_{c2} - \frac{(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1}{(2\alpha_1+b)} = \frac{1}{(2\alpha_1+b)(6\alpha_1+\alpha_2+4b)}$ $[-(2\alpha_1-\alpha_2)(2\alpha_1+\alpha_2+2b)(1-c_1)] < 0$. Thus, an equilibrium exists when $c_2 < \ddot{c}_{c2}$ with fee revenue

$$R_c^* = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)} = R_3^*. \qquad (A110)$$

Case 1(2)-1b: Suppose $r \geq \tilde{r}_c \equiv \frac{[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{(2\alpha_1-\alpha_2)}$. Then the problem in (43) becomes

$$\max_{r,\,f,\,\delta}\ 2f + r(q_{c1}^* + q_{c2}^*)$$
$$\text{s.t. } \delta_{c1} < \delta < \tilde{\tilde{\delta}}_c \text{ and } \tilde{r}_c \leq r < \bar{r}_c. \qquad (A111)$$

Denote L its Lagrange function with $L = \frac{\delta}{2\alpha_1}[(2\alpha_1-b)(1-r)-(2\alpha_1\alpha_2-b^2)\delta+bc_1-2\alpha_1c_2] + \frac{r}{2\alpha_1}[1+(2\alpha_1-b)\delta-c_1-r] + \lambda_1(\delta-\delta_{c1}) + \lambda_2(\tilde{\tilde{\delta}}_c-\delta) + \lambda_3(r-\tilde{r}_c) + \lambda_4(\bar{r}_c-r)$. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{(1-2r-c_1)}{2\alpha_1} + \frac{(2\alpha_1-b)\lambda_1}{4\alpha_1\alpha_2-b^2} - \frac{(2\alpha_1-b)\lambda_2}{2\alpha_1\alpha_2-b^2} + \lambda_3 - \lambda_4 \leq 0,\ r\cdot\frac{\partial L}{\partial r} = 0,\ (A112)$$

$$\frac{\partial L}{\partial \delta} = \frac{1}{2\alpha_1}[(2\alpha_1-b)-2(2\alpha_1\alpha_2-b^2)\delta+bc_1-2\alpha_1c_2] + \lambda_1 - \lambda_2 \leq 0,\ \delta\cdot\frac{\partial L}{\partial \delta} = 0,\ (A113)$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_{c1} \geq 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1} = 0,$$
$$\frac{\partial L}{\partial \lambda_2} = \tilde{\tilde{\delta}}_c - \delta \geq 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2} = 0, \qquad (A114)$$
$$\frac{\partial L}{\partial \lambda_3} = r - \tilde{r}_c \geq 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3} = 0,\ \text{and} \qquad (A115)$$
$$\frac{\partial L}{\partial \lambda_4} = \bar{r}_c - r \geq 0,\ \lambda_4\cdot\frac{\partial L}{\partial \lambda_4} = 0.$$

We have $\lambda_1^* = \lambda_2^* = \lambda_4^* = 0$ by the three strict inequalities in (A111). If $\lambda_3^* = 0$, we have $r_c^* = \frac{1-c_1}{2}$ and $\delta_c^* = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)}$ by (A112) and (A113). Some calculations show

$(\delta_c^* - \tilde{\delta}_c) = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)} - \frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1c_2}{(2\alpha_1\alpha_2-b^2)} = \frac{\alpha_1(c_2-c_1)}{(2\alpha_1\alpha_2-b^2)} > 0$. It is impossible to meet the requirement of $\delta_c^* < \tilde{\delta}_c$. Thus, no solution exists in this case.

By contrast, if $\lambda_3^* > 0$, we have $r_c^* = \tilde{r}_c = \frac{[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{(2\alpha_1-\alpha_2)}$, $\delta_c^* = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)}$, and $\lambda_3^* = \frac{(2\alpha_1-\alpha_2)+(2\alpha_1+\alpha_2+2b)c_1-2(2\alpha_1+b)c_2}{2\alpha_1(2\alpha_1-\alpha_2)}$ by (A112), (A113), and (A115). Note that $\lambda_3^* > 0$ iff $c_2 < \frac{(2\alpha_1-\alpha_2)+(2\alpha_1+\alpha_2+2b)c_1}{2(2\alpha_1+b)}$. On the other hand, (A114) requires $(\tilde{\delta}_c - \delta_c^*) > 0$. Some calculations show $(\tilde{\delta}_c - \delta_c^*) = \frac{1}{2(2\alpha_1-\alpha_2)(2\alpha_1\alpha_2-b^2)}[-(2\alpha_1 - \alpha_2)(2\alpha_1 - b) - (4\alpha_1\alpha_2 + 2b\alpha_1 - b\alpha_2 - 2b^2)c_1 + 2(\alpha_1\alpha_2 + 2\alpha_1^2 - b^2)c_2] > 0$ iff $c_2 > \frac{(2\alpha_1-\alpha_2)(2\alpha_1-b)+(4\alpha_1\alpha_2+2b\alpha_1-b\alpha_2-2b^2)c_1}{2(\alpha_1\alpha_2+2\alpha_1^2-b^2)}$, which contradicts $c_2 < \frac{(2\alpha_1-\alpha_2)+(2\alpha_1+\alpha_2+2b)c_1}{2(2\alpha_1+b)}$ derived above because $\frac{(2\alpha_1-\alpha_2)(2\alpha_1-b)+(4\alpha_1\alpha_2+2b\alpha_1-b\alpha_2-2b^2)c_1}{2(\alpha_1\alpha_2+2\alpha_1^2-b^2)} - \frac{(2\alpha_1-\alpha_2)+(2\alpha_1+\alpha_2+2b)c_1}{2(2\alpha_1+b)} = \frac{\alpha_1(2\alpha_1-\alpha_2)^2(1-c_1)}{2(2\alpha_1+b)(\alpha_1\alpha_2+2\alpha_1^2-b^2)} > 0$. Thus, no solution exists in this case.

Case 1(2)-2: Suppose $(2\alpha_1 - \alpha_2) \leq 0$, we have $\tilde{\delta}_c < \delta_{c2}$. Then the problem in (43) becomes

$$\begin{aligned} \max_{r,\, f,\, \delta} \quad & 2f + r(q_{c1}^* + q_{c2}^*) \\ \text{s.t.} \quad & \delta_{c1} < \delta < \tilde{\delta}_c \text{ and } 0 < r < \bar{r}_c. \end{aligned}$$

Solving this problem yields $r_c^* = \frac{1-c_1}{2}$ and $\delta_c^* = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)}$, the same as those in Case 1(2)-1b. Some calculations yield $(\delta_c^* - \tilde{\delta}_c) = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)} - \frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1c_2}{(2\alpha_1\alpha_2-b^2)} = \frac{\alpha_1(c_2-c_1)}{2\alpha_1\alpha_2-b^2} > 0$, which contradicts $\delta_c^* < \tilde{\delta}_c$. Thus, no solution exists in this case.

Case 1(3): Suppose $\delta \in (\delta_{c2}, \frac{1}{1+b}]$. Then, Lemma 12(ic) implies $\pi_{c1}^* > \pi_{c2}^*$ and $f_c^* = \pi_{c2}^* = \frac{1}{2}[1 - (\alpha_2 + b)\delta - c_2 - r]\delta$ with $f_c^* > 0$ iff $\delta < \frac{1-c_2-r}{(\alpha_2+b)}$ and $r < (1 - c_2)$. Note that $r < (1 - c_2)$ is implied by $r < \bar{r}_c \equiv \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]}{2\alpha_1-b}$. Accordingly, the problem in (43) becomes

$$\max_{r,\, \delta} \quad \delta[1 - (\alpha_2 + b)\delta - c_2 - r] + 2r\delta$$

$$\text{s.t. } \delta_{c2} < \delta < \frac{1 - c_2 - r}{(\alpha_2 + b)} \text{ and } 0 < r < \bar{r}_c. \qquad (A116)$$

Denote L its Lagrange function with $L = \delta[1 - (\alpha_2 + b)\delta - c_2 - r] + 2r\delta + \lambda_1(\delta - \delta_2) +$

$\lambda_2[\frac{1-c_2-r}{(\alpha_2+b)} - \delta] + \lambda_3(\bar{r}_c - r)$. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \delta + \frac{\lambda_1}{2\alpha_1+b} - \frac{\lambda_2}{\alpha_2+b} - \lambda_3 \le 0,\ r\cdot\frac{\partial L}{\partial r} = 0, \tag{A117}$$

$$\frac{\partial L}{\partial \delta} = 1 - 2(\alpha_2+b)\delta - c_2 + r + \lambda_1 - \lambda_2 \le 0,\ \delta\cdot\frac{\partial L}{\partial \delta} = 0,$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_{c2} \ge 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1} = 0,$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{1-c_2-r}{(\alpha_2+b)} - \delta \ge 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2} = 0,\ \text{and}$$

$$\frac{\partial L}{\partial \lambda_3} = \bar{r}_c - r \ge 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3} = 0.$$

Since all constraints in (A116) are strict inequalities, we must have $\lambda_1^* = \lambda_2^* = \lambda_3^* = 0$. However, some calculations show $r_c^* = 0$ by $\delta > 0$ and (A117), which contradicts $r_c^* > 0$. Thus, no solution exists in this case.

<u>Case 2</u>: Suppose $\alpha_1 > \alpha_2$ and $r \ge r_{12} \equiv \frac{2(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1-(2\alpha_1+b)c_2}{2(\alpha_1-\alpha_2)}$. Then, there are three sub-cases as follows.

<u>Case 2(1)</u>: Suppose $\delta \in [0,\ \delta_{c1}]$. Lemma 12(iia) implies $\pi_{c1}^* > \pi_{c2}^*$ and $f_c^* = \pi_{c2}^* = \frac{\alpha_2}{2}(q_{c2}^*)^2 > 0$. Then, the problem in (43) becomes

$$\max_{r,\ f,\ \delta}\ \ 2f + r(q_{c1}^* + q_{c2}^*)$$

$$\text{s.t. } 0 \le \delta \le \delta_{c1} \text{ and } r_{12} \le r < \bar{r}_c. \tag{A118}$$

Its Lagrange function is $L = \alpha_2(q_{c2}^*)^2 + r(q_{c1}^* + q_{c2}^*) + \lambda_1(\delta_{c1} - \delta) + \lambda_2(r - r_{12}) + \lambda_3(\bar{r}_c - r)$, where $\lambda_1,\ \lambda_2$, and λ_3 are the Lagrange multipliers for the inequality constraints in problem (A118). Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = 2\alpha_2 q_{c2}^*\frac{\partial q_{c2}^*}{\partial r} + r(\frac{\partial q_{c1}^*}{\partial r} + \frac{\partial q_{c2}^*}{\partial r}) + (q_{c1}^* + q_{c2}^*) + \lambda_1\frac{\partial \delta_1}{\partial r} + \lambda_2 - \lambda_3 \le 0,\ r\cdot\frac{\partial L}{\partial r} = 0, \tag{A119}$$

$$\frac{\partial L}{\partial \delta} = -\lambda_1 \le 0,\ \delta\cdot\frac{\partial L}{\partial \delta} = 0, \tag{A120}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta_{c1} - \delta \ge 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1} = 0, \tag{A121}$$

$$\frac{\partial L}{\partial \lambda_2} = r - r_{12} \geq 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0,\ \text{and} \tag{A122}$$

$$\frac{\partial L}{\partial \lambda_3} = \bar{r}_c - r \geq 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0. \tag{A123}$$

Constraint $r < \bar{r}_c$ suggests $\lambda_3^* = 0$ by (A123). If $\lambda_1^* > 0$, then (A121) suggests $\delta_c^* = \delta_{c1} > 0$. This in turn implies $\lambda_1^* = 0$ by (A120). It is a contradiction. Thus, we must have $\lambda_1^* = 0$. According to the values of λ_2, there are two situations.

Case 2(1)a: Suppose $\lambda_2^* = 0$. Then (A119) becomes $\frac{1}{(4\alpha_1\alpha_2-b^2)^2}[(2\alpha_2 - b)(4\alpha_1\alpha_2 + 2b\alpha_1 - 2b^2) - (8\alpha_1\alpha_2^2 - 4b^2\alpha_2 + b^3)c_1 + b^2(2\alpha_1 - b)c_2 - [16\alpha_1\alpha_2^2 + (8\alpha_1^2 - 8b\alpha_1 - 6b^2)\alpha_2 - 4b^2(\alpha_1 - b)]r] = 0$. Solving this equation yields $r_c^* = \frac{1}{2(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4b\alpha_1\alpha_2-3b^2\alpha_2-2b^2\alpha_1+2b^3)}$ $[(2\alpha_2 - b)(4\alpha_1\alpha_2 + 2b\alpha_1 - 2b^2) - (8\alpha_1\alpha_2^2 - 4b^2\alpha_2 + b^3)c_1 + b^2(2\alpha_1 - b)c_2]$. It remains to check whether $r_{12} \leq r_c^* < \bar{r}_c$ holds. By some calculations, we have $r_c^* < \bar{r}_c$ iff $c_2 < \hat{c}_{c2} \equiv \frac{(4\alpha_1^2+4\alpha_1\alpha_2-6b\alpha_1-2b\alpha_2+2b^2)+(4\alpha_1\alpha_2+2b\alpha_1+2b\alpha_2-3b^2)c_1}{(4\alpha_1^2+8\alpha_1\alpha_2-4b\alpha_1-b^2)}$, and $r_c^* \geq r_{12}$ iff $c_2 \geq \frac{2(\alpha_1-\alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1\alpha_2+b\alpha_1+2\alpha_2^2-2b^2)c_1}{(4\alpha_1\alpha_2+2\alpha_1^2-b\alpha_1+2b\alpha_2-2b^2)}$ with $\frac{2(\alpha_1-\alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1\alpha_2+b\alpha_1+2\alpha_2^2-2b^2)c_1}{(4\alpha_1\alpha_2+2\alpha_1^2-b\alpha_1+2b\alpha_2-2b^2)} < \hat{c}_{c2}$. In addition, (A121) implies both $\delta_c^* \in [0,\ \delta_{c1}]$ with $\delta_{c1} = \frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1c_2}{4\alpha_1\alpha_2-b^2}$ and $f_c^* = \frac{\alpha_2}{2}[\frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1c_2}{4\alpha_1\alpha_2-b^2}]^2 > 0$. Thus, under condition $\frac{1}{(4\alpha_1\alpha_2+2\alpha_1^2-b\alpha_1+2b\alpha_2-2b^2)}[2(\alpha_1 - \alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1\alpha_2+b\alpha_1+2\alpha_2^2-2b^2)c_1] \leq c_2 < \hat{c}_{c2}$, port authority's equilibrium fee revenue equals

$$R_c^* = \alpha_2[\frac{(2\alpha_1 - b)(1 - r_c^*) + bc_1 - 2\alpha_1c_2}{(4\alpha_1\alpha_2 - b^2)}]^2 + r_c^*[\frac{2(\alpha_1 + \alpha_2 - b)(1 - r_c^*) - (2\alpha_2 - b)c_1 - (2\alpha_1 - b)c_2}{(4\alpha_1\alpha_2 - b^2)}] = R_1^*. \tag{A124}$$

Case 2(1)b: Suppose $\lambda_2^* > 0$. Then, (A122) suggests $r_c^* = r_{12}$ and $\lambda_2^* = \frac{1}{(\alpha_1-\alpha_2)(4\alpha_1\alpha_2-b^2)}$ $[2(\alpha_1 - \alpha_2)(\alpha_1 + \alpha_2 - b) + (4\alpha_1\alpha_2 + b\alpha_1 + 2\alpha_2^2 - 2b^2)c_1 - (4\alpha_1\alpha_2 + 2\alpha_1^2 - b\alpha_1 + 2b\alpha_2 - 2b^2)c_2]$ by (A119). Note that $\lambda_2^* > 0$ iff $c_2 < \frac{2(\alpha_1-\alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1\alpha_2+b\alpha_1+2\alpha_2^2-2b^2)c_1}{(4\alpha_1\alpha_2+2\alpha_1^2-b\alpha_1+2b\alpha_2-2b^2)}$. On the other hand, $(\bar{r}_c - r_{12}) = \frac{(4\alpha_1\alpha_2-b^2)(c_2-c_1)}{2(\alpha_1-\alpha_2)(2\alpha_1-b)} > 0$. In addition, (A121) implies both $\delta_c^* \in [0,\ \delta_{c1}]$ with $\delta_{c1} = \frac{c_2-c_1}{2(\alpha_1-\alpha_2)}$ and $f_c^* = \frac{\alpha_2(c_2-c_1)^2}{8(\alpha_1-\alpha_2)^2} > 0$. Thus, if $c_2 < \frac{2(\alpha_1-\alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1\alpha_2+b\alpha_1+2\alpha_2^2-2b^2)c_1}{(4\alpha_1\alpha_2+2\alpha_1^2-b\alpha_1+2b\alpha_2-2b^2)}$, port authority's equilibrium fee revenue equals

$$R_c^* = \frac{(c_2 - c_1)[4(\alpha_1 - \alpha_2) + (3\alpha_2 + 2b)c_1 - (4\alpha_1 - \alpha_2 + 2b)c_2]}{4(\alpha_1 - \alpha_2)^2} = R_2^*. \tag{A125}$$

Case 2(2): Suppose $\delta \in (\delta_{c1},\ \delta_{c2}]$. Then, Lemma 12(iib) implies $\pi^*_{c1} > \pi^*_{c2}$ and $f^*_c = \pi^*_{c2} = \frac{\delta[(2\alpha_1-b)(1-r)-(2\alpha_1\alpha_2-b^2)\delta+bc_1-2\alpha_1c_2]}{4\alpha_1}$. We have $f^*_c > 0$ iff $\delta < \frac{(2\alpha_1-b)(1-r)+bc_1-2\alpha_1c_2}{(2\alpha_1\alpha_2-b^2)} \equiv \tilde{\delta}_c$ and $r \leq \bar{r}_c \equiv \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]}{2\alpha_1-b}$. In addition, $\tilde{\delta}_c >\ (\leq)\ \delta_{c2}$ iff $r <\ (\geq)\ \frac{1}{(2\alpha_1-\alpha_2)}[(2\alpha_1-\alpha_2)-(\alpha_2+b)c_1-(2\alpha_1+b)c_2] \equiv \tilde{r}_c$. By some calculations, we have $(\bar{r}_c - r_{12}) = \frac{(4\alpha_1\alpha_2-b^2)(c_2-c_1)}{2(\alpha_1-\alpha_2)(2\alpha_1-b)} > 0$, $(\bar{r}_c-\tilde{r}_c) = \frac{(2\alpha_1\alpha_2-b^2)(c_2-c_1)}{(2\alpha_1-b)(2\alpha_1-\alpha_2)} > 0$, $(\tilde{r}_c-r_{12}) = \frac{(2\alpha_1\alpha_2+b\alpha_2)(c_2-c_1)}{2(\alpha_1-\alpha_2)(2\alpha_1-\alpha_2)} > 0$ iff $\alpha_1 > \alpha_2$, and $(\delta_{c2}-\delta_{c1}) = \frac{2\alpha_1}{(2\alpha_1+b)(4\alpha_1\alpha_2-b^2)}[2(\alpha_2-\alpha_1)(1-r)-(2\alpha_2+b)c_1+(2\alpha_1+b)c_2] > 0$ iff $r > r_{12}$. Accordingly, there are another two sub-cases as follows.

Case 2(2)-1a: Suppose $r_{12} < r < \tilde{r}_c$. Then the problem in (43) becomes

$$\begin{aligned} \max_{r,\ f,\ \delta} \quad & 2f + r(q^*_{c1} + q^*_{c2}) \\ \text{s.t.} \quad & \delta_{c1} < \delta \leq \delta_{c2} \text{ and } r_{12} < r < \tilde{r}_c. \end{aligned}$$

Since this problem is similar to that in (A104), we just need to check whether $r_{12} < r$ holds at the equilibria of (A104). Since $(r^*_c-r_{12}) = \frac{-2(\alpha_1-\alpha_2)-(\alpha_1+2\alpha_2+2b)c_1+(3\alpha_1+2b)c_2}{2(\alpha_1-\alpha_2)(2\alpha_1+\alpha_2+2b)} > 0$ iff $c_2 > \frac{2(\alpha_1-\alpha_2)+(\alpha_1+2\alpha_2+2b)c_1}{(3\alpha_1+2b)}$, and $\frac{2(\alpha_1-\alpha_2)+(\alpha_1+2\alpha_2+2b)c_1}{(3\alpha_1+2b)} < \frac{2(2\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1}{(6\alpha_1+\alpha_2+4b)} \equiv \ddot{c}_{c2}$, an equilibrium exists when $\frac{2(\alpha_1-\alpha_2)+(\alpha_1+2\alpha_2+2b)c_1}{(3\alpha_1+2b)} < c_2 < \ddot{c}_{c2}$ with $r^*_c = \frac{1}{2(2\alpha_1+\alpha_2+2b)}[2(\alpha_2+b)-(2\alpha_1+2\alpha_2+3b)c_1+(2\alpha_1+b)c_2] > 0$, $\delta^*_c = \frac{2-c_1-c_2}{2(2\alpha_1+\alpha_2+2b)}$, and port authority's fee revenue

$$R^*_c = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)} = R^*_3. \tag{A126}$$

Case 2(2)-1b: Suppose $r \geq \tilde{r}_c \equiv \frac{[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{(2\alpha_1-\alpha_2)}$. Then the problem in (43) becomes

$$\begin{aligned} \max_{r,\ f,\ \delta} \quad & 2f + r(q^*_{c1} + q^*_{c2}) \\ \text{s.t.} \quad & \delta_{c1} < \delta < \tilde{\delta}_c \text{ and } \tilde{r}_c \leq r < \bar{r}_c. \end{aligned}$$

This problem is the same as that in (A111). Thus, no solution exists in this case.

Case 2(3): Suppose $\delta \in (\delta_{c2},\ \frac{1}{1+b})$. Then, Lemma 12(iic) implies $\pi^*_{c1} >\ (<)\ \pi^*_{c2}$ iff $\delta <\ (>)\ \frac{c_2-c_1}{\alpha_1-\alpha_2}$. Thus, there are two situations.

Case 2(3)-1: Suppose $\delta < \frac{c_2 - c_1}{\alpha_1 - \alpha_2}$. Then we have $f_c^* = \pi_{c2}^* = \frac{\delta[1-(\alpha_2+b)\delta - c_2 - r]}{2}$ with $f^* > 0$ iff $\delta < \frac{1-c_2-r}{(\alpha_2+b)}$ and $r < (1-c_2)$. Note that $r < (1-c_2)$ is implied by $r < \bar{r}_c = \frac{[(2\alpha_1 - b)+bc_1 - 2\alpha_1 c_2]}{2\alpha_1 - b}$. Accordingly, the problem in (43) becomes

$$\max_{r,\,\delta} \;\; \delta[1-(\alpha_2+b)\delta - c_2 - r] + 2r\delta$$

$$\text{s.t. } \delta_{c2} < \delta < \frac{(1-c_2-r)}{(\alpha_2+b)} \text{ and } r_{12} \leq r < \bar{r}_c. \tag{A127}$$

Its Lagrange function is $L = \delta[1-(\alpha_2+b)\delta - c_2 - r] + 2r\delta + \lambda_1(\delta - \delta_{c2}) + \lambda_2[\frac{1-c_2-r}{(\alpha_2+b)} - \delta] + \lambda_3(r - r_{12}) + \lambda_4(\bar{r}_c - r)$. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \delta + \frac{\lambda_1}{2\alpha_1 + b} - \frac{\lambda_2}{\alpha_2 + b} + \lambda_3 - \lambda_4 \leq 0, \; r \cdot \frac{\partial L}{\partial r} = 0, \tag{A128}$$

$$\begin{aligned} \frac{\partial L}{\partial \delta} &= 1 - 2(\alpha_2+b)\delta - c_2 + r + \lambda_1 - \lambda_2 \leq 0, \; \delta \cdot \frac{\partial L}{\partial \delta} = 0, \\ \frac{\partial L}{\partial \lambda_1} &= \delta - \delta_{c2} \geq 0, \; \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \\ \frac{\partial L}{\partial \lambda_2} &= \frac{1-c_2-r}{(\alpha_2+b)} - \delta \geq 0, \; \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \\ \frac{\partial L}{\partial \lambda_3} &= r - r_{12} \geq 0, \; \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \text{ and} \\ \frac{\partial L}{\partial \lambda_4} &= \bar{r}_c - r \geq 0, \; \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0. \end{aligned} \tag{A129}$$

Since the three constraints in (A127) are strict inequalities, we must have $\lambda_1^* = \lambda_2^* = \lambda_4^* = 0$. Suppose $\lambda_3^* = 0$. Then, we obtain $r_c^* = 0$ by $\delta > 0$ and (A128), which contradicts the requirement of $r_c^* > 0$. Thus, we must have $\lambda_3^* > 0$. Under the circumstance, $r_c^* = r_{12} = \frac{2(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1-(2\alpha_1+b)c_2}{2(\alpha_1-\alpha_2)}$ by (A129) and $\frac{\partial L}{\partial r} = \delta + \lambda_3 = 0$ due to (A128). In addition, $\delta_c^* < 0$ implied by $\frac{\partial L}{\partial r} = \delta + \lambda_3 = 0$, which contradicts $\delta_c^* \geq 0$. Thus, no solution exists in this case.

Case 2(3)-2: Suppose $\delta > \frac{(c_2-c_1)}{\alpha_1-\alpha_2}$. Then we have $f_c^* = \pi_{c1}^* = \frac{\delta[1-(\alpha_1+b)\delta - c_1 - r]}{2}$ with $f_c^* > 0$ iff $\delta < \frac{(1-c_1-r)}{(\alpha_1+b)}$ and $r < (1-c_1)$. As in Case 1(3), there exists no solution because (A128) does not hold.

Case 3: Suppose $\alpha_1 > \alpha_2$ and $r < r_{12} \equiv \frac{2(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1-(2\alpha_1+b)c_2}{2(\alpha_1-\alpha_2)}$. Since $(\bar{r}_c - r_{12}) = \frac{(4\alpha_1\alpha_2-b^2)(c_2-c_1)}{2(\alpha_1-\alpha_2)(2\alpha_1-b)} > 0$, $r < r_{12}$ implies $r < \bar{r}_c$. Accordingly, there are three sub-cases as follows.

Case 3(1): Suppose $\delta \in [0,\ \delta'_{c1}]$. Lemma 12(iiia) implies $\pi^*_{c2} > \pi^*_{c1}$ and $f^*_c = \pi^*_{c1} = \frac{1}{2}\alpha_1(q^*_{c1})^2 > 0$. Then the problem in (43) becomes

$$\max_{r,\ f,\ \delta}\ 2f + r(q^*_{c1} + q^*_{c2})$$

$$\text{s.t. } 0 \le \delta \le \delta'_{c1} \text{ and } r < r_{12}. \tag{A130}$$

Its Lagrange function is $L = \alpha_1(q^*_{c1})^2 + r(q^*_{c1} + q^*_{c2}) + \lambda_1(\delta'_{c1} - \delta) + \lambda_2(r_{12} - r)$ with the Lagrange multipliers λ_1 and λ_2. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = 2\alpha_2 q^*_{c2}\frac{\partial q^*_2}{\partial r} + r(\frac{\partial q^*_{c1}}{\partial r} + \frac{\partial q^*_{c2}}{\partial r}) + (q^*_{c1} + q^*_{c2}) + \lambda_1\frac{\partial \delta_{c1}}{\partial r} - \lambda_2 \le 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \tag{A131}$$

$$\frac{\partial L}{\partial \delta} = -\lambda_1 \le 0,\ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \tag{A132}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta'_{c1} - \delta \ge 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0,\ \text{and} \tag{A133}$$

$$\frac{\partial L}{\partial \lambda_2} = r_{12} - r \ge 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0. \tag{A134}$$

Constraint $r < r_{12}$ in (A130) suggests $\lambda^*_2 = 0$ by (A134). Based on the values of λ_1, there are two situations below.

Case 3(1)a: Suppose $\lambda^*_1 = 0$. Then (A131) becomes $\frac{1}{(4\alpha_1\alpha_2-b^2)^2}[(2\alpha_1 - b)(4\alpha_1\alpha_2 + 2b\alpha_2 - 2b^2) + b^2(2\alpha_2 - b)c_1 - (8\alpha_1^2\alpha_2 - 4b^2\alpha_1 + b^3)c_2 - 2(8\alpha_1^2\alpha_2 + 4\alpha_1\alpha_2^2 - 4b\alpha_1\alpha_2 - 3b^2\alpha_1 - 2b^2\alpha_2 + 2b^3)r] = 0$. Solving this equation yields $r^*_c = \frac{1}{2(8\alpha_1^2\alpha_2+4\alpha_1\alpha_2^2-4b\alpha_1\alpha_2-3b^2\alpha_1-2b^2\alpha_2+2b^3)}$ $[(2\alpha_1 - b)(4\alpha_1\alpha_2 + 2b\alpha_2 - 2b^2) + b^2(2\alpha_2 - b)c_1 - (8\alpha_1^2\alpha_2 - 4b^2\alpha_1 + b^3)c_2]$. It remains to check whether $r^*_c < r_{12}$ holds. By some calculations, we have $r^*_c < r_{12}$ iff $c_2 < \frac{[2(\alpha_1-\alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1\alpha_2+2b\alpha_1-b\alpha_2+2\alpha_2^2-2b^2)c_1]}{(4\alpha_1\alpha_2+2\alpha_1^2+b\alpha_2-2b^2)}$. Moreover, (A133) implies both $\delta^*_c \in [0,\ \delta'_{c1}]$ with $\delta'_{c1} = \frac{(2\alpha_2-b)(1-r^*_c)-2\alpha_2c_1+bc_2}{4\alpha_1\alpha_2-b^2}$ and $f^*_c = \frac{\alpha_1}{2}[\frac{(2\alpha_2-b)(1-r^*_c)-2\alpha_2c_1+bc_2}{4\alpha_1\alpha_2-b^2}]^2 > 0$. Thus, at the equilibrium, port authority's fee revenue equals

$$R^*_c = \alpha_1[\frac{(2\alpha_2 - b)(1 - r^*_c) - 2\alpha_2c_1 + bc_2}{4\alpha_1\alpha_2 - b^2}]^2 + r^*_c[\frac{2(\alpha_1 + \alpha_2 - b)(1 - r^*_c) - (2\alpha_2 - b)c_1 - (2\alpha_1 - b)c_2}{4\alpha_1\alpha_2 - b^2}] = R^*_4. \tag{A135}$$

Case 3(1)b: Suppose $\lambda_1^* > 0$. Then, (A133) suggests $\delta_c^* = \delta'_{c1} > 0$. This in turn implies $\lambda_1^* = 0$ by (A132). It is a contradiction. Thus, no solution exists in this case.

Case 3(2): Suppose $\delta \in (\delta'_{c1},\ \delta'_{c2}]$. Then, Lemma 12(iiib) implies $\pi_{c2}^* > \pi_{c1}^*$ and $f_c^* = \pi_{c1}^* = \frac{\delta[(2\alpha_2-b)(1-r)-(2\alpha_1\alpha_2-b^2)\delta+bc_2-2\alpha_2c_1]}{4\alpha_2}$. We have $f_c^* > 0$ iff $\delta < \frac{(2\alpha_2-b)(1-r)-2\alpha_2c_1+bc_2}{(2\alpha_1\alpha_2-b^2)} \equiv \tilde{\tilde{\delta}}_c$ and $r \leq \frac{(2\alpha_2-b)-2\alpha_2c_1+bc_2}{2\alpha_2-b}$ with $\frac{(2\alpha_2-b)-2\alpha_2c_1+bc_2}{2\alpha_2-b} > \bar{r}_c$. Moreover, if $(2\alpha_2-\alpha_1) > 0$, we have $\tilde{\tilde{\delta}}_c >\ (\leq)\ \delta'_{c2}$ iff $r <\ (\geq)\ \frac{[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2-\alpha_1)} \equiv \tilde{\tilde{r}}_c$. By contrast, if $(2\alpha_1-\alpha_2) < 0$, we have $\tilde{\tilde{\delta}}_c \leq\ (>)\ \delta'_{c2}$ iff $r \leq\ (>)\ \tilde{\tilde{r}}_c$. Since $(r_{12}-\tilde{\tilde{r}}_c) = \frac{-\alpha_1(2\alpha_2+b)(c_2-c_1)}{2(\alpha_1-\alpha_2)(2\alpha_2-\alpha_1)} >\ (<)\ 0$ iff $(2\alpha_2-\alpha_1) <\ (>)\ 0$, and $(\delta'_{c2}-\delta'_{c1}) = \frac{2\alpha_2[2(\alpha_1-\alpha_2)(1-r)+(2\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{(2\alpha_2+b)(4\alpha_1\alpha_2-b^2)} > 0$ iff $r < r_{12}$, we have $\tilde{\tilde{\delta}}_c > \delta'_{c2}$ if $(2\alpha_2-\alpha_1) \geq 0$, and $\tilde{\tilde{\delta}}_c \leq\ (>)\ \delta'_{c2}$ if $(2\alpha_2-\alpha_1) < 0$. Thus, we have the following two situations.

Case 3(2)-1: Suppose $(2\alpha_2-\alpha_1) \geq 0$. Then the problem in (43) becomes

$$\max_{r,\, f,\, \delta}\ 2f + r(q_{c1}^* + q_{c2}^*)$$

$$\text{s.t.}\quad \delta'_{c1} < \delta \leq \delta'_{c2} \text{ and } 0 < r < r_{12}. \tag{A136}$$

Its Lagrange function is $L = \frac{\delta}{2\alpha_2}[(2\alpha_2-b)(1-r)-(2\alpha_1\alpha_2-b^2)\delta+bc_2-2\alpha_2c_1] + \frac{r[1+(2\alpha_2-b)\delta-c_2-r]}{2\alpha_2} + \lambda_1(\delta-\delta'_{c1}) + \lambda_2(\delta'_{c2}-\delta) + \lambda_3[r_{12}-r]$, where λ_1, λ_2, and λ_3 are the Lagrange multipliers for the three constraints in (A136). Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{(1-2r-c_2)}{2\alpha_2} + \frac{(2\alpha_2-b)\lambda_1}{(4\alpha_1\alpha_2-b^2)} - \frac{\lambda_2}{(2\alpha_2+b)} - \lambda_3 \leq 0,\ r\cdot\frac{\partial L}{\partial r} = 0, \tag{A137}$$

$$\frac{\partial L}{\partial \delta} = \frac{1}{2\alpha_2}[(2\alpha_2-b)-2(2\alpha_1\alpha_2-b^2)\delta+bc_2-2\alpha_2c_1]+\lambda_1-\lambda_2 \leq 0,\ \delta\cdot\frac{\partial L}{\partial \delta} = 0, \tag{A138}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta-\delta'_{c1} \geq 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1} = 0, \tag{A139}$$

$$\frac{\partial L}{\partial \lambda_2} = \delta'_{c2}-\delta \geq 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2} = 0, \text{ and} \tag{A140}$$

$$\frac{\partial L}{\partial \lambda_3} = r_{12}-r \geq 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3} = 0. \tag{A141}$$

Constraints $\delta'_{c1} < \delta$ and $r < r_{12}$ suggest $\lambda_1^* = \lambda_3^* = 0$ by (A139) and (A141). If $\lambda_2^* = 0$, we have $r_c^* = \frac{(1-c_2)}{2}$ and $\delta_c^* = \frac{(2\alpha_2-b)+bc_2-2\alpha_2c_1}{2(2\alpha_1\alpha_2-b^2)}$ by (A137) and (A138). Since

$(\delta'_{c2} - \delta^*_c) = \frac{\alpha_2[-(2\alpha_2-\alpha_1)-(\alpha_1+b)c_2+(2\alpha_2+b)c_1]}{(2\alpha_2+b)(2\alpha_1\alpha_2-b^2)} < 0$, it is impossible to meet the requirement of $\delta^*_c \le \delta'_{c2}$. Thus, no solution exists in this case.

By contrast, if $\lambda^*_2 > 0$, then (A137), (A138), and (A140) suggest

$$\frac{(1-2r-c_2)}{2\alpha_2} - \frac{\lambda_2}{2\alpha_2+b} = 0, \ \frac{[(2\alpha_2-b)-2(2\alpha_1\alpha_2-b^2)\delta+bc_2-2\alpha_2c_1]}{2\alpha_2} - \lambda_2 = 0,$$
$$\text{and } (\delta'_{c2} - \delta) = 0.$$

Solving these equations yields $r^*_c = \frac{[2(\alpha_1+b)+(2\alpha_2+b)c_1-(2\alpha_1+2\alpha_2+3b)c_2]}{2(\alpha_1+2\alpha_2+2b)}$, $\delta^*_c = \delta'_{c2} = \frac{(2-c_1-c_2)}{2(\alpha_1+2\alpha_2+2b)}$, and $\lambda^*_2 = \frac{(2\alpha_2+b)}{2\alpha_2(\alpha_1+2\alpha_2+2b)}[(2\alpha_2 - \alpha_1) - (2\alpha_2 + b)c_1 + (\alpha_1 + b)c_2] > 0$. By some calculations, we have $(r_{12} - r^*_c) = \frac{(2\alpha_2+b)[2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1-(2\alpha_1+\alpha_2+2b)c_2]}{2(\alpha_1-\alpha_2)(\alpha_1+2\alpha_2+2b)} > 0$ iff $c_2 < \frac{2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1}{(2\alpha_1+\alpha_2+2b)}$, $r^*_c > 0$ iff $c_2 < \frac{2(\alpha_1+b)+(2\alpha_2+b)c_1}{(2\alpha_1+2\alpha_2+3b)}$, and $(\delta^*_c - \delta'_{c1}) > 0$ iff $r^*_c < r_{12}$. In addition, we have $\frac{2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1}{(2\alpha_1+\alpha_2+2b)} < \frac{2(\alpha_1+b)+(2\alpha_2+b)c_1}{(2\alpha_1+2\alpha_2+3b)}$. Thus, an equilibrium exists when $c_2 < \frac{2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1}{(2\alpha_1+\alpha_2+2b)}$ with port authority's fee revenue

$$R^*_c = \frac{(2 - c_1 - c_2)^2}{4(\alpha_1 + 2\alpha_2 + 2b)} = R^*_5. \tag{A142}$$

Case 3(2)-2: Suppose $(2\alpha_2 - \alpha_1) < 0$, we have $\tilde{\tilde{r}}_c < r_{12}$. Then there are two situations below.

Case 3(2)-2a: Suppose $r \le \frac{[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2-\alpha_1)} \equiv \tilde{\tilde{r}}_c$. Then, the problem in (43) becomes

$$\max_{r,\, f,\, \delta} \ 2f + r(q^*_{c1} + q^*_{c2})$$
$$\text{s.t. } \delta'_{c1} < \delta < \tilde{\tilde{\delta}}_c \text{ and } r \le \tilde{\tilde{r}}_c. \tag{A143}$$

Its Lagrange function is $L = \frac{\delta}{2\alpha_2}[(2\alpha_2 - b)(1 - r) - (2\alpha_1\alpha_2 - b^2)\delta + bc_2 - 2\alpha_2c_1] + \frac{r[1+(2\alpha_2-b)\delta-c_2-r]}{2\alpha_2} + \lambda_1(\delta - \delta'_{c1}) + \lambda_2(\tilde{\tilde{\delta}}_c - \delta) + \lambda_3(\tilde{\tilde{r}}_c - r)$. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{(1 - 2r - c_2)}{2\alpha_2} + \frac{(2\alpha_2 - b)}{(4\alpha_1\alpha_2 - b^2)}\lambda_1 - \frac{(2\alpha_2 - b)}{(2\alpha_1\alpha_2 - b^2)}\lambda_2 - \lambda_3 \le 0, \ r \cdot \frac{\partial L}{\partial r} = 0, \tag{A144}$$

$$\frac{\partial L}{\partial \delta} = \frac{1}{2\alpha_2}[(2\alpha_2 - b) - 2(2\alpha_1\alpha_2 - b^2)\delta + bc_2 - 2\alpha_2c_1] + \lambda_1 - \lambda_2 \le 0, \ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \tag{A145}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta'_{c1} \geq 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0,$$
$$\frac{\partial L}{\partial \lambda_2} = \tilde{\tilde{\delta}}_c - \delta \geq 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0,\ \text{and}$$
$$\frac{\partial L}{\partial \lambda_3} = \tilde{\tilde{r}}_c - r \geq 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \tag{A146}$$

where $\lambda_1,\ \lambda_2$, and λ_3 are the Lagrange multipliers for the three constraints in (A143). We have $\lambda_1^* = \lambda_2^* = 0$ by the three strict inequalities in (A143). According to the values of λ_3^*, there are two sub-cases as follows.

<u>Case 3(2)-2a-1</u>: Suppose $\lambda_3^* = 0$. Then $r_c^* = \frac{(1-c_2)}{2}$ and $\delta_c^* = \frac{(2\alpha_2-b)+bc_2-2\alpha_2 c_1}{2(2\alpha_1\alpha_2-b^2)}$ by (A144) and (A145). Some calculations yields $(\tilde{\tilde{\delta}}_c - \delta_c^*) = \frac{\alpha_2(c_2-c_1)}{2\alpha_1\alpha_2-b^2} > 0$ and $(\delta_c^* - \delta'_{c1}) = \frac{\alpha_2[\alpha_1(2\alpha_2-b)-b^2c_1-(2\alpha_1\alpha_2-b\alpha_1-b^2)c_2]}{(2\alpha_1\alpha_2-b^2)(4\alpha_1\alpha_2-b^2)} > 0$ iff $c_2 < \frac{\alpha_1(2\alpha_2-b)-b^2c_1}{(2\alpha_1\alpha_2-b\alpha_1-b^2)}$. Moreover, we have $(\tilde{\tilde{r}}_c - r_c^*) = \frac{[(2\alpha_2-\alpha_1)-2(2\alpha_2+b)c_1+(\alpha_1+2\alpha_2+2b)c_2]}{2(2\alpha_2-\alpha_1)} \geq 0$ iff $c_2 \leq \frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{(\alpha_1+2\alpha_2+2b)}$ due to $(2\alpha_2-\alpha_1) < 0$. Also, $\frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{(\alpha_1+2\alpha_2+2b)} - \frac{\alpha_1(2\alpha_2-b)-b^2c_1}{(2\alpha_1\alpha_2-b\alpha_1-b^2)} = \frac{-(8\alpha_1\alpha_2^2-b^2\alpha_1-2b^2\alpha_2)(1-c_1)}{(\alpha_1+2\alpha_2+2b)(2\alpha_1\alpha_2-b\alpha_1-b^2)} < 0$. Thus, an equilibrium exists when $c_2 \leq \frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{(\alpha_1+2\alpha_2+2b)}$ with port authority's fee revenue

$$R_c^* = \frac{(\alpha_1+2\alpha_2-2b)-2(2\alpha_2-b)c_1-2(\alpha_1-b)c_2-2bc_1c_2+2\alpha_2c_1^2+\alpha_1c_2^2}{4(2\alpha_1\alpha_2-b^2)} = R_6^*. \tag{A147}$$

<u>Case 3(2)-2a-2</u>: Suppose $\lambda_3^* > 0$. Then, $r_c^* = \tilde{\tilde{r}}_c = \frac{[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2-\alpha_1)}$, $\delta_c^* = \frac{(2\alpha_2-b)+bc_2-2\alpha_2c_1}{2(2\alpha_1\alpha_2-b^2)}$, and $\lambda_3^* = \frac{-(\alpha_1-2\alpha_2)-2(2\alpha_2+b)c_1+(\alpha_1+2\alpha_2+2b)c_2}{2\alpha_2(\alpha_1-2\alpha_2)}$ by (A144)-(A146). Note that $\lambda_3^* > 0$ iff $c_2 > \frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{\alpha_1+2\alpha_2+2b}$, and $r_c^* > 0$ iff $c_2 < \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{\alpha_1+b}$. On the other hand, condition $\delta'_{c1} < \delta^* < \tilde{\tilde{\delta}}_c$ is needed. Some calculations yield $(\tilde{\tilde{\delta}}_c - \delta_c^*) = \frac{-(\alpha_1-2\alpha_2)(2\alpha_2-b)-2(\alpha_1\alpha_2+2\alpha_2^2-b^2)c_1+(4\alpha_1\alpha_2-b\alpha_1+2b\alpha_2-2b^2)c_2}{2(\alpha_1-2\alpha_2)(2\alpha_1\alpha_2-b^2)} > 0$ iff $c_2 > \frac{(\alpha_1-2\alpha_2)(2\alpha_2-b+2(\alpha_1\alpha_2+2\alpha_2^2-b^2)c_1}{(4\alpha_1\alpha_2-b\alpha_1+2b\alpha_2-2b^2)}$, $(\delta_c^* - \delta'_{c1}) = \frac{(2\alpha_2-b)+bc_2-2\alpha_2c_1}{2(2\alpha_1\alpha_2-b^2)} - \frac{(2\alpha_1\alpha_2-b^2)(c_2-c_1)}{(4\alpha_1\alpha_2-b^2)(\alpha_1-2\alpha_2)} = \frac{A+Bc_1-Hc_2}{2(2\alpha_1\alpha_2-b^2)(4\alpha_1\alpha_2-b^2)(\alpha_1-2\alpha_2)} > 0$ iff $c_2 < \frac{A+Bc_1}{H}$, where $A = (2\alpha_2-b)(4\alpha_1\alpha_2-b^2)(\alpha_1-2\alpha_2) > 0,\ B = [2(2\alpha_1\alpha_2-b^2)^2 - 2\alpha_2(4\alpha_1\alpha_2-b^2)(\alpha_1-2\alpha_2)] > 0$, and $H = 2(2\alpha_1\alpha_2-b^2)^2 - b(4\alpha_1\alpha_2-b^2)(\alpha_1-2\alpha_2) > 0$. Moreover, we have $\frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{\alpha_1+b} < \frac{A+Bc_1}{H}$, and $\frac{(\alpha_1-2\alpha_2)(2\alpha_2-b)+2(\alpha_1\alpha_2+2\alpha_2^2-b^2)c_1}{(4\alpha_1\alpha_2-b\alpha_1+2b\alpha_2-2b^2)} < \frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{\alpha_1+2\alpha_2+2b} < \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{\alpha_1+b}$ because $\frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{\alpha_1+2\alpha_2+2b} - \frac{(\alpha_1-2\alpha_2)(2\alpha_2-b)+2(\alpha_1\alpha_2+2\alpha_2^2-b^2)c_1}{(4\alpha_1\alpha_2-b\alpha_1+2b\alpha_2-2b^2)} = \frac{2\alpha_2(\alpha_1-2\alpha_2)^2(1-c_1)}{(\alpha_1+2\alpha_2+2b)(4\alpha_1\alpha_2-b\alpha_1+2b\alpha_2-2b^2)} > 0$, $\frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{\alpha_1+b} - \frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{\alpha_1+2\alpha_2+2b} = \frac{(\alpha_1-2\alpha_2)(2\alpha_2+b)(1-c_1)}{(\alpha_1+b)(\alpha_1+2\alpha_2+2b)} > 0$, and $\frac{A+Bc_1}{H} - \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{\alpha_1+b} = \frac{b^2(\alpha_1-2\alpha_2)(2\alpha_1\alpha_2-b^2)(1-c_1)}{H(\alpha_1+b)} > 0$.

Thus, if $\frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{\alpha_1+2\alpha_2+2b} < c_2 < \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{\alpha_1+b}$, an equilibrium exists with port authority's fee revenue

$$R_c^* = \frac{\delta_c^*}{2\alpha_2}[(2\alpha_2-b)(1-r_c^*)-(2\alpha_1\alpha_2-b^2)\delta_c^*+bc_2-2\alpha_2c_1]+\frac{r_c^*}{2\alpha_2}[1+(2\alpha_2-b)\delta_c^*-c_2-r_c^*] = R_7^*. \tag{A148}$$

Case 3(2)-2b: Suppose $r > \tilde{\tilde{r}}_c$. Then the problem in (43) becomes

$$\max_{r,\,f,\,\delta}\; 2f + r(q_{c1}^* + q_{c2}^*)$$

$$\text{s.t.}\quad \delta_{c1}' < \delta \leq \delta_{c2}' \text{ and } \tilde{\tilde{r}}_c < r < r_{12}. \tag{A149}$$

Its Lagrange function is $L = \frac{\delta}{2\alpha_2}[(2\alpha_2-b)(1-r)-(2\alpha_1\alpha_2-b^2)\delta+bc_2-2\alpha_2c_1]+\frac{r}{2\alpha_2}[1+(2\alpha_2-b)\delta-c_2-r]+\lambda_1(\delta-\delta_{c1}')+\lambda_2(\delta_{c2}'-\delta)+\lambda_3(r-\tilde{\tilde{r}}_c)+\lambda_4(r_{12}-r)$. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{(1-2r-c_2)}{2\alpha_2}+\frac{(2\alpha_2-b)\lambda_1}{(4\alpha_1\alpha_2-b^2)}-\frac{\lambda_2}{(2\alpha_2+b)}+\lambda_3-\lambda_4 \leq 0,\; r\cdot\frac{\partial L}{\partial r}=0, \tag{A150}$$

$$\frac{\partial L}{\partial \delta} = \frac{[(2\alpha_2-b)-2(2\alpha_1\alpha_2-b^2)\delta+bc_2-2\alpha_2c_1]}{2\alpha_2}+\lambda_1-\lambda_2 \leq 0,\; \delta\cdot\frac{\partial L}{\partial \delta}=0, \tag{A151}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta-\delta_{c1}' \geq 0,\; \lambda_1\cdot\frac{\partial L}{\partial \lambda_1}=0, \tag{A152}$$

$$\frac{\partial L}{\partial \lambda_2} = \delta_{c2}'-\delta \geq 0,\; \lambda_2\cdot\frac{\partial L}{\partial \lambda_2}=0, \tag{A153}$$

$$\frac{\partial L}{\partial \lambda_3} = r-\tilde{\tilde{r}}_c \geq 0,\; \lambda_3\cdot\frac{\partial L}{\partial \lambda_3}=0, \text{ and} \tag{A154}$$

$$\frac{\partial L}{\partial \lambda_4} = r_{12}-r \geq 0,\; \lambda_4\cdot\frac{\partial L}{\partial \lambda_4}=0, \tag{A155}$$

where λ_1, λ_2, λ_3, and λ_4 are the Lagrange multipliers for the problem in (A149). Constraints $\delta_{c1}' < \delta$ and $\tilde{\tilde{r}}_c < r < r_{12}$ suggest $\lambda_1^* = \lambda_3^* = \lambda_4^* = 0$ by (A152), (A154), and (A155). According to the sign of λ_2, we have two situations below.

Case 3(2)-2b-1: Suppose $\lambda_2^* = 0$. Then, we have $r_c^* = \frac{(1-c_2)}{2}$ and $\delta_c^* = \frac{(2\alpha_2-b)+bc_2-2\alpha_2c_1}{2(2\alpha_1\alpha_2-b^2)}$ by (A150) and (A151). It remains to find the conditions under which $\delta_{c1}' < \delta_c^* \leq \delta_{c2}'$ and $\tilde{\tilde{r}}_c < r < r_{12}$ hold. By some calculations, we have $(\delta_{c2}'-\delta_c^*) = \frac{\alpha_2[(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1-(\alpha_1+b)c_2]}{(2\alpha_2+b)(2\alpha_1\alpha_2-b^2)}$

≥ 0 iff $c_2 \leq \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)}$, $(\delta_c^* - \delta_{c1}') = \frac{\alpha_2[\alpha_1(2\alpha_2-b)-b^2c_1-(2\alpha_1\alpha_2-b\alpha_1-b^2)c_2]}{(2\alpha_1\alpha_2-b^2)(4\alpha_1\alpha_2-b^2)} > 0$ iff $c_2 < \frac{\alpha_1(2\alpha_2-b)-b^2c_1}{(2\alpha_1\alpha_2-b\alpha_1-b^2)}$, $(\tilde{\tilde{r}}_c - r_c^*) = \frac{[(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1-(\alpha_1+2\alpha_2+2b)c_2]}{2(\alpha_1-2\alpha_2)} < 0$ iff $c_2 > \frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{(\alpha_1+2\alpha_2+2b)}$, and $(r_{12} - r_c^*) = \frac{(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1-(\alpha_1+\alpha_2+b)c_2}{2(\alpha_1-\alpha_2)} > 0$ iff $c_2 < \frac{(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+\alpha_2+b)}$. In addition, we have $\frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)} < \frac{\alpha_1(2\alpha_2-b)-b^2c_1}{(2\alpha_1\alpha_2-b\alpha_1-b^2)}$ because $\frac{\alpha_1(2\alpha_2-b)-b^2c_1}{(2\alpha_1\alpha_2-b\alpha_1-b^2)} - \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)} = \frac{2\alpha_2(2\alpha_1\alpha_2-b^2)(1-c_1)}{(\alpha_1+b)(2\alpha_1\alpha_2-b\alpha_1-b^2)} > 0$, $\frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)} < \frac{(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+\alpha_2+b)}$ due to $\frac{(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+\alpha_2+b)} - \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)} = \frac{\alpha_2(2\alpha_2+b)(1-c_1)}{(\alpha_1+b)(\alpha_1+\alpha_2+b)} > 0$, and $\frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)} > \frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{(\alpha_1+2\alpha_2+2b)}$.

Thus, if $\frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{(\alpha_1+2\alpha_2+2b)} < c_2 \leq \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)}$, an equilibrium exists with port authority's fee revenue

$$R_c^* = \frac{(\alpha_1+2\alpha_2-2b) - 2(2\alpha_2-b)c_1 - 2(\alpha_1-b)c_2 - 2bc_1c_2 + 2\alpha_2c_1^2 + \alpha_1c_2^2}{4(2\alpha_1\alpha_2-b^2)} = R_6^*. \quad (A156)$$

Case 3(2)-2b-2: Suppose $\lambda_2^* > 0$. Then (A150), (A151), and (A153) suggest $\frac{(1-2r-c_2)}{2\alpha_2} - \frac{\lambda_2}{2\alpha_2+b} = 0$, $\frac{1}{2\alpha_2}[(2\alpha_2-b)-2(2\alpha_1\alpha_2-b^2)\delta+bc_2-2\alpha_2c_1]-\lambda_2 = 0$, and $(\delta_{c2}'-\delta) = 0$. Solving these equations yields $r_c^* = \frac{[2(\alpha_1+b)+(2\alpha_2+b)c_1-(2\alpha_1+2\alpha_2+3b)c_2]}{2(\alpha_1+2\alpha_2+2b)}$, $\delta_c^* = \delta_{c2}' = \frac{2-c_1-c_2}{2(\alpha_1+2\alpha_2+2b)}$, and $\lambda_2^* = \frac{(2\alpha_2+b)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{2\alpha_2(\alpha_1+2\alpha_2+2b)}$. By some calculations, we have $(r_{12} - r_c^*) = \frac{(2\alpha_2+b)[2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1-(2\alpha_1+\alpha_2+2b)c_2]}{2(\alpha_1-\alpha_2)(\alpha_1+2\alpha_2+2b)} > 0$ iff $c_2 < \frac{2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1}{(2\alpha_1+\alpha_2+2b)}$, $(r_c^* - \tilde{\tilde{r}}_c) = \frac{(2\alpha_2+b)[-2(\alpha_1-2\alpha_2)-(\alpha_1+6\alpha_2+4b)c_1+(3\alpha_1+2\alpha_2+4b)c_2]}{2(\alpha_1-2\alpha_2)(\alpha_1+2\alpha_2+2b)} > 0$ iff $c_2 > \frac{2(\alpha_1-2\alpha_2)+(\alpha_1+6\alpha_2+4b)c_1}{(3\alpha_1+2\alpha_2+4b)}$, $\lambda_2^* > 0$ iff $c_2 > \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)}$, and $(\delta_c^* - \delta_{c1}') > 0$ iff $r_c^* < r_{12}$. Moreover, we have $\frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)} > \frac{2(\alpha_1-2\alpha_2)+(\alpha_1+6\alpha_2+4b)c_1}{(3\alpha_1+2\alpha_2+4b)}$ because $\frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)} - \frac{2(\alpha_1-2\alpha_2)+(\alpha_1+6\alpha_2+4b)c_1}{(3\alpha_1+2\alpha_2+4b)} = \frac{(\alpha_1-2\alpha_2)(\alpha_1+2\alpha_2+2b)(1-c_1)}{(\alpha_1+b)(3\alpha_1+2\alpha_2+4b)} > 0$, and $\frac{2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1}{(2\alpha_1+\alpha_2+2b)} > \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)}$ due to $\frac{2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1}{(2\alpha_1+\alpha_2+2b)} - \frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)} = \frac{\alpha_2(\alpha_1+2\alpha_2+2b)(1-c_1)}{(\alpha_1+b)(2\alpha_1+\alpha_2+2b)} > 0$.

Thus, an equilibrium exists when $\frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{(\alpha_1+b)} < c_2 < \frac{2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1}{(2\alpha_1+\alpha_2+2b)}$ with port authority's fee revenue

$$R_c^* = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)} = R_5^*. \quad (A157)$$

Case 3(3): Suppose $\delta \in (\delta_{c2}', \frac{1}{1+b})$. Then, Lemma 12(iiic) implies $\pi_{c1}^* >$ $(<)$ π_{c2}^* iff $\delta <$ $(>)$ $\frac{(c_2-c_1)}{\alpha_1-\alpha_2}$. Thus, we have two sub-cases below.

Case 3(3)-1: Suppose $\delta > \frac{(c_2-c_1)}{\alpha_1-\alpha_2}$ and $f_c^* = \pi_{c1}^* = \frac{\delta[1-(\alpha_1+b)\delta-c_1-r]}{2}$ with $f_c^* > 0$ iff $\delta < \frac{(1-c_1-r)}{(\alpha_1+b)}$ and $r < (1-c_1)$. Note that $r < (1-c_1)$ is implied by $r < r_{12}$. Accordingly, the problem in (43) becomes

$$\max_{r,\,\delta} \quad \delta[1-(\alpha_1+b)\delta-c_1-r]+2r\delta$$

$$\text{s.t. } \delta'_{c2} < \delta < \frac{(1-c_1-r)}{(\alpha_1+b)} \text{ and } 0 < r < r_{12}. \tag{A158}$$

Its Lagrange function is $L = \delta[1-(\alpha_1+b)\delta-c_1-r]+2r\delta+\lambda_1(\delta-\delta'_{c2})+\lambda_2[\frac{1-c_1-r}{(\alpha_1+b)} - \delta]+\lambda_3(r_{12}-r)$. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \delta + \frac{\lambda_1}{2\alpha_2+b} - \frac{\lambda_2}{\alpha_1+b} - \lambda_3 \le 0,\ r\cdot\frac{\partial L}{\partial r} = 0, \tag{A159}$$

$$\frac{\partial L}{\partial \delta} = 1-2(\alpha_1+b)\delta - c_1 + r + \lambda_1 - \lambda_2 \le 0,\ \delta\cdot\frac{\partial L}{\partial \delta} = 0,$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta'_{c2} \ge 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1} = 0,$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{1-c_1-r}{(\alpha_1+b)} - \delta \ge 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2} = 0, \text{ and}$$

$$\frac{\partial L}{\partial \lambda_3} = r_{12} - r \ge 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3} = 0,$$

where $\lambda_1,\ \lambda_2$, and λ_3 are the Lagrange multipliers. Since all constraints in (A158) are strict inequalities, we must have $\lambda_1^* = \lambda_2^* = \lambda_3^* = 0$. However, by some calculations, we obtain $r_c^* = 0$ by $\delta > 0$ and (A159), which contradicts the requirement of $r_c^* > 0$. Thus, no solution exists in this case.

Case 3(3)-2: Suppose $\delta \le \frac{(c_2-c_1)}{\alpha_1-\alpha_2}$ and $f_c^* = \pi_{c2}^* = \frac{\delta[1-(\alpha_2+b)\delta-c_2-r]}{2}$ with $f_c^* > 0$ iff $\delta < \frac{(1-c_2-r)}{(\alpha_2+b)}$ and $r < (1-c_2)$. As in Case 1(3), no solution exists here because (A159) does not hold.

Based on the above and the values of α_1 and α_2, we can obtain optimal two-part tariff contracts as follows.

Case (i): Suppose $\alpha_1 \le \frac{\alpha_2}{2}$. The proofs are the same as those of Case (ii) below. They verify Lemma 13(i).

Case (ii): Suppose $\frac{\alpha_2}{2} < \alpha_1 \le \alpha_2$. Since $\ddot{c}_2 \equiv \frac{2(2\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1}{(6\alpha_1+\alpha_2+4b)} < \hat{c}_2$, we have two cases as follows.

First, if $c_2 \in (c_1,\ \ddot{c}_{c2})$, then equilibria of R_1^* in (A103) of Case 1(1)a and R_3^* in (A110) of Case 1(2)-1a exist. Define $M_1 = (R_3^* - R_1^*)$. Some calculations show $\frac{\partial^2 M_1}{\partial c_2^2} = \frac{-(4\alpha_1^3+8\alpha_1^2\alpha_2+8b\alpha_1\alpha_2+2b^2\alpha_2-2b^2\alpha_1)}{(2\alpha_1+\alpha_2+2b)(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4b\alpha_1\alpha_2-3b^2\alpha_2-2b^2\alpha_1+2b^3)} < 0,\ M_1 = \frac{(2\alpha_1^2+3\alpha_1\alpha_2-\alpha_2^2-2b\alpha_1)(\alpha_2-\alpha_1)(1-c_1)^2}{(2\alpha_1+\alpha_2+2b)(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4b\alpha_1\alpha_2-3b^2\alpha_2-2b^2\alpha_1+2b^3)} > 0$ at $c_2 = c_1$ due to $(2\alpha_1^2+3\alpha_1\alpha_2-\alpha_2^2-2b\alpha_1) > 0$ by $\alpha_1 \le \alpha_2 < 2\alpha_1$, and $M_1 = \frac{(-4\alpha_1^4+12\alpha_1^3\alpha_2+11\alpha_1^2\alpha_2^2+2\alpha_1\alpha_2^3+4b\alpha_1\alpha_2^2-8b^2\alpha_1\alpha_2-2b\alpha_2^3-\alpha_2^4)(1-c_1)^2}{(6\alpha_1+\alpha_2+4b)^2(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4b\alpha_1\alpha_2-3b^2\alpha_2-2b^2\alpha_1+2b^3)}$ > 0 at $c_2 = \ddot{c}_2$ due to $(-4\alpha_1^4+12\alpha_1^3\alpha_2+11\alpha_1^2\alpha_2^2+2\alpha_1\alpha_2^3+4b\alpha_1\alpha_2^2-8b^2\alpha_1\alpha_2-2b\alpha_2^3-\alpha_2^4) > (-4\alpha_1^4+12\alpha_1^4+11\alpha_1^2\alpha_2^2+2\alpha_1\alpha_2^3+4b\alpha_1\alpha_2^2-8b^2\alpha_1\alpha_2-2b\alpha_2^3-4\alpha_1^2\alpha_2^2) = (8\alpha_1^4+7\alpha_1^2\alpha_2^2+2\alpha_1\alpha_2^3+4b\alpha_1\alpha_2^2-8b^2\alpha_1\alpha_2-2b\alpha_2^3) > 0$. These imply $R_3^* > R_1^*$.

Thus, for $c_2 \in (c_1,\ \ddot{c}_{c2})$, port authority's optimal two-part tariff contract and minimum throughput guarantee are those in Case1(2)-1a with $r_c^* = \frac{2(\alpha_2+b)-(2\alpha_1+2\alpha_2+3b)c_1+(2\alpha_1+b)c_2}{2(2\alpha_1+\alpha_2+2b)}$, $\delta_c^* = \frac{(2-c_1-c_2)}{2(2\alpha_1+\alpha_2+2b)}$, $f_c^* = \frac{(2-c_1-c_2)[2(2\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1-(6\alpha_1+\alpha_2+4b)c_2]}{8(2\alpha_1+\alpha_2+2b)^2}$, and $R_c^* = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$.

For $c_2 \in [\ddot{c}_{c2},\ \hat{c}_{c2})$, a unique solution in Case 1(1)a exists. Thus, port authority's optimal two-part tariff contract and minimum throughput guarantee are those in Case 1(1)a with $r_c^* = \frac{(2\alpha_2-b)(4\alpha_1\alpha_2+2b\alpha_1-2b^2)-(8\alpha_1\alpha_2^2-4b^2\alpha_2+b^3)c_1+b^2(2\alpha_1-b)c_2}{2(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4b\alpha_1\alpha_2-3b^2\alpha_2-2b^2\alpha_1+2b^3)}$, $\delta_c^* \in [0,\ \frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1c_2}{4\alpha_1\alpha_2-b^2}]$, $f_c^* = \frac{\alpha_2}{2}[\frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1c_2}{4\alpha_1\alpha_2-b^2}]^2$, and $R_c^* = 2f_c^* + r_c^*[\frac{2(\alpha_1+\alpha_2-b)(1-r^*)-(2\alpha_2-b)c_1-(2\alpha_1-b)c_2}{4\alpha_1\alpha_2-b^2}]$. Then, Lemma 13(ii) is proved.

Case (iii): Suppose $\alpha_2 < \alpha_1 \le 2\alpha_2$. We need to know relative sizes of critical points $c_{c2}^{t1} \equiv \frac{2(\alpha_1-\alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1\alpha_2+b\alpha_1+2\alpha_2^2-2b^2)c_1}{(4\alpha_1\alpha_2+2\alpha_1^2-b\alpha_1+2b\alpha_2-2b^2)}$ and $\hat{c}_{c2} \equiv \frac{1}{(4\alpha_1^2+8\alpha_1\alpha_2-4b\alpha_1-b^2)}[(4\alpha_1^2+4\alpha_1\alpha_2-6b\alpha_1-2b\alpha_2+2b^2)+(4\alpha_1\alpha_2+2b\alpha_1+2b\alpha_2-3b^2)c_1]$ in Case 2(1)a, $c_{c2}^{t2} \equiv \frac{2(\alpha_1-\alpha_2)+(\alpha_1+2\alpha_2+2b)c_1}{(3\alpha_1+2b)}$ and $\ddot{c}_{c2} \equiv \frac{2(2\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1}{(6\alpha_1+\alpha_2+4b)}$ in Case 2(2)-1a, $c_{c2}^{t3} \equiv \frac{1}{(4\alpha_1\alpha_2+2\alpha_1^2+b\alpha_2-2b^2)}[2(\alpha_1-\alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1\alpha_2+2b\alpha_1-b\alpha_2+2\alpha_2^2-2b^2)c_1]$ in Case 3(1)a, and $\dot{c}_2 \equiv \frac{2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1}{(2\alpha_1+\alpha_2+2b)}$ in Case 3(2)-1. Some calculations show $(\ddot{c}_2-\dot{c}_2) = \frac{2(5\alpha_1\alpha_2-2\alpha_1^2+2b\alpha_2)(1-c_1)}{(6\alpha_1+\alpha_2+4b)(2\alpha_1+\alpha_2+2b)} > 0,\ (\dot{c}_{c2}-c_{c2}^{t1}) = \frac{2(\alpha_1-\alpha_2)^2(\alpha_2-b)(1-c_1)}{(2\alpha_1+\alpha_2+2b)(4\alpha_1\alpha_2+2\alpha_1^2-b\alpha_1+2b\alpha_2-2b^2)} > 0,\ (c_{c2}^{t1}-c_{c2}^{t3}) = \frac{2b(\alpha_1-\alpha_2)^2(\alpha_1+\alpha_2-b)(1-c_1)}{(4\alpha_1\alpha_2+2\alpha_1^2+b\alpha_2-2b^2)(4\alpha_1\alpha_2+2\alpha_1^2-b\alpha_1+2b\alpha_2-2b^2)}$ > 0, and $(c_{c2}^{t3} - c_{c2}^{t2}) = \frac{2(\alpha_1-\alpha_2)^2(\alpha_1-b)(1-c_1)}{(4\alpha_1\alpha_2+2\alpha_1^2+b\alpha_2-2b^2)(3\alpha_1+2b)} > 0$. Thus, we get $c_{c2}^{t2} < c_{c2}^{t3} < c_{c2}^{t1} <$

$\dot{c}_{c2} < \ddot{c}_{c2} < \hat{c}_{c2}$, and optimal two-part tariff contracts can be derived by the following steps.

First, for $c_2 < c_{c2}^{t3}$, we need to comparing R_4^* in (A135) of Case 3(1)a and R_5^* in (A142) of Case 3(2)-1. Define $M_2 = (R_5^* - R_4^*)$. Since $\frac{\partial^2 M_2}{\partial c_2^2} = \frac{-2(\alpha_1^3 - \alpha_1\alpha_2^2 + 2b\alpha_1^2 + b\alpha_1\alpha_2 + b^2\alpha_1 + b^2\alpha_2)}{(\alpha_1+2\alpha_2+2b)(8\alpha_1^2\alpha_2+4\alpha_1\alpha_2^2-4b\alpha_1\alpha_2-3b^2\alpha_1-2b^2\alpha_2+2b^3)} < 0$ and $\frac{\partial M_2}{\partial c_2} = \frac{(2\alpha_1^3-4\alpha_1^2\alpha_2-4\alpha_1\alpha_2^2+2b\alpha_1^2-b\alpha_1\alpha_2-2b\alpha_2^2+2b^2\alpha_2)(1-c_1)}{(\alpha_1+2\alpha_2+2b)(8\alpha_1^2\alpha_2+4\alpha_1\alpha_2^2-4b\alpha_1\alpha_2-3b^2\alpha_1-2b^2\alpha_2+2b^3)} < 0$ at $c_2 = c_1$, we have $\frac{\partial M_2}{\partial c_2} < 0$. Moreover, since $M_2 = \frac{\alpha_2^2(\alpha_1-\alpha_2)^2(1-c_1)^2}{(\alpha_1+2\alpha_2+2b)(4\alpha_1\alpha_2+2\alpha_1^2+b\alpha_2-2b^2)^2} > 0$ at $c_2 = c_{c2}^{t3}$, we get $R_5^* > R_4^*$ when $c_2 < c_{c2}^{t3}$.

Second, for $c_2 < c_{c2}^{t1}$, we need to comparing R_2^* in (A125) of Case 2(1)b and R_5^* in (A142) of Case 3(2)-1. Define $M_3 = (R_5^* - R_2^*)$. Since $\frac{\partial^2 M_3}{\partial c_2^2} = \frac{(5\alpha_1^2-\alpha_2^2+5\alpha_1\alpha_2+2b\alpha_2+10b\alpha_1+4b^2)}{2(\alpha_1+2\alpha_2+2b)(\alpha_1-\alpha_2)^2} > 0$, and $M_3 = \frac{(\alpha_1-\alpha_2)(1-c_1)^2}{(5\alpha_1^2-\alpha_2^2+5\alpha_1\alpha_2+2b\alpha_2+10b\alpha_1+4b^2)} > 0$ at $c_2 = \frac{(4\alpha_1^2-2\alpha_2^2-2\alpha_1\alpha_2+4b\alpha_1-4b\alpha_2)+(\alpha_1^2+\alpha_2^2+7\alpha_1\alpha_2+6b\alpha_1+6b\alpha_2+4b^2)c_1}{(5\alpha_1^2-\alpha_2^2+5\alpha_1\alpha_2+2b\alpha_2+10b\alpha_1+4b^2)}$, which is the solution of $\frac{\partial M_3}{\partial c_2} = 0$. Accordingly, the minimum value of M_3 is $\frac{(\alpha_1-\alpha_2)(1-c_1)^2}{(5\alpha_1^2-\alpha_2^2+5\alpha_1\alpha_2+2b\alpha_2+10b\alpha_1+4b^2)} > 0$, and $R_5^* > R_2^*$ for all c_2.

Third, we always have $(R_5^* - R_3^*) = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)} - \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)} = \frac{(\alpha_1-\alpha_2)(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)(2\alpha_1+\alpha_2+2b)} > 0$.

Fourth, we have $R_3^* > R_1^*$ for $c_2 \in (c_1, \ , \ddot{c}_{c2})$ from Case (i). Moreover, $\frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)} > \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$ and $\dot{c}_{c2} < \ddot{c}_{c2}$. Thus, for $c_2 < \dot{c}_{c2}$, we have $R_5^* > R_1^*$.

In sum, for $c_2 \in (c_1, \ \dot{c}_{c2})$, the above results imply that the optimal two-part tariff contract and the minimum throughput guarantee are those in Case 3(2)-1 with $r_c^* = \frac{[2(\alpha_1+b)+(2\alpha_2+b)c_1-(2\alpha_1+2\alpha_2+3b)c_2]}{2(\alpha_1+2\alpha_2+2b)}$, $f_c^* = \frac{(2-c_1-c_2)[2(2\alpha_2-\alpha_1)-(\alpha_1+6\alpha_2+4b)c_1+(3\alpha_1+2\alpha_2+4b)c_2]}{8(\alpha_1+2\alpha_2+2b)^2}$, $\delta_c^* = \frac{(2-c_1-c_2)}{2(\alpha_1+2\alpha_2+2b)}$, and $R^* = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$. For $c_2 \in [\dot{c}_{c2}, \ \ddot{c}_{c2})$, the fourth result implies that the optimal two-part tariff contract and the minimum throughput guarantee are those in Case 2(2)-1a with $r_c^* = \frac{2(\alpha_2+b)-(2\alpha_1+2\alpha_2+3b)c_1+(2\alpha_1+b)c_2}{2(2\alpha_1+\alpha_2+2b)}$, $\delta_c^* = \frac{(2-c_1-c_2)}{2(2\alpha_1+\alpha_2+2b)}$, $f_c^* = \frac{(2-c_1-c_2)[2(2\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1-(6\alpha_1+\alpha_2+4b)c_2]}{8(2\alpha_1+\alpha_2+2b)^2}$, and $R_c^* = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$. For $c_2 \in [\ddot{c}_{c2}, \ \hat{c}_{c2})$, a unique equilibrium exists in Case 2(1)a with $r_c^* = \frac{1}{2(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4b\alpha_1\alpha_2-3b^2\alpha_2-2b^2\alpha_1+2b^3)}$

$[(2\alpha_2-b)(4\alpha_1\alpha_2+2b\alpha_1-2b^2)-(8\alpha_1\alpha_2^2-4b^2\alpha_2+b^3)c_1+b^2(2\alpha_1-b)c_2]$, δ_c^* $\in [0,\ \frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1c_2}{4\alpha_1\alpha_2-b^2}]$, $f_c^*=\frac{\alpha_2}{2}[\frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1c_2}{4\alpha_1\alpha_2-b^2}]^2$, and $R_c^*=2f_c^*+r_c^*\cdot$ $[\frac{2(\alpha_1+\alpha_2-b)(1-r_c^*)-(2\alpha_2-b)c_1-(2\alpha_1-b)c_2}{4\alpha_1\alpha_2-b^2}]$. These prove Lemma 13(iii).

Case (iv): Suppose $\alpha_1>2\alpha_2$. We need to know relative sizes of critical points $c_{c2}^{t1}\equiv\frac{2(\alpha_1-\alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1\alpha_2+b\alpha_1+2\alpha_2^2-2b^2)c_1}{(4\alpha_1\alpha_2+2\alpha_1^2-b\alpha_1+2b\alpha_2-2b^2)}$ and $\hat{c}_{c2}\equiv\frac{1}{(4\alpha_1^2+8\alpha_1\alpha_2-4b\alpha_1-b^2)}[(4\alpha_1^2+4\alpha_1\alpha_2-6b\alpha_1-2b\alpha_2+2b^2)+(4\alpha_1\alpha_2+2b\alpha_1+2b\alpha_2-3b^2)c_1]$ in Case 2(1)a, $c_{c2}^{t2}\equiv\frac{2(\alpha_1-\alpha_2)+(\alpha_1+2\alpha_2+2b)c_1}{(3\alpha_1+2b)}$ and $\ddot{c}_{c2}\equiv\frac{2(2\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1}{(6\alpha_1+\alpha_2+4b)}$ in Case 2(2)-1a, $c_{c2}^{t3}\equiv\frac{1}{(4\alpha_1\alpha_2+2\alpha_1^2+b\alpha_2-2b^2)}[2(\alpha_1-\alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1\alpha_2+2b\alpha_1-b\alpha_2+2\alpha_2^2-2b^2)c_1]$ in Case 3(1)a, $c_{c2}^{t4}\equiv\frac{(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1}{(\alpha_1+2\alpha_2+2b)}$ and $c_2''\equiv\frac{(\alpha_1-2\alpha_2)+(2\alpha_2+b)c_1}{\alpha_1+b}$ in Case 3(2)-2a-2. Some calculations yield $c_2''>c_{c2}^{t4}$, $(\dot{c}_{c2}-c_{c2}^{t1})=\frac{2(\alpha_1-\alpha_2)^2(\alpha_2-b)(1-c_1)}{(2\alpha_1+\alpha_2+2b)(4\alpha_1\alpha_2+2\alpha_1^2-b\alpha_1+2b\alpha_2-2b^2)}>0$, $(c_{c2}^{t1}-c_{c2}^{t3})=\frac{2b(\alpha_1-\alpha_2)^2(\alpha_1+\alpha_2-b)(1-c_1)}{(4\alpha_1\alpha_2+2\alpha_1^2+b\alpha_2-2b^2)(4\alpha_1\alpha_2+2\alpha_1^2-b\alpha_1+2b\alpha_2-2b^2)}$ >0, $(c_{c2}^{t3}-c_{c2}^{t2})=\frac{2(\alpha_1-\alpha_2)^2(\alpha_1-b)(1-c_1)}{(4\alpha_1\alpha_2+2\alpha_1^2+b\alpha_2-2b^2)(3\alpha_1+2b)}>0$, and $(c_{c2}^{t3}-c_{c2}'')=\frac{\alpha_2(6\alpha_1\alpha_2+b\alpha_1-2b^2)(1-c_1)}{(\alpha_1+b)(4\alpha_1\alpha_2+2\alpha_1^2+b\alpha_2-2b^2)}$ >0. However, the relative size of c_{c2}'' and c_{c2}^{t2} is uncertain, and we are not sure whether $\ddot{c}_{c2}$ is larger than the values of c_{c2}^{t2}, c_{c2}^{t3}, c_{c2}^{t1}, and $\dot{c}_{c2}$. Thus, optimal two-part tariff contracts can be derived by the following steps.

First, if $c_2\in(c_1,\ c_{c2}'']$, then equilibria of R_3^* in (A126) of Case 2(2)-1a for $c_{c2}^{t2}<c_{c2}''$, R_4^* in (A135) of Case 3(1)a, R_2^* in (A125) of Case 2(1)b, R_6^* in (A147) of Case 3(2)-2a-1 and (A156) of Case 3(2)-2b-1, and R_7^* in (A148) of Case 3(2)-2a-2 may exist. Define $M_4=(R_3^*-R_4^*)$. Since $\frac{\partial^2M_4}{\partial c_2^2}=\frac{4\alpha_1^2\alpha_2+4\alpha_1\alpha_2^2-8\alpha_1^3-8b\alpha_1^2-4b\alpha_1\alpha_2-5b^2\alpha_1-3b^2\alpha_2}{2(2\alpha_1+\alpha_2+2b)(8\alpha_1^2\alpha_2+4\alpha_1\alpha_2^2-4b\alpha_1\alpha_2-3b^2\alpha_1-2b^2\alpha_2+2b^3)}<0$, M_4 is a strictly concave function of c_2, and has the maximum value of $\frac{-(\alpha_1^2-\alpha_1\alpha_2-\alpha_2^2)(1-c_1)^2}{8\alpha_1^3-4\alpha_1^2\alpha_2-4\alpha_1\alpha_2^2+8b\alpha_1^2+4b\alpha_1\alpha_2+5b^2\alpha_1+3b^2\alpha_2}<0$ if $(2\alpha_2-\alpha_1)<0$. Thus, we have $R_3^*<R_4^*$ for all c_2.

Next, defining $M_5=(R_6^*-R_4^*)$. Since $\frac{\partial^2M_5}{\partial c_2^2}=\frac{(2\alpha_1\alpha_2-b\alpha_1-b^2)^2}{2(2\alpha_1\alpha_2-b^2)(8\alpha_1^2\alpha_2+4\alpha_1\alpha_2^2-4b\alpha_1\alpha_2-3b^2\alpha_1-2b^2\alpha_2+2b^3)}$ >0, M_5 is a strictly convex function of c_2, and has the minimum value 0 at $c_2=\frac{(2\alpha_1\alpha_2-b\alpha_1-b^2c_1)}{(2\alpha_1\alpha_2-b\alpha_1-b^2)}$ with $\frac{(2\alpha_1\alpha_2-b\alpha_1-b^2c_1)}{(2\alpha_1\alpha_2-b\alpha_1-b^2)}>c_{c2}''$. Thus, $R_6^*>R_4^*$ for $c_2\in(c_1,\ c_{c2}'']$. Then, we define $M_6=(R_6^*-R_2^*)$. Since $\frac{\partial^2M_6}{\partial c_2^2}=\frac{(4\alpha_1-\alpha_2+2b)}{2(\alpha_1-\alpha_2)^2}+\frac{\alpha_1}{2(2\alpha_1\alpha_2-b^2)}>0$, M_6 is a strictly convex function of c_2, and has the minimum value $\frac{(\alpha_1^2+\alpha_1\alpha_2-\alpha_2^2-2b\alpha_1\alpha_2+2b\alpha_2)(1-c_1)^2}{4(\alpha_1^3+6\alpha_1^2\alpha_2-\alpha_1\alpha_2^2+4b\alpha_1\alpha_2-4b^2\alpha_1+b^2\alpha_2-2b^3)}>0$. Thus, we get $R_6^*>R_2^*$ for all c_2. In addition, $(R_6^*-R_7^*)=\frac{[(\alpha_1-2\alpha_2)+2(2\alpha_2+b)c_1-(\alpha_1+2\alpha_2+2b)c_2]^2}{8\alpha_2(\alpha_1-2\alpha_2)^2}$

> 0. Therefore, for $c_2 \in (c_1,\ c''_{c2}]$, port authority's optimal two-part tariff contract and the minimum throughput guarantee are those in Case 3(2)-2a-1 and in Case 3(2)-2b-1 with $r_c^* = \frac{(1-c_2)}{2}$, $\delta_c^* = \frac{(2\alpha_2-b)+bc_2-2\alpha_2 c_1}{2(2\alpha_1\alpha_2-b^2)}$, $f_c^* = \frac{(2\alpha_2-b-2\alpha_2 c_1+bc_2)(c_2-c_1)}{8(2\alpha_1\alpha_2-b^2)}$, and $R_c^* = \frac{(\alpha_1+2\alpha_2-2b)-2(2\alpha_2-b)c_1-2(\alpha_1-b)c_2-2bc_1c_2+2\alpha_2 c_1^2+\alpha_1 c_2^2}{4(2\alpha_1\alpha_2-b^2)}$.

Second, for $c_2 \in (c''_{c2},\ \dot{c}_{c2})$, equilibria of R_1^* in (A124) of Case 2(1)a for $c_2 \in [c_{c2}^{t1},\ \hat{c}_{c2})$, R_2^* in (A125) of Case 2(1)b for $c_2 < c_{c2}^{t1}$, R_3^* in (A126) of Case 2(1)-1a for $c_2 \in [c_{c2}^{t2},\ \dot{c}_{c2})$, R_4^* in (A135) of Case 3(1)a for $c_2 < c_{c2}^{t3}$, and R_5^* in (A157) of Case 3(2)-2b-2 for $c_2 \in (c''_{c2},\ \dot{c}_{c2})$ may exist. Since $R_5^* > R_1^*$, $R_5^* > R_2^*$, $R_5^* > R_3^*$, and $R_5^* > R_4^*$ are shown by Case (iii), R_5^* is the best. Thus, port authority's optimal two-part tariff contract and the minimum throughput guarantee are those in Case 3(2)-2b-2 with $r_c^* = \frac{[2(\alpha_1+b)+(2\alpha_2+b)c_1-(2\alpha_1+2\alpha_2+3b)c_2]}{2(\alpha_1+2\alpha_2+2b)}$, $f_c^* = \frac{(2-c_1-c_2)[2(2\alpha_2-\alpha_1)-(\alpha_1+6\alpha_2+4b)c_1+(3\alpha_1+2\alpha_2+4b)c_2]}{8(\alpha_1+2\alpha_2+2b)^2}$, $\delta_c^* = \frac{(2-c_1-c_2)}{2(\alpha_1+2\alpha_2+2b)}$, and $R_c^* = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$.

Third, suppose $\dot{c}_{c2} < \ddot{c}_{c2}$. Then, for $c_2 \in [\dot{c}_{c2},\ \ddot{c}_{c2})$, the outcomes in Case (iii) show that port authority's optimal two-part tariff contract and minimum throughput guarantee are those in Case 2(2)-1a with $r_c^* = \frac{2(\alpha_2+b)-(2\alpha_1+2\alpha_2+3b)c_1+(2\alpha_1+b)c_2}{2(2\alpha_1+\alpha_2+2b)}$, $\delta_c^* = \frac{(2-c_1-c_2)}{2(2\alpha_1+\alpha_2+2b)}$, $f_c^* = \frac{(2-c_1-c_2)[2(2\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1-(6\alpha_1+\alpha_2+4b)c_2]}{8(2\alpha_1+\alpha_2+2b)^2}$, and $R_c^* = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$.

Fourth, if $\dot{c}_{c2} < \ddot{c}_{c2}$ and $c_2 \in [\ddot{c}_{c2},\ \hat{c}_{c2})$, a unique equilibrium exists in Case 2(1)a. Thus, port authority's optimal two-part tariff contract and minimum throughput guarantee are those in Case 2(1)a with $r_c^* = \frac{1}{2(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4b\alpha_1\alpha_2-3b^2\alpha_2-2b^2\alpha_1+2b^3)}[(2\alpha_2 - b)(4\alpha_1\alpha_2+2b\alpha_1-2b^2)-(8\alpha_1\alpha_2^2-4b^2\alpha_2+b^3)c_1+b^2(2\alpha_1-b)c_2]$, $\delta_c^* \in [0,\ \frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1 c_2}{4\alpha_1\alpha_2-b^2}]$, $f_c^* = \frac{\alpha_2}{2}[\frac{(2\alpha_1-b)(1-r_c^*)+bc_1-2\alpha_1 c_2}{4\alpha_1\alpha_2-b^2}]^2$, and $R_c^* = 2f_c^* + r_c^*[\frac{2(\alpha_1+\alpha_2-b)(1-r_c^*)-(2\alpha_2-b)c_1-(2\alpha_1-b)c_2}{4\alpha_1\alpha_2-b^2}]$.

By contrast, if $\dot{c}_{c2} > \ddot{c}_{c2}$, then the outcomes in the third and fourth parts above will change. For $c_2 \in [\dot{c}_{c2},\ \hat{c}_{c2})$, the optimal contract is the unique solution in Case 2(1)a. These prove Lemma 13(iv). □

Lemma 14. *Suppose the conditions in (40) hold. Then we have the following.*

(i) *Suppose* $\alpha_1 \le \alpha_2$.

(ia) *If* $c_2 \in (c_1,\ \ddot{c}_{c2}]$ *with* $\ddot{c}_{c2} = \frac{1}{(4\alpha_1\alpha_2+4\alpha_1^2-3b^2)}[(2\alpha_1\alpha_2 + 4\alpha_1^2 - 2b\alpha_1 + b\alpha_2 - 2b^2) + (2\alpha_1\alpha_2 + 2b\alpha_1 - b\alpha_2 - b^2)c_1 - (1-c_1)\sqrt{2(\alpha_2 + b)(2\alpha_1 + \alpha_2 - 2b)(2\alpha_1\alpha_2 - b^2)}]$, *then port authority's optimal unit-fee contract and minimum throughput requirement are* $(r_c^u,\ \delta_c^u)$ *with* $r_c^u = \frac{1-c_2}{2}$ *and* $\delta_c^u = \frac{1-c_2}{2(\alpha_2+b)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{ci}^u = \delta_c^u$ *for* $i = 1,\ 2$, *and port authority's fee revenue equals* $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)}$.

(ib) *If* $c_2 \in (\ddot{c}_{c2},\ \tilde{c}_{c2}]$ *with* $\tilde{c}_{c2} \equiv \frac{(2\alpha_1-b)(2\alpha_1+\alpha_2-2b)+(2\alpha_1\alpha_2+2b\alpha_1+b\alpha_2-3b^2)c_1}{(4\alpha_1\alpha_2+4\alpha_1^2-4b\alpha_1-b^2)}$, *then port authority's optimal unit-fee contract and minimum throughput requirement are* $(r_c^u,\ \delta_c^u)$ *with* $r_c^u = \frac{(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2}{2(2\alpha_1+\alpha_2-2b)}$ *and* $\delta_c^u = \frac{(2\alpha_1-b)(1-r_c^u)+bc_1-2\alpha_1c_2}{(2\alpha_1\alpha_2-b^2)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^u = \frac{1-b\delta_c^u-c_1-r_c^u}{2\alpha_1}$ *and* $q_{c2}^u = \delta_c^u$, *and port authority's fee revenue equals* $R_c^u = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$.

(ic) *If* $c_2 \in (\tilde{c}_{c2},\ \bar{c}_{c2})$ *with* $\bar{c}_{c2} = \frac{(2\alpha_1-b)+bc_1}{2\alpha_1}$, *then port authority's optimal unit-fee contract and minimum throughput requirement are* $(r_c^u,\ \delta_c^u)$ *with* $r_c^u = \bar{r}_c \equiv \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2\alpha_1-b}$ *and* $\delta_c^u = 0$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^u = \frac{c_2-c_1}{2\alpha_1-b}$ *and* $q_{c2}^u = 0$, *and port authority's fee revenue equals* $R_c^u = \frac{(c_2-c_1)[(2\alpha_1-b)+bc_1-2\alpha_1c_2]}{(2\alpha_1-b)^2}$.

(ii) *Suppose* $\alpha_2 < \alpha_1 \le 2\alpha_2$.

(iia) *If* $c_2 \in (c_1,\ c_{c2}^{u11})$ *with* $c_{c2}^{u11} = \frac{(2\alpha_1-\alpha_2)+(2\alpha_1+\alpha_2+2b)c_1}{2(2\alpha_1+b)}$, *then port authority's optimal unit-fee contract and minimum throughput requirement are* $(r_c^u,\ \delta_c^u)$ *with* $r_c^u = \frac{1-c_1}{2}$ *and* $\delta_c^u = \frac{1-c_1}{2(\alpha_1+b)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{ci}^u = \delta_c^u$ *for* $i = 1,\ 2$, *and port authority's fee revenue equals* $R_c^u = \frac{(1-c_1)^2}{2(\alpha_1+b)}$.

(iib) *If* $c_2 \in [c_{c2}^{u11},\ c_{c2}^{u12}]$ *with* $c_{c2}^{u12} = \frac{(\alpha_1-\alpha_2)+2(\alpha_2+b)c_1}{(\alpha_1+\alpha_2+2b)}$, *then we have two sub-cases as follows.*

(iib-1) *Suppose* $\alpha_1 > \frac{2\alpha_2^2+5b\alpha_2+4b^2}{(2\alpha_2+b)}$. *For* $c_2 \in [c_{c2}^{u11},\ c'_{c2})$ *with* $c'_{c2} = \frac{1}{(2\alpha_1+b)}[(2\alpha_1 + b) - (1 - c_1)\sqrt{(2\alpha_1 + b)(2\alpha_2 + b)}]$, *port authority's optimal unit-fee contract and minimum throughput requirement are* $(r_c^u,\ \delta_c^u)$ *with* $r_c^u = \frac{1-c_2}{2}$ *and* $\delta_c^u = \frac{1-c_2}{2(2\alpha_2+b)}$. *At the equi-*

librium, operators' cargo-handling amounts are $q_{c1}^u = \delta_c^u$ *and* $q_{c2}^u = \frac{1-b\delta_c^u-c_2-r_c^u}{2\alpha_2}$, *and port authority's fee revenue equals* $R_c^u = \frac{(1-c_2)^2}{2(2\alpha_2+b)}$. *By contrast, if* $c_2 \in [c'_{c2},\ c_{c2}^{u12}]$, *then port authority's optimal unit-fee contract and minimum throughput requirement are* $(r_c^u,\ \delta_c^u)$ *with* $r_c^u = \frac{1-c_1}{2}$ *and* $\delta_c^u = \frac{1-c_1}{2(2\alpha_1+b)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^u = \frac{1-b\delta_c^u-c_1-r_c^u}{2\alpha_1}$ *and* $q_{c2}^u = \delta_c^u$, *and port authority's fee revenue equals* $R_c^u = \frac{(1-c_1)^2}{2(2\alpha_1+b)}$.

(iib-2) *Suppose* $\alpha_1 < \frac{2\alpha_2^2+5b\alpha_2+4b^2}{(2\alpha_2+b)}$. *If* $c_2 \in [c_{c2}^{u11},\ c_{c2}^{u12}]$, *then port authority's optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(iib-1).*

(iic) *If* $c_2 \in (c_{c2}^{u12},\ \ddot{c}_{c2}]$, *then port authority's optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(ia).*

(iid) *If* $c_2 \in (\ddot{c}_{c2},\ \tilde{c}_2]$, *then port authority's optimal unit-fee contract and minimum throughput requirement are the same as those in Lemma 14(ib).*

(iie) *If* $c_2 \in (\tilde{c}_{c2},\ \bar{c}_{c2})$, *then port authority's optimal unit-fee contract and minimum throughput requirement are the same as those in Lemma 14(ic).*

(iii) *Suppose* $(2\alpha_2 - \alpha_1) < 0$.

(iiia) *If* $c_2 \in (c_1,\ c_{c2}^{u11})$. *Then we have two sub-cases.*

(iiia-1) *Suppose* $(\alpha_1^3 + 6b\alpha_1\alpha_2 - 9b\alpha_2^2 - 7\alpha_1\alpha_2^2 + 8b^2\alpha_2 - 6\alpha_1^2\alpha_2 - b\alpha_1^2) < 0$. *For* $c_2 \in (c_1,\ c'''_{c2}]$ *with* $c'''_{c2} = \frac{1}{(\alpha_1+b)(\alpha_1-b)}[(\alpha_1^2 + 2\alpha_1\alpha_2 - b\alpha_1 + 2b\alpha_2 - 2b^2) - (2\alpha_1\alpha_2 - b\alpha_1 + 2b\alpha_2 - b^2)c_1 - (1-c_1)\sqrt{2(\alpha_1+b)(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}]$, *port authority's optimal unit-fee contract and minimum throughput requirement are* $(r_c^u,\ \delta_c^u)$ *with* $r_c^u = \frac{(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2}{2(\alpha_1+2\alpha_2-2b)}$ *and* $\delta_c^u = \frac{(2\alpha_2-b)(1-r_c^u)+bc_2-2\alpha_2c_1}{(2\alpha_1\alpha_2-b^2)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^u = \delta_c^u$ *and* $q_{c2}^u = \frac{1-b\delta_c^u-c_2-r_c^u}{2\alpha_2}$, *and port authority's fee revenue equals* $R_c^u = \frac{[(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2]^2}{4(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$. *By contrast, for* $c_2 \in (c'''_{c2},\ c_{c2}^{u11})$, *port authority's optimal unit-fee contract and minimum throughput* *requirement are the same as those in Lemma 14(iia).*

(iiia-2) *Suppose* $(\alpha_1^3 + 6b\alpha_1\alpha_2 - 9b\alpha_2^2 - 7\alpha_1\alpha_2^2 + 8b^2\alpha_2 - 6\alpha_1^2\alpha_2 - b\alpha_1^2) > 0$. *Then, for* $c_2 \in (c_1,\ c_{c2}^{u11})$, *port authority's optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(iiia-1).*

(iiib) *If* $c_2 \in [c_{c2}^{u11},\ ,c_{c2}^{u12}]$, *then the optimal unit-fee contract and minimum throughput requirement depend on relative values of* $\frac{(1-c_2)^2}{2(2\alpha_2+b)}$, $\frac{(1-c_1)^2}{2(2\alpha_1+b)}$ *and* $\frac{[(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2]^2}{4(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$. *If* $\frac{(1-c_2)^2}{2(2\alpha_2+b)}$ *is the largest, then port authority's optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(iib-1). If* $\frac{(1-c_1)^2}{2(2\alpha_1+b)}$ *is the largest, then port authority's optimal unit-fee contract and minimum throughput requirement are the same as those in the second part of Lemma 14(iib-1). Finally, if* $\frac{[(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2]^2}{4(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ *is the largest, then port authority's optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(iiia-1).*

(iiic) *If* $c_2 \in (c_{c2}^{u12},\ \ddot{c}_{c2}]$, *then port authority's optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(ia).*

(iiid) *If* $c_2 \in (\ddot{c}_{c2},\ \tilde{c}_{c2}]$, *then port authority's optimal unit-fee contract and minimum throughput requirement are the same as those in Lemma 14(ib).*

(iiie) *If* $c_2 \in (\tilde{c}_{c2},\ \bar{c}_{c2})$, *then port authority's the optimal unit-fee contract and minimum throughput requirement are the same as those in Lemma 14(ic).*

Proof of Lemma 14: The proofs are similar to those of Lemma 13, and thus omitted. □

Lemma 15. *Suppose the conditions in (40) hold. Then we have the following.*

(i) *Suppose* $\alpha_1 \le \alpha_2 < 2\alpha_1$. *Then we have the following.*

(ia) *Suppose* $(4\alpha_1^2 - 4\alpha_1\alpha_2 + b^2) > 0$. *Then there are two sub-cases.*

(ia-1) *If* $c_2 \in (c_1,\ c_{c2}^{f1})$ *with* $c_{c2}^{f1} = \frac{(4\alpha_1^2-4\alpha_1\alpha_2+b^2)+(4\alpha_1\alpha_2+2b\alpha_1-b^2)c_1}{2\alpha_1(2\alpha_1+b)}$, *then port authority's optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{(1-c_1)[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{2(2\alpha_1+b)^2}$ *and* $\delta_c^f = \frac{1-c_1}{2\alpha_1+b}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{ci}^f = \delta_c^f$ *for* $i = 1,\ 2$, *and port authority's fee revenue equals* $R_c^f = \frac{(1-c_1)[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{(2\alpha_1+b)^2}$.

(ia-2) *If* $c_2 \in [c_{c2}^{f1},\ \bar{c}_{c2})$ *with* $\bar{c}_{c2} = \frac{(2\alpha_1-b)+bc_1}{2\alpha_1}$, *then port authority's optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{16\alpha_1(2\alpha_1\alpha_2-b^2)}$ *and* $\delta_c^f = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^f = \frac{1-c_1-b\delta_c^f}{2\alpha_1}$ *and* $q_{c2}^f = \delta_c^f$, *and port authority's fee revenue equals* $R_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$.

(ib) *Suppose* $(4\alpha_1^2-4\alpha_1\alpha_2+b^2) < 0$. *Then, for* $c_2 \in (c_1,\ \bar{c}_{c2})$ *with* $\bar{c}_2 = \frac{(2\alpha_1-b)+bc_1}{2\alpha_1}$, *port authority's optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{16\alpha_1(2\alpha_1\alpha_2-b^2)}$ *and* $\delta_c^f = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^f = \frac{1-c_1-b\delta_c^f}{2\alpha_1}$ *and* $q_{c2}^f = \delta_c^f$, *and port authority's fee revenue equals* $R_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$.

(ii) *Suppose* $(2\alpha_1 - \alpha_2) \le 0$. *Then, for* $c_2 \in (c_1,\ \bar{c}_{c2})$ *with* $\bar{c}_{c2} = \frac{(2\alpha_1-b)+bc_1}{2\alpha_1}$, *port authority's optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{16\alpha_1(2\alpha_1\alpha_2-b^2)}$ *and* $\delta_c^f = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^f = \frac{1-c_1-b\delta_c^f}{2\alpha_1}$ *and* $q_{c2}^f = \delta_c^f$, *and port authority's fee revenue equals* $R_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$.

(iii) *Suppose* $\alpha_2 < \alpha_1 \le 2\alpha_2$. *Then there are four sub-cases as follows.*

(iiia) *If* $(4\alpha_1\alpha_2-4\alpha_2^2-b^2) > 0$ *and* $c_2 \in (c_1,\ c_{c2}^{f4}]$ *with* $c_{c2}^{f4} = \frac{(4\alpha_1\alpha_2-4\alpha_2^2-b^2)+2\alpha_2(2\alpha_2+b)c_1}{(4\alpha_1\alpha_2+2b\alpha_2-b^2)}$, *then port authority's optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{[(2\alpha_2-b)-2\alpha_2c_1+bc_2]^2}{16\alpha_2(2\alpha_1\alpha_2-b^2)}$ *and* $\delta_c^f = \frac{(2\alpha_2-b)-2\alpha_2c_1+bc_2}{2(2\alpha_1\alpha_2-b^2)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^f = \delta_c^f$ *and* $q_{c2}^f = \frac{1-c_2-b\delta^f}{2\alpha_2}$, *and port authority's fee revenue equals* $R_c^f = \frac{[(2\alpha_2-b)-2\alpha_2c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)}$.

(iiib) *If* $c_2 \in (\max\{c_1,\ c_{c2}^{f4}\},\ c_{c2}^{f3})$ *with* $c_{c2}^{f3} = \frac{2(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1}{(2\alpha_1+b)}$, *then port authority's optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{2(2\alpha_2+b)^2}$ *and* $\delta_c^f = \frac{1-c_2}{2\alpha_2+b}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^f = q_{c2}^f = \delta_c^f$, *and port authority's fee revenue equals* $R_c^f = \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2+b)^2}$.

(iiic) *If* $c_2 \in [c_{c2}^{f3},\ c_{c2}^{f1})$ *with* $c_{c2}^{f1} = \frac{(4\alpha_1^2-4\alpha_1\alpha_2+b^2)+(4\alpha_1\alpha_2+2b\alpha_1-b^2)c_1}{2\alpha_1(2\alpha_1+b)}$, *then port authority's optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{(1-c_1)[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{2(2\alpha_1+b)^2}$ *and* $\delta_c^f = \frac{1-c_1}{2\alpha_1+b}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^f = q_{c2}^f = \delta_c^f$, *and port authority's fee revenue equals* $R_c^f = 2f_c^f$.

(iiid) *If* $c_2 \in [c_{c2}^{f1},\ \bar{c}_{c2})$, *then port authority's optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{16\alpha_1(2\alpha_1\alpha_2-b^2)}$ *and* $\delta_c^f = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^f = \frac{1-c_1-b\delta_c^f}{2\alpha_1}$ *and* $q_{c2}^f = \delta_c^f$, *and port authority's fee revenue equals* $R_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$.

(iv) *Suppose* $(2\alpha_2-\alpha_1) < 0$. *Then there are four sub-cases as follows.*

(iva) *If* $(4\alpha_1\alpha_2-4\alpha_2^2-b^2) > 0$ *and* $c_2 \in (c_1,\ c_{c2}^{f4}]$ *with* $c_{c2}^{f4} = \frac{(4\alpha_1\alpha_2-4\alpha_2^2-b^2)+2\alpha_2(2\alpha_2+b)c_1}{(4\alpha_1\alpha_2+2b\alpha_2-b^2)}$, *then port authority's optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{[(2\alpha_2-b)-2\alpha_2c_1+bc_2]^2}{16\alpha_2(2\alpha_1\alpha_2-b^2)}$ *and* $\delta_c^f = \frac{(2\alpha_2-b)-2\alpha_2c_1+bc_2}{2(2\alpha_1\alpha_2-b^2)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^f = \delta_c^f$ *and* $q_{c2}^f = \frac{1-c_2-b\delta_c^f}{2\alpha_2}$, *and port authority's fee revenue equals* $R_c^f = \frac{[(2\alpha_2-b)-2\alpha_2c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)}$.

(ivb) *If* $c_2 \in (c_{c2}^{f4},\ c_{c2}^{f3})$ *with* $c_{c2}^{f3} = \frac{2(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1}{(2\alpha_1+b)}$, *then port authority's optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{2(2\alpha_2+b)^2}$ *and* $\delta_c^f = \frac{1-c_2}{2\alpha_2+b}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^f = q_{c2}^f = \delta_c^f$, *and port authority's fee revenue equals* $R_c^f = \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2+b)^2}$.

(ivc) *If* $c_2 \in [c_{c2}^{f3},\ c_{c2}^{f1})$ *with* $c_{c2}^{f1} = \frac{(4\alpha_1^2-4\alpha_1\alpha_2+b^2)+(4\alpha_1\alpha_2+2b\alpha_1-b^2)c_1}{2\alpha_1(2\alpha_1+b)}$, *then port authority's* *optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{(1-c_1)[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{2(2\alpha_1+b)^2}$ *and* $\delta_c^f = \frac{1-c_1}{2\alpha_1+b}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^f = q_{c2}^f = \delta_c^f$, *and port authority's fee revenue equals* $R_c^f = 2f_c^f$.

(ivd) *If* $c_2 \in [c_{c2}^{f1},\ \bar{c}_{c2})$, *then port authority's optimal fixed-fee contract and minimum throughput requirement are* $(f_c^f,\ \delta_c^f)$ *with* $f_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{16\alpha_1(2\alpha_1\alpha_2-b^2)}$ *and* $\delta_c^f = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)}$. *At the equilibrium, operators' cargo-handling amounts are* $q_{c1}^f = \frac{1-c_1-b\delta_c^f}{2\alpha_1}$ *and* $q_{c2}^f = \delta_c^f$, *and port authority's fee revenue equals* $R_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$.

<u>*Proof of Lemma 15*</u>: The proofs are similar to those of Lemma 13, and thus omitted. □

<u>*Proof of Proposition 4*</u>: Note that $\max\{c_1,\ c_{c2}^{f1}\} < \ddot{c}_{c2} < \dddot{c}_{c2} < \hat{c}_{c2} < \tilde{c}_{c2} < \bar{c}_{c2}$. In each of the following four cases, we will first compare the two-part tariff scheme with the unit-fee scheme, and then compare the better of these two with the fixed-fee scheme.

<u>Case 1</u>: Suppose $\alpha_1 \le \alpha_2 < 2\alpha_1$.

<u>**Comparing the two-part tariff scheme with the unit-fee scheme**</u>:

First, for $c_2 \in (c_1,\ \ddot{c}_{c2})$, we have $R_c^* = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$ by Lemma 13 (ia), and $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)}$ by Lemma 14 (ia). Define $H_1 = (R_c^u - R_c^*)$. Since $\frac{\partial^2 H_1}{\partial c_2^2} = \frac{(4\alpha_1+\alpha_2+3b)}{2(\alpha_2+b)(2\alpha_1+\alpha_2+2b)} > 0$, we have $\frac{\partial H_1}{\partial c_2} = \frac{-2(2\alpha_1+b)-(\alpha_2+b)c_1+(4\alpha_1+\alpha_2+3b)c_2}{2(\alpha_2+b)(2\alpha_1+\alpha_2+2b)} < \frac{-2(2\alpha_1+b)-(\alpha_2+b)c_1+(4\alpha_1+\alpha_2+3b)\ddot{c}_{c2}}{2(\alpha_2+b)(2\alpha_1+\alpha_2+2b)} = \frac{-(2\alpha_1+\alpha_2+2b)(1-c_1)}{(\alpha_2+b)(6\alpha_1+\alpha_2+4b)} < 0$, and $H_1 > \frac{(1-\ddot{c}_{c2})^2}{2(\alpha_2+b)} - \frac{(2-c_1-\ddot{c}_{c2})^2}{4(2\alpha_1+\alpha_2+2b)} = \frac{(2\alpha_1-\alpha_2)^2(1-c_1)^2}{2(\alpha_2+b)(6\alpha_1+\alpha_2+4b)^2} > 0$. They imply $R_c^u > R_c^*$.

Second, for $c_2 \in [\ddot{c}_{c2},\ \hat{c}_{c2})$, we have $R_c^* = 2f_c^* + r_c^*[\frac{2(\alpha_1+\alpha_2-b)(1-r_c^*)-(2\alpha_2-b)c_1-(2\alpha_1-b)c_2}{4\alpha_1\alpha_2-b^2}]$ by Lemma 13(ib), and $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)}$ by Lemma 14(ia), and $R_c^u = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ by Lemma 14(ib) with $\frac{(1-c_2)^2}{2(\alpha_2+b)} \ge\ (<)\ \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ iff $c_2 \le\ (>)\ \dddot{c}_{c2}$. Define $H_2 = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)} - R_c^*$. Since $\frac{\partial^2 H_2}{\partial c_2^2} = \frac{\alpha_2(-8\alpha_1^2\alpha_2^2-4\alpha_1^3\alpha_2+4b\alpha_1^2\alpha_2+4b^2\alpha_1^2+7b^2\alpha_1\alpha_2-4b^3\alpha_1-b^4)}{(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4b\alpha_1\alpha_2-2b^2\alpha_1-3b^2\alpha_2+2b^2)} < 0$ and $\frac{\partial H_2}{\partial c_2} = \frac{-b^2\alpha_2(2\alpha_1-b)(1-c_1)}{2(2\alpha_1\alpha_2-b^2)(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4b\alpha_1\alpha_2-2b^2\alpha_1-3b^2\alpha_2+2b^2)} < 0$ at $c_2 = c_1$, we have $\frac{\partial H_2}{\partial c_2} < 0$ for

$c_2 \in [\ddot{c}_{c2},\ \hat{c}_{c2})$. Moreover, we have $H_2 = \frac{\alpha_2^2(2\alpha_1-b)^4(1-c_1)^2}{4(2\alpha_1\alpha_2-b^2)(2\alpha_1+\alpha_2-2b)(8\alpha_1\alpha_2+4\alpha_1^2-4b\alpha_1-b^2)^2} > 0$ at $c_2 = \hat{c}_{c2}$. These imply $\frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)} > R_c^*$ for $c_2 \in [\ddot{c}_{c2},\ \hat{c}_{c2})$. Thus, we get $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)} > R_c^*$ for $c_2 \in [\ddot{c}_{c2},\ \dddot{c}_c]$, and $R_c^u = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)} > R_c^*$ for $c_2 \in [\dddot{c}_{c2},\ \hat{c}_{c2})$.

Third, for $c_2 \in [\hat{c}_{c2},\ \bar{c}_{c2})$, no equilibrium two-part tariff scheme exists. Thus, the unit-fee scheme is the better one.

Comparing the better scheme above with the fixed-fee scheme:

First, for $c_2 \in (c_1,\ c_{c2}^{f1})$, we have $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)}$ by Lemma 14(ia), and $R_c^f = \frac{(1-c_1)[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{(2\alpha_1+b)^2}$ by Lemma 15(ia). Define $H_3 = (R_c^u - R_c^f)$. Since $\frac{\partial H_3}{\partial c_2} = \frac{-(2\alpha_1-\alpha_2)-(\alpha_2+b)c_1+(2\alpha_1+b)c_2}{(\alpha_2+b)(2\alpha_1+b)}$, we can obtain $\frac{\partial H_3}{\partial c_2} < \frac{-(2\alpha_1-\alpha_2)-(\alpha_2+b)c_1+(2\alpha_1+b)c_{c2}^{f1}}{(\alpha_2+b)(2\alpha_1+b)} = \frac{-(2\alpha_1\alpha_2-b^2)(1-c_1)}{2\alpha_1(\alpha_2+b)(2\alpha_1+b)} < 0$, and $H_3 > \frac{(1-c_{c2}^{f1})^2}{2(\alpha_2+b)} - \frac{(1-c_1)[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_{c2}^{f1}]}{(2\alpha_1+b)^2} = \frac{(8\alpha_1^2\alpha_2^2+8b\alpha_1^2\alpha_2+4b^2\alpha_1^2-4b^2\alpha_1\alpha_2+b^4)(1-c_1)^2}{8\alpha_1^2(\alpha_2+b)(2\alpha_1+b)^2} > 0$ for $c_2 \in (c_1,\ c_{c2}^{f1})$. These imply $R_c^u > R_c^f$.

Second, for $c_2 \in [c_{c2}^{f1},\ \tilde{c}_{c2}]$, we have $R_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$ by Lemma 15(ib), $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)}$ by Lemma 14(ia), and $R_c^u = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ by Lemma 14(ib) with $\frac{(1-c_2)^2}{2(\alpha_2+b)} \geq\ (<)\ \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ iff $c_2 \leq\ (>)\ \dddot{c}_{c2}$. Define $H_4 = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)} - \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$. Since $\frac{\partial H_4}{\partial c_2} = \frac{-(c_2-c_1)}{2(2\alpha_1+\alpha_2-2b)} < 0$, we have $H_4 > \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)\tilde{c}_{c2}]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)} - \frac{[(2\alpha_1-b)+bc_1-2\alpha_1\tilde{c}_{c2}]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)} = \frac{(2\alpha_1\alpha_2-b^2)(12\alpha_1^2+8\alpha_1\alpha_2-12b\alpha_1-b^2)(1-c_1)^2}{8\alpha_1(4\alpha_1^2+4\alpha_1\alpha_2-4b\alpha_1-b^2)^2} > 0$, and hence $\frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)} > \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$ for $c_2 < \tilde{c}_{c2}$. Moreover, $\frac{(1-c_2)^2}{2(\alpha_2+b)} \geq\ (<) \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ iff $c_2 \leq\ (>) \dddot{c}_{c2}$. Thus, we have $\frac{(1-c_2)^2}{2(\alpha_2+b)} > \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$ for $c_2 \in [c_{c2}^{f1},\ \dddot{c}_{c2}]$, and $\frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)} > \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$ for $c_2 \in (\dddot{c}_{c2},\ \tilde{c}_{c2}]$. These suggest that the unit-fee scheme is optimal for $c_2 \in [c_{c2}^{f1},\ \tilde{c}_{c2}]$.

Third, for $c_2 \in (\tilde{c}_{c2},\ \bar{c}_{c2})$, we have $R_c^u = \frac{(c_2-c_1)[(2\alpha_1-b)+bc_1-2\alpha_1c_2]}{(2\alpha_1-b)^2}$ by Lemma 14(ic), and $R_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$ by Lemma 15(ib). Define $H_5 = \frac{(c_2-c_1)[(2\alpha_1-b)+bc_1-2\alpha_1c_2]}{(2\alpha_1-b)^2} - \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$. Since $\frac{\partial^2 H_5}{\partial c_2^2} = \frac{-\alpha_1(4\alpha_1^2+8\alpha_1\alpha_2-4b\alpha_1-3b^2)}{(2\alpha_1-b)^2(2\alpha_1\alpha_2-b^2)} < 0$ and $\frac{\partial H_5}{\partial c_2} = \frac{-(2\alpha_1-b)(1-c_1)}{2(4\alpha_1^2+4\alpha_1\alpha_2-4b\alpha_1-b^2)} < 0$ at $c_2 = \tilde{c}_{c2}$, we have $\frac{\partial H_5}{\partial c_2} < 0$ for $c_2 \in (\tilde{c}_{c2},\ \bar{c}_{c2})$, and $H_5 > \frac{(\bar{c}_{c2}-c_1)[(2\alpha_1-b)+bc_1-2\alpha_1\bar{c}_2]}{(2\alpha_1-b)^2} -$

$\frac{[(2\alpha_1-b)+bc_1-2\alpha_1\bar{c}_{c2}]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}=0$. These imply $R_c^u > R_c^f$, and thus the unit-fee scheme is optimal.

Case 2: Suppose $(2\alpha_1-\alpha_2)\leq 0$. As in Case 1, we can show that the unit-fee scheme is port authority's best choice.

Case 3: Suppose $\alpha_2<\alpha_1\leq 2\alpha_2$.

Comparing the two-part tariff scheme with the unit-fee scheme :

Lemma 14(ii) shows that $R_c^u=\frac{(1-c_1)^2}{2(\alpha_1+b)}$ for $c_2\in(c_1,\ c_{c2}^{u11})$, $R_c^u=\max\{\frac{(1-c_2)^2}{2(2\alpha_2+b)},\ \frac{(1-c_1)^2}{2(2\alpha_1+b)}\}$ for $c_2\in[c_{c2}^{u11},\ c_{c2}^{u12}]$, $R_c^u=\frac{(1-c_2)^2}{2(\alpha_2+b)}$ for $c_2\in(c_{c2}^{u12},\ \dddot{c}_{c2}]$, $R_c^u=\frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ for $c_2\in(\dddot{c}_{c2},\ \tilde{c}_{c2}]$, and $R_c^u=\frac{(c_2-c_1)[(2\alpha_1-b)+bc_1-2\alpha_1c_2]}{(2\alpha_1-b)^2}$ for $c_2\in(\tilde{c}_{c2},\ \bar{c}_{c2})$. On the other hand, Lemma 13(iii)shows that $R_c^*=\frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$ for $c_2\in(c_1,\ \dot{c}_{c2})$, $R_c^*=\frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$ for $c_2\in[\dot{c}_{c2},\ \ddot{c}_{c2})$, and $R_c^*=2f_c^*+r_c^*\cdot$ $[\frac{2(\alpha_1+\alpha_2-b)(1-r_c^*)-(2\alpha_2-b)c_1-(2\alpha_1-b)c_2}{4\alpha_1\alpha_2-b^2}]$ for $c_2\in[\ddot{c}_{c2},\ \hat{c}_{c2})$.

By some calculations, we have $c_{c2}^{u11}<c_{c2}^{u12}<\dot{c}_{c2}<\ddot{c}_{c2}<\dddot{c}_{c2}<\hat{c}_{c2}<\tilde{c}_{c2}<\bar{c}_{c2}$. Thus, there are six intervals to be discussed.

First, for $c_2\in(c_1,\ c_{c2}^{u11})$, we have $R_c^*=\frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$ and $R_c^u=\frac{(1-c_1)^2}{2(\alpha_1+b)}$. Define $H_6=(R_c^u-R_c^*)=\frac{(1-c_1)^2}{2(\alpha_1+b)}-\frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$. Since $\frac{\partial H_6}{\partial c_2}=\frac{(2-c_1-c_2)}{2(\alpha_1+2\alpha_2+2b)}>0$, we have $H_6>\frac{(1-c_1)^2}{2(\alpha_1+b)}-\frac{(2-c_1-c_1)^2}{4(\alpha_1+2\alpha_2+2b)}=\frac{(2\alpha_2-\alpha_1)(1-c_1)^2}{2(\alpha_1+b)(\alpha_1+2\alpha_2+2b)}>0$. Thus, the unit-fee scheme is better than the two-part tariff scheme.

Second, for $c_2\in[c_{c2}^{u11},\ c_{c2}^{u12}]$, we have $R_c^u=\max\{\frac{(1-c_2)^2}{2(2\alpha_2+b)},\ \frac{(1-c_1)^2}{2(2\alpha_1+b)}\}$ and $R_c^*=\frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$. Define $H_7=\frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}-\frac{(1-c_2)^2}{2(2\alpha_2+b)}$. Since $\frac{\partial^2 H_7}{\partial c_2^2}=\frac{-(2\alpha_1+2\alpha_2+3b)}{2(2\alpha_2+b)(\alpha_1+2\alpha_2+2b)}<0$ and $\frac{\partial H_7}{\partial c_2}=\frac{(2\alpha_1\alpha_2+2\alpha_2^2+3b\alpha_1+5b\alpha_2+4b^2)(1-c_1)}{2(2\alpha_2+b)(\alpha_1+2\alpha_2+2b)(\alpha_1+\alpha_2+2b)}>0$ at $c_2=c_{c2}^{u12}$, we have $\frac{\partial H_7}{\partial c_2}>0$ for $c_2\in[c_{c2}^{u11},\ c_{c2}^{u12}]$, and $H_7>\frac{(2-c_1-c_{c2}^{u11})^2}{4(\alpha_1+2\alpha_2+2b)}-\frac{(1-c_{c2}^{u11})^2}{2(2\alpha_2+b)}=\frac{1}{16(2\alpha_2+b)(\alpha_1+b)^2(\alpha_1+2\alpha_2+2b)}[(-2\alpha_1^3-2\alpha_2^3+10\alpha_1^2\alpha_2+2\alpha_1\alpha_2^2+22b\alpha_1\alpha_2-3b\alpha_1^2-3b\alpha_2^2+8b^2\alpha_2)(1-c_1)^2]>0$, which imply $\frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}>\frac{(1-c_2)^2}{2(2\alpha_2+b)}$. On the other hand, define $H_8=\frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}-\frac{(1-c_1)^2}{2(2\alpha_1+b)}$. Since $\frac{\partial H_8}{\partial c_2}=\frac{-(2-c_1-c_2)}{2(\alpha_1+2\alpha_2+2b)}<0$, we have $H_8>\frac{(2-c_1-c_{c2}^{u12})^2}{4(\alpha_1+2\alpha_2+2b)}-\frac{(1-c_1)^2}{2(2\alpha_1+b)}=$ $\frac{(4\alpha_1^2\alpha_2+16b^2\alpha_1+5b\alpha_1^2-8b^2\alpha_2-11b\alpha_2^2+8\alpha_1\alpha_2^2+22b\alpha_1\alpha_2-4\alpha_2^3)(1-c_1)^2}{4(2\alpha_1+b)(\alpha_1+2\alpha_2+2b)(\alpha_1+\alpha_2+2b)^2}>0$. Thus, the two-part tariff scheme is better than the unit-fee scheme for $c_2 \in [c_{c2}^{u11},\ c_{c2}^{u12}]$.

Third, for $c_2 \in (c_{c2}^{u12},\ \dot{c}_{c2})$, we have $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)}$ and $R_c^* = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$. Define $H_9 = (R_c^u - R_c^*) = \frac{(1-c_2)^2}{2(\alpha_2+b)} - \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$. Since $\frac{\partial^2 H_9}{\partial c_2^2} = \frac{(2\alpha_1+3\alpha_2+3b)}{2(\alpha_2+b)(\alpha_1+2\alpha_2+2b)} > 0$ and $\frac{\partial H_9}{\partial c_2} = \frac{-(2\alpha_2+b)(1-c_1)}{(\alpha_2+b)(2\alpha_1+\alpha_2+2b)} < 0$ at $c_2 = \dot{c}_{c2}$, we have $\frac{\partial H_9}{\partial c_2} < 0$ for $c_2 \in (c_{c2}^{u12},\ \dot{c}_{c2})$, and $H_9 > \frac{(1-\dot{c}_{c2})^2}{2(\alpha_2+b)} - \frac{(2-c_1-\dot{c}_{c2})^2}{4(\alpha_1+2\alpha_2+2b)} = \frac{(5\alpha_2^2-2\alpha_1\alpha_2-2b\alpha_1+4b\alpha_2)}{2(\alpha_2+b)(2\alpha_1+\alpha_2+2b)^2} > 0$. Thus, the unit-fee scheme is better than the two-part tariff scheme for $c_2 \in (c_{c2}^{u12},\ \dot{c}_{c2})$.

Fourth, for $c_2 \in [\dot{c}_{c2},\ \ddot{c}_{c2})$, we have $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)}$ and $R_c^* = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$. As in Case 1, we have $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)} > R_c^* = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$.

Fifth, for $c_2 \in [\ddot{c}_{c2},\ \hat{c}_{c2})$, the unit-fee scheme is better than the two-part tariff scheme as shown in Case 1.

Sixth, for $c_2 \in [\hat{c}_{c2},\ \bar{c}_{c2})$, there exists no equilibrium two-part tariff scheme. Thus, the unit-fee scheme is better.

The outcomes of the six parts above are summarized below. The unit-fee scheme is better than the two-part tariff scheme for $c_2 \in (c_1,\ c_{c2}^{u11})$ with $R_c^u = \frac{(1-c_1)^2}{2(\alpha_1+b)}$. The two-part tariff scheme is better than the unit-fee scheme for $c_2 \in [c_{c2}^{u11},\ c_{c2}^{u12}]$ with $R_c^* = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$. The unit-fee scheme is better than the two-part tariff scheme for $c_2 \in (c_{c2}^{u12},\ \dddot{c}_{c2}]$ with $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)}$. The unit-fee scheme is better than the two-part tariff scheme for $c_2 \in (\dddot{c}_{c2},\ \tilde{c}_{c2}]$ with $R_c^u = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$. Finally, the unit-fee scheme is better than the two-part tariff scheme for $c_2 \in (\tilde{c}_{c2},\ \bar{c}_{c2})$ with $R_c^u = \frac{(c_2-c_1)[(2\alpha_1-b)+bc_1-2\alpha_1 c_2]}{(2\alpha_1-b)^2}$.

Comparing the better scheme above with the fixed-fee scheme:

Lemma 15(iii) shows $R_c^f = \frac{[(2\alpha_2-b)-2\alpha_2 c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)}$ for $c_2 \in (c_1,\ c_{c2}^{f4}]$ iff $c_{c2}^{f4} > c_1$, $R_c^f = \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2+b)^2}$ for $c_2 \in (c_{c2}^{f4},\ c_{c2}^{f3})$, $R_c^f = \frac{(1-c_1)[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{(2\alpha_1+b)^2}$ for $c_2 \in [c_{c2}^{f3},\ c_{c2}^{f1})$, and $R_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1 c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$ for $c_2 \in [c_{c2}^{f1},\ \bar{c}_{c2})$.

Some calculations show $\min\{c_{c2}^{u11},\ c_{c2}^{f4}\} < \max\{c_{c2}^{f4},\ c_{c2}^{u12}\} < c_{c2}^{f3} < c_{c2}^{f1} < \ddot{c}_{c2} < \tilde{c}_{c2} < \bar{c}_{c2}$. Thus, we need to discuss the following five intervals.

First, for $c_2 \in (c_1,\ c_{c2}^{u11})$, one equilibrium unit-fee scheme exists with $R_c^u = \frac{(1-c_1)^2}{2(\alpha_1+b)}$ and two equilibrium fixed-fee schemes may exist with $R_c^f = \frac{[(2\alpha_2-b)-2\alpha_2 c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)}$ or $R_c^f = \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2+b)^2}$. Define $H_{10} = \frac{(1-c_1)^2}{2(\alpha_1+b)} - \frac{[(2\alpha_2-b)-2\alpha_2 c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)}$. Since $\frac{\partial H_{10}}{\partial c_2} = \frac{-b[(2\alpha_2-b)-2\alpha_2 c_1+bc_2]}{4\alpha_2 2\alpha_1\alpha_2-b^2)} < 0$, we have $H_{10} = \frac{(1-c_1)^2}{2(\alpha_1+b)} - \frac{[(2\alpha_2-b)-2\alpha_2 c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)} > \frac{(1-c_1)^2}{2(\alpha_1+b)} - \frac{[(2\alpha_2-b)-2\alpha_2 c_1+bc_{c2}^{u11}]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)} = \frac{(16\alpha_1^2\alpha_2^2+8b\alpha_1\alpha_2^2+8b\alpha_1^2\alpha_2+6b^2\alpha_1\alpha_2-b^2\alpha_1^2-9b^2\alpha_2^2-4b^3\alpha_1-4b^3\alpha_2-4b^4)(1-c_1)^2}{32\alpha_2(2\alpha_1\alpha_2-b^2)(\alpha_1+b)^2} > 0$. These imply $R_c^u = \frac{(1-c_1)^2}{2(\alpha_1+b)} > R_c^f = \frac{[(2\alpha_2-b)-2\alpha_2 c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)}$.

Define $H_{11} = \frac{(1-c_1)^2}{2(\alpha_1+b)} - \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2+b)^2}$. Since $\frac{\partial H_{11}}{\partial c_2} = \frac{1}{(2\alpha_2+b)^2}[-(2\alpha_1-2\alpha_2+b)-(2\alpha_2+b)c_1+2(\alpha_1+b)c_2] < \frac{-(2\alpha_1-2\alpha_2+b)-(2\alpha_2+b)c_1+2(\alpha_1+b)c_{c2}^{u11}}{(2\alpha_2+b)^2} = \frac{-(\alpha_1-\alpha_2+b)(1-c_1)}{(2\alpha_2+b)^2} < 0$, we have $H_{11} > \frac{(1-c_1)^2}{2(\alpha_1+b)} - \frac{(1-c_{c2}^{u11})[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_{c2}^{u11}]}{(2\alpha_2+b)^2} = \frac{1}{4(\alpha_1+b)(2\alpha_2+b)^2}[(1-c_{c2}^{u11})(\alpha_1^2+5\alpha_2^2-2\alpha_1\alpha_2+2b\alpha_1+2b\alpha_2+2b^2)(1-c_1)^2] > 0$. Thus, for $c_2 \in (c_1,\ c_{c2}^{u11})$, the unit-fee scheme is better than the fixed-fee scheme, and thus the optimal contract.

Second, for $c_2 \in [c_{c2}^{u11},\ c_{c2}^{u12}]$, one equilibrium two-part tariff scheme exists with $R_c^* = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$ and two equilibrium fixed-fee schemes may exist with $R_c^f = \frac{1}{8\alpha_2(2\alpha_1\alpha_2-b^2)}[(2\alpha_2-b)-2\alpha_2 c_1+bc_2]^2$ or $R_c^f = \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2+b)^2}$. Define $H_{12} = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)} - \frac{[(2\alpha_2-b)-2\alpha_2 c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)}$. Since $\frac{\partial H_{12}}{\partial c_2} = \frac{1}{8\alpha_2(\alpha_1+b)(\alpha_1+2\alpha_2+2b)(2\alpha_1\alpha_2-b^2)}[-(2\alpha_2+b)(6\alpha_1^2\alpha_2+2\alpha_1\alpha_2^2+11b\alpha_1\alpha_2-b\alpha_1^2-4b^2\alpha_1+2b^2\alpha_2-4b^3)(1-c_1)] < 0$ at $c_2 = c_{c2}^{u11}$, and $\frac{\partial H_{12}}{\partial c_2} = \frac{-(2\alpha_2+b)(\alpha_1^2\alpha_2+3\alpha_1\alpha_2^2+4b\alpha_1\alpha_2-b^2\alpha_1+b\alpha_2^2-2b^3)(1-c_1)}{2\alpha_2(\alpha_1+\alpha_2+2b)(\alpha_1+2\alpha_2+2b)(2\alpha_1\alpha_2-b^2)} < 0$ at $c_2 = c_{c2}^{u12}$, we have $H_{12} > \frac{(2-c_1-c_{c2}^{u12})^2}{4(\alpha_1+2\alpha_2+2b)} - \frac{[(2\alpha_2-b)-2\alpha_2 c_1+bc_{c2}^{u12}]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)} = \frac{1}{4\alpha_2(\alpha_1+\alpha_2+2b)^2(\alpha_1+2\alpha_2+2b)(2\alpha_1\alpha_2-b^2)}[(2\alpha_1^2\alpha_2^2+4\alpha_1\alpha_2^3-2\alpha_2^4+3b\alpha_1^2\alpha_2+12b\alpha_1\alpha_2^2-5b\alpha_2^3+8b^2\alpha_1\alpha_2-4b^2\alpha_2^2-2b^3\alpha_1-4b^3\alpha_2-4b^4)(1-c_1)^2] > 0$ due to $\frac{\partial G}{\partial \alpha_1} = 4\alpha_1\alpha_2^2+4\alpha_2^3+12b\alpha_2^2+8b^2\alpha_2+6b\alpha_1\alpha_2-2b^3 > 0$ and $G = 4\alpha_2^4+10b\alpha_2^3+4b^2\alpha_2^2-6b^3\alpha_2-4b^4 > 0$ at $\alpha_1 = \alpha_2$, where $G = (2\alpha_1^2\alpha_2^2+4\alpha_1\alpha_2^3-2\alpha_2^4+3b\alpha_1^2\alpha_2+12b\alpha_1\alpha_2^2-5b\alpha_2^3+8b^2\alpha_1\alpha_2-4b^2\alpha_2^2-2b^3\alpha_1-4b^3\alpha_2-4b^4)$. Moreover, since $\frac{[(2\alpha_2-b)-2\alpha_2 c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)} - \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2+b)^2} = \frac{[(4\alpha_1\alpha_2-4\alpha_2^2-b^2)+2\alpha_2(2\alpha_2+b)c_1-(4\alpha_1\alpha_2+2b\alpha_2-b^2)c_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)(2\alpha_2+b)^2} > 0$, we have $\frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)} > \frac{[(2\alpha_2-b)-2\alpha_2 c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)} > \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2+b)^2}$ for $c_2 \in [c_{c2}^{u11},\ c_{c2}^{u12}]$. Thus, for $c_2 \in [c_{c2}^{u11},\ c_{c2}^{u12}]$, the two-part tariff scheme is optimal.

Third, for $c_2 \in (c_{c2}^{u12},\ \ddot{c}_{c2}]$, one equilibrium unit-fee scheme exists with $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)}$, and four equilibrium fixed-fee schemes may exist with $R_c^f = \frac{[(2\alpha_2-b)-2\alpha_2c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)}$, $R_c^f = \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2+b)^2}$, $R_c^f = \frac{(1-c_1)[(2\alpha_1-\alpha_2)+(\alpha_2+b)c_1-(2\alpha_1+b)c_2]}{(2\alpha_1+b)^2}$, or $R_c^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$.

Define $H_{13} = \frac{(1-c_2)^2}{2(\alpha_2+b)} - \frac{[(2\alpha_2-b)-2\alpha_2c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)}$. Since $\frac{\partial^2 H_{13}}{\partial c_2^2} = \frac{(8\alpha_1\alpha_2^2-5b^2\alpha_2-b^3)}{4\alpha_2(\alpha_2+b)(2\alpha_1\alpha_2-b^2)} > 0$ and $\frac{\partial H_{13}}{\partial c_2} = \frac{-(4\alpha_2^2+3b\alpha_2+b^2)(1-c_1)}{(\alpha_2+b)(4\alpha_1\alpha_2+2b\alpha_2-b^2)} < 0$ at $c_2 = c_{c2}^{f4}$, we have $\frac{\partial H_{13}}{\partial c_2} < 0$ for $c_2 \le c_{c2}^{f4}$, and $H_{13} = \frac{(1-c_2)^2}{2(\alpha_2+b)} - \frac{[(2\alpha_2-b)-2\alpha_2c_1+bc_2]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)} > \frac{(1-c_2^{f4})^2}{2(\alpha_2+b)} - \frac{[(2\alpha_2-b)-2\alpha_2c_1+bc_{c2}^{f4}]^2}{8\alpha_2(2\alpha_1\alpha_2-b^2)} = \frac{1}{(\alpha_2+b)(4\alpha_1\alpha_2+2b\alpha_2-b^2)^2}[2(4\alpha_2^3 + 4b\alpha_2^2 - 2\alpha_1\alpha_2^2 - 2b\alpha_1\alpha_2 + 2b^2\alpha_2 + b^3)(1-c_1)^2] > 0$ for $c_2 \le c_{c2}^{f4}$ with $c_{c2}^{f4} < \ddot{c}_{c2}$. These imply $R_c^u > R_c^f$.

Defining $H_{14} = \frac{(1-c_2)^2}{2(\alpha_2+b)} - \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_2]}{(2\alpha_2+b)^2}$. Since $\frac{\partial^2 H_{14}}{\partial c_2^2} = \frac{1}{(\alpha_2+b)(2\alpha_2+b)^2}[4\alpha_2^2 + 2\alpha_1\alpha_2 + 2b\alpha_1 + 6b\alpha_2 + 3b^2] > 0$ and $\frac{\partial H_{14}}{\partial c_2} = \frac{-(4\alpha_2^2+5b\alpha_2+2b^2)(1-c_1)}{(\alpha_2+b)(2\alpha_2+b)(2\alpha_1+b)} < 0$ at $c_2 = c_{c2}^{f3}$, we have $\frac{\partial H_{14}}{\partial c_2} < 0$ for $c_2 < c_{c2}^{f3}$, and $H_{14} > \frac{(1-c_{x2}^{f3})^2}{2(\alpha_2+b)} - \frac{(1-c_2)[(2\alpha_2-\alpha_1)-(2\alpha_2+b)c_1+(\alpha_1+b)c_{c2}^{f3}]}{(2\alpha_2+b)^2} = \frac{(4\alpha_2^2-2\alpha_1\alpha_2+4b\alpha_2-2b\alpha_1+b^2)(1-c_1)^2}{2(\alpha_2+b)(2\alpha_1+b)^2} > 0$. Moreover, we have $\frac{(1-c_2)^2}{2(\alpha_2+b)} > \frac{1}{(2\alpha_1+b)^2}\{(1-c_1)[(2\alpha_1 - \alpha_2) + (\alpha_2+b)c_1 - (2\alpha_1+b)c_2]\}$ for $c_2 \in (c_1,\ c_{c2}^{f1})$ and $\frac{(1-c_2)^2}{2(\alpha_2+b)} > \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$ for $c_2 \in [c_{c2}^{f1},\ \ddot{c}_{c2}]$ by the outcomes of Case 1. Thus, for $c_2 \in (c_{c2}^{u12},\ \ddot{c}_{c2}]$, the unit-fee scheme is optimal.

Fourth, for $c_2 \in (\ddot{c}_{c2},\ \tilde{c}_{c2}]$, one equilibrium unit-fee scheme exists with $R_c^u = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$, and one equilibrium fixed-fee scheme exists with $R^f = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$. As in Case 1, we have $\frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)} > \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$. These suggest that the unit-fee scheme is optimal.

Fifth, for $c_2 \in (\tilde{c}_{c2},\ \bar{c}_{c2})$, one equilibrium unit-fee scheme exists with $R_c^u = \frac{(c_2-c_1)[(2\alpha_1-b)+bc_1-2\alpha_1c_2]}{(2\alpha_1-b)^2}$, and one equilibrium fixed-fee scheme exists with $R_c^f = \frac{1}{8\alpha_1(2\alpha_1\alpha_2-b^2)}[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2$. As in Case 1, we have $R_c^u = \frac{(c_2-c_1)[(2\alpha_1-b)+bc_1-2\alpha_1c_2]}{(2\alpha_1-b)^2} > \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b^2)}$. Thus, the unit-fee scheme is optimal.

<u>Case 4</u>: Suppose $(2\alpha_2 - \alpha_1) < 0$.

<u>**Comparing the two-part tariff scheme with the unit-fee scheme**</u> :

Lemma 14(iii) shows that $R_c^u = \frac{[(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2]^2}{4(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ for $c_2 \in (c_1,\ c_{c2}^{u11})$ if $(\alpha_1^3+6b\alpha_1\alpha_2-9b\alpha_2^2-7\alpha_1\alpha_2^2+8b^2\alpha_2-6\alpha_1^2\alpha_2-b\alpha_1^2) > 0$, $R_c^u = \frac{[(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2]^2}{4(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ for $c_2 \in (c_1,\ c_{c2}''')$, $R_c^u = \frac{(1-c_1)^2}{2(\alpha_1+b)}$ for $c_2 \in (c_{c2}''',\ c_{c2}^{u11})$ if $(\alpha_1^3+6b\alpha_1\alpha_2-9b\alpha_2^2-7\alpha_1\alpha_2^2+8b^2\alpha_2-6\alpha_1^2\alpha_2 - b\alpha_1^2) < 0$, $R_c^u = \max\{\frac{(1-c_2)^2}{2(2\alpha_2+b)},\ \frac{(1-c_1)^2}{2(2\alpha_1+b)},\ \frac{[(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2]^2}{4(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}\}$ for $c_2 \in [c_{c2}^{u11},\ c_{c2}^{u12}]$, $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)}$ for $c_2 \in (c_{c2}^{u12},\ \dddot{c}_{c2}]$, $R_c^u = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2t]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ for $c_2 \in (\dddot{c}_{c2},\ \tilde{c}_{c2}]$, and $R_c^u = \frac{(c_2-c_1)[(2\alpha_1-b)+bc_1-2\alpha_1c_2]}{(2\alpha_1-b)^2}$ for $c_2 \in (\tilde{c}_{c2},\ \bar{c}_{c2})$.

On the other hand, Lemma 13(iii) shows that $R_c^* = \frac{1}{4(2\alpha_1\alpha_2-b^2)}[(\alpha_1 + 2\alpha_2 - 2b) - 2(2\alpha_2 - b)c_1 - 2(\alpha_1 - b)c_2 - 2bc_1c_2 + 2\alpha_2c_1^2 + \alpha_1c_2^2]$ for $c_2 \in (c_1,\ c_{c2}'']$, $R_c^* = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$ for $c_2 \in (c_{c2}'',\ \dot{c}_{c2})$, $R_c^* = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$ for $c_2 \in [\dot{c}_{c2},\ \ddot{c}_2)$ iff $\dot{c}_{c2} < \ddot{c}_{c2}$, and $R_c^* = 2f_c^* + r_c^*[\frac{2(\alpha_1+\alpha_2-b)(1-r_c^*)-(2\alpha_2-b)c_1-(2\alpha_1-b)c_2}{4\alpha_1\alpha_2-b^2}]$ for $c_2 \in [\ddot{c}_{c2},\ \hat{c}_{c2})$.

By some calculations, we have $c_{c2}^{u10} < c_{c2}^{u11}$ and $c_{c2}^{u11} < c_{c2}^{u12} < \min\{\dot{c}_{c2},\ \ddot{c}_{c2}\} < \max\{\dot{c}_{c2},\ \ddot{c}_{c2}\} < \dddot{c}_{c2} < \hat{c}_{c2} < \tilde{c}_{c2} < \bar{c}_{c2}$ as shown in Case 3. However, relative sizes of c_{c2}'' and c_{c2}^{u12} and of c_{c2}'' and c_{c2}^{u11} are unknown. Thus, we need to discuss the following five intervals.

First, if $c_2 \in (c_1,\ c_{c2}^{u11})$ and $(\alpha_1^3+6b\alpha_1\alpha_2-9b\alpha_2^2-7\alpha_1\alpha_2^2+8b^2\alpha_2-6\alpha_1^2\alpha_2-b\alpha_1^2) > 0$, then we have $c_{c2}^{u11} < c_{c2}''$. Under the circumstance, one equilibrium unit-fee scheme exists with $R_c^u = \frac{[(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2]^2}{4(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$, and one equilibrium two-part tariff scheme exists with $R_c^* = \frac{(\alpha_1+2\alpha_2-2b)-2(2\alpha_2-b)c_1-2(\alpha_1-b)c_2-2bc_1c_2+2\alpha_2c_1^2+\alpha_1c_2^2}{4(2\alpha_1\alpha_2-b^2)}$. Since $(R_c^u - R_c^*) = \frac{-(c_2-c_1)^2}{4(\alpha_1+2\alpha_2-2b)} < 0$, the two-part tariff scheme is better.

By contrast, if $(\alpha_1^3 + 6b\alpha_1\alpha_2 - 9b\alpha_2^2 - 7\alpha_1\alpha_2^2 + 8b^2\alpha_2 - 6\alpha_1^2\alpha_2 - b\alpha_1^2) < 0$, then c_2'' can be less than c_{c2}^{u11}. Two equilibrium unit-fee schemes may exist with $R_c^u = \frac{[(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2]^2}{4(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ or $R_c^u = \frac{(1-c_1)^2}{2(\alpha_1+b)}$ for $c_2 \in (c_{c2}''',\ c_{c2}^{u11})$. On the other hand, two equilibrium two-part tariff schemes may exist with $R_c^* = \frac{1}{4(2\alpha_1\alpha_2-b^2)}[(\alpha_1+2\alpha_2-2b) - 2(2\alpha_2-b)c_1 - 2(\alpha_1-b)c_2 - 2bc_1c_2 + 2\alpha_2c_1^2 + \alpha_1c_2^2]$ or $R_c^* = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$ for $c_2 \in (c_{c2}'',\ \dot{c}_{c2})$ iff $c_{c2}'' < c_{c2}^{u11}$. As in Case 3, we can show $\frac{(1-c_1)^2}{2(\alpha_1+b)} > \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$ for $c_2 < c_{c2}^{u11}$, and

$\frac{(\alpha_1+2\alpha_2-2b)-2(2\alpha_2-b)c_1-2(\alpha_1-b)c_2-2bc_1c_2+2\alpha_2c_1^2+\alpha_1c_2^2}{4(2\alpha_1\alpha_2-b^2)} > \frac{[(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2]^2}{4(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$.

Define $H_{15} = \frac{(1-c_1)^2}{2(\alpha_1+b)} - \frac{(\alpha_1+2\alpha_2-2b)-2(2\alpha_2-b)c_1-2(\alpha_1-b)c_2-2bc_1c_2+2\alpha_2c_1^2+\alpha_1c_2^2}{4(2\alpha_1\alpha_2-b^2)}$. Since $\frac{\partial H_{15}}{\partial c_2} = \frac{(\alpha_1-b)+bc_1-\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)} > \frac{(\alpha_1-b)+bc_1-\alpha_1c_{c2}^{u11}}{2(2\alpha_1\alpha_2-b^2)} = \frac{(\alpha_1^2+2\alpha_1\alpha_2-2b^2)(1-c_1)}{4(\alpha_1+b)(2\alpha_1\alpha_2-b^2)} > 0,\ H_{15} = \frac{-(\alpha_1-b)(\alpha_1-2\alpha_2)(1-c_1)^2}{4(\alpha_1+b)(2\alpha_1\alpha_2-b^2)}$ < 0 at $c_2 = c_1,\ H_{15} = \frac{(\alpha_1-2\alpha_2)(1-c_1)^2}{4(\alpha_1+b)^2} > 0$ at $c_2 = c''_{c2},\ H_{15} = \frac{(6\alpha_1^2\alpha_2-\alpha_1^3-\alpha_1\alpha_2^2-4b^2\alpha_2)(1-c_1)^2}{16(\alpha_1+b)^2(2\alpha_1\alpha_2-b^2)} >$ $(<)$ 0 iff $(6\alpha_1^2\alpha_2 - \alpha_1^3 - \alpha_1\alpha_2^2 - 4b^2\alpha_2) >$ $(<)$ 0 at $c_2 = c_{c2}^{u11}$, and $H_{15} = 0$ at $c_2 = \frac{(\alpha_1^2-b^2)+b(\alpha_1+b)c_1-(1-c_1)\sqrt{(\alpha_1^2-b^2)(2\alpha_1\alpha_2-b^2)}}{\alpha_1(\alpha_1+b)} \equiv \breve{c}_{c2}$, we have $H_{15} < 0$ for $c_2 = c_{c2}^{u11}$ if $(6\alpha_1^2\alpha_2 - \alpha_1^3 - \alpha_1\alpha_2^2 - 4b^2\alpha_2) < 0,\ H_{15} \leq 0$ for $c_2 \leq \breve{c}_{c2}$, and $H_{15} > 0$ for $\breve{c}_{c2} < c_2 < c_{c2}^{u11}$ if $(6\alpha_1^2\alpha_2 - \alpha_1^3 - \alpha_1\alpha_2^2 - 4b^2\alpha_2) > 0$.

Thus, if $(6\alpha_1^2\alpha_2 - \alpha_1^3 - \alpha_1\alpha_2^2 - 4b^2\alpha_2) < 0$, the two-part tariff scheme is better than the unit-fee scheme, and so is when $(6\alpha_1^2\alpha_2 - \alpha_1^3 - \alpha_1\alpha_2^2 - 4b^2\alpha_2) > 0$ and $c_2 \leq \breve{c}_{c2}$. However, the unit-fee scheme will be better if $\breve{c}_{c2} < c_2 < c_{c2}^{u11}$.

Second, for $c_2 \in [c_{c2}^{u11},\ c_{c2}^{u12}]$, one equilibrium unit-fee scheme exists with $R_c^u = \max\{\frac{(1-c_2)^2}{2(2\alpha_2+b)},\ \frac{(1-c_1)^2}{2(2\alpha_1+b)},\ \frac{[(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2]^2}{4(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}\}$, and two equilibrium two-part tariff schemes may exist with $R_c^* = \frac{(\alpha_1+2\alpha_2-2b)-2(2\alpha_2-b)c_1-2(\alpha_1-b)c_2-2bc_1c_2+2\alpha_2c_1^2+\alpha_1c_2^2}{4(2\alpha_1\alpha_2-b^2)}$ or $R_c^* = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$. Since $\frac{(\alpha_1+2\alpha_2-2b)-2(2\alpha_2-b)c_1-2(\alpha_1-b)c_2-2bc_1c_2+2\alpha_2c_1^2+\alpha_1c_2^2}{4(2\alpha_1\alpha_2-b^2)}$ $> \frac{[(\alpha_1+2\alpha_2-2b)-(2\alpha_2-b)c_1-(\alpha_1-b)c_2]^2}{4(\alpha_1+2\alpha_2-2b)(2\alpha_1\alpha_2-b^2)},\ \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)} > \max\{\frac{(1-c_2)^2}{2(2\alpha_2+b)},\ \frac{(1-c_1)^2}{2(2\alpha_1+b)}\}$ for $c_2 \in [c_{c2}^{u11},\ c_{c2}^{u12}]$, and the results derived in Case 3, the two-part tariff scheme is better than the unit-fee scheme.

Third, for $c_2 \in (c_{c2}^{u12},\ \ddot{c}_{c2}]$, one equilibrium unit-fee scheme exists with $R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)}$, and four equilibrium two-part tariff schemes may exist with $R_c^* = \frac{(\alpha_1+2\alpha_2-2b)-2(2\alpha_2-b)c_1-2(\alpha_1-b)c_2-2bc_1c_2+2\alpha_2c_1^2+\alpha_1c_2^2}{4(2\alpha_1\alpha_2-b^2)}$ iff $c''_{c2} > c_{c2}^{u12},\ R_c^* = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)},\ R_c^* = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$ iff $\dot{c}_{c2} < \ddot{c}_{c2}$, and $R_c^* = 2f_c^* + r_c^*[\frac{2(\alpha_1+\alpha_2-b)(1-r_c^*)-(2\alpha_2-b)c_1-(2\alpha_1-b)c_2}{4\alpha_1\alpha_2-b^2}]$. As in Case 3, we can show $\frac{(1-c_2)^2}{2(\alpha_2+b)} > \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)},\ \frac{(1-c_2)^2}{2(\alpha_2+b)} > \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$, and $\frac{(1-c_2)^2}{2(\alpha_2+b)} > 2f_c^* + r_c^*[\frac{2(\alpha_1+\alpha_2-b)(1-r_c^*)-(2\alpha_2-b)c_1-(2\alpha_1-b)c_2}{4\alpha_1\alpha_2-b^2}]$.

Note that $(c''_{c2} - c_{c2}^{u12}) = \frac{(b\alpha_1-2\alpha_1^2-3b\alpha_2)(1-c_1)}{(\alpha_1+b)(\alpha_1+\alpha_2+2b)} > 0$ iff $(b\alpha_1 - 2\alpha_1^2 - 3b\alpha_2) > 0$. Define $H_{16} = \frac{(1-c_2)^2}{2(\alpha_2+b)} - \frac{(\alpha_1+2\alpha_2-2b)-2(2\alpha_2-b)c_1-2(\alpha_1-b)c_2-2bc_1c_2+2\alpha_2c_1^2+\alpha_1c_2^2}{4(2\alpha_1\alpha_2-b^2)}$. Since $\frac{\partial^2 H_{16}}{\partial c_2^2} =$

$\frac{(3\alpha_1\alpha_2-b\alpha_1-2b^2)}{2(\alpha_2+b)(2\alpha_1\alpha_2-b^2)} > 0$ and $\frac{\partial H_{16}}{\partial c_2} = \frac{-(3\alpha_2+b)(1-c_1)}{2(\alpha_2+b)(\alpha_1+b)} < 0$ at $c_2 = c''_{c2}$, we have $\frac{\partial H_{16}}{\partial c_2} < 0$ for $c_2 < c''_{c2}$, and H_{16} has the minimum value $\frac{(6\alpha_2^2+4b\alpha_2-\alpha_1\alpha_2-b\alpha_1)(1-c_1)^2}{4(\alpha_2+b)(\alpha_1+b)^2} < 0$ by $(b\alpha_1 - 2\alpha_1^2 - 3b\alpha_2) > 0$ at $c_2 = c''_{c2}$. However, since the sign of H_{16} at $c_2 = c_{c2}^{u12}$ is uncertain, we have two situations below.

If $(b\alpha_1 - 2\alpha_1^2 - 3b\alpha_2) >, 0$ then the unit-fee scheme is better (worse) than the two-part tariff scheme if $\frac{(1-c_2)^2}{2(\alpha_2+b)}$ is larger (smaller) than $\frac{1}{4(2\alpha_1\alpha_2-b^2)}[(\alpha_1 + 2\alpha_2 - 2b) - 2(2\alpha_2 - b)c_1 - 2(\alpha_1 - b)c_2 - 2bc_1c_2 + 2\alpha_2 c_1^2 + \alpha_1 c_2^2]$ for $c_2 \in (c_{c2}^{u12},\ c''_{c2}]$, and the unit-fee scheme is better for $c_2 \in (c''_2,\ \ddot{c}_{c2}]$. By contrast, if $(b\alpha_1 - 2\alpha_1^2 - 3b\alpha_2) < 0$, then the unit-fee scheme is better than the two-part tariff scheme for $c_2 \in (c_{c2}^{u12},\ \ddot{c}_{c2}]$.

Fourth, for $c_2 \in (c_{c2},\ \tilde{c}_{c2}]$, the unit-fee scheme is better than the two-part tariff scheme with $R_c^u = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b^2)}$ as shown in Case 3.

Fifth, for $c_2 \in (\tilde{c}_{c2},\ \bar{c}_{c2})$, the unit-fee scheme is better than the two-part tariff scheme with $R_c^u = \frac{(c_2-c_1)[(2\alpha_1-b)+bc_1-2\alpha_1 c_2]}{(2\alpha_1-b)^2}$ as shown in Case 3 again.

Comparing the better scheme above with the fixed-fee scheme :

Since the equilibria of Lemma 15(iv) are parts of those of Lemma 15(iii) as shown in Case 3, the better contract obtained in the above steps is optimal for the port authority.

Finally, the outcomes of Cases 1 and 2 prove Proposition 4(i), the results of Case 3 show Proposition 4(ii), and the outcomes of Case 4 verify Proposition 4(iii). □

Lemma 16. *Given fee contract and minimum throughput requirement* $(r_i,\ f_i,\ \delta_i)$ *offered to operator* $i,\ i = 1,\ 2$, *operators' optimal behaviors are as follows.*

(i) *For* $\delta_1 \in [0,\ \dot{\delta}_{d1}]$ *with* $\dot{\delta}_{d1} = \frac{(2-b)-2(r_1+c_1)+b(r_2+c_2)}{4-b^2}$ *and* $\delta_2 \in [0,\ \dot{\delta}_{d2}]$ *with* $\dot{\delta}_{d2} = \frac{(2-b)-2(r_2+c_2)+b(r_1+c_1)}{4-b^2}$, *both operators' equilibrium cargo-handling amounts are* $q_{d1}^* = \dot{\delta}_{d1}$ *and* $q_{d2}^* = \dot{\delta}_{d2}$, *the equilibrium service prices are* $p_{di}^* = c_i + r_i + q_{di}^* > 0$, *and their* *equilibrium profits are* $\pi^*_{di} = (q^*_{di})^2 - f_i$ *for* $i = 1,\ 2$.

(ii) *For* $\delta_1 \in [0,\ \frac{1-b\delta_2-c_1-r_1}{2}]$ *and* $\delta_2 \in (\dot{\delta}_{d2},\ \frac{(2-b)+b(c_1+r_1)}{(2-b^2)})$, *both operators' equilibrium cargo-handling amounts are* $q^*_{d1} = \frac{1-b\delta_2-c_1-r_1}{2}$ *and* $q^*_{d2} = \delta_2$, *the equilibrium prices are* $p^*_{d1} = \frac{(1-b\delta_2+c_1+r_1)}{2} > 0$ *and* $p^*_{d2} = \frac{[(2-b)-(2-b^2)\delta_2+bc_1+br_1]}{2} > 0$, *and their equilibrium profits are* $\pi^*_{d1} = (q^*_{d1})^2 - f_1$ *and* $\pi^*_{d2} = \frac{\delta_2[(2-b)-(2-b^2)\delta_2-2(r_2+c_2)+b(r_1+c_1)]}{2} - f_2$.

(iii) *For* $\delta_1 \in (\dot{\delta}_{d1},\ \frac{(2-b)+b(c_2+r_2)}{(2-b^2)})$ *and* $\delta_2 \in [0,\ \frac{1-b\delta_1-c_2-r_2}{2}]$, *both operators' equilibrium cargo-handling amounts are* $q^*_{d1} = \delta_1$ *and* $q^*_{d2} = \frac{1-b\delta_1-c_2-r_2}{2}$, *the equilibrium prices are* $p^*_{d1} = \frac{[(2-b)-(2-b^2)\delta_1+bc_2+br_2]}{2} > 0$ *and* $p^*_{d2} = \frac{(1-b\delta_1+c_2+r_2)}{2} > 0$, *and their equilibrium profits are* $\pi^*_{d1} = \frac{\delta_1[(2-b)-(2-b^2)\delta_1-2(r_1+c_1)+b(r_2+c_2)]}{2} - f_1$ *and* $\pi^*_{d2} = (q^*_{d2})^2 - f_2$.

(iv) *For* $\delta_1 \in (\frac{(1-b\delta_2-c_1-r_1)}{2},\ 1-b\delta_2)$ *and* $\delta_2 \in (\frac{(1-b\delta_1-c_2-r_2)}{2},\ 1-b\delta_1)$, *both operators' equilibrium cargo-handling amounts are* $q^*_{d1} = \delta_1$ *and* $q^*_{d2} = \delta_2$, *the equilibrium service prices are* $p^*_{d1} = (1-\delta_1-b\delta_2) > 0$ *and* $p^*_{d2} = (1-b\delta_1-\delta_2) > 0$, *and their equilibrium profits are* $\pi^*_{di} = \delta_i[1-\delta_i-b\delta_j-c_i-r_i] - f_i$ *for* $i,\ j \in \{1,\ 2 \mid i \neq j\}$.

Proof of Lemma 16: Denote L_1 and L_2 the Lagrange functions of operators 1 and 2 in problem (46) with $L_1 = (1-q_1-bq_2-c_1-r_1)q_1 - f_1 + \lambda_1(q_1-\delta_1)$ and $L_2 = (1-q_2-bq_1-c_2-r_2)q_2 - f_2 + \lambda_2(q_2-\delta_2)$, where λ_1 and λ_2 are the Lagrange multipliers for the operators. Then, the Kuhn-Tucker conditions for operator 1 are

$$\frac{\partial L_1}{\partial q_1} = 1-2q_1-bq_2-c_1-r_1+\lambda_1 \leq 0,\ q_1 \cdot \frac{\partial L_1}{\partial q_1} = 0, \tag{A160}$$

$$\frac{\partial L_1}{\partial \lambda_1} = q_1-\delta_1 \geq 0,\ \lambda_1 \cdot \frac{\partial L_1}{\partial \lambda_1} = 0, \tag{A161}$$

and for operator 2 are

$$\frac{\partial L_2}{\partial q_2} = 1-2q_2-bq_1-c_2-r_2+\lambda_2 \leq 0,\ q_2 \cdot \frac{\partial L_2}{\partial q_2} = 0,\ \text{and} \tag{A162}$$

$$\frac{\partial L_2}{\partial \lambda_2} = q_2-\delta_2 \geq 0,\ \lambda_2 \cdot \frac{\partial L_2}{\partial \lambda_2} = 0. \tag{A163}$$

Based on the values of λ_1 and λ_2, we have four cases as follows.

Case 1: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then (A160) and (A162) suggest $(1 - 2q_1 - bq_2 - c_1 - r_1) = 0$ and $(1 - 2q_2 - bq_1 - c_2 - r_2) = 0$. Solving these equations yields $q_{d1}^* = \frac{(2-b)-2(r_1+c_1)+b(r_2+c_2)}{4-b^2}$ and $q_{d2}^* = \frac{(2-b)-2(r_2+c_2)+b(r_1+c_1)}{4-b^2}$. To guarantee $q_{d1}^* \geq \delta_1$ and $q_{d2}^* \geq \delta_2$, conditions $0 \leq \delta_1 \leq \dot{\delta}_{d1} \equiv \frac{(2-b)-2(r_1+c_1)+b(r_2+c_2)}{4-b^2}$ and $0 \leq \delta_2 \leq \dot{\delta}_{d2} \equiv \frac{(2-b)-2(r_2+c_2)+b(r_1+c_1)}{4-b^2}$ are imposed. Moreover, we have $q_{d1}^* \geq 0$ iff $r_1 \leq \bar{r}_{d1} \equiv \frac{1}{2}[(2-b) - 2c_1 + b(r_2+c_2)]$, and $q_{d2}^* \geq 0$ iff $r_2 \leq \bar{r}_{d2} \equiv \frac{1}{2}[(2-b) - 2c_2 + b(r_1+c_1)]$. Substituting q_{d1}^* and q_{d2}^* into (1)-(2) yields $p_{di}^* = c_i + r_i + q_{di}^* > 0$, and into (4) yields $\pi_{di}^* = (q_{di}^*)^2 - f_i$ for $i = 1,\ 2$. These prove Lemma 16(i).

Case 2: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then (A160), (A162), and (A163) suggest $(1 - 2q_1 - bq_2 - c_1 - r_1) = 0$, $(1 - 2q_2 - bq_1 - c_2 - r_2 + \lambda_2) = 0$, and $(q_2 - \delta_2) = 0$. Solving these equations yields $q_{d1}^* = \frac{(1-b\delta_2-c_1-r_1)}{2}$, $q_{d2}^* = \delta_2$, and $\lambda_2^* = \frac{(4-b^2)(\delta_2-\dot{\delta}_{d2})}{2}$. To have $\lambda_2^* > 0$, conditions $\delta_2 > \dot{\delta}_{d2}$ and $r \leq \bar{r}_{d2}$ are needed. On the other hand, to have $q_{d1}^* \geq \delta_1$, condition $\delta_1 \leq \frac{(1-b\delta_2-c_1-r_1)}{2}$ is needed. Moreover, we have $q_{d1}^* \geq 0$ iff $r_1 \leq (1-b\delta_2-c_1)$. Substituting q_{d1}^* and q_{d2}^* into (1)-(2) produces $p_{d1}^* = \frac{(1-b\delta_2+c_1+r_1)}{2} > 0$ and $p_{d2}^* = \frac{[(2-b)-(2-b^2)\delta_2+bc_1+br_1]}{2} > 0$ iff $\delta_2 < \frac{(2-b)+b(c_1+r_1)}{(2-b^2)}$, and into (4) gives $\pi_{d1}^* = (q_{d1}^*)^2 - f_1$ and $\pi_{d2}^* = \frac{\delta_2[(2-b)-(2-b^2)\delta_2-2(r_2+c_2)+b(r_1+c_1)]}{2} - f_2$. Thus, the plausible range for δ_1 is $\delta_1 \in [0,\ \frac{1-b\delta_2-c_1-r_1}{2}]$, and for δ_2 is $\delta_2 \in (\dot{\delta}_{d2},\ \frac{(2-b)+b(c_1+r_1)}{(2-b^2)})$. These prove Lemma 16(ii).

Case 3: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then (A160)-(A162) suggest $(q_1 - \delta_1) = 0$, $(1 - 2q_1 - bq_2 - c_1 - r_1 + \lambda_1) = 0$, and $(1 - 2q_2 - bq_1 - c_2 - r_2) = 0$. Solving these equations yields $q_{d1}^* = \delta_1$, $q_{d2}^* = \frac{(1-b\delta_1-c_2-r_2)}{2}$, and $\lambda_1^* = \frac{1}{2}(4-b^2)(\delta_1 - \dot{\delta}_{d1})$. To guarantee $\lambda_1^* > 0$, conditions $\delta_1 > \dot{\delta}_{d1}$ and $r \leq \bar{r}_{d1}$ are needed. On the other hand, to have $q_{d2}^* \geq \delta_2$, condition $\delta_2 \leq \frac{(1-b\delta_1-c_2-r_2)}{2}$ is needed. Moreover, we have $q_{d2}^* \geq 0$ iff $r_2 \leq (1 - b\delta_1 - c_2)$. Substituting q_{d1}^* and q_{d2}^* into (1)-(2) produces $p_{d2}^* = \frac{(1-b\delta_1+c_2+r_2)}{2} > 0$ and $p_{d1}^* = \frac{[(2-b)-(2-b^2)\delta_1+bc_2+br_2]}{2} > 0$ iff $\delta_1 < \frac{(2-b)+b(c_2+r_2)}{(2-b^2)}$, and into (4) gives $\pi_{d2}^* = (q_{d2}^*)^2 - f_2$ and $\pi_{d1}^* = \frac{\delta_1[(2-b)-(2-b^2)\delta_1-2(r_1+c_1)+b(r_2+c_2)]}{2} - f_1$. Thus, the plausible range for δ_2 is $\delta_2 \in [0,\ \frac{(1-b\delta_1-c_2-r_2)}{2}]$, and for δ_1 is $\delta_1 \in (\dot{\delta}_{d1},\ \frac{(2-b)+b(c_2+r_2)}{(2-b^2)})$. These prove Lemma

16(iii).

Case 4: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. Then (A160)-(A163) suggest $q_{d1}^* = \delta_1$, $q_{d2}^* = \delta_2$, $\lambda_1^* = -1 + 2\delta_1 + b\delta_2 + c_1 + r_1$, and $\lambda_2^* = -1 + 2\delta_2 + b\delta_1 + c_2 + r_2$. To have $\lambda_1^* > 0$ and $\lambda_2^* > 0$, conditions $\delta_1 > \frac{(1-b\delta_2-c_1-r_1)}{2}$ and $\delta_2 > \frac{(1-b\delta_1-c_2-r_2)}{2}$ are needed. Substituting $q_{d1}^* = \delta_1$ and $q_{d2}^* = \delta_2$ into (1)-(2) yields $p_{d1}^* = (1 - \delta_1 - b\delta_2) > 0$ iff $\delta_1 < (1 - b\delta_2)$ and $p_{d2}^* = (1-b\delta_1-\delta_2) > 0$ iff $\delta_2 < (1-b\delta_1)$, and into (4) gives $\pi_{di}^* = \delta_i[1-\delta_i-b\delta_j-c_i-r_i]-f_i$ for $i,\ j \in \{1,\ 2|\ i \neq j\}$. These prove Lemma 16(iv). □

Proof of Proposition 5: Suppose $\delta_1 \in [0,\ \dot{\delta}_{d1}]$ with $\dot{\delta}_{d1} = \frac{(2-b)-2(r_1+c_1)+b(r_2+c_2)}{4-b^2}$ and $\delta_2 \in [0,\ \dot{\delta}_{d2}]$ with $\dot{\delta}_{d2} = \frac{(2-b)-2(r_2+c_2)+b(r_1+c_1)}{4-b^2}$. Lemma 16(i) implies $f_{d1}^* = \pi_{d1}^* = \frac{1}{2}(q_{d1}^*)^2 \geq 0$ and $f_{d2}^* = \pi_{d2}^* = \frac{1}{2}(q_{d2}^*)^2 \geq 0$. Thus, the problem in (49) becomes

$$\max_{r_1,\, f_1,\, \delta_1,\, r_2,\, f_2,\, \delta_2} \quad f_1 + f_2 + r_1 q_{d1}^* + r_2 q_{d2}^*$$
$$\text{s.t.} \quad 0 \leq \delta_1 \leq \dot{\delta}_{d1},\ 0 \leq \delta_2 \leq \dot{\delta}_{d2},\ 0 \leq r_1 \leq \bar{r}_{d1},\ \text{and } 0 \leq r_2 \leq \bar{r}_{d2}.$$

Its Lagrange function is

$$L = \frac{1}{2}(q_{d1}^*)^2 + \frac{1}{2}(q_{d2}^*)^2 + r_1 \cdot q_{d1}^* + r_2 \cdot q_{d2}^* + \lambda_1(\dot{\delta}_{d1} - \delta_1) + \lambda_2(\dot{\delta}_{d2} - \delta_2) + \lambda_3(\bar{r}_{d1} - r_1) + \lambda_4(\bar{r}_{d2} - r_2),$$

where $\lambda_1,\ \lambda_2,\ \lambda_3$, and λ_4 are the Lagrange multipliers for the four inequality constraints. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r_1} = \frac{1}{(4-b^2)^2}[(1+b)(2-b)^2 - (4-3b^2)c_1 - b^3 c_2 - (12-5b^2)r_1 + 2b(2-b^2)r_2]$$
$$-\frac{2\lambda_1}{(4-b^2)} + \frac{b\lambda_2}{(4-b^2)} - \lambda_3 + \frac{b\lambda_4}{2} \leq 0,\ r_1 \cdot \frac{\partial L}{\partial r_1} = 0, \quad (A164)$$
$$\frac{\partial L}{\partial r_2} = \frac{1}{(4-b^2)^2}[(1+b)(2-b)^2 - b^3 c_1 - (4-3b^2)c_2 + 2b(2-b^2)r_1 - (12-5b^2)r_2]$$
$$+\frac{b\lambda_1}{(4-b^2)} - \frac{2\lambda_2}{(4-b^2)} + \frac{b\lambda_3}{2} - \lambda_4 \leq 0,\ r_2 \cdot \frac{\partial L}{\partial r_2} = 0, \quad (A165)$$
$$\frac{\partial L}{\partial \delta_1} = -\lambda_1 \leq 0,\ \delta_1 \cdot \frac{\partial L}{\partial \delta_1} = 0, \quad (A166)$$
$$\frac{\partial L}{\partial \delta_2} = -\lambda_2 \leq 0,\ \delta_2 \cdot \frac{\partial L}{\partial \delta_2} = 0, \quad (A167)$$

$$\frac{\partial L}{\partial \lambda_1} = \dot{\delta}_{d1} - \delta_1 \geq 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A168)$$

$$\frac{\partial L}{\partial \lambda_2} = \dot{\delta}_{d2} - \delta_2 \geq 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad (A169)$$

$$\frac{\partial L}{\partial \lambda_3} = \bar{r}_{d1} - r_1 \geq 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \quad (A170)$$

$$\frac{\partial L}{\partial \lambda_4} = \bar{r}_{d2} - r_2 \geq 0,\ \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0. \quad (A171)$$

If $\lambda_1^* > 0$, then $\frac{\partial L}{\partial \delta_1} = -\lambda_1 < 0$ and $\delta_1^* = 0$ by (A166). They in turn suggest $\frac{\partial L}{\partial \lambda_1} = \dot{\delta}_{d1} > 0$ and $\lambda_1^* = 0$ by (A168). It is a contradiction. Thus, we must have $\lambda_1^* = 0$. Similarly, we have $\lambda_2^* = 0$ by (A167). Then, based on the values of λ_3 and λ_4, there are four sub-cases as follows.

<u>Case 1a</u>: Suppose $\lambda_3^* = 0$ and $\lambda_4^* = 0$. Then (A164) and (A165) suggest

$$\tfrac{1}{(4-b^2)^2}[(1+b)(2-b)^2 - (4-3b^2)c_1 - b^3c_2 - (12-5b^2)r_1 + 2b(2-b^2)r_2] = 0, \text{ and}$$

$$\tfrac{1}{(4-b^2)^2}[(1+b)(2-b)^2 - b^3c_1 - (4-3b^2)c_2 + 2b(2-b^2)r_1 - (12-5b^2)r_2] = 0.$$

Solving these equations yields $r_{d1}^* = \frac{(1+b)(3-2b)-(3-2b^2)c_1-bc_2}{9-4b^2} > 0$ and $r_{d2}^* = \frac{1}{9-4b^2}[(1+b)(3-2b) - bc_1 - (3-2b^2)c_2] > 0$. It remains to check whether $r_{d1}^* < \bar{r}_{d1}$ and $r_{d2}^* < \bar{r}_{d2}$ hold. By some calculations, we have $(\bar{r}_{d1} - r_{d1}^*) = \frac{(4-b^2)[(3-2b)-3c_1+2bc_2]}{2(9-4b^2)} > \frac{(4-b^2)(3-2b)(1-c_2)}{2(9-4b^2)} > 0$, and $(\bar{r}_{d2} - r_{d2}^*) = \frac{(4-b^2)[(3-2b)+2bc_1-3c_2]}{2(9-4b^2)} \geq 0$ iff $c_2 \leq \hat{c}_{d2} \equiv \frac{(3-2b)+2bc_1}{3}$. In addition, (A168) and (A169) imply $\delta_{d1}^* \in [0,\ \dot{\delta}_{d1}]$, $\delta_{d2}^* \in [0,\ \dot{\delta}_{d2}]$, $f_{d1}^* = \frac{1}{2}[\frac{(3-2b)-3c_1+2bc_2}{(9-4b^2)}]^2 > 0$, and $f_{d2}^* = \frac{1}{2}[\frac{(3-2b)+2bc_1-3c_2}{(9-4b^2)}]^2 > 0$ iff $c_2 \leq \hat{c}_{d2}$. Thus, for $c_2 \leq \hat{c}_{d2}$, port authority's equilibrium fee revenue equals

$$R_d^* = \frac{2(3-2b)(1-c_1-c_2) - 4bc_1c_2 + 3(c_1^2+c_2^2)}{2(9-4b^2)} \equiv R_1^*. \quad (A172)$$

<u>Case 1b</u>: Suppose $\lambda_3^* = 0$ and $\lambda_4^* > 0$. Then (A164), (A165), and (A171) suggest

$$\tfrac{1}{(4-b^2)^2}[(1+b)(2-b)^2 - (4-3b^2)c_1 - b^3c_2 - (12-5b^2)r_1 + 2b(2-b^2)r_2] + \tfrac{b}{2}\lambda_4^* = 0,$$

$$\tfrac{1}{(4-b^2)^2}[(1+b)(2-b)^2 - b^3c_1 - (4-3b^2)c_2 + 2b(2-b^2)r_1 - (12-5b^2)r_2] - \lambda_4^* = 0, \text{ and}$$

$$r_2 - \tfrac{1}{2}[(2-b) - 2c_2 + b(r_1+c_1)] = 0.$$

Solving these equations yields $r^*_{d1} = \frac{1-c_1}{3} > 0,\ r^*_{d2} = \frac{(3-b)+bc_1-3c_2}{3}$, and $\lambda^*_4 = \frac{2[-(3-2b)-2bc_1+3c_2]}{3(4-b^2)}$. Note that $r^*_{d2} \geq 0$ iff $c_2 \leq \frac{(3-b)+bc_1}{3},\ (\bar{r}_{d1} - r^*_{d1}) = \frac{(4-b^2)(1-c_1)}{6} > 0,\ \lambda^*_4 > 0$ iff $c_2 > \hat{c}_{d2}$ with $\frac{(3-b)+bc_1}{3} - \hat{c}_{d2} = \frac{2b(3-b^2)(1-c_1)}{3(6-b^2)} > 0,\ f^*_{d2} = 0$, and $\delta^*_{d2} = 0$ due to $\dot{\delta}_{d2} = \frac{(2-b)-2(r^*_{d2}+c_2)+b(r^*_{d1}+c_1)}{4-b^2} = 0$. In addition, (A168) implies $\delta^*_{d1} \in [0,\ \dot{\delta}_{d1}]$ with $\dot{\delta}_{d1} = \frac{1-c_1}{3}$ and $f^*_{d1} = \frac{(1-c_1)^2}{18} > 0$. Thus, for $\hat{c}_{d2} < c_2 \leq \frac{(3-b)+bc_1}{3}$, port authority's equilibrium fee revenue equals

$$R^*_d = f^*_{d1} + r^*_{d1} \cdot q^*_{d1} = \frac{(1-c_1)^2}{6} \equiv R^*_2. \qquad (A173)$$

Case 1c: Suppose $\lambda^*_3 > 0$ and $\lambda^*_4 = 0$. Then, (A164), (A165), and (A170) suggest$r^*_{d1} = \bar{r}_{d1} = \frac{(3-b)-3c_1+bc_2}{3} > 0,\ r^*_{d2} = \frac{1-c_2}{3} > 0$, and $\lambda^*_3 = \frac{2[-(3-2b)+3c_1-2bc_2]}{3(4-b^2)} < 0$. It is a contradiction. Thus, no solution exists in this case.

Case 1d: Suppose $\lambda^*_3 > 0$ and $\lambda^*_4 > 0$. Then, (A164), (A165), (A170), and (A171) suggest $r^*_{d1} = \bar{r}_{d1} = 1-c_1,\ r^*_{d2} = \bar{r}_{d2} = 1-c_2,\ \lambda^*_3 = \frac{-2(1-c_1)}{(4-b^2)} < 0$, and $\lambda^*_4 = \frac{-2(1-c_2)}{(4-b^2)} < 0$. However, $\lambda^*_3 < 0$ is a contradiction. Thus, no solution exists in this case.

Case 2: Suppose $\delta_1 \in [0,\ \frac{1-b\delta_2-c_1-r_1}{2}]$ and $\delta_2 \in (\dot{\delta}_{d2},\ \frac{(2-b)+b(c_1+r_1)}{(2-b^2)})$. Then, Lemma 16(ii) implies $\pi^*_{d1} = (\frac{1-b\delta_2-c_1-r_1}{2})^2 - f^*_{d1}$ and $\pi^*_{d2} = \frac{\delta_2}{2}[(2-b)-(2-b^2)\delta_2-2(r_2+c_2)+b(r_1+c_1)] - f^*_{d2}$ with $f^*_{d1} = \frac{1}{2}(\frac{1-b\delta_2-c_1-r_1}{2})^2$ and $f^*_{d2} = \frac{\delta_2}{4}[(2-b)-(2-b^2)\delta_2-2(r_2+c_2)+b(r_1+c_1)]$. We have $f^*_{d1} \geq 0$ iff $r_1 \leq (1-b\delta_2-c_1)$ with $(1-b\delta_2-c_1) < \bar{r}_{d1},\ f^*_{d2} \geq 0$ iff $\delta_2 \leq \frac{(2-b)-2(r_2+c_2)+b(r_1+c_1)}{(2-b^2)} \equiv \tilde{\delta}_{d2}$ and $r_2 \leq \bar{r}_{d2}$. In addition, we have $\frac{(2-b)-2(r_2+c_2)+b(r_1+c_1)}{(2-b^2)} < \frac{(2-b)+b(c_1+r_1)}{(2-b^2)}$. Thus, the problem in (49) becomes

$$\max_{r_1,\, f_1,\, \delta_1,\, r_2,\, f_2,\, \delta_2} \quad f^*_{d1} + f^*_{d2} + r_1 q^*_{d1} + r_2 q^*_{d2}$$

$$\text{s.t. } 0 \leq \delta_1 \leq \frac{1-b\delta_2-c_1-r_1}{2},\ \dot{\delta}_{d2} < \delta_2 \leq \tilde{\delta}_{d2},\ 0 \leq r_1 \leq (1-b\delta_2-c_1),\ \text{and } r_2 \leq \bar{r}_{d2}. \qquad (A174)$$

Its Lagrange function is

$$\begin{aligned} L &= \frac{r_1(1-b\delta_2-c_1-r_1)}{2}+r_2\delta_2+\frac{1}{2}(\frac{1-b\delta_2-c_1-r_1}{2})^2+\frac{\delta_2}{4}[(2-b)-(2-b^2)\delta_2- \\ & 2(r_2+c_2)+b(r_1+c_1)]+\lambda_1(\frac{1-b\delta_2-c_1-r_1}{2}-\delta_1)+\lambda_2(\delta_2-\dot{\delta}_{d2}) \\ & +\lambda_3(\tilde{\delta}_{d2}-\delta_2)+\lambda_4[(1-b\delta_2-c_1)-r_1]+\lambda_5(\bar{r}_{d2}-r_2). \end{aligned}$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r_1}=\frac{(1-3r_1-c_1)}{4}-\frac{\lambda_1}{2}-\frac{b\lambda_2}{(4-b^2)}+\frac{b\lambda_3}{(2-b^2)}-\lambda_4+\frac{b\lambda_5}{2}\leq 0,\ r_1\cdot\frac{\partial L}{\partial r_1}=0, \quad (A175)$$

$$\frac{\partial L}{\partial r_2}=\frac{\delta_2}{2}+\frac{2\lambda_2}{(4-b^2)}-\frac{2\lambda_3}{(2-b^2)}-\lambda_5\leq 0,\ r_2\cdot\frac{\partial L}{\partial r_2}=0, \quad (A176)$$

$$\frac{\partial L}{\partial \delta_1}=-\lambda_1\leq 0,\ \delta_1\cdot\frac{\partial L}{\partial \delta_1}=0, \quad (A177)$$

$$\frac{\partial L}{\partial \delta_2}=\frac{1}{4}[2(1-b)+2bc_1-2c_2+2r_2-(4-3b^2)\delta_2]-\frac{b}{2}\lambda_1+\lambda_2-\lambda_3-b\lambda_4\leq 0,\ \delta_2\frac{\partial L}{\partial \delta_2}=0, \quad (A178)$$

$$\frac{\partial L}{\partial \lambda_1}=\frac{1-b\delta_2-c_1-r_1}{2}-\delta_1\geq 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1}=0, \quad (A179)$$

$$\frac{\partial L}{\partial \lambda_2}=\delta_2-\dot{\delta}_{d2}\geq 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2}=0, \quad (A180)$$

$$\frac{\partial L}{\partial \lambda_3}=\tilde{\delta}_{d2}-\delta_2\geq 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3}=0, \quad (A181)$$

$$\frac{\partial L}{\partial \lambda_4}=(1-b\delta_2-c_1)-r_1\geq 0,\ \lambda_4\cdot\frac{\partial L}{\partial \lambda_4}=0,\ \text{and} \quad (A182)$$

$$\frac{\partial L}{\partial \lambda_5}=\bar{r}_{d2}-r_2\geq 0,\ \lambda_5\cdot\frac{\partial L}{\partial \lambda_5}=0. \quad (A183)$$

Constraint $\delta_2 > \dot{\delta}_{d2}$ in (A174) suggests $\lambda_2^* = 0$ by (A180). If $\lambda_1^* > 0$, then $\frac{\partial L}{\partial \delta_1} = -\lambda_1 < 0$ and $\delta_{d1}^* = 0$ by (A177). They in turn suggest $\frac{\partial L}{\partial \lambda_1} = \frac{1-b\delta_2-c_1-r_1}{2} > 0$ and $\lambda_1^* = 0$ by (A179). This is a contradiction. Thus, we must have $\lambda_1^* = 0$. Similarly, if $\lambda_5^* > 0$, we have $r_{d2}^* = \bar{r}_{d2}$ by (A183) and $\dot{\delta}_{d2} = \tilde{\delta}_{d2} = 0$, which contradicts $\dot{\delta}_{d2} < \delta_2 \leq \tilde{\delta}_{d2}$. Thus, we must have $\lambda_5^* = 0$. Moreover, if $\lambda_1^* = \lambda_5^* = 0$, (A176) becomes $\frac{\partial L}{\partial r_2} = \frac{\delta_2}{2} - \frac{2\lambda_3}{(2-b^2)} \leq 0$. Thus, we must have $\lambda_3^* > 0$.

Based on the values of λ_4, there are two sub-cases as follows.

Case 2a: Suppose $\lambda_4^* = 0$. Then (A175), (A176), (A178), and (A181) suggest $[\frac{(1-3r_1-c_1)}{4} + \frac{b\lambda_3}{(2-b^2)}] = 0$, $[\frac{\delta_2}{2} - \frac{2\lambda_3}{(2-b^2)}] = 0$, $\frac{1}{4}[2(1-b) + 2bc_1 - 2c_2 + 2r_2 - (4-3b^2)\delta_2] - \lambda_3 = 0$, and $\delta_2 = \tilde{\delta}_{d2} = \frac{(2-b)-2(r_2+c_2)+b(r_1+c_1)}{(2-b^2)}$. Solving these equations yields $r_{d1}^* = \frac{(2+b-2b^2)-2(1-b^2)c_1-bc_2}{2(3-2b^2)} > 0$, $r_{d2}^* = \frac{1-c_2}{2} > 0$, $\delta_{d2}^* = \frac{(3-2b)+2bc_1-3c_2}{2(3-2b^2)}$, and $\lambda_3^* = \frac{(2-b^2)[(3-2b)+2bc_1-3c_2]}{8(3-2b^2)}$. Note that $f_{d2}^* = 0$ due to $\delta_{d2}^* = \frac{(3-2b)+2bc_1-3c_2}{2(3-2b^2)}$, $(1-b\delta_{d2}^*-c_1)-r_{d1}^* = \frac{2[(1-b)-c_1+bc_2]}{(3-2b^2)} > 0$, $(\bar{r}_{d2} - r_{d2}^*) = \frac{(2-b^2)[(3-2b)+2bc_1-3c_2]}{4(3-2b^2)} \geq 0$ iff $c_2 \leq \hat{c}_{d2} \equiv \frac{(3-2b)+2bc_1}{3}$, and $\lambda_3^* > 0$ iff $c_2 < \hat{c}_{d2}$. In addition, (A179) implies $\delta_{d1}^* \in [0, \frac{(1-b)-c_1+bc_2}{(3-2b^2)}]$ and $f_{d1}^* = \frac{1}{2}[\frac{(1-b)-c_1+bc_2}{(3-2b^2)}]^2 > 0$. Thus, for $c_2 < \hat{c}_{d2}$, port authority's equilibrium fee revenue equals

$$R_d^* = \frac{(5-4b)-4(1-b)c_1-2(3-2b)c_2-4bc_1c_2+2c_1^2+3c_2^2}{4(3-2b^2)} \equiv R_3^*. \qquad (A184)$$

Case 2b: Suppose $\lambda_4^* > 0$. Then (A181) and (A182) suggest $\delta_{d2}^* = \tilde{\delta}_{d2}$ and $r_{d1}^* = (1-b\delta_{d2}^*-c_1) > 0$. Substituting δ_{d2}^* and r_{d1}^* into equations $[\frac{(1-3r_1-c_1)}{4} + \frac{b\lambda_3}{(2-b^2)} - \lambda_4] = 0$, $[\frac{\delta_2}{2} - \frac{2\lambda_3}{(2-b^2)}] = 0$ and $\frac{1}{4}[2(1-b)+2bc_1-2c_2+2r_2-(4-3b^2)\delta_2] - \lambda_3 - b\lambda_4 = 0$ yields $\lambda_4^* = \frac{1}{2}[-(1-b)+c_1-bc_2] < 0$, which contradicts $\lambda_4^* > 0$. Thus, no solution exists in this case.

Case 3: Suppose $\delta_1 \in (\dot{\delta}_{d1}, \frac{(2-b)+b(c_2+r_2)}{(2-b^2)})$ and $\delta_2 \in [0, \frac{1-b\delta_1-c_2-r_2}{2}]$. Then, Lemma 16(iii) implies $\pi_{d1}^* = \frac{\delta_1[(2-b)-(2-b^2)\delta_1-2(r_1+c_1)+b(r_2+c_2)]}{2} - f_{d1}^*$ and $\pi_{d2}^* = (q_{d2}^*)^2 - f_{d2}^*$ with $f_{d1}^* = \frac{\delta_1}{4}[(2-b)-(2-b^2)\delta_1-2(r_1+c_1)+b(r_2+c_2)]$ and $f_{d2}^* = \frac{1}{2}[\frac{1-b\delta_1-c_2-r_2}{2}]^2$. We have $f_{d1}^* \geq 0$ iff $\delta_1 \leq \frac{(2-b)-2(r_1+c_1)+b(r_2+c_2)}{(2-b^2)} \equiv \tilde{\delta}_{d1}$, and $r_1 \leq \bar{r}_{d1}$ and $f_{d2}^* \geq 0$ iff $r_2 \leq (1-b\delta_1-c_2)$ with $(1-b\delta_1-c_2) < \bar{r}_{d2}$. In addition, we have $\frac{(2-b)-2(r_1+c_1)+b(r_2+c_2)}{(2-b^2)} < \frac{(2-b)+b(c_2+r_2)}{(2-b^2)}$. Thus, the problem in (49) becomes

$$\max_{r_1,\, f_1,\, \delta_1,\, r_2,\, f_2,\, \delta_2} \quad f_{d1}^* + f_{d2}^* + r_1 q_{d1}^* + r_2 q_{d2}^*$$

$$\text{s.t. } 0 \leq \delta_2 \leq \frac{1-b\delta_1-c_2-r_2}{2},\ \dot{\delta}_{d1} < \delta_1 \leq \tilde{\delta}_{d1},\ r_2 \leq (1-b\delta_1-c_2),\ \text{and } r_1 \leq \bar{r}_{d1}. \qquad (A185)$$

Its Lagrange function is

$$
\begin{aligned}
L \quad = \quad & \frac{r_2}{2}(1-b\delta_1-c_2-r_2)+r_1\delta_1+\frac{1}{2}[\frac{1-b\delta_1-c_2-r_2}{2}]^2+\frac{\delta_1}{4}[(2-b)-(2-b^2)\delta_1 \\
& -2(r_1+c_1)+b(r_2+c_2)]+\lambda_1(\frac{1-b\delta_1-c_2-r_2}{2}-\delta_2)+\lambda_2(\delta_1-\dot{\delta}_{d1}) \\
& +\lambda_3(\tilde{\delta}_{d1}-\delta_1)+\lambda_4[(1-b\delta_1-c_2)-r_2]+\lambda_5(\bar{r}_{d1}-r_1).
\end{aligned}
$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r_1}=\frac{\delta_1}{2}+\frac{2\lambda_2}{(4-b^2)}-\frac{2\lambda_3}{(2-b^2)}-\lambda_5\leq 0,\ r_1\cdot\frac{\partial L}{\partial r_1}=0, \qquad (A186)$$

$$\frac{\partial L}{\partial r_2}=\frac{(1-3r_2-c_2)}{4}-\frac{\lambda_1}{2}-\frac{b\lambda_2}{(4-b^2)}+\frac{b\lambda_3}{(2-b^2)}-\lambda_4+\frac{b\lambda_5}{2}\leq 0,\ r_2\cdot\frac{\partial L}{\partial r_2}=0, \quad (A187)$$

$$\frac{\partial L}{\partial \delta_1}=\frac{1}{4}[2(1-b)+2bc_2-2c_1+2r_1-(4-3b^2)\delta_1]-\frac{b\lambda_1}{2}+\lambda_2-\lambda_3-b\lambda_4\leq 0,\ \delta_1\frac{\partial L}{\partial \delta_1}=0, \qquad (A188)$$

$$\frac{\partial L}{\partial \delta_2}=-\lambda_1\leq 0,\ \delta_2\cdot\frac{\partial L}{\partial \delta_2}=0, \qquad (A189)$$

$$\frac{\partial L}{\partial \lambda_1}=\frac{1-b\delta_1-c_2-r_2}{2}-\delta_2\geq 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1}=0, \qquad (A190)$$

$$\frac{\partial L}{\partial \lambda_2}=\delta_1-\dot{\delta}_{d1}\geq 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2}=0, \qquad (A191)$$

$$\frac{\partial L}{\partial \lambda_3}=\tilde{\delta}_{d1}-\delta_1\geq 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3}=0, \qquad (A192)$$

$$\frac{\partial L}{\partial \lambda_4}=(1-b\delta_1-c_2)-r_2\geq 0,\ \lambda_4\cdot\frac{\partial L}{\partial \lambda_4}=0,\ \text{and} \qquad (A193)$$

$$\frac{\partial L}{\partial \lambda_5}=\bar{r}_{d1}-r_1\geq 0,\ \lambda_5\cdot\frac{\partial L}{\partial \lambda_5}=0. \qquad (A194)$$

Constraint $\delta_1 > \dot{\delta}_{d1}$ in (A185) suggests $\lambda_2^* = 0$ by (A191). If $\lambda_1^* > 0$, then $\frac{\partial L}{\partial \delta_2} = -\lambda_1 < 0$ and $\delta_{d1}^* = 0$ by (A189). They in turn suggest $\frac{\partial L}{\partial \lambda_1} = \frac{1-b\delta_1-c_2-r_2}{2} > 0$ and $\lambda_1^* = 0$ by (A190). It is a contradiction. Thus, we must have $\lambda_1^* = 0$. On the other hand, if $\lambda_5^* > 0$, we have $r_{d1}^* = \bar{r}_{d1}$ by (A194) and $\dot{\delta}_{d1} = \tilde{\delta}_{d1} = 0$, which contradicts $\dot{\delta}_{d1} < \delta_2 \leq \tilde{\delta}_{d1}$. Thus, we must have $\lambda_5^* = 0$. Moreover, if $\lambda_2^* = \lambda_5^* = 0$, (A186) becomes $\frac{\partial L}{\partial r_1} = \frac{\delta_1}{2} - \frac{2\lambda_3}{(2-b^2)} \leq 0$, which implies $\lambda_3^* > 0$.

Based on the values of λ_4, we have two sub-cases as follows.

Case 3a: Suppose $\lambda_4^* = 0$. Then (A186)-(A188) and (A192) suggest $[\frac{(1-3r_2-c_2)}{4} + \frac{b\lambda_3}{(2-b^2)}] = 0$, $[\frac{\delta_1}{2} - \frac{2\lambda_3}{(2-b^2)}] = 0$, $\frac{1}{4}[2(1-b)+2bc_2-2c_1+2r_1-(4-3b^2)\delta_1]-\lambda_3 = 0$, and $\delta_1 = \tilde{\delta}_{d1} = \frac{(2-b)-2(r_1+c_1)+b(r_2+c_2)}{(2-b^2)}$. Solving these equations yields $r_{d2}^* = \frac{(2+b-2b^2)-2(1-b^2)c_2-bc_1}{2(3-2b^2)} > 0$, $r_{d1}^* = \frac{1-c_1}{2} > 0$, $\delta_{d1}^* = \frac{(3-2b)+2bc_2-3c_1}{2(3-2b^2)} > 0$, and $\lambda_3^* = \frac{(2-b^2)[(3-2b)+2bc_2-3c_1]}{8(3-2b^2)} > 0$. Note that $f_{d1}^* = 0$ by $\delta_{d1}^* = \frac{(3-2b)+2bc_2-3c_1}{2(3-2b^2)}$, $(1-b\delta_{d1}^*-c_2)-r_{d2}^* = \frac{2[(1-b)-c_2+bc_1]}{(3-2b^2)} \geq 0$ iff $c_2 \leq (1-b)+bc_1$, and $(\bar{r}_{d1} - r_{d1}^*) = \frac{(2-b^2)[(3-2b)+2bc_2-3c_1]}{4(3-2b^2)} > 0$. Moreover, (A190) implies $\delta_{d2}^* \in [0, \frac{(1-b)-c_2+bc_1}{(3-2b^2)}]$ and $f_{d2}^* = \frac{1}{2}[\frac{(1-b)-c_2+bc_1}{(3-2b^2)}]^2 > 0$ iff $c_2 \leq (1-b+bc_1)$. Thus, for $c_2 \leq (1-b)+bc_1$, port authority's equilibrium fee revenue equals

$$R_d^* = \frac{(5-4b)-4(1-b)c_2-2(3-2b)c_1-4bc_1c_2+3c_1^2+2c_2^2}{4(3-2b^2)} \equiv R_4^*. \qquad (A195)$$

Case 3b-1: Suppose $\lambda_4^* > 0$, $r_{d1}^* > 0$, and $r_{d2}^* > 0$. Then (A186)-(A188) and (A192)-(A193) suggest $[\frac{1}{4}(1-3r_2-c_2)+\frac{b\lambda_3}{(2-b^2)}-\lambda_4] = 0$, $[\frac{\delta_1}{2} - \frac{2\lambda_3}{(2-b^2)}] = 0$, $\frac{1}{4}[2(1-b)+2bc_2-2c_1+2r_1-(4-3b^2)\delta_1]-\lambda_3-b\lambda_4 = 0$, $\delta_1 = \tilde{\delta}_{d1} = \frac{(2-b)-2(r_1+c_1)+b(r_2+c_2)}{(2-b^2)}$, and $r_2 = (1-b\delta_1-c_2)$. Solving these equations yields $r_{d1}^* = \frac{1-c_1}{2} > 0$, $r_{d2}^* = \frac{[(2-b)+bc_1-2c_2]}{2}$, $\delta_{d1}^* = \frac{1-c_1}{2} > 0$, $\lambda_3^* = \frac{(2-b^2)(1-c_1)}{8} > 0$, and $\lambda_4^* = \frac{[-(1-b)-bc_1+c_2]}{2}$. Note that $f_{d1}^* = 0$ due to $\delta_{d1}^* = \tilde{\delta}_{d1} = \frac{1-c_1}{2}$, $f_{d2}^* = 0$ due to $q_{d2}^* = 0$, $\lambda_4^* > 0$ iff $c_2 > (1-b)+bc_1$, $r_{d2}^* > 0$ iff $c_2 < \frac{1}{2}[(2-b)+bc_1]$ with $\frac{[(2-b)+bc_1]}{2} > (1-b)+bc_1$, and $(\bar{r}_{d1} - r_{d1}^*) = \frac{(2-b^2)(1-c_1)}{4} > 0$. Thus, for $[(1-b)+bc_1] < c_2 < \frac{1}{2}[(2-b)+bc_1]$, port authority's equilibrium fee revenue equals

$$R_d^* = r_{d1}^* \cdot \delta_{d1}^* = \frac{(1-c_1)^2}{4} \equiv R_5^*. \qquad (A196)$$

Obviously, we have $r_{d2}^* < 0$ iff $c_2 > \frac{[(2-b)+bc_1]}{2}$ from the above. Thus, $r_{d2}^* = 0$ for large c_2, and we have the following sub-cases.

Case 3b-2: Suppose $\lambda_4^* > 0$, $r_{d1}^* > 0$, and $r_{d2}^* = 0$. Then (A186)-(A188) and (A192)-(A193) suggest $[\frac{(1-3r_2-c_2)}{4} + \frac{b\lambda_3}{(2-b^2)} - \lambda_4] \leq 0$, $[\frac{1}{2}\delta_1 - \frac{2\lambda_3}{(2-b^2)}] = 0$, $\frac{1}{4}[2(1-b)+2bc_2-2c_1+2r_1-(4-3b^2)\delta_1]-\lambda_3-b\lambda_4 = 0$, $\delta_1 = \tilde{\delta}_{d1} = \frac{(2-b)-2(r_1+c_1)+b(r_2+c_2)}{(2-b^2)}$, and $r_2 = (1-b\delta_1-c_2) = 0$. Solving these equations yields $r_{d1}^* = \frac{[-(1-b)-bc_1+c_2]}{b}$, $r_{d2}^* = 0$, $\delta_{d1}^* = \frac{1-c_2}{b} > 0$, $\lambda_3^* = \frac{(2-b^2)(1-c_2)}{4b} > 0$, and $\lambda_4^* = \frac{[-(4-2b-b^2)-2bc_1+(4-b^2)c_2]}{2b^2}$. Note that $f_{d1}^* = 0$ due to $\delta_{d1}^* = \tilde{\delta}_{d1} = \frac{1-c_2}{b}$, $f_{d2}^* = 0$ due to $q_{d2}^* = 0$, $\lambda_4^* > 0$ iff $c_2 > \frac{(4-2b-b^2)+2bc_1}{(4-b^2)}$, $[\frac{(1-3r_2-c_2)}{4} + \frac{b\lambda_3}{(2-b^2)} - \lambda_4] = \frac{[(2-b)+bc_1-2c_2]}{b^2} \leq 0$ iff $c_2 \geq \frac{[(2-b)+bc_1]}{2}$ with $\frac{[(2-b)+bc_1]}{2} > \frac{(4-2b-b^2)+2bc_1}{(4-b^2)} > (1-b)+bc_1$, and $(\bar{r}_{d1} - r_{d1}^*) = \frac{(2-b^2)(1-c_1)}{4} > 0$. Thus, for $c_2 \geq \frac{[(2-b)+bc_1]}{2}$, port authority's equilibrium fee revenue equals

$$R_d^* = r_{d1}^* \cdot \delta_{d1}^* = \frac{1}{b^2}(1-c_2)[-(1-b) - bc_1 + c_2] \equiv R_6^*. \tag{A197}$$

<u>Case 4</u>: Suppose $\delta_1 \in (\frac{(1-b\delta_2-c_1-r_1)}{2},\ (1-b\delta_2))$ and $\delta_2 \in (\frac{(1-b\delta_1-c_2-r_2)}{2},\ (1-b\delta_1))$. Then, Lemma 16(iv) implies $\pi_{di}^* = \delta_i[1-\delta_i-b\delta_j-c_i-r_i] - f_i$ for $i,\ j \in \{1,\ 2 \mid i \neq j\}$. Accordingly, we have $f_{d1}^* = \frac{\delta_1[1-\delta_1-b\delta_2-(r_1+c_1)]}{2}$ and $f_{d2}^* = \frac{\delta_2[1-\delta_2-b\delta_1-(r_2+c_2)]}{2}$. Moreover, $f_{d1}^* \geq 0$ iff $\delta_1 \leq [1-b\delta_2-r_1-c_1]$ and $r_1 \leq \bar{r}_{d1}$, and $f_{d2}^* \geq 0$ iff $\delta_2 \leq [1-b\delta_1-(r_2+c_2)]$ and $r_2 \leq \bar{r}_{d2}$. Then, the problem in (49) becomes

$$\max_{r_1,\, f_1,\, \delta_1,\, r_2,\, f_2,\, \delta_2} \quad f_{d1}^* + f_{d2}^* + r_1 q_{d1}^* + r_2 q_{d2}^*$$

$$\text{s.t. } \frac{(1-b\delta_2-c_1-r_1)}{2} < \delta_1 \leq [1-b\delta_2-r_1-c_1],\ r_1 \leq \bar{r}_{d1},$$
$$\frac{(1-b\delta_1-c_2-r_2)}{2} < \delta_2 \leq [1-b\delta_1-(r_2+c_2)],\ \text{and } r_2 \leq \bar{r}_{d2}. \tag{A198}$$

Its Lagrange function is

$$\begin{aligned} L &= r_1\delta_1 + r_2\delta_2 + \frac{\delta_1[1-\delta_1-b\delta_2-(r_1+c_1)]}{2} + \frac{\delta_2[1-\delta_2-b\delta_1-(r_2+c_2)]}{2} \\ &\quad +\lambda_1[\delta_1 - \frac{(1-b\delta_2-c_1-r_1)}{2}] + \lambda_2[(1-b\delta_2-r_1-c_1)-\delta_1] + \lambda_3[\delta_2 \\ &\quad -\frac{(1-b\delta_1-c_2-r_2)}{2}] + \lambda_4[1-b\delta_1-(r_2+c_2)-\delta_2] + \lambda_5(\bar{r}_{d1}-r_1) + \lambda_6(\bar{r}_{d2}-r_2). \end{aligned}$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r_1} = \frac{1}{2}\delta_1 + \frac{1}{2}\lambda_1 - \lambda_2 - \lambda_5 + \frac{b}{2}\lambda_6 \leq 0,\ r_1 \cdot \frac{\partial L}{\partial r_1} = 0, \tag{A199}$$

$$\frac{\partial L}{\partial r_2} = \frac{1}{2}\delta_2 + \frac{1}{2}\lambda_3 - \lambda_4 + \frac{b}{2}\lambda_5 - \lambda_6 \leq 0,\ r_2 \cdot \frac{\partial L}{\partial r_2} = 0, \tag{A200}$$

$$\frac{\partial L}{\partial \delta_1} = \frac{1}{2}[1-2\delta_1-2b\delta_2-c_1+r_1] + \lambda_1 - \lambda_2 + \frac{b}{2}\lambda_3 - b\lambda_4 \leq 0,\ \delta_1\frac{\partial L}{\partial \delta_1} = 0, \tag{A201}$$

$$\frac{\partial L}{\partial \delta_2} = \frac{1}{2}[1 - 2\delta_2 - 2b\delta_1 - c_2 + r_2] + \frac{b}{2}\lambda_1 - b\lambda_2 + \lambda_3 - \lambda_4 \leq 0,\ \delta_2 \cdot \frac{\partial L}{\partial \delta_2} = 0, \quad (A202)$$

$$\frac{\partial L}{\partial \lambda_1} = \delta_1 - \frac{(1 - b\delta_2 - c_1 - r_1)}{2} \geq 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A203)$$

$$\frac{\partial L}{\partial \lambda_2} = (1 - b\delta_2 - r_1 - c_1) - \delta_1 \geq 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad (A204)$$

$$\frac{\partial L}{\partial \lambda_3} = \delta_2 - \frac{(1 - b\delta_1 - c_2 - r_2)}{2} \geq 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \quad (A205)$$

$$\frac{\partial L}{\partial \lambda_4} = 1 - b\delta_1 - (r_2 + c_2) - \delta_2 \geq 0,\ \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0, \quad (A206)$$

$$\frac{\partial L}{\partial \lambda_5} = \bar{r}_{d1} - r_1 \geq 0,\ \lambda_5 \cdot \frac{\partial L}{\partial \lambda_5} = 0, \text{ and} \quad (A207)$$

$$\frac{\partial L}{\partial \lambda_6} = \bar{r}_{d2} - r_2 \geq 0,\ \lambda_6 \cdot \frac{\partial L}{\partial \lambda_6} = 0. \quad (A208)$$

Constraints $\frac{(1-b\delta_2-c_1-r_1)}{2} < \delta_1$ and $\frac{(1-b\delta_1-c_2-r_2)}{2} < \delta_2$ in (A198) suggest $\lambda_1^* = \lambda_3^* = 0$ by (A203) and (A205). If $\lambda_5^* > 0$, we have $r_{d1}^* = \bar{r}_{d1}$ by (A207) and $\dot{\delta}_{d1} = 0$. Then, we get $\delta_1 \leq (1 - b\delta_2 - \bar{r}_{d1} - c_1) < 1 - \frac{b(1-b\delta_1-c_2-r_2)}{2} - \bar{r}_{d1} - c_1 = \frac{b^2\delta_1}{2}$. This is a contraction. Thus, we must have $\lambda_5^* = 0$. Similarly, $\lambda_6^* = 0$ can be shown by (A208). Moreover, we have $\frac{\partial L}{\partial r_1} = \frac{\delta_1}{2} - \lambda_2 \leq 0$ by (A199), $\frac{\partial L}{\partial r_2} = \frac{\delta_2}{2} - \lambda_4 \leq 0$ by (A200), $\delta_1 > 0$ and $\delta_2 > 0$ due to $r_{d1}^* < \bar{r}_{d1}$ and $r_2 < \bar{r}_{d2}$. Thus, we must have $\lambda_2^* > 0$ and $\lambda_4^* > 0$.

Then, (A199)-(A202), (A204) and (A206) suggest $(\frac{\delta_1}{2} - \lambda_2) = 0$, $(\frac{\delta_2}{2} - \lambda_4) = 0$, $\frac{1}{2}[1 - 2\delta_1 - 2b\delta_2 - c_1 + r_1] - \lambda_2 - b\lambda_4 = 0$, $\frac{1}{2}[1 - 2\delta_2 - 2b\delta_1 - c_2 + r_2] - b\lambda_2 - \lambda_4 = 0$, $[(1 - b\delta_2 - r_1 - c_1) - \delta_1] = 0$, and $[1 - b\delta_1 - (r_2 + c_2) - \delta_2] = 0$. Solving these equations yields $r_{d1}^* = \frac{1-c_1}{2} > 0$, $r_{d2}^* = \frac{1-c_2}{2}$, $\delta_{d1}^* = \frac{(1-b)-c_1+bc_2}{2(1-b^2)} > 0$, $\delta_{d2}^* = \frac{(1-b)+bc_1-c_2}{2(1-b^2)}$, $\lambda_2^* = \frac{(1-b)-c_1+bc_2}{4(1-b^2)} > 0$, and $\lambda_4^* = \frac{(1-b)+bc_1-c_2}{4(1-b^2)}$. Note that $f_{d1}^* = 0$ due to $\delta_{d1}^* = (1 - b\delta_{d2}^* - r_{d1}^* - c_1)$, $f_{d2}^* = 0$ due to $\delta_{d2}^* = (1 - b\delta_{d1}^* - r_{d2}^* - c_2)$, $\lambda_4^* > 0$ iff $c_2 < (1 - b + bc_1)$, $(\bar{r}_{d1} - r_{d1}^*) = \frac{(2-b)-2c_1+bc_2}{4} > 0$, and $(\bar{r}_{d2} - r_{d2}^*) = \frac{(2-b)+bc_1-2c_2}{4} \geq 0$ iff $c_2 \leq \frac{1}{2}[(2 - b) + bc_1]$ with $\frac{1}{2}[(2 - b) + bc_1] > (1 - b + bc_1)$. Thus, for $c_2 < [(1 - b) + bc_1]$, port authority's equilibrium fee revenue equals

$$R_d^* = r_{d1}^* \cdot \delta_{d1}^* + r_{d2}^* \cdot \delta_{d2}^* = \frac{(1 - c_1)[(1 - b) - c_1 + bc_2] + (1 - c_2)[(1 - b) + bc_1 - c_2]}{4(1 - b^2)} = R_7^*. \quad (A209)$$

By comparing port authority's equilibrium fee revenues derived in Cases 1-4, we can obtain optimal concession contracts. We first compare critical points $\hat{c}_{d2} \equiv \frac{(3-2b)+2bc_1}{3}$ in Case 1a, $\dddot{c}_{d2} \equiv \frac{(3-b)+bc_1}{3}$ in Case 1b, $\dot{c}_{d2} \equiv (1-b)+bc_1$ in Case 3a, and $\ddot{c}_{d2} \equiv \frac{1}{2}[(2-b)+bc_1]$ in Case 3b. Since $(\dddot{c}_{d2} - \ddot{c}_{d2}) = \frac{b(1-c_1)}{6} > 0$, $(\ddot{c}_{d2} - \hat{c}_{d2}) = \frac{b(1-c_1)}{6} > 0$, and $(\hat{c}_{d2} - \dot{c}_{d2}) = \frac{b(1-c_1)}{3} > 0$, we have $\dot{c}_{d2} < \hat{c}_{d2} < \ddot{c}_{d2} < \dddot{c}_{d2}$. Thus, there are four situations below.

First, for $c_2 < \dot{c}_{d2}$, equilibria of R_1^* in (A172) for $c_2 \leq \hat{c}_{d2}$ of Case 1a, R_3^* in (A184) for $c_2 < \hat{c}_{d2}$ of Case 2a, R_4^* in (A195) for $c_2 \leq \dot{c}_{d2}$ of Case 3a, and R_7^* in (A209) for $c_2 < \dot{c}_{d2}$ of Case 4 exist. Since $(R_7^* - R_3^*) = \frac{[(1-b)-c_1+bc_2]^2}{4(1-b^2)(3-2b^2)} > 0$, $(R_7^* - R_4^*) = \frac{[(1-b)+bc_1-c_2]^2}{4(1-b^2)(3-2b^2)} > 0$, and $(R_3^* - R_1^*) = \frac{[(3-2b)+2bc_1-3c_2]^2}{4(9-4b^2)(3-2b^2)} > 0$, R_7^* is optimal. Thus, port authority's best choices are $r_{d1}^* = \frac{1-c_1}{2}$, $r_{d2}^* = \frac{1-c_2}{2}$, $\delta_{d1}^* = \frac{(1-b)-c_1+bc_2}{2(1-b^2)}$, $\delta_{d2}^* = \frac{(1-b)+bc_1-c_2}{2(1-b^2)}$, $f_{d1}^* = 0$, $f_{d2}^* = 0$, and $R_d^* = \frac{(1-c_1)[(1-b)-c_1+bc_2]+(1-c_2)[(1-b)+bc_1-c_2]}{4(1-b^2)}$ in (A209). These prove Proposition 5(i).

Second, for $\dot{c}_{d2} \leq c_2 \leq \hat{c}_{d2}$, equilibria of R_1^* in (A172) for $c_2 \leq \hat{c}_{d2}$ in Case 1a, R_3^* in (A184) for $c_2 < \hat{c}_{d2}$ in Case 2a, and R_5^* in (A196) for $\dot{c}_{d2} < c_2 < \ddot{c}_{d2}$ of Case 3b-1 exist. Since $(R_5^* - R_3^*) = \frac{(1-c_1)^2}{4} - \frac{(5-4b)-4(1-b)c_1-2(3-2b)c_2-4bc_1c_2+2c_1^2+3c_2^2}{4(3-2b^2)}$ and $\frac{\partial(R_5^*-R_3^*)}{\partial c_2} = \frac{(3-2b)+2bc_1-3c_2}{4(3-2b^2)} \geq 0$ for $\dot{c}_{d2} \leq c_2 \leq \hat{c}_{d2}$, we have $(R_5^* - R_3^*) > \frac{(1-c_1)^2}{4} - \frac{(5-4b)-4(1-b)c_1-2(3-2b\dot{c}_{d2}-4bc_1\dot{c}_{d2}+2c_1^2+3\dot{c}_{d2}^2}{4(3-2b^2)} = \frac{(1-b^2)(1-c_1)^2}{4(3-2b^2)} > 0$. In addition, $(R_3^* - R_1^*) = \frac{[(3-2b)+2bc_1-3c_2]^2}{4(9-4b^2)(3-2b^2)} > 0$. Thus, R_5^* is optimal, and port authority's best choices are $r_{d1}^* = \frac{1-c_1}{2}$, $r_{d2}^* = \frac{1}{2}[(2-b)+bc_1-2c_2]$, $\delta_{d1}^* = \frac{1-c_1}{2}$, $f_{d1}^* = 0$, $f_{d2}^* = \delta_{d2}^* = 0$, and $R_d^* = r_{d1}^* \cdot \delta_{d1}^* = \frac{(1-c_1)^2}{4}$ in (A196). These prove Proposition 5(ii) with $c_2 \in [\dot{c}_{d2},\ \hat{c}_{d2}]$.

Third, for $\hat{c}_{d2} < c_2 < \ddot{c}_{d2}$, equilibria of R_2^* in (A173) for $\hat{c}_{d2} < c_2 \leq \dddot{c}_{d2}$ of Case 1b and R_5^* in (A196) for $\dot{c}_{d2} < c_2 < \ddot{c}_{d2}$ of Case 3b-1 exist. Since $(R_5^* - R_2^*) = \frac{(1-c_1)^2}{4} - \frac{(1-c_1)^2}{6} > 0$, R_5^* is optimal, the same as the second situation. These prove Proposition 5(ii) with $c_2 \in (\hat{c}_{d2},\ \ddot{c}_{d2})$.

Fourth, for $c_2 \geq \ddot{c}_{d2}$, equilibria of R_2^* in (A173) for $\hat{c}_{d2} < c_2 \leq \dddot{c}_{d2}$ of Case 1b and R_6^* in (A197) for $c_2 \geq \ddot{c}_{d2}$ of Case 3b-2 exist. Since $(R_6^* - R_2^*) = \frac{1}{b^2}(1 -$

$c_2)[-(1-b)-bc_1+c_2]-\frac{(1-c_1)^2}{6}$ and $\frac{\partial(R_6^*-R_2^*)}{\partial c_2}=\frac{(2-b)+bc_1-2c_2}{b^2}\leq 0$ for $c_2\geq \ddot{c}_{d2}$, we have $(R_6^*-R_2^*)>\frac{1}{b^2}(1-c_2)[-(1-b)-bc_1+c_2]-\frac{(1-c_1)^2}{6}=\frac{(1-c_1)^2}{18}>0$ for $\ddot{c}_{d2}\leq c_2\leq \dddot{c}_{d2}$. Thus, for $c_2\geq \ddot{c}_{d2}$, the port authority will only offer the unit-fee contract to operator 1 with $r_{d1}^*=\frac{1}{b}[-(1-b)-bc_1+c_2]$, $\delta_{d1}^*=\frac{1-c_2}{b}$, $f_{d1}^*=0$, and $R_d^*=r_{d1}^*\cdot\delta_{d1}^*=\frac{1}{b^2}(1-c_2)[-(1-b)-bc_1+c_2]$ in (A197). These prove Proposition 5(iii). □

Lemma 17. *Given two-part tariff contract and minimum throughput requirement* $(r,\ f,\ \delta)$, *operators' optimal behaviors are as follows.*
(i) *If* $\delta\leq\frac{(1-c-r)}{(1+n)}$, *the equilibrium cargo-handling amount of operator* k *is* $q_{gk}^*=\frac{(1-c-r)}{(1+n)}$, *its equilibrium service price is* $p_{gk}^*=\frac{(1+nc+nr)}{(1+n)}>0$, *and its equilibrium profit is* $\pi_{gk}^*=(q_{gk}^*)^2-f$ *for* $k=1,\ 2,\ldots,n$.

(ii) *If* $\delta>\frac{(1-c-r)}{(1+n)}$, *the equilibrium cargo-handling amount of operator* k *is* $q_{gk}^*=\delta$, *its equilibrium service price is* $p_{gk}^*=(1-n\delta)$, *and its equilibrium profit is* $\pi_{gk}^*=(1-n\delta-c-r)\delta-f$ *for* $k=1,\ 2,\ldots,n$.

Proof of Lemma 17: The proofs are similar to those of Lemma 1, and thus omitted. □

Lemma 18. *Suppose the conditions in (53) hold. Then we have* $r_g^*=\frac{n(1-c)}{1+2n}$, $f_g^*=\frac{(1-c)^2}{2(1+2n)^2}$, *and* $\delta_g^*\in[0,\ \frac{(1-c)}{1+2n}]$. *At the equilibrium, operator* k *will handle cargo amount* $q_{gk}^*=\frac{1-c}{1+2n}>0$, *charge price* $p_{gk}^*=\frac{1+n+cn}{1+2n}>0$, *and obtain profit* $\pi_{gk}^*=\frac{(1-c)^2}{2(1+2n)^2}>0$ *for* $k=1,\ 2,\ldots,n$. *Moreover, port authority's fee revenue is* $R_g^*=\frac{n(1-c)^2}{2(1+2n)}>0$.

Proof of Lemma 18: According to Lemma 17, we have two cases as follow.

Case 1: Suppose $\delta\leq\frac{1-c-r}{1+n}$. Since $\pi_{g1}^*=\pi_{g2}^*=\ldots=\pi_{gn}^*=(q_{gk}^*)^2-f$, we have $f\leq(q_{gk}^*)^2-f$, and hence $f_g^*=\frac{(q_{gk}^*)^2}{2}$ and $\pi_{gk}^*=\frac{(q_{gk}^*)^2}{2}>0$. Then, the problem in (54) becomes

$$\begin{aligned}&\max_r \quad \frac{n(q_{gk}^*)^2}{2}+rn[\frac{(1-c-r)}{(1+n)}]\\&\text{s.t.} \quad 0<r\leq\bar{r}_g\equiv(1-c).\end{aligned}$$

Its Lagrange function is $L=\frac{n}{2}[\frac{(1-c-r)}{(1+n)}]^2+rn[\frac{(1-c-r)}{(1+n)}]+\lambda(\bar{r}_g-r)$, and the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{n(1-c-r)}{1+n} - \frac{n(1-c-r)}{(1+n)^2} - \frac{rn}{1+n} - \lambda \leq 0,\ r \cdot \frac{\partial L}{\partial r} = 0 \text{ and} \qquad (A210)$$

$$\frac{\partial L}{\partial \lambda} = (1-c-r) \geq 0,\ \lambda \cdot \frac{\partial L}{\partial \lambda} = 0. \qquad (A211)$$

According to the values of λ, there are two sub-cases.

Case 1a: Suppose $\lambda^* = 0$. Then (A210) implies $r_g^* = \frac{n(1-c)}{1+2n}$ with $(1-c-r_g^*) = \frac{(1+n)(1-c)}{1+2n} > 0$. Since no constraint is imposed on δ_g^*, it can be any number in interval $[0,\ \frac{(1-c)}{1+2n}]$. Substituting r_g^* into Lemma 17(i) yields $q_{gk}^* = \frac{(1-c)}{1+2n} > 0$, into (50) yields $p_{gk}^* = \frac{(1+n+cn)}{1+2n} > 0$, and into (51) yields $\pi_{gk}^* = f_{gk}^* = \frac{(1-c)^2}{2(1+2n)^2} > 0$. At the equilibrium, port authority's fee revenue is $R_g^* = \frac{n(1-c)^2}{2(1+2n)} > 0$.

Case 1b: Suppose $\lambda^* > 0$. Then (A211) implies $r_g^* = (1-c) > 0$ and $\lambda^* = \frac{n(c-1)}{1+n} < 0$, which contradicts $\lambda^* > 0$. Thus, no solution exists in this case.

Case 2: Suppose $\delta > \frac{1-c-r}{1+n}$. Then Lemma 17(ii) implies $q_{gk}^* = \delta$ with $\delta < \frac{1}{n}$ and $\pi_{gk}^* = (1-n\delta-c-r)\delta - f$. On the other hand, since $\pi_{g1}^* = \pi_{g2}^* = \ldots = \pi_{gn}^* = (1-n\delta-c-r)\delta - f$, we have $f \leq (1-n\delta-c-r)\delta - f$, and thus $f_g^* = \frac{(1-n\delta-c-r)\delta}{2}$ and $\pi_{gk}^* = \frac{(1-n\delta-c-r)\delta}{2} > 0$. Accordingly, the problem in (54) becomes

$$\begin{aligned} &\max_{r,\ \delta} \quad \frac{n\delta(1-n\delta-c-r)}{2} + nr\delta \\ &\text{s.t.} \quad 0 < r \leq (1-c) \text{ and } \frac{1-c-r}{(1+n)} < \delta \leq \frac{1-c-r}{n}. \end{aligned}$$

Since $\delta \leq \frac{1-c-r}{n}$ implies $r \leq (1-c-n\delta)$ and $\delta > \frac{1-c-r}{(1+n)}$ implies $r > [1-c-(n+1)\delta]$, this problem can be reduced to

$$\begin{aligned} &\max_{r,\ \delta} \quad R \equiv \frac{n\delta(1-n\delta-c-r)}{2} + nr\delta \\ &\text{s.t.} \quad [1-c-(n+1)\delta] < r \leq [1-c-n\delta]. \end{aligned}$$

Due to $\frac{\partial R}{\partial r} = \frac{n\delta}{2} > 0$, we have $r_g^* = (1-c-n\delta)$. Accordingly, at r_g^*, we have $f_g^* = \pi_g^* = \frac{1-c-r_g^*-n\delta}{2} = 0$, which contradicts $f > 0$. Thus, no solution exists in this case.

The solutions in Case 1 prove Lemma 18. □

Lemma 19. *Suppose the conditions in (53) hold. Then we have* $r_g^u = \frac{(1-c)}{2}$ *and* $\delta_g^u = \frac{(1-c)}{2n}$. *At the equilibrium, operator* k *will handle cargo amount* $q_{gk}^u = \frac{(1-c)}{2n} > 0$, *charge price* $p_{gk}^u = \frac{1+c}{2} > 0$, *and obtain profit* $\pi_{gk}^u = 0$ *for* $k = 1,\ 2, \ldots, n$. *Moreover, port authority's fee revenue is* $R_g^u = \frac{(1-c)^2}{4} > 0$.

Proof of Lemma 19: The proofs are similar to those of Lemma 18. □

Lemma 20. *Suppose the conditions in (53) hold. Then we have* $f_g^f = \frac{(1-c)^2}{2(n+1)^2}$ *and* $\delta_g^f \in [0,\ \frac{(1-c)}{1+n}]$. *At the equilibrium, operator* k *will handle cargo amount* $q_{gk}^f = \frac{(1-c)}{1+n} > 0$, *charge price* $p_{gk}^f = \frac{1+cn}{1+n} > 0$, *and obtain profit* $\pi_{gk}^f = \frac{(1-c)^2}{2(1+n)^2} > 0$ *for* $k = 1,\ 2, \ldots, n$. *Moreover, port authority's fee revenue is* $R_g^f = \frac{n(1-c)^2}{2(1+n)^2} > 0$. □

Proof of Lemma 20: The proofs are similar to those of Lemma 18. □

Proof of Footnote 8: By letting $c_1 = c_2 = c$ and $b = 0$ in Lemma 1, we get optimal cargo-handling amount q^m and equilibrium service price p^m for a monopolistic operator under the two-part tariff scheme $(r,\ f)$ as follows.

Lemma 21. *Given two-part tariff scheme* $(r,\ f)$ *and minimum throughput guarantee* δ, *we obtain the following optimal behaviors for a monopolistic operator.*

(i) *For* $\delta \in [0,\ \hat{\delta}_m]$ *with* $\hat{\delta}_m \equiv \frac{1-r-c}{2}$, *the operator's equilibrium cargo-handling amount is* $q^m = \frac{1-r-c}{2} = \hat{\delta}_m$, *equilibrium service price is* $p^m = \frac{1+r+c}{2} > 0$, *and equilibrium profit is* $\pi^m = (q^m)^2 - f$.

(ii) *For* $\delta \in (\hat{\delta}_m,\ 1)$, *the operator's equilibrium cargo-handling amount is* $q^m = \delta$, *equilibrium service price is* $p^m = (1-\delta) > 0$, *and equilibrium profit is* $\pi^m = (1 - \delta - c - r)\delta - f$.

Based on these outcomes, the port authority will choose $(r^m,\ f^m,\ \delta^m)$ to solve the problem of

$$\max_{r,\ f,\ \delta} \quad f + rq^m$$
$$\text{s.t.} \quad 0 \le \delta < 1,\ 0 \le r \le (1-c),\ \pi^m \ge 0, \ \text{ and } 0 \le f \le \pi^m.$$

The solutions are listed in Lemma 22.

Lemma 22. *Suppose* $r \le (1-c)$. *Given* $(q^m,\ \pi^m)$ *in Lemma 21, the port authority's optimal concession contract is the unit-fee contract with* $r^m = \frac{(1-c)}{2}$ *and* $\delta^m = \frac{(1-c)}{2}$. *At the equilibrium, port authority's equilibrium fee revenue is* $R^m = \frac{(1-c)^2}{4}$.

Proof. According to the sizes of δ, we have two cases.

Case 1: Suppose $\delta \in [0,\ \hat{\delta}_m]$. Lemma 21(i) implies $f^m = \frac{1}{2}(q^m)^2$. Thus, port authority's problem becomes

$$\begin{aligned} \max_{r,\, f,\, \delta} \quad & \tfrac{1}{2}(q^m)^2 + rq^m \\ \text{s.t.} \quad & 0 \le \delta < \hat{\delta}_m \text{ and } 0 \le r \le (1-c). \end{aligned}$$

Its Lagrange function is $L = \frac{1}{2}(q^m)^2 + rq^m + \lambda_1(\hat{\delta}_m - \delta) + \lambda_2[(1-c) - r]$, where λ_1 and λ_2 are the Lagrange multipliers associated with the two inequalities. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{(1-3r-c)}{4} - \frac{\lambda_1}{2} - \lambda_2 \le 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \tag{A212}$$

$$\frac{\partial L}{\partial \delta} = -\lambda_1 \le 0,\ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \tag{A213}$$

$$\frac{\partial L}{\partial \lambda_1} = \hat{\delta}_m - \delta \ge 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \text{ and} \tag{A214}$$

$$\frac{\partial L}{\partial \lambda_2} = (1-c) - r \ge 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0. \tag{A215}$$

Based on the values of λ_1 and λ_2, we have four sub-cases.

Case 1a: Suppose $\lambda_1^* = \lambda_2^* = 0$. Then, $r^m = \frac{(1-c)}{3} > 0$ by (A212). It remains to check whether $r^m \le (1-c)$ holds. We can show $r^m \le (1-c)$ by some calculations. Accordingly, $\delta^m \in [0,\ \hat{\delta}_m]$ with $\delta^m = \frac{(1-c)}{3}$ and $f^m = \frac{(1-c)^2}{18} > 0$, and port authority's equilibrium fee revenue is

$$R^m = f^m + r^m q^m = \frac{(1-c)^2}{6}. \tag{A216}$$

Case 1b: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then, $r^m = (1-c) > 0$ by (A215). However, we have $\lambda_2^* = \frac{-(1-c)}{2} < 0$ by (A212), which leads to a contradiction. Thus, no solution exists in this case.

Case 1c: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then, $\delta^m = \hat{\delta}_m > 0$ by (A214). This in turn implies $\lambda_1^* = 0$ by (A213), which leads to a contradiction. Thus, no solution exists in this case.

Case 1d: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. As in Case 1b, we have $\lambda_1^* = 0$ by (A212), and no solution exists in this case.

Case 2: Suppose $\delta \in (\hat{\delta}_m,\ 1)$. Lemma 21(ii) implies $f^m = \frac{\delta[1-\delta-r-c]}{2}$. Thus, port authority's problem becomes

$$\begin{aligned} \max\nolimits_{r,\ f,\ \delta} \quad & r\delta + \tfrac{\delta[1-\delta-r-c]}{2} \\ \text{s.t.} \quad & \hat{\delta}_m < \delta < 1 \text{ and } 0 \le r \le (1-c). \end{aligned}$$

Its Lagrange function is $L = r\delta + \frac{\delta[1-\delta-r-c]}{2} + \lambda_1(\delta - \hat{\delta}_m) + \lambda_2[(1-r-c)-\delta] + \lambda_3[(1-c)-r]$, where $\lambda_1,\ \lambda_2$, and λ_3 are the Lagrange multipliers associated with the three inequalities. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{\delta}{2} + \frac{\lambda_1}{2} - \lambda_2 - \lambda_3 \le 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \tag{A217}$$

$$\frac{\partial L}{\partial \delta} = \frac{(1-2\delta-c+r)}{2} + \lambda_1 - \lambda_2 \le 0,\ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \tag{A218}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \hat{\delta}_m \ge 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \tag{A219}$$

$$\frac{\partial L}{\partial \lambda_2} = (1-r-c) - \delta \ge 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \text{ and} \tag{A220}$$

$$\frac{\partial L}{\partial \lambda_3} = (1-c) - r \ge 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0. \tag{A221}$$

Since $\hat{\delta}_m < \delta$, we must have $\lambda_1^* = 0$ by (A219). If $\lambda_3^* > 0$, then $r^m = (1-c) > 0$ by (A221) and $\delta^m = 0$ by (A219) and (A220). However, (A217) suggests $r^m = 0$ due to $\frac{\partial L}{\partial r} < 0$, which leads to a contradiction. Thus, we must have $\lambda_3^* = 0$. On the other hand, if $\lambda_2^* > 0$, then $\frac{\partial L}{\partial r} = \frac{\delta}{2} \leq 0$, which contradicts the requirement of $\delta > \hat{\delta}_m > 0$. Thus, we must have $\lambda_2^* = 0$. Under the circumstance, (A217), (A218) and (A220) imply $\frac{\delta}{2} = \lambda_2,\ \lambda_2 = \frac{(1-2\delta-c+r)}{2}$, and $(1-r-c) = \delta$. Solving the three equations yields $r^m = \frac{(1-c)}{2} < (1-c),\ \delta^m = \frac{(1-c)}{2}$, and $\lambda_2^* = \frac{(1-c)}{4} > 0$ with $(\delta^m - \hat{\delta}_m) = \frac{(1-c)}{4} > 0$ and $f^m = 0$. Thus, port authority's equilibrium fee revenue is

$$R^m = f^m + r^m q^m = \frac{(1-c)^2}{4}. \tag{A222}$$

Comparing $R^m = \frac{(1-c)^2}{6}$ in (A216) and $R^m = \frac{(1-c)^2}{4}$ in (A222), we obtain that port authority's best choice is the unit-fee contract defined in Case 2 with $R^m = \frac{(1-c)^2}{4}$. □

Finally, since port authority' equilibrium fee revenue in Proposition 1 with $c_1 = c_2 = c$ is $R^u = \frac{(1-c)^2}{2(1+b)}$, we have $(R^u - R^m) = \frac{(1-c)^2}{2(1+b)} - \frac{(1-c)^2}{4} = \frac{(1-b)(1-c)^2}{4(1+b)} > 0$. This implies that the port authority is better off when there are two terminal operators, instead of one, in the market.