Appendix

Proof of Lemma 1: Denote $L_1$ and $L_2$ the Lagrange functions of operators 1 and 2 in problem (9) with

$$L_1 = (1 - q_1 - bq_2)q_1 - (c_1 + r)q_1 - f + \lambda_1(q_1 - \delta)$$

and

$$L_2 = (1 - q_2 - bq_1)q_2 - (c_2 + r)q_2 - f + \lambda_2(q_2 - \delta),$$

where $\lambda_1$ and $\lambda_2$ are the respective Lagrange multipliers of operators 1 and 2. Then, the Kuhn-Tucker conditions for operator 1 are

$$\frac{\partial L_1}{\partial q_1} = 1 - 2q_1 - bq_2 - c_1 - r + \lambda_1 \leq 0, \quad q_1 \frac{\partial L_1}{\partial q_1} = 0$$

and

$$\frac{\partial L_1}{\partial \lambda_1} = q_1 - \delta \geq 0, \quad \lambda_1 \frac{\partial L_1}{\partial \lambda_1} = 0; \quad (A1)$$

and for operator 2 are

$$\frac{\partial L_2}{\partial q_2} = 1 - 2q_2 - bq_1 - c_2 - r + \lambda_2 \leq 0, \quad q_2 \frac{\partial L_2}{\partial q_2} = 0$$

and

$$\frac{\partial L_2}{\partial \lambda_2} = q_2 - \delta \geq 0, \quad \lambda_2 \frac{\partial L_2}{\partial \lambda_2} = 0. \quad (A2)$$

If $\frac{\partial L_1}{\partial q_1} < 0$, we have $q_1^* = 0$ by (A1). Then $q_1^* \geq \delta$ will not hold unless $\delta = 0$. Since this is not an interesting solution, we focus on solution $q_1^*$, which satisfies $\frac{\partial L_1}{\partial q_1} = 0$ in (A1). Similarly, we focus on solution $q_2^*$, which satisfies $\frac{\partial L_2}{\partial q_2} = 0$ in (A3). Based on the values of $\lambda_1$ and $\lambda_2$, there are four cases as follows.

Case 1: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then (A1) and (A3) suggest

$$1 - 2q_1 - bq_2 - c_1 - r = 0$$

and

$$1 - 2q_2 - bq_1 - c_2 - r = 0.$$

Solving these equations yields $q_1^* = \frac{1 - r}{2 + b} + \frac{bc_2 - 2c_1}{4 - b^2}$ and $q_2^* = \frac{1 - r}{2 + b} + \frac{bc_1 - 2c_2}{4 - b^2}$. The conditions in (12) imply non-negative $q_i^*$ for $i = 1, 2$. To guarantee $q_1^* \geq \delta$ and $q_2^* \geq \delta$, condition
0 \leq \delta \leq \delta_1 \equiv \frac{1-r}{2+b} + \frac{bc_1-2c_2}{4-b^2} = q_2^* should be met, because \( c_1 < c_2 \) implies \( q_1^* \geq q_2^* \) and \( q_2^* \geq \delta \) implies \( q_1^* \geq \delta \). Substituting \( q_1^* \) and \( q_2^* \) into (1)-(2) yields 
\( p_2^* = \frac{1+r(1+b)}{2+b} + \frac{2c_2}{4-b^2} > 0 \), and into (4) yields \( \pi_i^* = (q_i^*)^2 - f \) for \( i = 1, 2 \). These prove Lemma 1(i).

**Case 2:** Suppose \( \lambda_1^* = 0 \) and \( \lambda_2^* > 0 \). Then, (A1), (A3), and (A4) suggest

\( (1 - 2q_1 - bq_2 - c_1 - r) = 0, \ (q_2 - \delta) = 0, \ (1 - 2q_2 - bq_1 - c_2 - r + \lambda_2) = 0. \)

They in turn imply \( q_1^* = \frac{1-r-c_1-b\delta}{2}, \ q_2^* = \delta, \) and \( \lambda_2^* = \frac{(4-b^2)(\delta-\delta_1)}{2} \). To guarantee \( \lambda_2^* > 0 \), conditions \( \delta > \delta_1 \equiv \frac{1+r+c_1-2c_2-r}{3}, \ r \leq \bar{r}, \) and \( c_2 \leq \bar{c}_2 \) are needed. On the other hand, to have \( q_1^* \geq \delta \), conditions \( \delta \leq \delta_2 \equiv \frac{1-r-c_1-r}{2+b} \) and \( r \leq (1 - c_1) \) are needed. Thus, the plausible range for \( \delta \) is \( \delta \in (\delta_1, \delta_2) \). Substituting \( q_1^* \) and \( q_2^* \) into (1)-(2) produces

\( p_2^* = \frac{1}{2}[(2-b)(2-b^2)\delta + bc_1 + br] > p_1^* = \frac{1}{2}(1-b\delta + c_1 + r) > 0 \) if \( \delta \leq \delta_2 \), and into (4) gives \( \pi_1^* = (q_1^*)^2 - f \) and \( \pi_2^* = \frac{\delta}{2}[(2-b)(1-r) - (2-b^2)\delta + bc_1 - 2c_2] - f. \) These prove Lemma 1(ii).

**Case 3:** Suppose \( \lambda_1^* > 0 \) and \( \lambda_2^* = 0 \). Then (A1)-(A3) suggest

\( (q_1 - \delta) = 0, \ (1 - 2q_2 - bq_1 - c_2 - r) = 0, \ (1 - 2q_1 - bq_2 - c_1 - r + \lambda_1) = 0. \)

Solving these equations yields

\( q_1^* = \delta, \ q_2^* = \frac{1-r-c_2-b\delta}{2}, \) and \( \lambda_1^* = \frac{(4-b^2)}{2} [\delta - (\frac{1-r}{2+b} + \frac{bc_2-2c_1}{4-b^2})]. \)

To guarantee \( \lambda_1^* > 0 \), conditions

\[ \delta > \frac{1-r}{2+b} + \frac{bc_2-2c_1}{4-b^2} \quad \text{and} \quad r \leq 1 - \frac{2c_1-bc_2}{2-b} \] \hspace{1cm} (A5)

are needed. On the other hand, \( q_2^* \geq \delta \) is guaranteed by assuming

\[ \delta \leq \frac{1-c_2-r}{2+b}. \] \hspace{1cm} (A6)

However, (A5) and (A6) are incompatible with each other because

\[ \frac{1-c_2-r}{2+b} - \frac{1-r}{2+b} + \frac{bc_2-2c_1}{4-b^2} = -\frac{2(c_2-c_1)}{4-b^2} < 0. \] Thus, no solution exists in this case.
Case 4: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. Then, (A2) and (A4) suggest $q_1^* = \delta$ and $q_2^* = \delta$, and (A1) and (A3) imply $\lambda_1^* = -1 + (2 + b)\delta + c_1 + r$, and $\lambda_2^* = -1 + (2 + b)\delta + c_2 + r$. To have $\lambda_1^* > 0$ and $\lambda_2^* > 0$, conditions $\delta > \bar{\delta} \equiv \frac{1-c_1-r}{2-b}$ and $r \leq (1-c_1)$ are needed.

Note that $r \leq (1-c_1)$ is inferred from $r \leq \bar{r} \equiv \frac{(2-b)+bc_1-2c_2}{2-b}$. Substituting $q_1^* = q_2^* = \delta$ into (1)-(2) produces $p_1^* = p_2^* = 1 - (1 + b)\delta > 0$ if $\delta < \frac{1}{1+b}$, and into (4) gives $\pi_i^* = \delta[1 - (1 + b)\delta - c_i - r] - f$ for $i = 1, 2$. These prove Lemma 1(iii).

**Proof of Lemma 2:** The proofs are similar to those of Lemma 1, and thus omitted.

**Proof of Lemma 3:** The proofs are similar to those of Lemma 1, and thus omitted.

**Proof of Lemma 4:** Lemma 1 shows that operators’ optimal choices depend on the values of $\delta$. Thus, we have the following cases.

Case 1: Suppose $\delta \in [0, \delta_1]$. Then Lemma 1(i) implies $\pi_1^* > \pi_2^*$, and $f^* = \pi_2^* = \frac{1}{2}(q_2^*)^2 > 0$. The problem in (15) thus becomes

$$\max_{r,f,\delta} 2f + r(q_1^* + q_2^*)$$

s.t. $0 \leq \delta \leq \delta_1$ and $0 < r \leq \bar{r}$.

Its Lagrange function is

$$L = (q_2^*)^2 + r(q_1^* + q_2^*) + \lambda_1(\delta_1 - \delta) + \lambda_2(\bar{r} - r),$$

where $\lambda_1$ and $\lambda_2$ are the Lagrange multipliers associated with the inequality constraints.

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = 2q_2^* \frac{\partial q_2^*}{\partial r} + r(\frac{\partial q_1^*}{\partial r} + \frac{\partial q_2^*}{\partial r}) + (q_1^* + q_2^*) + \lambda_1 \frac{\partial \delta_1}{\partial r} - \lambda_2 \leq 0, r \cdot \frac{\partial L}{\partial r} = 0, \quad (A7)$$

$$\frac{\partial L}{\partial \delta} = -\lambda_1 \leq 0, \delta \frac{\partial L}{\partial \delta} = 0, \quad (A8)$$

$$\frac{\partial L}{\partial \lambda_1} = \delta_1 - \delta \geq 0, \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \text{ and} \quad (A9)$$

$$\frac{\partial L}{\partial \lambda_2} = \bar{r} - r \geq 0, \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0. \quad (A10)$$
Based on the values of $\lambda_1$ and $\lambda_2$, we have four sub-cases.

**Case 1a:** Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then, (A7) becomes
$$\frac{1}{(2+b)^2}[(2+2b) - r(4b+6) - \frac{(4+2b-b^2)c_1}{(2-b)} + \frac{b^2c_2}{(2-b)}] = 0,$$
which implies $r^* = \frac{[(2+2b) + \frac{b^2c_2}{(2-b)} - \frac{(4+2b-b^2)c_1}{(2-b)}]}{(4b+b^3)} > 0$. It remains to check whether $r^* < \bar{r}$ holds. By some calculations, we have $r^* < \bar{r}$ iff $c_2 < \hat{c}_2 \equiv \frac{2(2-b)+(2+3b)c_1}{(b+6)}$. On the other hand, (A9) implies both $\delta^* \in [0, \delta_1]$ with
$$\delta_1 = \frac{1-r^*+\frac{bc_1-2c_2}{4-b^2} = \frac{2(2-b)+(2+3b)c_1-(6+b)c_2}{2(2-b)(3+2b)}}{b^2}$$
and $f^* = \frac{1}{2}\left[\frac{2(2-b)+(2+3b)c_1-(6+b)c_2}{2(2-b)(3+2b)}\right]^2 > 0$. Thus, at the equilibrium, port authority’s fee revenue equals
$$R^* = \frac{[\frac{2(2-b)+(2+3b)c_1-(6+b)c_2}{2(2-b)(3+2b)}]^2 + r^*\left[\frac{2(1-r^*)-(c_1+c_2)}{2+b}\right]}{2(2-b)(3+2b)}.$$ 
(A11)

**Case 1b:** Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then, (A10) suggests $r^* = \bar{r} \equiv \frac{[(2-b)+bc_1-2c_2]}{(2-b)}$. However, at $r^*$, we have $f^* = 0$ due to $q_2^* = 0$, which contradicts the requirement of positive fixed fee. Thus, no solution exists in this case.

**Case 1c:** Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then, (A9) suggests $\delta^* = \delta_1 > 0$. This in turn implies $\lambda_1^* = 0$ by (A8). It is a contradiction. Thus, no solution exists in this case.

**Case 1d:** Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. As in Case 1b, we have $f^* = 0$. Again, no solution exists here.

**Case 2:** Suppose $\delta \in (\delta_1, \delta_2)$. Then Lemma 1(ii) implies $\pi_1^* > \pi_2^*$, and $f^* = \pi_2^* = \frac{\delta}{4}[(2-b)(1-r) - (2-b^2)\delta + bc_1 - 2c_2].$ We have $f^* > 0$ iff $\delta < \frac{(2-b)(1-r)+bc_1-2c_2}{(2-b^2)} \ge \frac{(2-b)(1-r)+bc_1-2c_2}{(2-b^2)}$. In addition, $\frac{(2-b)(1-r)+bc_1-2c_2}{(2-b^2)} < \delta$ iff $r < (\ge ) 1 + (1+b)c_1 - (2+b)c_2$. Thus, we have two sub-cases as follows.

**Case 2a:** Suppose $r \ge [1+(1+b)c_1-(2+b)c_2].$ Then, the problem in (15) becomes
$$\max_{r, f, \delta} 2f + r(q_1^* + q_2^*)$$
s.t. $\delta < \frac{(2-b)(1-r)+bc_1-2c_2}{(2-b^2)}$ and $[1+(1+b)c_1-(2+b)c_2] \le r < \bar{r}$. (A12)
Its Lagrange function is
\[
L = \frac{\delta}{2}[(2 - b)(1 - r) - (2 - b^2)\delta + bc_1 - 2c_2] + \frac{r}{2}[1 + (2 - b)\delta - c_1 - r] + \lambda_1(\delta - \delta_1) + \lambda_2\left(\frac{(2 - b)(1 - r) + bc_1 - 2c_2}{(2 - b^2)} - \delta\right) + \lambda_3\{r - [1 + (1 + b)c_1 - (2 + b)c_2]\} + \lambda_4(\bar{r} - r).
\]

Then, the Kuhn-Tucker conditions are
\[
\begin{align*}
\frac{\partial L}{\partial r} &= \frac{1}{2}(1 - 2r - c_1) + \frac{1}{2 + b}\lambda_1 - \frac{2 - b}{(2 - b^2)}\lambda_2 + \lambda_3 - \lambda_4 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \\
\frac{\partial L}{\partial \delta} &= \frac{1}{2}[(2 - b) - 2(2 - b^2)\delta + bc_1 - 2c_2] + \lambda_1 - \lambda_2 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \\
\frac{\partial L}{\partial \lambda_1} &= \delta - \delta_1 \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \\
\frac{\partial L}{\partial \lambda_2} &= \frac{(2 - b)(1 - r) + bc_1 - 2c_2}{(2 - b^2)} - \delta \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \\
\frac{\partial L}{\partial \lambda_3} &= r - [1 + (1 + b)c_1 - (2 + b)c_2] \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \quad \text{and} \\
\frac{\partial L}{\partial \lambda_4} &= \bar{r} - r \geq 0, \quad \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0,
\end{align*}
\]

where \(\lambda_1, \lambda_2, \lambda_3,\) and \(\lambda_4\) are the Lagrange multipliers for the four constraints in \((A12)\).

Since three of the constraints are strict inequalities, we must have \(\lambda_1^* = \lambda_2^* = \lambda_3^* = 0\). If \(\lambda_3^* = 0\), we have \(\delta^* = \frac{(2 - b) + bc_1 - 2c_2}{2(2 - b^2)}\). By some calculations, we have \(\frac{(2 - b)(1 - r^*) + bc_1 - 2c_2}{(2 - b^2)} = \frac{2(c_1 - c_2)}{2(2 - b^2)} < 0\), which contradicts \(\delta^* < \frac{(2 - b)(1 - r^*) + bc_1 - 2c_2}{(2 - b^2)}\) required by problem \((A12)\). Thus, no solution exists in this case. By contrast, if \(\lambda_3^* > 0\), we have \(r^* = 1 + (1 + b)c_1 - (2 + b)c_2\), \(\delta^* = \frac{(2 - b) + bc_1 - 2c_2}{2(2 - b^2)}\), and \(\lambda_3^* = \frac{1}{2}[1 + (3 + 2b)c_1 - 2(2 + b)c_2]\). Note that \(\lambda_3^* > 0\) iff \(c_2 < \frac{1 + (3 + 2b)c_1}{2(2 + b)}\). On the other hand, \(\frac{\partial L}{\partial \lambda_3} \geq 0\) requires \(\frac{(2 - b)(1 - r^*) + bc_1 - 2c_2}{(2 - b^2)} - \delta^* \geq 0\).

By some calculations, we have \(\frac{(2 - b)(1 - r^*) + bc_1 - 2c_2}{(2 - b^2)} - \delta^* = \frac{(2 - b) + (4 + 2b - 2b^2)c_1 + 2(3 - b^2)c_2}{2(2 - b^2)} \geq 0\) iff \(c_2 \geq \frac{(2 - b) + (4 + 2b - 2b^2)c_1}{2(3 - b^2)}\), which contradicts \(c_2 < \frac{1 + (3 + 2b)c_1}{2(2 + b)}\) required by \(\lambda_3^* > 0\). That is because \(\frac{(2 - b) + (4 + 2b - 2b^2)c_1}{2(3 - b^2)} > \frac{1 + (3 + 2b)c_1}{2(2 + b)}\). Thus, no solution exists in this case.

Case 2b: Suppose \(r < [1 + (1 + b)c_1 - (2 + b)c_2]\). Then the problem in \((15)\) becomes
\[
\begin{align*}
\max_{r, \delta} & \quad 2f^* + r(q_1^* + q_2^*) \\
\text{s.t.} & \quad \delta_1 \leq \delta \leq \delta_2 \quad \text{and} \quad 0 < r < [1 + (1 + b)c_1 - (2 + b)c_2]. \quad (A13)
\end{align*}
\]
Its Lagrange function is

\[ L = \frac{\delta}{2} [(2-b)(1-r) - (2-b^2)\delta + bc_1 - 2c_2] + \frac{r}{2} [1 + (2-b)\delta - c_1 - r] \]

\[ + \lambda_1 (\delta - \delta_1) + \lambda_2 (\delta_2 - \delta) + \lambda_3 [1 + (1+b)c_1 - (2+b)c_2 - r]. \]

Then, the Kuhn-Tucker conditions are

\[ \frac{\partial L}{\partial \lambda_1} = \frac{1}{2} (1-2r - c_1) + \frac{1}{2+b} \lambda_1 - \frac{1}{2+b} \lambda_2 - \lambda_3 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \quad (A14) \]

\[ \frac{\partial L}{\partial \lambda_2} = \frac{1}{2} [(2-b) - 2(2-b^2)\delta + bc_1 - 2c_2] + \lambda_1 - \lambda_2 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad (A15) \]

\[ \frac{\partial L}{\partial \lambda_1} = \delta - \delta_1 \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A16) \]

\[ \frac{\partial L}{\partial \lambda_2} = \delta_2 - \delta \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad \text{and} \quad (A17) \]

\[ \frac{\partial L}{\partial \lambda_3} = 1 + (1+b)c_1 - (2+b)c_2 - r \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \quad (A18) \]

where \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are the Lagrange multipliers for the three constraints in (A13).

Constraints \( \delta_1 < \delta \) and \( r < [1 + (1+b)c_1 - (2+b)c_2] \) suggest \( \lambda_1^* = \lambda_3^* = 0 \) by (A16) and (A18). If \( \lambda_2^* > 0 \), (A14), (A15) and (A17) suggest \( \frac{1-(2r-c_1)}{2} - \frac{\lambda_2}{2+b} = 0, \quad \frac{(2-b-2(2-b^2)\delta+bc_1-2c_2)}{2(3+2b)} - \lambda_2 = 0, \) and \( (\delta_2 - \delta) = 0. \) Solving these equations yields

\[ r^* = \frac{2(1+b)-(4+3b)c_1+(2+b)c_2}{2(3+2b)}, \quad \delta^* = \frac{2-c_1-c_2}{4-b^2}, \quad \text{and} \quad \lambda_2^* = \frac{2+b}{2(3+2b)}[(2+b)1-(1+b)c_1-(2+b)c_2]. \]

By some calculations, we have \( (\delta^* - \delta_1) = \frac{2(c_2-c_1)}{4-b^2} > 0, \) and \( \lambda_2^* > 0 \) iff \( c_2 < \frac{1+(1+b)c_1}{2+b}. \) On the other hand, we have \( r^* - 1 + (1+b)c_1 - (2+b)c_2 \leq \frac{(2+b)}{2(3+2b)}[-2 - (5+4b)c_1 + (7+4b)c_2]. \)

Thus, we get \( r^* < 1 + (1+b)c_1 - (2+b)c_2 \) iff \( c_2 < \frac{2+(5+4b)c_1}{7+4b}. \)

By contrast, if \( \lambda_2^* = 0, \) we have \( r^* = \frac{1-c_1}{2} \) and \( \delta^* = \frac{2-b+bc_1-2c_2}{2(2-b^2)} \) by (A15) and (A16). Note that \( \delta^* \leq \delta_2 \) iff \( c_2 > \frac{1+(1+b)c_1}{2+b}. \) It in turn implies \( (1+b)c_1 - (2+b)c_2 < 0 \) \( r^*, \) which contradicts \( r^* < [1 + (1+b)c_1 - (2+b)c_2]. \) Thus, no solution exists in this case.

In sum, if \( c_2 \leq \frac{2+(5+4b)c_1}{7+4b}, \) an equilibrium with \( r^* = \frac{2(1+b)-(4+3b)c_1+(2+b)c_2}{2(3+2b)} \) \( > 0 \) and \( \delta^* = \frac{(2-c_1-c_2)}{2(3+2b)} \) exists. At the equilibrium, port authority’s fee revenues equals

\[ R^* = \frac{(2-c_1-c_2)^2}{4(3+2b)}. \quad (A19) \]
Case 3: Suppose $\delta \in (\delta_2, \frac{1}{(1+b)})$. Then, Lemma 1(iii) implies $\pi_1^* > \pi_2^*$, and $f^* = \pi_2^* = \frac{1}{2}[1-(1+b)\delta-c_2-r] \delta$ with $f^* > 0$ iff $\delta < \frac{1-c_2-r}{1+b}$ and $r < (1-c_2)$. Note that $r < (1-c_2)$ is implied by $r \leq \bar{r} = \frac{(2-b)+bc_1-2c_2}{2-b}$. Accordingly, the problem in (15) becomes

$$\max_{r, \delta} \delta[1-(1+b)\delta-c_2-r] + 2r\delta$$

s.t. $\delta < \frac{1-c_2-r}{1+b}$ and $r < \bar{r}$. (A20)

Its Lagrange function is

$$L = \delta[1-(1+b)\delta-c_2-r] + 2r\delta + \lambda_1(\delta - \delta_2) + \lambda_2\left[1-c_2-r \over (1+b)\right] - \delta_3(r - r)$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \delta + \frac{1}{2+b}\lambda_1 - \frac{1}{1+b}\lambda_2 - \lambda_3 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0,$$

$$\frac{\partial L}{\partial \delta} = 1 - 2(1+b)\delta - c_2 + r + \lambda_1 - \lambda_2 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0,$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_2 \geq 0, \quad \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0,$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{1-c_2-r}{(1+b)} - \delta \geq 0, \quad \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0, \text{ and}$$

$$\frac{\partial L}{\partial \lambda_3} = \bar{r} - r \geq 0, \quad \lambda_3 \frac{\partial L}{\partial \lambda_3} = 0.$$

Since all constraints in problem (A20) are strict inequalities, we must have $\lambda_1^* = \lambda_2^* = \lambda_3^* = 0$. However, by some calculations, we discover that when $\lambda_1^* = \lambda_2^* = \lambda_3^* = 0$, $r^* = -1 + c_2 < 0$ contradicts $r^* > 0$. Thus, no solution exists in this case.

Based on the above, we can obtain the ensuing optimal two-part tariff contracts. Since $\hat{c}_2 \equiv \frac{2+ (5+4b)c_1}{(7+4b)} < \hat{c}_2 \equiv \frac{2(2-b)+(2+3b)c_1}{(b+6)}$, we have two cases. First, if $c_2 \in (c_1, \hat{c}_2)$, one solution exists in Case 1a with $R^* = 2f^* + r^*[\frac{2(1-r^*)-(c_1+c_2)}{(2+b)}]$ in (A11), and one solution exists in Case 2b with $R^* = \frac{(2-c_1-c_2)^2}{4(3+2b)}$ in (A19). Because $\frac{(2-c_1-c_2)^2}{4(3+2b)} - 2f^* - r^*[\frac{2(1-r^*)-(c_1+c_2)}{(2+b)}] = \frac{(c_2-c_1)(2-b)+bc_1-2c_2}{(3+2b)(2-b)^2} > 0$ by $c_2 < \hat{c}_2$, port authority’s optimal two-part tariff contract and the minimum throughput guarantee are those in Case 2b. These prove Lemma 4(i). Second, if $c_2 \in (\hat{c}_2, \hat{c}_2)$, there exists a unique solution in Case 1a, which is the optimal contract. Lemma 4(ii) is then proved. □
Proof of Lemma 5: Lemma 2 shows that \( q_1^u \) and \( q_2^u \) depend on the values of \( \delta \). Thus, we have three cases and their sub-cases as follows.

Case 1: Suppose \( \delta \in [0, \delta_1] \). Lemma 2(i) implies \( \pi_1^u > \pi_2^u \geq 0 \) due to \( r \leq \bar{r} \) and \( c_2 > c_1 \). Accordingly, the problem in (16) becomes

\[
\max_{r, \delta} \ r(q_1^u + q_2^u) \\
\text{s.t. } 0 \leq \delta \leq \delta_1 \text{ and } 0 < r \leq \bar{r}.
\]

Its Lagrange function is

\[
L = r\left[\frac{2(1 - r) - c_1 - c_2}{2 + b}\right] + \lambda_1(\delta_1 - \delta) + \lambda_2(\bar{r} - r).
\]

Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = \frac{2 - c_1 - c_2 - 4r}{(2 + b)} - \frac{1}{(2 + b)}\lambda_1 - \lambda_2 \leq 0, \quad r \frac{\partial L}{\partial r} = 0, \quad (A21)
\]

\[
\frac{\partial L}{\partial \delta} = -\lambda_1 \leq 0, \quad \frac{\partial L}{\partial \delta} = 0, \quad (A22)
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta_1 - \delta \geq 0, \quad \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \quad \text{and} \quad (A23)
\]

\[
\frac{\partial L}{\partial \lambda_2} = \bar{r} - r \geq 0, \quad \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0, \quad (A24)
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the Lagrange multipliers for the two inequality constraints. If \( \lambda_1^* > 0 \), then \( \frac{\partial L}{\partial \delta} = -\lambda_1^* < 0 \) and \( \delta^* = 0 \) by (A22). They in turn suggest \( \frac{\partial L}{\partial \lambda_1} = \delta_1 > 0 \) and \( \lambda_1^* = 0 \) by (A23). This is a contradiction. Thus, we must have \( \lambda_1^* = 0 \), and two sub-cases below.

Case 1a: Suppose \( \lambda_2^* = 0 \). Then, we have \( r^u = \frac{2 - c_1 - c_2}{4} \) by (A21). In addition, \( (\bar{r} - r^u) = \frac{[2(2 - b) + (2 + 3b)c_1 - (b + 6)c_2]}{4(2 - b)} \geq 0 \) iff \( c_2 \leq \hat{c}_2 \equiv \frac{[2(2 - b) + (2 + 3b)c_1]}{(b + 6)} \). Thus, port authority’s equilibrium unit-fee revenue equals

\[
R^u = \frac{(2 - c_1 - c_2)^2}{8(2 + b)}, \quad (A25)
\]
and the optimal minimum throughput guarantee is \( \delta^u \in \left[ 0, \frac{2(2-b) + (2+3b)c_1 - (b+6)c_2}{4(4-b^2)} \right] \) if \( c_2 \leq \hat{c}_2 \).

**Case 1b:** Suppose \( \lambda_2^* > 0 \). Then \( (24) \) suggests \( \bar{r} = \frac{(2-b) + bc_1 - 2c_2}{2-b} \), and \( (21) \) suggests \( \lambda_2^* = \frac{[-2(2-b) - (2+3b)c_1 + (b+6)c_2]}{4(4-b^2)} \). We have \( r^u \geq 0 \) iff \( c_2 \leq \bar{c}_2 \equiv \frac{(2-b) + bc_1}{2} \), \( \lambda_2^* > 0 \) iff \( c_2 > \hat{c}_2 \), and \( \delta^u = 0 \) due to \( \delta_1 = \frac{1}{2 + b} + \frac{bc_1 - 2c_2}{(4-b^2)} = 0 \). At the equilibrium, port authority’s fee revenue equals

\[
R^u = \frac{(c_2 - c_1)[(2 - b) + bc_1 - 2c_2]}{(2 - b)^2}.
\]

**Case 2:** Suppose \( \delta \in (\delta_1, \delta_2) \). Then, Lemma 2(ii) implies \( \pi^u_1 > \pi^u_2 = \frac{\delta}{2}[(2-b)(1-r) - (2-b^2)\delta + bc_1 - 2c_2] \) and \( \pi^u_2 \geq 0 \) iff \( \delta \leq \frac{[(2-b)(1-r) + bc_1 - 2c_2]}{(2-b^2)} \) and \( r \leq \bar{r} \equiv \frac{(2-b) + bc_1 - 2c_2}{2-b} \). Moreover, because \( \frac{[(2-b)(1-r) + bc_1 - 2c_2]}{(2-b^2)} \geq (\langle \rangle) \delta_2 \) iff \( r \leq (\rangle) [1 + (1+b)c_1 - (2+b)c_2] \), and \( [1 + (1+b)c_1 - (2+b)c_2] < \bar{r} \) by \( c_1 < c_2 \), we have two sub-cases below.

**Case 2a:** Suppose \( 0 < r \leq [1 + (1+b)c_1 - (2+b)c_2] \). Then the problem in (16) becomes

\[
\max_{r, \delta} r(q_1^u + q_2^u) \\
\text{s.t. } \delta_1 < \delta \leq \delta_2 \text{ and } 0 < r \leq [1 + (1+b)c_1 - (2+b)c_2].
\]

Its Lagrange function is

\[
L = \frac{r[1 + (2-b)\delta - c_1 - r]}{2} + \lambda_1(\delta - \delta_1) + \lambda_2(\delta_2 - \delta) + \lambda_3[1 + (1+b)c_1 - (2+b)c_2 - r].
\]

Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = \frac{[1 + (2-b)\delta - c_1 - 2r]}{2} + \frac{\lambda_1}{2 + b} - \frac{\lambda_2}{2 + b} - \lambda_3 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \quad (A27)
\]

\[
\frac{\partial L}{\partial \delta} = \frac{(2-b)r}{2} + \lambda_1 - \lambda_2 \leq 0, \quad \delta \frac{\partial L}{\partial \delta} = 0, \quad (A28)
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta - \delta_1 \geq 0, \quad \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \quad (A29)
\]

\[
\frac{\partial L}{\partial \lambda_2} = \delta_2 - \delta \geq 0, \quad \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0, \quad (A30)
\]


\[ \frac{\partial L}{\partial \lambda_3} = 1 + (1 + b)c_1 - (2 + b)c_2 - r \geq 0, \ \lambda_3 \frac{\partial L}{\partial \lambda_3} = 0. \quad (A31) \]

Since \( \delta > \delta_1 \), we have \( \lambda_1^* = 0 \) suggested by (A29). Thus, \( \lambda_2^* > 0 \) is inferred from (A28) and \( r > 0 \), and two sub-cases are as follows.

**Case 2a-1:** Suppose \( \lambda_3^* > 0 \). Then (A30) and (A31) suggest 
\( r^u = 1 + (1 + b)c_1 - (2 + b)c_2 \) and \( \delta^u = \delta_2 = (c_2 - c_1) > 0 \). Moreover, (A28) implies \( \lambda_2^* = \frac{(2 - b)r^u}{2} = \frac{(2 - b)[1 + (1 + b)c_1 - (2 + b)c_2]}{2(2 + b)} \), and (A27) implies \( \lambda_3^* = \frac{-2[1 + (3 + 2b)c_1 - 2(2 + b)c_2]}{(2 + b)} \). By some calculations, we have \( \delta^u = \delta_2 > \delta_1, \ r^u > 0, \ \lambda_2^* > 0 \) iff 
\( c_2 > \frac{1 + (3 + 2b)c_1}{2(2 + b)} \) with \( \frac{1 + (3 + 2b)c_1}{2(2 + b)} < \frac{1 + (1 + b)c_1}{(2 + b)} \). Thus, under \( \frac{1 + (3 + 2b)c_1}{2(2 + b)} < c_2 < \frac{1 + (1 + b)c_1}{(2 + b)} \), the equilibrium exists and port authority’s fee revenue equals

\[ R^u = 2(c_2 - c_1)[1 + (1 + b)c_1 - (2 + b)c_2]. \quad (A32) \]

**Case 2a-2:** Suppose \( \lambda_3^* = 0 \). Then (A27)-(A31) suggest 
\( r^u = \frac{1 - c_1}{2} > 0 \), \( \delta^u = \delta_2 = \frac{1 - c_1}{2(2 + b)} > 0 \), and \( \lambda_2^* = \frac{(2 - b)(1 - c_1)}{4} > 0 \). By some calculations, we have \( [1 + (1 + b)c_1 - (2 + b)c_2] - \frac{1 - c_1}{2} = \frac{1}{2}[1 + (3 + 2b)c_1 - 2(2 + b)c_2] \geq 0 \) iff 
\( c_2 \leq \bar{c}_2 \equiv \frac{1 + (3 + 2b)c_1}{2(2 + b)} \). Thus, under condition 
\( c_2 \leq \bar{c}_2 \), we have \( r^u \leq [1 + (1 + b)c_1 - (2 + b)c_2] \), and the equilibrium exists with port authority’s fee revenue equal to

\[ R^u = \frac{(1 - c_1)^2}{2(2 + b)}. \quad (A33) \]

**Case 2b:** Suppose \( r > [1 + (1 + b)c_1 - (2 + b)c_2] \). Then, the problem in (16) becomes

\[
\max_{r, \delta} r(q_1^u + q_2^u)
\]

s.t. \( \delta_1 < \delta \leq \frac{[(2 - b)(1 - r) + bc_1 - 2c_2]}{(2 - b^2)} \) and \( [1 + (1 + b)c_1 - (2 + b)c_2] < r \leq \bar{r} \).

Its Lagrange function is

\[
L = \frac{r}{2}[1 + (2 - b)\delta - c_1 - r] + \lambda_1(\delta - \delta_1) + \lambda_2\left\{\frac{[(2 - b)(1 - r) + bc_1 - 2c_2]}{(2 - b^2)} - \delta\right\}
\]
\[ + \lambda_3\{r - [1 + (1 + b)c_1 - (2 + b)c_2]\} + \lambda_4(\bar{r} - r). \]
Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = \frac{[1 + (2 - b)\delta - c_1 - 2r]}{2} + \frac{\lambda_1}{2 + b} - \frac{(2 - b)\lambda_2}{(2 - b^2)} + \lambda_3 - \lambda_4 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \quad (A34) \\
\frac{\partial L}{\partial \delta} = \frac{r(2 - b)}{2} + \lambda_1 - \lambda_2 \leq 0, \quad \delta \frac{\partial L}{\partial \delta} = 0, \quad (A35) \\
\frac{\partial L}{\partial \lambda_1} = \delta - \delta_1 \geq 0, \quad \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \quad (A36) \\
\frac{\partial L}{\partial \lambda_2} = \frac{(2 - b)(1 - r) + bc_1 - 2c_2}{(2 - b^2)} - \delta \geq 0, \quad \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0, \quad (A37) \\
\frac{\partial L}{\partial \lambda_3} = r - [1 + (1 + b)c_1 - (2 + b)c_2] \geq 0, \quad \lambda_3 \frac{\partial L}{\partial \lambda_3} = 0, \quad \text{and} \quad (A38) \\
\frac{\partial L}{\partial \lambda_4} = \bar{r} - r \geq 0, \quad \lambda_4 \frac{\partial L}{\partial \lambda_4} = 0. \quad (A39)
\]

Since \(\delta > \delta_1\) and \(r > [1 + (1 + b)c_1 - (2 + b)c_2]\), \(\lambda_1^* = \lambda_2^* = 0\) are implied by (A36) and (A38). Thus, (A35) suggests \(\lambda_2^* > 0\), and two sub-cases are as follows.

**Case 2b-1:** Suppose \(\lambda_4^* > 0\). Then, \(r^u = \bar{r}\) is implied by (A39), and \(\delta^u = 0\) by (A37). However, since \(\delta_1 = \frac{1 - r^u}{2 + b} + \frac{bc_1 - 2c_2}{4 - b^2} = 0\), we have \(\delta^u = \delta_1 = 0\), which contradicts \(\delta > \delta_1\). Thus, no solution exists in this case.

**Case 2b-2:** Suppose \(\lambda_4^* = 0\). Then (A34)-(A35), (A37), and \(\lambda_2^* > 0\) suggest \(r^u = \frac{[3 - (2b - 1)(c_1 - 2b)c_2]}{2(2 - b)}\), \(\delta^u = \frac{(2 - b)(1 - r^u) + bc_1 - 2c_2}{2(2 - b)}\), and \(\lambda_2^* = \frac{(2 - b)r^u}{2}\). By some calculations, we have \(r^u - [1 + (1 + b)c_1 - (2 + b)c_2] = \frac{-[(3 - 2b - (7 + b - b^2)c_1 + (10 - b - b^2)c_2]}{2(3 - 2b)}\) and \((\bar{r} - r^u) = \frac{[(2 - b)(3 - 2b) + (2 + 3b - 3b^2)c_1 - (8 - 4b - b^2)c_2]}{2(2 - b)(3 - 2b)}\). Thus, \(r^u > [1 + (1 + b)c_1 - (2 + b)c_2] \iff c_2 > c_2^* \equiv \frac{(3 - 2b) + (7 + b - b^2)c_1}{(2 - b)}\) with \(\frac{(3 - 2b) + (7 + b - b^2)c_1}{(10 - 6b - b^2)} < c_2^* \equiv \frac{(2 - b)(3 - 2b) + (2 + 3b - 3b^2)c_1}{(8 - 4b - b^2)} < \frac{(3 - 2b) - (1 - b)c_1}{(2 - b)}\). Moreover, we have \(\lambda_2^* > 0\) and \(\delta^u > \delta_1\) implied by \(r^u > 0\) and \(r^u \leq \bar{r}\), respectively. Thus, under condition \(\frac{(3 - 2b) + (7 + b - b^2)c_1}{(10 - 6b - b^2)} < c_2 \leq \frac{(2 - b)(3 - 2b) + (2 + 3b - 3b^2)c_1}{(8 - 4b - b^2)}\), the equilibrium exists and port authority’s fee revenue equals

\[
R^u = \frac{[(3 - 2b) - (1 - b)c_1 - (2 - b)c_2]^2}{4(3 - 2b)(2 - b^2)}. \quad (A40)
\]
Case 3: Suppose $\delta \in (\delta_2, \frac{1}{(1+b)})$. Then, Lemma 2(iii) suggests $\pi^u_1 > \pi^u_2 = \delta[1 - (1 + b)\delta - c_2 - r] \geq 0$ iff $\delta \leq \frac{1-c_2 - r}{(1+b)}$ and $r < (1-c_2)$ with $(1-c_2) > \bar{r}$. Thus, the problem in (16) becomes

$$\max_{r, \delta} 2r\delta$$

s.t. $\delta_2 < \delta \leq \frac{(1-c_2 - r)}{(1+b)}$ and $0 < r \leq \bar{r}$.

Its Lagrange function is

$$L = 2r\delta + \lambda_1[\delta - \delta_2] + \lambda_2\left[\frac{(1-c_2 - r)}{(1+b)} - \delta\right] + \lambda_3(\bar{r} - r).$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = 2\delta + \frac{\lambda_1}{(2+b)} - \frac{\lambda_2}{(1+b)} - \lambda_3 \leq 0, \quad r \frac{\partial L}{\partial r} = 0,$$

$$\frac{\partial L}{\partial \delta} = 2r + \lambda_1 - \lambda_2 \leq 0, \quad \delta \frac{\partial L}{\partial \delta} = 0,$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_2 \geq 0, \quad \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0,$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{1 - c_2 - r}{(1+b)} - \delta \geq 0, \quad \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0, \quad \text{and}$$

$$\frac{\partial L}{\partial \lambda_3} = \bar{r} - r \geq 0, \quad \lambda_3 \frac{\partial L}{\partial \lambda_3} = 0.$$

Since $\delta > \delta_2$, we have $\lambda^*_1 = 0$ by (A43), and $\lambda^*_2 > 0$ by $r > 0$ and (A42). Thus, there are two sub-cases as follows.

Case 3a: Suppose $\lambda^*_3 = 0$. Then, (A41), (A42), and (A44) suggest $r^u = \frac{1-c_2}{2} > 0$, $\delta^u = \frac{1-c_2}{2(1+b)} > 0$, and $\lambda^*_2 = \frac{2}{2(1+b)} > 0$. Under the circumstance, we have $(\bar{r} - r^u) = \frac{(2-b) + 2bc_1 - 2c_2 - \frac{1-c_2}{2}}{(2-b)} - \frac{1-c_2}{2} = \frac{(2-b) + 2bc_1 - (2+b)c_2}{2(2-b)}$ and $(\delta^u - \delta_2) = \frac{1-c_2}{2(1+b)} - \frac{(1-c_1 - r^u)}{(2+b)} = \frac{1+2(1+b)c_1 - (3+2b)c_2}{2(1+b)(2+b)}$. Thus, $\bar{r} \geq r^u$ iff $c_2 \leq \frac{(2-b) + 2bc_1}{(2+b)}$, and $\delta^u > \delta_2$ iff $c_2 < c_2'' \equiv \frac{1+2(1+b)c_1}{(3+2b)}$ with $\frac{(2-b) + 2bc_1}{(2+b)} > \frac{1+2(1+b)c_1}{(3+2b)}$. For $c_2 < \frac{1+2(1+b)c_1}{(3+2b)}$, the equilibrium exists with $r^u = \frac{1-c_2}{2}$ and $\delta^u = \frac{1-c_2}{2(1+b)}$. At the equilibrium, port authority’s fee revenue equals

$$R^u = \frac{(1-c_2)^2}{2(1+b)}.$$
Case 3b: Suppose \( \lambda_3^* > 0 \). Then we have \( r^u = \bar{r} \) by (A45) and \( \delta^u = \frac{b(c_2-c_1)}{(1+b)(2-b)} \) by (A41). However, condition \( (\delta^u - \delta_2) = -\frac{b(c_2-c_1)}{(1+b)(2-b)} - 1 - \frac{\bar{r} - \bar{r}^u}{(2+b)} < 0 \) contradicts \( \delta > \delta_2 \). Thus, no solution exists in this case.

Based on the above, we can derive optimal unit-fee schemes as follows. We first compare the values of \( \hat{c}_2 \) of Case 3a exist. In this parameter range, we have (A46). First, for \( c_2 \in (c_1, \hat{c}_2] \), equilibria of \( R^u = \frac{(2-c_1-c_2)^2}{8(2+b)} \) in (A25) of Case 1a, \( R^u = \frac{(1-c_2)^2}{2(2+b)} \) in (A33) of Case 2a-2, and \( R^u = \frac{(1-c_2)^2}{2(1+b)} \) in (A46) of Case 3a exist. Defining \( M_1 = \frac{(1-c_2)^2 - (2-c_1-c_2)^2}{2(1+b)} \) and \( M_2 = \frac{(1-c_2)^2 - (1-c_2)^2}{2(2+b)} \). Since \( \frac{\partial M_2}{\partial c_2} = \frac{-(1-c_2)^2}{(1+b)^2} + \frac{(2-c_1-c_2)^2}{2(2+b)} < 0 \) and \( \frac{\Delta^2 M_2}{\Delta c_2^2} = \frac{(7+3b)}{4(1+b)(2+b)^2} > 0 \), we have \( M_1 = \frac{(1-c_2)^2 - (2-c_1-c_2)^2}{2(1+b)} > \frac{(1-c_2)^2}{(2+b)} > \frac{(1-c_2)^2}{2(1+b)} \) and thus \( (1-c_2)^2 > \frac{(2-c_1-c_2)^2}{2(1+b)} \). Similarly, since \( \frac{\partial M_2}{\partial c_2} = \frac{-(1-c_2)^2}{(1+b)^2} < 0 \) and \( \frac{\Delta^2 M_2}{\Delta c_2^2} = \frac{1}{1+b} > 0 \), we have \( M_2 = \frac{(1-c_2)^2 - (2-c_1-c_2)^2}{2(1+b)} > \frac{(1-c_2)^2}{(2+b)} > \frac{(1-c_2)^2}{2(1+b)} \). Hence, for \( c_2 \in (c_1, \hat{c}_2] \), the port authority will choose the unit-fee scheme in Case 3a with \( r^u = \frac{(1-c_2)^2}{2(1+b)} \) and \( \delta^u = \frac{(1-c_2)^2}{2(1+b)} \), and obtain the equilibrium fee revenue \( R^u = \frac{(1-c_2)^2}{2(1+b)} \) in (A46).

Second, for \( c_2 \in (\hat{c}_2, \hat{c}_2'] \), equilibria of \( R^u = \frac{(2-c_1-c_2)^2}{8(2+b)} \) in (A25) of Case 1a, \( R^u = 2(c_2 - c_1)[1 + (1 + b)c_1 - (2 + b)c_2] \) in (A32) of Case 2a-1, and \( R^u = \frac{(1-c_2)^2}{2(1+b)} \) in (A46) of Case 3a exist. In this parameter range, we have \( \frac{\partial M_2}{\partial c_2} = \frac{-(1-c_2)^2}{(1+b)^2} + \frac{(2-c_1-c_2)^2}{2(2+b)} < 0 \) and \( \frac{\Delta^2 M_2}{\Delta c_2^2} = \frac{(7+3b)}{4(1+b)(2+b)^2} > 0 \). Accordingly, \( M_1 = \frac{(1-c_2)^2 - (2-c_1-c_2)^2}{2(1+b)} > \frac{(1-c_2)^2}{(2+b)} > \frac{(1-c_2)^2}{2(1+b)} > \frac{(2-c_1-c_2)^2}{8(2+b)} \).

\[1\] That is because \( (c_2' - \hat{c}_2) = \frac{(1-c_2)(1+b)}{2(2+b)(10-b-4b')}, \quad (c_2 - \hat{c}_2'') = \frac{(1-c_2)(1+b)}{2(2+b)(10-b-4b')}, \quad (c_2' - \hat{c}_2') = \frac{(1-c_2)(1+b)}{2(2+b)(10-b-4b')}, \quad (c_2'' - c_2) = \frac{(1-c_2)(1+b)}{2(2+b)(10-b-4b')}, \quad (c_2' - c_2') = \frac{(1-c_2)(1+b)}{2(2+b)(10-b-4b')}, \quad (c_2' - c_2'') = \frac{(1-c_2)(1+b)}{2(2+b)(10-b-4b')}, \quad (c_2' - c_2''') = \frac{(1-c_2)(1+b)}{2(2+b)(10-b-4b')}, \quad (c_2' - c_2''') = \frac{(1-c_2)(1+b)}{2(2+b)(10-b-4b')}, \quad (c_2' - c_2''') = \frac{(1-c_2)(1+b)}{2(2+b)(10-b-4b')}, \quad (c_2' - c_2''') = \frac{(1-c_2)(1+b)}{2(2+b)(10-b-4b')}.
Accordingly, we have equilibrium fee revenue
\[
\frac{(1-c_1)^2(7+8b)}{8(2+b)(3+2b)^2} > 0, \quad \text{and thus } \frac{(1-c_2)^2}{2(1+b)} > \frac{(2-c_1-c_2)^2}{8(2+b)}.
\]
On the other hand, we have \(\frac{(1-c_2)^2}{2(1+b)} - 2(c_2-c_1)[1+(1+b)c_1-2c_2] > 0\). Accordingly, for \(c_2 \in (c_1, c_2]\), the port authority will choose the unit-fee scheme in Case 3a with \(r^u = \frac{1-c_2}{2}\) and \(\delta^u = \frac{1-c_2}{2(1+b)}\), and obtain the equilibrium fee revenue \(R^u = \frac{(1-c_2)^2}{2(1+b)}\) in (A46).

Third, for \(c_2 \in (c_2', c_2'')\), equilibria of \(R^u = \frac{(2-c_1-c_2)^2}{8(2+b)}\) in (A25) of Case 1a, \(R^u = 2(c_2-c_1)[1+(1+b)c_1-2c_2]\) in (A32) of Case 2a-1, \(R^u = \frac{|(3-2b)-(1-b)c_1-(2-b)c_2|^2}{4(3-2b)(2-b^2)}\) in (A40) of Case 2b-2, and \(R^u = \frac{(1-c_2)^2}{2(1+b)}\) in (A46) of Case 3a exist. Similarly, we can show \(\frac{(1-c_2)^2}{2(1+b)} > 2(c_2-c_1)[1+(1+b)c_1-2c_2]\) and \(\frac{(1-c_2)^2}{2(1+b)} > \frac{(2-c_1-c_2)^2}{8(2+b)}\). Thus, it remains to compare \(\frac{(1-c_2)^2}{2(1+b)}\) and \(\frac{|(3-2b)-(1-b)c_1-(2-b)c_2|^2}{4(3-2b)(2-b^2)}\). Define \(M_3 = \frac{(1-c_2)^2}{2(1+b)} - \frac{|(3-2b)-(1-b)c_1-(2-b)c_2|^2}{4(3-2b)(2-b^2)}\).

Since \(\frac{\partial M_3}{\partial c_2} = -\frac{(1-c_2)}{2(1+b)} + \frac{(2-b)[(3-2b)-(1-b)c_1-(2-b)c_2]}{4(3-2b)(2-b^2)}\) and \(\frac{\partial^2 M_3}{\partial c_2^2} = \frac{(1-c_2)}{2(1+b)} + \frac{(2-b)[(3-2b)-(1-b)c_1-(2-b)c_2]}{4(3-2b)(2-b^2)} > 0\), we have \(\frac{\partial M_3}{\partial c_2} < 0\) and \(\frac{\partial^2 M_3}{\partial c_2^2} > 0\). Thus, \(M_3 < 0\), and \(M_3 > \frac{(1-c_2)^2}{2(1+b)} - \frac{|(3-2b)-(1-b)c_1-(2-b)c_2|^2}{4(3-2b)(2-b^2)}\) \(= -\frac{(1-c_2)^2}{4(3-2b)(2-b^2)} < 0\). These imply that \(M_3\) can be positive or negative for \(c_2 \in (c_2', c_2'')\), and there must exist some \(\hat{c}_2, c'_2 < \hat{c}_2 < c''_2\), at which \(M_3 = 0\). Solving \(M_3 = 0\) yields \(\hat{c}_2 = \frac{1}{8(3-2b^2)}\left\{[(2+b)(3-2b)+(1+b)(2-b)]c_1 - \sqrt{2(1+b)(3-2b)(2-b^2)(1-c_1)^2}\right\}\). Accordingly, we have \(\frac{(1-c_2)^2}{2(1+b)} \geq \frac{|(3-2b)-(1-b)c_1-(2-b)c_2|^2}{4(3-2b)(2-b^2)}\) for \(c_2 < c_2 \leq \hat{c}_2\) \((\hat{c}_2 < c_2 < c''_2)\). For \(c_2 \in (c_2', c''_2)\), there are two sub-cases. If \(c_2 \in (c_2', \hat{c}_2\), the port authority will choose the unit-fee scheme in Case 3a with \(r^u = \frac{1-c_2}{2}\) and \(\delta^u = \frac{1-c_2}{2(1+b)}\), and obtain the equilibrium fee revenue \(R^u = \frac{(1-c_2)^2}{2(1+b)}\) in (A46). If \(c_2 \in (\hat{c}_2, c''_2)\), the port authority will choose the unit-fee scheme in Case 2b-2 with \(r^u = \frac{|(3-2b)-(1-b)c_1-(2-b)c_2|^2}{2(3-2b)}\) and \(\delta^u = \frac{(2-b)(1-r^u)+bc_1-2c_2}{(2-b^2)}\), and obtain the equilibrium fee revenue \(R^u = \frac{|(3-2b)-(1-b)c_1-(2-b)c_2|^2}{4(3-2b)(2-b^2)}\) in (A40).

Fourth, we have either \(c_2'' = \frac{1+(1+b)c_1}{2(1+b)} < \hat{c}_2 \equiv \frac{1}{(b+6)}[2(2-b) + (2 + 3b)c_1]\) or \(\hat{c}_2 < c''_2\). If \(c_2'' < \hat{c}_2\), then \((2 - b - 2b^2) > 0\). For \(c_2 \in (c_2'', c''_2)\), equilibria of \(R^u = \frac{(2-c_1-c_2)^2}{8(2+b)}\) in (A25) of Case 1a, \(R^u = 2(c_2-c_1)[1 + (1+b)c_1 - 2c_2]\) in (A32) of Case 2a-1, and \(R^u = \frac{|(3-2b)-(1-b)c_1-(2-b)c_2|^2}{4(3-2b)(2-b^2)}\) in (A40) of Case 2b-2 exist. Define \(M_4 = \frac{|(3-2b)-(1-b)c_1-(2-b)c_2|^2}{4(3-2b)(2-b^2)} - \frac{(2-c_1-c_2)^2}{8(2+b)}\). Since \(\frac{\partial M_4}{\partial c_2} = \frac{(2-b)[(3-2b)-(1-b)c_1-(2-b)c_2]}{2(3-2b)(2-b^2)} + \frac{(2-c_1-c_2)^2}{4(2+b)}\) and
\( \frac{\partial^2 M_4}{\partial c_2^2} = \frac{10-4b-b^2}{4(2+b)(3-2b)(2-b^2)} > 0 \), so we have \( \frac{\partial M_4}{\partial c_2} = \frac{-7(1-c_1)}{4(3-2b)(2-b^2)} < 0 \) and \( M_4 > \frac{[3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} - \frac{2(c_2-c_1)^2}{8(2+b)} = \frac{(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > 0 \) by 
\((5-2b-4b^2) > (2-b-2b^2) > 0 \). Then, we have 
\[ \frac{[3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > \frac{(2-c_2-c_1)^2}{8(2+b)} \].

On the other hand, we have 
\[ \frac{[3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} - 2(c_2-c_1)[1 + (1+b)c_1 - (2 + b)c_2] = \frac{([3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > 0 \]. Thus, \( R^u = \frac{[3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} \) in (A40) is optimal for the port authority if \( c_2 \in [c_2', c_2''] \) with \( c_2'' < c_2' \).

For \( c_2 \in [c_2'', \hat{c}_2] \), equilibria of \( R^u = \frac{(2-c_2-c_1)^2}{8(2+b)} \) in (A25) of Case 1a and \( R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} \) in (A40) of Case 2b-2 exist. Since 
\( \frac{\partial^2 M_4}{\partial c_2^2} = \frac{10-4b-b^2}{4(2+b)(3-2b)(2-b^2)} > 0 \), and 
\( \frac{\partial M_4}{\partial c_2} = \frac{-7(1-c_1)}{4(3-2b)(2-b^2)} < 0 \), we have \( M_4 > \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} - \frac{2(c_2-c_1)^2}{8(2+b)} = \frac{(2-c_2-c_1)^2}{8(2+b)} > 0 \). Thus, 
\[ \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > \frac{(2-c_2-c_1)^2}{8(2+b)} \]. It implies that for \( c_2 \in [c_2'', \hat{c}_2] \) with \( c_2'' < c_2' \), the port authority will choose the unit-fee scheme in Case 2b-2 with 
\( r^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{2(3-2b)} \) (8(2+b)) and \( \delta^u = \frac{(2-b)(1-r^u)+bc_1-2c_2}{(2-b^2)} \), and get the equilibrium fee revenue 
\( R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} \) in (A40).

By contrast, if \( c_2'' > \hat{c}_2 \), then \( (2-b-2b^2) < 0 \). For \( c_2 \in [c_2'', \hat{c}_2] \), equilibria of 
\( R^u = \frac{(2-c_2-c_1)^2}{8(2+b)} \) in (A25) of Case 1a, \( R^u = 2(c_2-c_1)[1 + (1+b)c_1 - (2 + b)c_2] \) in (A32) of Case 2a-1, and 
\( R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} \) in (A40) of Case 2b-2 exist. Similarly, we can show 
\[ \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > \frac{(2-c_2-c_1)^2}{8(2+b)} \), and \( \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > 2(c_2-c_1)[1 + (1+b)c_1 - (2 + b)c_2] \). For \( c_2 \in (c_2', c_2'') \), equilibria of 
\( R^u = 2(c_2-c_1)[1 + (1+b)c_1 - (2 + b)c_2] \) in (A32) of Case 2a-1 and 
\( R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} \) in (A40) of Case 2b-2 exist. We can show 
\[ \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} > 2(c_2-c_1)[1 + (1+b)c_1 - (2 + b)c_2] \). These imply that for \( c_2 \in [c_2'', c_2'''] \) with \( c_2'' < c_2' \), the port authority will choose the unit-fee scheme in Case 2b-2 with 
\( \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} \) and \( \delta^u = \frac{(2-b)(1-r^u)+bc_1-2c_2}{(2-b^2)} \), and get the equilibrium fee revenue 
\( R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} \) in (A40).

Fifth, we have either \( c_2 \in [\hat{c}_2, \hat{c}_2] \) with \( c_2'' < \hat{c}_2 \) or \( c_2 \in [c_2'', \hat{c}_2] \) with \( c_2'' > \hat{c}_2 \). In both cases, equilibria of 
\( R^u = \frac{(c_2-c_1)[(2-b)+bc_1-2c_2]}{(2-b^2)} \) in (A26) of Case 1b and 
\( R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b^2)} \) in (A40) of Case 2b-2 always exist. By some calculations, we can
Lemma 3 shows that suppose we have \((2-b)^2 \geq f\). Thus, the port authority will choose the unit-fee scheme in Case 2b-2 with \(r^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]}{2(3-2b)}\) and \(\delta^u = \frac{(2-b)(1-r^u)+bc_1-2c_2}{(2-b)^2}\), and obtain the equilibrium fee revenue \(R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b)^2}\) in (A40).

Sixth, for \(c_2 \in [\bar{c}_2, \overline{c}_2]\), there is a unique equilibrium in Case 1b. Thus, the port authority will choose the unit-fee scheme with \(r^u = \tilde{r} \equiv \frac{(2-b)+bc_1-2c_2}{2-b}\) and \(\delta^u = 0\), and obtain the equilibrium fee revenue \(R^u = \frac{(c_2-c_1)[(2-b)+bc_1-2c_2]}{(2-b)^2}\) in (A26).

In sum, the first, second and third cases yield Lemma 5(i), the third, fourth and fifth cases provide Lemma 5(ii), and the sixth case gives Lemma 5(iii). \(\square\)

**Proof of Lemma 6:** Lemma 3 shows that \(q_1^f\) and \(q_2^f\) depend on the values of \(\delta\). Thus, we have three cases below.

**Case 1:** Suppose \(\delta \in (0, \delta_3]\) with \(\delta_3 = \frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}\). Then Lemma 3(i) suggests \(\pi_1^f > \pi_2^f = (q_2^f)^2 - f\), and \(f^f = \frac{1}{2}[(\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2})]^2\). The problem in (17) thus becomes

\[
\max_{\delta} \left[ \frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2} \right]^2
\]

s.t. \(\delta \leq \delta_3 = \frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}\). \(\quad (A47)\)

Since the constraint in (A47) is independent of the objective function of this problem, equilibrium \(\delta^f\) can be any value within \([0, \delta_3]\), and port authority’s equilibrium fee revenue equals

\[
R^f = \left[ \frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2} \right]^2. \quad (A48)
\]

**Case 2:** Suppose \(\delta \in (\delta_3, \delta_4]\) with \(\delta_4 = \frac{1-c_1}{2+b}\). Then Lemma 3(ii) suggests \(\pi_1^f > \pi_2^f = \frac{\delta}{2}[(2-b)-(2-b^2)\delta + bc_1 - 2c_2] - f\), and \(f^f = \pi_2^f = \frac{\delta}{4}[(2-b)-(2-b^2)\delta + bc_1 - 2c_2]\).

We will have \(f^f > 0\) iff \(\delta < \frac{(2-b)+bc_1-2c_2}{(2-b^2)}\) and \(c_2 < \bar{c}_2 \equiv \frac{(2-b)+bc_1}{2}\). On the other hand, we have \(\frac{(2-b)+bc_1-2c_2}{(2-b^2)} > (\leq) \delta_4 \) iff \(c_2 < (\geq) \frac{1+(1+b)c_1}{(2+b)}\) with \(\frac{1+(1+b)c_1}{(2+b)} < \bar{c}_2\). Thus, there are two sub-cases as follows.
Case 2a: Suppose $c_2 \leq \frac{1+(1+b)c_1}{(2+b)}$. Then the problem in (17) becomes

$$\max_\delta \frac{\delta}{2}[(2-b) - (2-b^2)\delta + bc_1 - 2c_2]$$

s.t. $\delta_3 < \delta \leq \delta_4 = \frac{1-c_1}{2+b}$.

Its Lagrange function is

$$L = \frac{\delta}{2}[(2-b) - (2-b^2)\delta + bc_1 - 2c_2] + \lambda_1(\delta - \delta_3) + \lambda_2(\delta_4 - \delta).$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial \delta} = \frac{1}{2}[(2-b) - 2(2-b^2)\delta + bc_1 - 2c_2] + \lambda_1 - \lambda_2 \leq 0, \ \delta \frac{\partial L}{\partial \delta} = 0, \ \text{(A49)}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_3 \geq 0, \ \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \ \text{and}$$

$$\frac{\partial L}{\partial \lambda_2} = \delta_4 - \delta \geq 0, \ \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0. \ \text{(A50)}$$

Because $\delta > \delta_3$, we have $\lambda_1^* = 0$ by (A50), and the following two sub-cases.

Case 2a-1: Suppose $\lambda_2^* > 0$. Then, (A51) implies $\delta^f = \delta_4 = \frac{1-c_1}{2+b}$ and (A49) suggests $\lambda_2^* = \frac{b^2+(4+2b-b^2)c_1-2(2+b)c_2}{2(2+b)}$. To have $\lambda_2^* > 0$, we need $c_2 < \frac{b^2+(4+2b-b^2)c_1}{2(2+b)}$. In addition, we have $(\delta^f - \delta_3) = \frac{2(c_2-c_1)}{4-b^2} > 0$. With condition $c_2 < \frac{b^2+(4+2b-b^2)c_1}{2(2+b)}$, port authority’s equilibrium fee revenue equals

$$R^f = \frac{(1-c_1)[1+(1+b)c_1 - (2+b)c_2]}{(2+b)^2}. \ \text{(A52)}$$

Case 2a-2: Suppose $\lambda_2^* = 0$. Then we have $\delta^f = \frac{(2-b)+bc_1-2c_2}{2(2+b)}$ by (A49). By some calculations, we have $(\delta_4 - \delta^f) = \frac{[-b^2-(4+2b-b^2)c_1+2(2+b)c_2]}{2(2+b)(2-b^2)} \geq 0$ iff $c_2 \geq \frac{b^2+(4+2b-b^2)c_1}{2(2+b)}$, and $\delta^f > \delta_3$ iff $c_2 < \tilde{c}_2$. Combining this condition with $c_2 < \frac{1+(1+b)c_1}{(2+b)}$ and $\frac{b^2+(4+2b-b^2)c_1}{2(2+b)} < \frac{1+(1+b)c_1}{(2+b)}$, we assume $\frac{b^2+(4+2b-b^2)c_1}{2(2+b)} \leq c_2 < \frac{1+(1+b)c_1}{(2+b)}$. Under the circumstance, port authority’s equilibrium fee revenue equals

$$R^f = \frac{[(2-b) + bc_1 - 2c_2]^2}{8(2-b^2)}. \ \text{(A53)}$$
Case 2b: Suppose \( \frac{1+(1+b)c_1}{(2+b)} \leq c_2 < \tilde{c}_2 \equiv \frac{(2-b)+bc_1}{2} \). Then, the problem in (17) becomes

\[
\max_{\delta} \quad \frac{\delta}{2} \left[ (2-b) - (2-b^2)\delta + bc_1 - 2c_2 \right]
\]

s.t. \( \delta_3 < \delta \leq \frac{(2-b)+bc_1-2c_2}{(2-b^2)} \).

Its Lagrange function is

\[
L = \frac{\delta}{2} \left[ (2-b) - (2-b^2)\delta + bc_1 - 2c_2 \right] + \lambda_1 (\delta - \delta_3) + \lambda_2 \left[ \frac{(2-b)+bc_1-2c_2}{(2-b^2)} - \delta \right].
\]

Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial \delta} = \frac{1}{2} \left[ (2-b) - 2(2-b^2)\delta + bc_1 - 2c_2 \right] + \lambda_1 - \lambda_2 \leq 0, \quad \delta \frac{\partial L}{\partial \delta} = 0,
\]

(A54)

\[
\frac{\partial L}{\partial \lambda_1} = \delta - \delta_3 \geq 0, \quad \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \quad \text{and}
\]

(A55)

\[
\frac{\partial L}{\partial \lambda_2} = \left( \frac{2-b+b c_1-2c_2}{(2-b^2)} - \delta \right) \geq 0, \quad \lambda_2 \left( \frac{2-b+b c_1-2c_2}{(2-b^2)} - \delta \right) = 0.
\]

(A56)

Since \( \delta_3 < \delta \leq \frac{(2-b)+bc_1-2c_2}{(2-b^2)} \), we have \( \lambda_1^* = \lambda_2^* = 0 \) by (A55)-(A56) and \( \delta^f = \frac{(2-b)+bc_1-2c_2}{2(2-b^2)} \).

Then, the problem becomes

\[
\max_{\delta} \quad \frac{1}{2} \left[ (2-b) - (2-b^2)\delta + bc_1 - 2c_2 \right]
\]

s.t. \( \frac{1}{2} \left[ (2-b)+bc_1-2c_2 \right] < \delta < \frac{2-b+b c_1-2c_2}{(2-b^2)} \).

Its Lagrange function is

\[
L = \frac{1}{2} \left[ (2-b) - (2-b^2)\delta + bc_1 - 2c_2 \right] + \lambda_1 \left( \delta - \frac{1}{2} \left[ (2-b)+bc_1-2c_2 \right] \right) + \lambda_2 \left( \frac{2-b+b c_1-2c_2}{(2-b^2)} - \delta \right).
\]

Case 3: Suppose \( \delta \in \left( \delta_1, \frac{1}{1+b} \right) \). Then, Lemma 3(iii) suggests \( \pi_i^f = [1 - (1+b)\delta - c_i] \delta - f \)

for \( i = 1, 2 \), and thus \( f^f = \frac{\delta}{2} [1 - (1+b)\delta - c_2] \). To have \( f^f > 0 \), condition \( \delta < \frac{(1-c_2^f)}{(1+b)} \) is needed. The problem in (17) thus becomes

\[
\max_{\delta} \quad [1 - (1+b)\delta - c_2] \delta
\]

s.t. \( \frac{1-c_1}{2+b} < \delta < \frac{1-c_2}{1+b} \).

Its Lagrange function is

\[
L = [1 - (1+b)\delta - c_2] \delta + \lambda_1 (\delta - \frac{1-c_1}{2+b}) + \lambda_2 \left( \frac{1-c_2}{1+b} - \delta \right).
\]
Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial \delta} = 1 - c_2 - 2(1+b)\delta + \lambda_1 - \lambda_2 \leq 0, \quad \frac{\partial L}{\partial \delta} = 0, \tag{A58}
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta - \frac{1-c_1}{(2+b)} \geq 0, \quad \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \quad \text{and} \tag{A59}
\]

\[
\frac{\partial L}{\partial \lambda_2} = 1 - \frac{c_2}{(1+b)} - \delta \geq 0, \quad \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0. \tag{A60}
\]

Because \(\frac{1-c_1}{(2+b)} < \delta < \frac{1-c_2}{(1+b)}\), we have \(\lambda_1^* = \lambda_2^* = 0\) by (A59)-(A60). We can also obtain \(\delta f = \frac{1-c_2}{2(1+b)} > 0\) by (A58). However, since \((\delta f - \delta_4) = -\frac{b(1-c_1)-(2+b)(c_2-c_1)}{2(1+b)(2+b)} < 0\) due to \(0 < c_1 < c_2 < 1\), no solution exists in this case.

Based on the above, we can derive optimal fixed-fee schemes as follows. Since \(\tilde{c}_2 \equiv \frac{b^2+(1+2b-b^2)c_1}{2(1+b)} < 1+(1+b)c_1 \quad \text{and} \quad \tilde{c}_2 \equiv \frac{(2-b)+bc_1}{2(1+b)}\), we have three cases below.

First, for \(c_2 \in (c_1, \tilde{c}_2)\), equilibria of \(R^f = \frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}\) in (A48) of Case 1, and \(R^f = \frac{(1-c_1)(1+(1+b)c_1-(2+b)c_2)}{2(1+b)^2}\) in (A52) of Case 2a-1 exist. Define \(M_5 = \frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}\) when \(c_2 < \tilde{c}_2\). Since \(\frac{\partial M_5}{\partial c_2} = 8c_2(1-c_1)-8(1-c_1)(2+b)^2(2+b)^2 \) and \(\frac{\partial^2 M_5}{\partial c_2^2} = 8 \frac{4(2-b)^2(2+b)^2}{2(1+b)^2}\), we have \(M_5 < \max\{M_5|_{c_2=c_1}, M_5|_{c_2=\tilde{c}_2}\} = 0, \quad \frac{4(2-b)^2(2+b)^2}{2(1+b)^2}\) and thus \(\frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}\) if \(c_2 \in (c_1, \, \tilde{c}_2)\). The port authority will choose the fixed-fee scheme in Case 2a-1 with \(f^f = \frac{(1-c_1)(1+(1+b)c_1-(2+b)c_2)}{2(1+b)^2}\) and \(\delta f = \delta_4 = \frac{1-c_1}{2+b}\), and get the equilibrium fee revenue \(R^f = \frac{(1-c_1)(1+(1+b)c_1-(2+b)c_2)}{2(1+b)^2}\). These prove Lemma 6(i).

Second, for \(c_2 \in [\tilde{c}_2, \, \frac{1+(1+b)c_1}{(2+b)}]\), an equilibrium exists in Case 1 with \(R^f = \frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}\) in (A48), and an equilibrium exists in Case 2a-2 with \(R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}\) in (A53). Since \(\frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)} - \frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2} = \frac{4[(2-b)+bc_1-2c_2]^2}{8(2-b^2)(2-b)^2} > 0\), the port authority will choose the fixed-fee scheme in Case 2a-2 with \(f^f = \frac{[(2-b)+bc_1-2c_2]^2}{16(2-b^2)}\) and \(\delta f = \frac{(2-b)+bc_1-2c_2}{2(2-b)^2}\), and obtain the equilibrium fee revenue \(R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}\). These prove Lemma 6(ii).

Third, for \(c_2 \in [\frac{1+(1+b)c_1}{(2+b)}, \, \tilde{c}_2]\), an equilibrium exists in Case 1 with \(R^f = \frac{1}{2+b} + \frac{bc_1-2c_2}{4-b^2}\) in (A48), and an equilibrium exists in Case 2b with \(R^f = \frac{[(2-b)+bc_1-2c_2]^2}{8(2-b^2)}\) in
We can obtain optimal concession contracts by comparing the port authority’s equilibrium fee revenues derived in Lemmas 4-6. Since \( \tilde{c}_2 - \bar{c}_2 = \frac{(8+4b-7b^2-4b^3)(1-c_1)}{2(2+b)(7+4b)} > 0 \), \( c'_2 - \bar{c}_2 = \frac{(1-c_1)}{(7+4b)(10-b-4b^2)} > 0 \), and \( c'_2 < \tilde{c}_2 < \hat{c}_2 < \bar{c}_2 < \bar{c}_2 \) as shown in the proof of Lemma 5, we have \( \tilde{c}_2 < \hat{c}_2 < \tilde{c}_2 < \hat{c}_2 < \tilde{c}_2 \). Thus, we have the following six cases.

**Case 1:** Suppose \( c_2 \in [\hat{c}_2, \bar{c}_2] \). Then Lemma 5(iii) shows \( R^u = \frac{([c_2-c_1])((2-b)+bc_1-2c_2]}{(2-b)^s} \) in (A26), and Lemma 6(ii) displays \( R^f = \frac{([2-b]+bc_1-2c_2]}{8(2-b^2)} \) in (A57). Some calculations yield \( (R^u - R^f) = \frac{([2-b]+bc_1-2c_2]}{8(2-b^2)}((2-b)^s) \) > 0 due to \( (-2-b)^3 - (16 + 4b - 12b^2 + b^3)c_1 + (24 - 8b - 6b^2)c_2 \) > 0 by \( c_2 \geq \bar{c}_2 \) and \( \bar{c}_2 - \bar{c}_2 = \frac{(2-b)(2-b)^s(20-12b-2b^2)(1-c_1)}{(24-8b-6b^2)(24-8b-6b^2)} > 0 \), and \( (2-b) + bc_1 - 2c_2 > 0 \) by \( c_2 < \bar{c}_2 \). Thus, \( R^u > R^f \). Then, port authority’s best choice is the unit-fee contract with \( r^u = \frac{(2-b)+bc_1-2c_2}{2-b} \), \( \delta^u = 0 \), and \( R^u = \frac{([c_2-c_1])((2-b)+bc_1-2c_2]}{(2-b)^s} \).

**Case 2:** Suppose \( c_2 \in [\hat{c}_2, \bar{c}_2] \). Then Lemma 5(iii) shows \( R^u = \frac{([3-2b]-(1-b)c_1-(2-b)c_2]}{4(3-2b)(2-b^2)} \) in (A40), and Lemma 6(ii) displays \( R^f = \frac{([2-b]+bc_1-2c_2]}{8(2-b^2)} \) in (A57). Define \( M_6 = \frac{([3-2b]-(1-b)c_1-(2-b)c_2]}{4(3-2b)(2-b^2)} \) - \( \frac{([2-b]+bc_1-2c_2]}{8(2-b^2)} = \frac{([3-2b]-(2-3b)c_1+4c_2(1-b)c_1-(1-b)c_1^2-2c_2]}{8(3-2b)} \). Since \( \frac{\partial M_6}{\partial c_2} = \frac{(c_1-c_2)}{2(3-2b)} < 0 \), we have \( M_6 > \frac{([3-2b]-(1-b)c_1-(2-b)c_2]}{4(3-2b)(2-b^2)} - \frac{([2-b]+bc_1-2c_2]}{8(2-b^2)} = \frac{(-2b^2)(20-12b-2b^2)(1-c_1)^2}{8(8-4b-6b^2)^2} > 0 \). Thus, \( R^u > R^f \), which implies that port authority’s best choice is the unit-fee contract with \( r^u = \frac{([3-2b]-(1-b)c_1-(2-b)c_2]}{2(3-2b)} \), \( \delta^u = \frac{([2-b](1-r^u)+bc_1-2c_2]}{(2-b)^s} \), and \( R^u = \frac{1}{4(3-2b)} ((3-2b) - (1-b)c_1 - (2-b)c_2]^2 \).

**Case 3:** Suppose \( c_2 \in (\bar{c}_2, \hat{c}_2) \). Then Lemma 4(ii) demonstrates \( R^* = \frac{([2-b]+(2+3b)c_1-(6+b)c_2]}{2(2-b)(3+2b)} \) in (A11), Lemma 5(ii) shows \( R^u = \frac{([3-2b]-(1-b)c_1-(2-b)c_2]}{4(3-2b)(2-b^2)} \) in (A40), and Lemma 6(ii) displays \( R^f = \frac{([2-b]+bc_1-2c_2]}{8(2-b^2)} \) in (A57). Define \( M_7 = \frac{1}{4(3-2b)(2-b^2)} ([3-2b] - (1-b)c_1 - (2-b)c_2]^2 - \frac{r^*([2(1-r^u)-(c_1+c_2)]}{2+b} \) = \( \frac{1}{4(2-b^2)(3+2b)} \) in (A57).
\[[-12 + 20b + 2b^3 - 11b^2] + (24 - 40b + 10b^2 + 10b^3 - 4b^4)c_1 + (12b^2 - 14b^3 + 4b^4)c_2 + (-48 + 16b + 32b^2 - 2b^3 - 8b^4)c_1 c_2 + (12 + 12b - 21b^2 - 4b^3 + 6b^4)c_2^2 + (24 - 8b - 2b^2 + 8b^3 + 2b^4)c_2^3\]

with \(\frac{\partial M_7}{\partial c_2} = \frac{b^2(12 - 4b + 4b^2)(1 - c_1) + 4(12 - 4b - 11b^2 + 4b^3 + b^4)(c_2 - c_1)}{(-4(1 - 4b^2))(b^2)(2 - b^2)}\). Since \((12 - 14b + 4b^2) > 0\) by \(\frac{\partial (12 - 14b + 4b^2)}{\partial b} = -14 + 8b < 0\), and \((12 - 4b - 11b^2 + 4b^3 + b^4) > 0\) by \(\frac{\partial (12 - 4b - 11b^2 + 4b^3 + b^4)}{\partial b} = -4 - 22b + 12b^2 + 4b^3 < 0\), we have \(\frac{\partial M_7}{\partial c_2} < 0\) and \(M_7 > \frac{[(3 - 2b) - (1 - b)c_1 - (2 - b)c_2]^2 - [2(2 - b) + (3 + b)c_1 - (6 + b)c_2]^2 - \frac{2(1 - r - (c_1 + \tilde{c}_2))}{2 + b}}{4(3 - 2b)(2 - b^2)}\) \(\geq (2b - (1 - r_{u}) + bc_1 - 2c_2)(2 - b^2)\) and \(R^u = \frac{[(3 - 2b) - (1 - b)c_1 - (2 - b)c_2]^2}{4(3 - 2b)(2 - b^2)}\), and \(R^u = \frac{(3 - 2b) - (1 - b)c_1 - (2 - b)c_2}{2 + b}\).

**Case 4:** Suppose \(c_2 \in [\tilde{c}_2, \hat{c}_2]\). Then Lemma 4(iii) demonstrates \(R^u = \frac{(1 - c_2)^2}{2 + b}\) in (A11), Lemma 5(i) shows \(R^u = \frac{(1 - c_2)^2}{2 + b}\) in (A46), and Lemma 6(ii) displays \(R^f = \frac{[(2 - b) + bc_1 - 2c_2]^2}{8(2 - b)^2}\) in (A57). We can show \(R^u > R^f\) by \(c_2 < \hat{c}_2 < \tilde{c}_2\) as in Case 2, \(R^u > R^s\) due to \(\frac{(1 - c_2)^2}{2 + b} > \frac{[(3 - 2b) - (1 - b)c_1 - (2 - b)c_2]^2}{4(3 - 2b)(2 - b^2)}\) by \(c_2 \leq \hat{c}_2 < \tilde{c}_2\) as in the proof of Case 3 of Lemma 5, and \(\frac{[(3 - 2b) - (1 - b)c_1 - (2 - b)c_2]^2}{4(3 - 2b)(2 - b^2)} > R^s\) by \(c_2 < \hat{c}_2\) as shown in Case 3. Thus, \(R^u > R^s\) and \(R^u > R^f\). Then, port authority’s best choice is the unit-fee contract with \(r^u = \frac{1 - c_2}{2 + b}\), \(\delta^u = \frac{1 - c_2}{2 + b}\), and \(R^u = \frac{(1 - c_2)^2}{2 + b}\).

**Case 5:** Suppose \(c_2 \in [\hat{c}_2, \tilde{c}_2]\). Then Lemma 4(i) demonstrates \(R^s = \frac{(2 - c_1 - c_2)^2}{4(3 + 2b)}\) in (A11), Lemma 5(i) shows \(R^u = \frac{(1 - c_2)^2}{2 + b}\) in (A46), and Lemma 6(ii) displays \(R^f = \frac{[(2 - b) + bc_1 - 2c_2]^2}{8(2 - b)^2}\) in (A57). As in Case 4, we have \(R^u > R^f\) by \(c_2 < \hat{c}_2 < \tilde{c}_2\). It remains to compare \(R^s\) and \(R^u\). Define \(M_8 = \frac{(1 - c_2)^2}{2 + b} - \frac{(2 - c_1 - c_2)^2}{4(3 + 2b)}\). Since \(\frac{\partial M_8}{\partial c_2} = \frac{[-2(2 + b)(1 - b)c_1 + (5 + 3b)c_2]}{2(1 + b)(3 + 2b)}\) and \(\frac{\partial^2 M_8}{\partial c_2^2} = \frac{(5 + 3b)}{2(1 + b)(3 + 2b)} > 0\), we have \(\frac{\partial M_8}{\partial c_2} < \frac{[-2(2 + b)(1 - b)c_1 + (5 + 3b)c_2]}{2(1 + b)(3 + 2b)} < 0\) and \(M_8 > \frac{(1 - c_2)^2}{2 + b} - \frac{(2 - c_1 - c_2)^2}{4(3 + 2b)} = \frac{(1 - c_2)^2}{2 + b} > 0\). These imply \(R^u > R^s\). Thus, the port authority will choose the unit-fee scheme with \(r^u = \frac{1 - c_2}{2 + b}\), \(\delta^u = \frac{1 - c_2}{2 + b}\), and \(R^u = \frac{(1 - c_2)^2}{2 + b}\).

**Case 6:** Suppose \(c_2 \in (c_1, \hat{c}_2)\). Then Lemma 4(i) demonstrates \(R^s = \frac{(2 - c_1 - c_2)^2}{4(3 + 2b)}\) in (A19), Lemma 5(i) shows \(R^u = \frac{(1 - c_2)^2}{2 + b}\) in (A46), and Lemma 6(i) displays \(R^f = \frac{[(1 - c_1)(1 + b)c_1 - (2 + b)c_2]}{(2 + b)^2}\) in (A52). As in Case 5, we have \(R^u > R^s\) by \(c_2 < \hat{c}_2 < \tilde{c}_2\).
It remains to compare $R_u$ and $R_f$. Define $M_9 = \frac{(1-c_2)^2}{2(1+b)} - \frac{(1-c_1)[1+(1+b)c_1-(2+b)c_2]}{(2+b)^2}$. Since 
\[
\frac{\partial M_9}{\partial c_2} = \frac{-1-(1+b)c_1+(2+b)c_2}{(1+b)(2+b)} = \frac{1}{(1+b)} > 0,
\]
and 
\[
\frac{\partial^2 M_9}{\partial c_2^2} = \frac{-2(2-b)(1-c_1)}{(4+6b+2b^2)} < 0 \quad \text{and} \quad M_9 > \frac{(1-c_2)^2}{2(1+b)} - \frac{(1-c_1)[1+(1+b)c_1-(2+b)c_2]}{(2+b)^2} = \frac{8+8b+b^3}{8(1+b)(2+b)^2} > 0.
\]
These imply $R_u > R_f$. Thus, the port authority will choose the unit-fee scheme with $r_u = \frac{1-c_2}{2}$, $\delta_u = \frac{1-c_2}{2(1+b)}$, and $R_u = \frac{(1-c_2)^2}{2(1+b)}$.

In sum, Cases 4-6, Cases 2-3, and Case 1 show Proposition 1(i), Proposition 1(ii), and Proposition 1(iii), respectively. □

**Proof of Proposition 2.** Denote $R_1^u \equiv \frac{(1-c_2)^2}{2(1+b)}$, $R_2^u \equiv \frac{(3-2b)-(1-b)c_1-(2-b)c_2}{4(3-2b)(2-b)^2}$, and $R_3^u \equiv \frac{(c_2-c_1)(2-b)+bc_1-2c_2}{(2-b)^2}$ port authority’s equilibrium fee revenues in Lemma 5(i), Lemma 5(ii), and Lemma 5(iii), respectively. In addition, denote $R_1^* \equiv \frac{(2-c_1-c_2)^2}{4(3+2b)}$ and $R_2^* \equiv \frac{[(3-2b)+(2+b)c_1-(6+b)c_2]}{2(2-b)(3+2b)} + r^*[2(1-\gamma)\alpha] \cdot R_3^u$ port authority’s equilibrium fee revenues in Lemma 4(i) and Lemma 4(ii), respectively. Note that $R^u = R_2^u$ by (24) and $R^u = R_3^u$ by (25). Recall that $c_1 < \tilde{c}_2 < \hat{c}_2 < \bar{c}_2 < \ldots < \bar{c}_2 < \hat{c}_2 < \tilde{c}_2$. First, for $c_2 \in (c_1, \hat{c}_2)$, Lemma 4(i) and Proposition 1(i) show $R_1^* > R_2^* = R^*$ and $R_1^u > R_1^*$, and thus $R_1^u > R^*$. Second, for $c_2 \in [\hat{c}_2, \bar{c}_2)$, Lemma 4(ii) and Proposition 1(i) show $R_1^u > R_2^* = R^*$. Third, for $c_2 \in [\bar{c}_2, \hat{c}_2)$, Lemma 4(ii) and Proposition 1(ii) show $R_2^u > R_2^* = R^*$. Fourth, for $c_2 \in [\hat{c}_2, \bar{c}_2)$, no optimal two-part tariff contract exists by Lemma 4, and Proposition 1(ii) shows $R_3^u > R_2^u$. Finally, for $c_2 \in [\bar{c}_2, \hat{c}_2)$, no optimal two-part tariff contract exists by Lemma 4, and $R_3^u = R^u$ is the optimal concession contract by Proposition 1(iii). In sum, for $c_2 \in (c_1, \hat{c}_2)$, the port authority will be better off by imposing the minimum throughput requirements on operators, while it will have the same equilibrium fee revenues in both scenarios if $c_2 \in [\hat{c}_2, \bar{c}_2)$. These prove Proposition 2. □

**Proof of Lemma 7:** The proofs are straightforward, and thus omitted. □

**Lemma 8.** Given two-part tariff scheme $(r, f)$ and minimum throughput guarantee $\delta$, optimal behaviors of the two operators are as follows.
(i) For $\delta \in [0, \delta_p]$ with $\delta_{p1} = \frac{(1-r)}{1+b} + \frac{b_c(2-b^2)c_1}{(1-b^2)(4-b^2)}$, both operators’ equilibrium service prices are $p^*_1 = \frac{1-b+r}{2-b} + \frac{2c_1+b_c}{4-b^2} > 0$ and $p^*_2 = \frac{1-b+r}{2-b} + \frac{b_c(1+2c_2)}{4-b^2} > 0$, and the equilibrium cargo-handling amounts are $q^*_1 = \frac{1-r}{(1+b)(2-b)} + \frac{b_c(2-b^2)c_1}{(1-b^2)(4-b^2)} > \delta_{p1}$ and $q^*_2 = \delta_{p1}$. The equilibrium profit of operator $i$ is $\pi^*_i = (1-b^2)(p^*_i)^2 - f$ for $i = 1, 2$.

(ii) For $\delta \in (\delta_{p1}, \delta_{p2}]$ with $\delta_{p2} \equiv \frac{1-c}{1+b} + \frac{2c_1-b_c}{4-b^2}$, both operators’ equilibrium service prices are $p^*_1 = \frac{(1-b^2)(1-b^2)+c_1}{2-b^2}$ and $p^*_2 = \frac{[1-b(2-b)-2(1-b^2)\delta + b_c + b c_1]}{(2-b^2)} > 0$, their equilibrium cargo-handling amounts are $q^*_1 = \frac{[1-b^2-c_1]}{2-b^2}$ and $q^*_2 = \delta$, and their equilibrium profits are $\pi^*_1 = (1-b^2)(p^*_1)^2 - f$ and $\pi^*_2 = \frac{\delta}{(2-b^2)}[(1-b)(2+b)(1-r) - 2(1-b^2)\delta + bc_1 - (2-b^2)c_2] - f$.

(iii) For $\delta \in (\delta_{p2}, \frac{1}{1+b}]$, operators’ equilibrium service prices are $p^*_1 = p^*_2 = 1 - (1+b)\delta > 0$, their equilibrium cargo-handling amounts are $q^*_1 = q^*_2 = \delta$, and operator $i$’s equilibrium profit is $\pi^*_i = \delta[1 - (1+b)\delta - r - c_i] - f$ for $i = 1, 2$.

Proof of Lemma 8: Denote $L_1$ and $L_2$ the respective Lagrange functions of operators 1 and 2 in problem (30) with

$$L_1 = (p_1 - c_1 - r)[\frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2}] - f + \lambda_1[\frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} - \delta]$$

and

$$L_2 = (p_2 - c_2 - r)[\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2}] - f + \lambda_2[\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2} - \delta],$$

where $\lambda_1$ and $\lambda_2$ are the Lagrange multipliers associated with the operators. Then, the Kuhn-Tucker conditions for operator 1 are

$$\frac{\partial L_1}{\partial p_1} = \left[ \frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} \right] - \frac{(p_1 - c_1 - r)}{1-b^2} - \frac{\lambda_1}{1-b^2} \leq 0, \quad p_1 \cdot \frac{\partial L_1}{\partial p_1} = 0, \quad (A61)$$

and

$$\frac{\partial L_1}{\partial \lambda_1} = \frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} - \delta \geq 0, \quad \lambda_1 \cdot \frac{\partial L_1}{\partial \lambda_1} = 0, \quad (A62)$$

and for operator 2 are

$$\frac{\partial L_2}{\partial p_2} = \left[ \frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2} \right] - \frac{(p_2 - c_2 - r)}{1-b^2} - \frac{\lambda_2}{1-b^2} \leq 0, \quad p_2 \cdot \frac{\partial L_2}{\partial p_2} = 0, \quad (A63)$$

and

$$\frac{\partial L_2}{\partial \lambda_2} = \frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2} - \delta \geq 0, \quad \lambda_2 \cdot \frac{\partial L_2}{\partial \lambda_2} = 0. \quad (A64)$$
According to the values of $\lambda_1$ and $\lambda_2$, there are four cases as follows.

**Case 1:** Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then (A61) and (A63) become

\[
\left[ \frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} \right] - \frac{(p_1-c_1-r)}{1-b^2} = 0 \quad \text{and} \\
\left[ \frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2} \right] - \frac{(p_2-c_2-r)}{1-b^2} = 0.
\]

Solving these equations yields $p_{p1}^* = \frac{1-b+r}{2-b} + \frac{2c_1+b c_2}{4-b^2} > 0$ and $p_{p2}^* = \frac{1-b+r}{2-b} + \frac{b c_1+2 c_2}{4-b^2} > 0$.

Substituting $p_{p1}^*$ and $p_{p2}^*$ into (27)-(28) yields $q_{p1}^* = \frac{1-r}{(1+b)(2-b)} + \frac{b c_2-(b-c_2)c_1}{(1-b^2)(4-b^2)}$ and $q_{p2}^* = \frac{1-r}{(1+b)(2-b)} + \frac{b c_1-(b-c_1)c_2}{(1-b^2)(4-b^2)}$. To guarantee $q_{p1}^* \geq \delta$ and $q_{p2}^* \geq \delta$, condition $0 \leq \delta \leq \delta_{p1} \equiv \frac{1-r}{(1+b)(2-b)} + \frac{b c_1-(b-c_1)c_2}{(1-b^2)(4-b^2)}$ is needed. Substituting $p_{p1}^*$ and $p_{p2}^*$ into (29) yields $\pi_{p1}^* = (1-b^2)(q_{p1}^*)^2 - f$ for $i = 1, 2$. These prove Lemma 8(i).

**Case 2:** Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then (A61), (A63), and (A64) suggest

\[
\left[ \frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} \right] - \frac{(p_1-c_1-r)}{1-b^2} = 0, \\
\left[ \frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2} \right] - \frac{(p_2-c_2-r)}{1-b^2} - \frac{\lambda_2}{1-b^2} = 0, \quad \text{and} \quad \\
\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2} - \delta = 0.
\]

Solving these equations yields $\lambda_2^* = \frac{(1-b^2)(4-b^2)(\delta-\delta_1)}{2-b^2}$, $p_{p1}^* = \frac{(1-b^2)(1-bd)+c_1+r}{(2-b^2)}$, and $p_{p2}^* = \frac{[(1-b)(2+b)-2(1-b^2)c_1+b c_1+b r]}{2-b^2}$. Substituting $p_{p1}^*$ and $p_{p2}^*$ into (27)-(28) yields $q_{p1}^* = \frac{1-bd-c_1-r}{(2-b^2)}$ and $q_{p2}^* = \delta$. To guarantee $\lambda_2^* > 0$, conditions $\delta > \delta_{p1}$, $r \leq \bar{r}_p$, and $c_2 < \bar{c}_{p2}$ are needed; and condition $\delta \leq \delta_{p2} \equiv \frac{1-r}{(1+b)(2-b)}$ is needed to guarantee $q_{p1}^* \geq \delta$. Thus, the plausible range for $\delta$ is $\delta \in (\delta_{p1}, \delta_{p2})$. Under the circumstance, $p_{p1}^* = \frac{(1-b^2)(1-bd)+c_1+r}{(2-b^2)} > p_{p2}^* = \frac{[(1-b)(2+b)-2(1-b^2)c_1+b c_1+b r]}{2-b^2} > 0$ if $\delta \leq \delta_{p2}$. Substituting $p_{p1}^*$ and $p_{p2}^*$ into (29) gives $\pi_{p1}^* = (1-b^2)(q_{p1}^*)^2 - f$ and $\pi_{p2}^* = \frac{\delta}{(2-b^2)[(1-b)(2+b)(1-r)-2(1-b^2)b c_1-(2-b^2)c_2]} - f$. These prove Lemma 8(ii).

**Case 3:** Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then (A61)-(A63) suggest

\[
\frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} - \delta = 0, \\
\left[ \frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} \right] - \frac{(p_1-c_1-r)}{1-b^2} - \frac{\lambda_1}{1-b^2} = 0, \quad \text{and} \quad \\
\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2} - \frac{(p_2-c_2-r)}{1-b^2} = 0.
\]
Solving these equations yields $p_{p1}^* = \frac{(1-b)(2+b)-2(1-b^2)\delta+bc_1+br}{(2-b^2)}$, $p_{p2}^* = \frac{(1-b^2)(1-b\delta)+c_2+r}{2-b^2}$, and $
abla^* = \frac{(1-b^2)(4-b^2)}{2-b^2}[\delta - \frac{1-r}{(1+b)(2-b)} - \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)}]$. Substituting $p_{p1}^*$ and $p_{p2}^*$ into (27)-(28) yields $q_{p1}^* = \delta$ and $q_{p2}^* = \frac{(1-b^2-c_2-r)}{2-b^2}$. To guarantee $\nabla^* > 0$, condition $\delta > \frac{1-r}{(1+b)(2-b)} + \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)}$ is needed. On the other hand, $q_{p2}^* \geq \delta$ is guaranteed if $\delta \leq \frac{1-c_2-r}{(1+b)(2-b)}$. However, these two conditions are incompatible with each other because $\frac{1-c_2-r}{(1+b)(2-b)} - \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)} = \frac{(2-b^2)c_2-c_1}{(1-b^2)(4-b^2)} < 0$. Thus, no solution exists in this case.

Case 4: Suppose $\nabla^*_1 > 0$ and $\nabla^*_2 > 0$. Then (A61)-(A64) suggest

$$\frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} - \delta = 0,$$

$$\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2} - \delta = 0,$$

$$[\frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2}] - \frac{(p_1-c_1-r)}{1-b^2} - \frac{\nabla^*_1}{1-b^2} = 0,$$

$$[\frac{1}{1+b} - \frac{p_2}{1-b^2} + \frac{bp_1}{1-b^2}] - \frac{(p_2-c_2-r)}{1-b^2} - \frac{\nabla^*_2}{1-b^2} = 0.$$
\[
\frac{2(1-b)(2+b)+(2+3b-b^2)c_1-(6+b-3b^2)c_2}{2(2+b)(3-b)(1-b^2)}. \text{At the equilibrium, operators’ cargo-handling amounts are} q^*_p = \frac{1-r_p^u}{(1+b)(2-b)} + \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)} \text{ and } q^*_p = \frac{1-r_p^u}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)} \text{ as in Lemma 8(i), and port authority’s fee revenue equals} R_p^* = (1-b^2)\left[\frac{2(1-b)(2+b)+(2+3b-b^2)c_1-(6+b-3b^2)c_2}{2(2+b)(3-b)(1-b^2)}\right]^2 + r_p^u\left[\frac{2(1-r_p^u)-(c_1+c_2}{(1+b)(2-b)}\right].
\]

**Lemma 10.** Suppose conditions in (31) hold. Given \((q^u_p, q^w_p, \pi^u_p, \pi^w_p)\) derived in problem (32), port authority’s optimal unit-fee scheme and minimum throughout requirement \((r^u_p, \delta^u_p)\) are as follows.

(i) If \(c_2 \in (0, \bar{c}_p)\) with \(\bar{c}_p = \frac{[6-b-b^2] + (2+b)c_1 - 2\sqrt{(3+b)(1-2c_1+c_2^2)}}{(8-b^2)}\), then we have \(r^u_p = \frac{(1-c_2)}{2} > 0\) and \(\delta^u_p = \frac{(1-c_2)}{2(1+b)} > 0\). At the equilibrium, operator i’s cargo-handling amount is \(q^u_{pi} = \delta^u_p > 0\) for \(i = 1, 2\), and port authority’s fee revenue equals \(R_p^u = \frac{(1-c_2)^2}{2(1+b)}\).

(ii) If \(c_2 \in (\bar{c}_p, \tilde{c}_p)\) with \(\tilde{c}_p = \frac{(1-b)(2+b)(3+b)+(2+3b-b^2)c_1}{(8+4b-3b^2-b^4)}\) and \(\bar{c}_p < \tilde{c}_p\), then we have \(r^u_p = \frac{[(3+b)-c_1-(2-b^2)c_2]}{2(3+b)}\) and \(\delta^u_p = \frac{(1-b)(2+b)(1-r^u_p)+bc_1-(2-b^2)c_2}{2(1-b^2)}\). At the equilibrium, operators’ cargo-handling amounts are \(q^u_{p1} = \frac{(1-bc_1-c_1-r^u_p)}{(2-b^2)}\) and \(q^u_{p2} = \delta^u_p\), and port authority’s fee revenue equals \(R_p^u = \frac{[3+b-c_1-(2-b^2)]}{8(1+b)(3+b)}\).

(iii) If \(c_2 \in (\tilde{c}_p, \bar{c}_p)\) with \(\bar{c}_p = \frac{(1-b)(2+b)+bc_1}{(2-b^2)}\), then we have \(r^u_p = \bar{r}_p = \frac{(1-b)(2+b)+bc_1-(2-b^2)c_2}{(1-b)(2+b)}\) > 0 and \(\delta^u_p = 0\). At the equilibrium, operators’ cargo-handling amounts are \(q^u_{p1} = \frac{(c_2-c_1)}{(1-b)(2+b)}\) and \(q^u_{p2} = 0\), and port authority’s fee revenue equals \(R_p^u = \frac{(c_2-c_1)}{(1-b)(2+b)^2}\).

**Lemma 11.** Suppose conditions in (31) hold. Then, given \((q^f_p, q^w_p, \pi^f_p, \pi^w_p)\) derived in problem (33) and \(c_2 \in (c_1, \bar{c}_p)\) with \(\bar{c}_p = \frac{(1-b)(2+b)+bc_1}{(2-b^2)}\), we have optimal fixed fee \(f^*_p = \frac{1-b^2}{2} + \frac{bc_1-(2-b^2)c_2}{(2-b^2)}\), and optimal minimum throughput guarantee \(\delta^f_p \in [0, \frac{1}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)}]\). At the equilibrium, operators’ cargo-handling amounts are \(q^f_{p1} = \frac{1}{(1+b)(2-b)} + \frac{bc_2-(2-b^2)c_1}{(1-b^2)(4-b^2)}\) and \(q^f_{p2} = \frac{1}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)}\), and port authority’s fee revenue equals \(R^*_p = (1-b^2)\left[\frac{1}{(1+b)(2-b)} + \frac{bc_1-(2-b^2)c_2}{(1-b^2)(4-b^2)}\right]^2\).

**Proof of Lemmas 9-11 and Proposition 3:** Denote \(L\) the following problem

\[
\max_{r, f, \delta} \ 2f + r(q^*_p + q^*_p)
\]
s.t. $0 \leq \delta < \frac{1}{1+b}$, $0 \leq r \leq \bar{r}_p$, $\pi_{pi}^* \geq 0$, $\pi_{p2}^* \geq 0$, and $0 \leq f \leq \min\{\pi_{p1}^*, \pi_{p2}^*\}$. (A65)

Moreover, denote $L_1$, $L_2$, and $L_3$ the problems given in (34), (35), and (36), respectively. Let $S_L$ be the set of $(r, f, \delta)$ satisfying all constraints in problem $L$, and $S_{L_i}$ the set of $(r, f, \delta)$ satisfying all constraints in problem $L_i$. Similarly, $S_{L_2}$ is the set of $(r, \delta)$ satisfying all constraints in problem $L_2$, and $S_{L_3}$ is the set of $(f, \delta)$ satisfying all constraints in problem $L_3$.

First, since $q_{pi}^* = q_{pi}^u$ for all $(r, f, \delta)$, we have $\pi_{pi}^* \leq \pi_{pi}^u$ with equality held at $f = 0$, and $\pi_{pi}^* \geq 0$ will hold if operators handle nonzero cargo amounts for $i = 1, 2$. Thus, we have $S_{L_1} \subset S_L$, and the maximum fee revenue in problem $L$ is not less than that in problem $L_1$. Second, we have $\pi_{pi}^* = (\pi_{pi}^u - f)$ for all $(r, f, \delta)$ because $q_{pi}^* = q_{pi}^u$ for $i = 1, 2$. Thus, for any $(r, \delta)$ with $\pi_{pi}^u \geq 0$, we have $\pi_{pi}^* = (\pi_{pi}^u - f) \geq \pi_{pi}^u - \min\{\pi_{p1}^*, \pi_{p2}^*\} = \pi_{pi}^u - \min\{(\pi_{p1}^u - f), (\pi_{p2}^u - f)\} \geq f \geq 0$ by $0 < f \leq \min\{\pi_{p1}^*, \pi_{p2}^*\}$. These imply $S_{L_2} \subset S_L$. On the other hand, we have $2f + r(q_{p1}^* + q_{p2}^*) \geq r(q_{p1}^u + q_{p2}^u)$ for any given $(r, f, \delta)$. Thus, the maximum fee revenue in problem $L$ is not less than that in problem $L_2$. They will be equal if the solutions in problem $L$ have $f^* = 0$. Third, for any solution $(r^*, f^*, \delta^*)$ of problem $L$ with $r^* = 0$, it must also be the solution of problem $L_2$. However, if $r^* > 0$, then the solution of problem $L$ will have higher value than that of problem $L_3$. Thus, the maximum fee revenue in problem $L$ is not less than that in problem $L_3$ as well. In sum, deriving optimal concession contracts is equivalent to solving problem $L$ in (A65). If the associated solution $(r^*, f^*, \delta^*)$ has nonzero $(r^*, f^*)$, then the two-part tariff scheme is port authority’s best choice, the unit-fee scheme is the best if $f^* = 0$, and the fixed-fee scheme is the best if $r^* = 0$. Thus, all the solutions of problem $L$ are derived below.

Case 1: Suppose $\delta \in [0, \delta_{p1}]$. Then, Lemma 8(i) implies $\pi_{p1}^* > \pi_{p2}^*$ and $f_p^* = \pi_{p2}^* = (1 - b^2)\frac{1}{2}(q_{p2}^*)^2 > 0$. Thus, the problem in (A65) becomes

$$\max_{r, f, \delta} 2f + r[q_{p1}^* + q_{p2}^*]$$

s.t. $0 \leq \delta \leq \delta_{p1}$ and $0 < r \leq \bar{r}_p$. 

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Its Lagrange function is

\[ L = (1 - b^2)(q_{p2}^*)^2 + r[q_{p1} + q_{p2}^*] + \lambda_1(\delta - \delta) + \lambda_2(r_p - r), \]

where \( \lambda_1 \) and \( \lambda_2 \) are the Lagrange multipliers associated with the inequality constraints of this problem. The Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = 2(1 - b^2)q_{p2}^* \frac{\partial q_{p2}^*}{\partial r} + r[\frac{\partial q_{p1}}{\partial r} + \frac{\partial q_{p2}^*}{\partial r}] + (q_{p1} + q_{p2}^*) + \lambda_1 \frac{\partial \delta}{\partial r} - \lambda_2 \leq 0, \ r \cdot \frac{\partial L}{\partial r} = 0, \quad (A66)
\]

\[
\frac{\partial L}{\partial \lambda_1} = -\lambda_1 \leq 0, \ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad (A67)
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta - \delta \geq 0, \ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A68)
\]

\[
\frac{\partial L}{\partial \lambda_2} = r_p - r \geq 0, \ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0. \quad (A69)
\]

Based on the values of \( \lambda_1 \) and \( \lambda_2 \), there are four cases as follows.

**Case 1a:** Suppose \( \lambda_1^* = 0 \) and \( \lambda_2^* = 0 \). Then (A66) becomes

\[
\frac{[2(2 + b) - 2(2 + b)(3 - b)r - (4 + 2b - b^2)c_1 - b^2c_2]}{(1 + b)(2b - b^2)} = 0.
\]

Solving this equation yields \( r_p^* = \frac{1 - b}{3 - b} - \frac{4 + 2b - b^2}{2(2 + b)(3 - b)} c_1 > 0 \). It remains to check whether \( r_p^* < r_p \) holds. By some calculations, we have \( r_p^* < r_p \) iff \( c_2 = (1 - b)(2 + b) + (2 + 2b - b^2)c_1 \equiv 0 \). In addition, (A68) implies \( \delta^* \in [0, \delta_{p1}] \) with \( \delta_{p1} = \frac{1}{2(2 + b)(3 - b)(1 - b^2)} \)

\[
\frac{[2(2 + b) + (2 + 3b - b^2)c_1 - (6 + b - b^2)c_2]}{2(2 + b)(3 - b)(1 - b^2)} > 0.
\]

Accordingly, port authority’s equilibrium fee revenue equals

\[
\bar{R}_p^* = 2f_p^* + r_p^* \left[ \frac{2(1 - r_p^*) - (c_1 + c_2)}{(1 + b)(2 - b)} \right]. \quad (A70)
\]

**Case 1b:** Suppose \( \lambda_1^* = 0 \) and \( \lambda_2^* > 0 \). Then we have \( r_p^* = \bar{r}_p \equiv \frac{(1-b)(2+b) + bc_1 - (2-b^2)c_2}{(1-b)(2+b)} \) by (A66) and \( \lambda_2^* = \left[ -2(1-b)(2+b) - (2+3b - b^2)c_1 + (6 + b - 3b^2)c_2 \right] \) by (A66). Note that \( r_p^* > 0 \) iff \( c_2 < \bar{c}_p \equiv \frac{(1-b)(2+b) + bc_1}{2(b^2)} \), \( \lambda_2^* > 0 \) iff \( c_2 > \bar{c}_p, \ f_p^* = 0 \), and \( \delta_p^* = 0 \) due to \( \delta_{p1} = \frac{1 - r_p^*}{(1 + b)(2 - b)} + \frac{bc_1 - (2 - b^2)c_2}{(1 - b^2)(4 - b^2)} = 0 \). At the equilibrium, port authority’s fee revenue equals

\[
\bar{R}_p^* = \bar{r}_p q_{p1}^* \left[ (c_2 - c_1)(1-b)(2+b) + bc_1 - (2-b^2)c_2 \right], \quad (A71)
\]

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Case 1c: Suppose $\lambda^*_1 > 0$ and $\lambda^*_2 = 0$. Then, (A68) suggests $\delta^*_p = \delta^*_p > 0$. This in turn implies $\lambda^*_1 = 0$ by (A67). It is a contradiction. Thus, no solution exists in this case.

Case 1d: Suppose $\lambda^*_1 > 0$ and $\lambda^*_2 > 0$. As in Case 1b, we have $f^*_p = 0$, $r^*_p = \bar{r}_p$, $\delta^*_p = 0$, and $R^*_p = \bar{r}_p q^*_p = \frac{(c_2 - c_1)(1-b)(2+b)bc - (2-b^2)c_2}{(1-b^2)(2+b)^2}$.

Case 2: Suppose $\delta \in (\delta_{p1}, \delta_{p2})$. Then, Lemma 8(ii) implies $\pi^*_{p1} > \pi^*_{p2}$, and $f^*_p = \frac{\delta(1-b)(2+b)(1-r) - 2(1-b^2)\delta + bc_1 - (2-b^2)c_2}{2(1-b^2)}$ with $f^*_p \geq 0$ iff $\delta \leq \frac{(1-b)(2+b)(1-r) + bc_1 - (2-b^2)c_2}{2(1-b^2)}$ and $r \leq \frac{(1-b)(2+b)bc - (2-b^2)c_2}{1-b}$. In addition, $\frac{(1-b)(2+b)(1-r) + bc_1 - (2-b^2)c_2}{2(1-b^2)} \geq (>) \delta_{p2}$ iff $r \leq (>) \frac{(1-b)c_1 - (2-b)c_2}{1-b}$. Thus, we have two sub-cases.

Case 2a: Suppose $r \leq \frac{(1-b)c_1 - (2-b)c_2}{1-b}$. Then the problem in (A65) becomes

$$\max_{r, f, \delta} \quad 2f + r[q^*_p + q^*_p]$$

s.t. $\delta_{p1} < \delta \leq \delta_{p2}$ and $0 \leq r \leq \frac{(1-b) + c_1 - (2-b)c_2}{1-b}$. \hfill (A72)

Its Lagrange function is

$$L = \frac{\delta}{2-b^2}[(1-b)(2+b)(1-r) - 2(1-b^2)\delta + bc_1 - (2-b^2)c_2]$$

$$+ \frac{r(1+b)(2+b) - c_1 - r}{2 - b^2} + \lambda_1(\delta - \delta_{p1}) + \lambda_2(\delta_{p2} - \delta)$$

$$+ \lambda_3 \left\{ \frac{1}{1-b}[(1-b) + c_1 - (2-b)c_2] - r \right\}.$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{1-2r - c_1}{2-b^2} + \frac{\lambda_1}{(1+b)(2-b)} - \frac{\lambda_2}{(1+b)(2-b)} - \lambda_3 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \quad \hfill (A73)$$

$$\frac{\partial L}{\partial \delta} = \frac{1}{2-b^2}[(1-b)(2+b) - 4(1-b^2)\delta + bc_1 - (2-b^2)c_2] + \lambda_1 - \lambda_2 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad \hfill (A74)$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_{p1} \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0,$$

$$\frac{\partial L}{\partial \lambda_2} = \delta_{p2} - \delta \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad \hfill (A75)$$

and

$$\frac{\partial L}{\partial \lambda_3} = 0, \quad \hfill (A76)$$
\[
\frac{\partial L}{\partial \lambda_3} = \frac{1}{1-b}[(1-b) + c_1 - (2-b)c_2] - r \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \tag{A77}
\]

where \(\lambda_1\), \(\lambda_2\) and \(\lambda_3\) are the Lagrange multipliers associated with the inequality constraints in (A72). Constraint \(\delta_{p1} < \delta\) suggests \(\lambda^*_1 = 0\) by (A75). If \(\lambda^*_2 = 0\), we have \(\delta^*_p = \frac{1-b)(2+b)+bc_1-(2-b)c_2}{4(1-b^2)}\) by (A74). Note that \(\delta_{p2} - \delta^*_p = \frac{(2-b^2)(1-b-c_1+2-b)c_2}{4(1-b^2)(2-b)} \geq 0\) iff \(c_2 \geq \frac{(1-b)+c_1}{(2-b)}\). However, since \(r \leq \frac{[(1-b)+c_1-(2-b)c_2]}{1-b} \leq 0\) if \(c_2 \geq \frac{(1-b)+c_1}{(2-b)}\), it is a contradiction. Thus, we have \(\lambda^*_2 > 0\). Based on the values of \(\lambda_3\), there are two sub-cases.

Case 2a-1: Suppose \(\lambda^*_3 = 0\). Then (A73), (A74), and (A76) suggest \(\frac{1-b-c_1}{2-b^2} - \frac{\lambda_2}{(1+b)(2-b)} = 0, \quad \frac{[(1-b)(2+b)-4(1-b^2)c_1+(2-b)c_2]}{2-b^2} - \lambda_2 = 0, \quad \text{and} \quad \delta_{p2} - \delta = 0\). Solving these equations yields \(r^*_p = \frac{2-(1-b)c_1+(2-b)c_2}{2(3-b)} > 0\), \(\delta^*_p = \delta_{p2} = \frac{2-c_1-c_2}{2(1+b)(3-b)} > 0\), and \(\lambda^*_2 = \frac{1-b)(2+b)+(1-b)c_1-(2-b)c_2}{(3-b)(2-b^2)}\). Since \(\frac{[(1-b)+c_1-(2-b)c_2]}{1-b} - r^*_p = \frac{(2-b)(1-b)+5-b)c_1-(7-b)c_2}{2(1+b)(3-b)} \geq 0\) iff \(c_2 \leq \frac{2(1-b)+(5-b)c_1}{7-3b}\), we have \(r^*_p \leq \frac{[(1-b)+c_1-(2-b)c_2]}{2-b}\) if \(c_2 \leq \frac{2(1-b)+(5-b)c_1}{7-3b}\). On the other hand, \(\lambda^*_2 > 0\) iff \(c_2 < \frac{(1-b)+c_1}{2-b}\). Since \(\frac{(1-b)+c_1}{2-b} - \frac{2(1-b)+(5-b)c_1}{7-3b} = \frac{(1-b)(3-b)(1-c_1)}{(2-b)(7-3b)} > 0\), the equilibrium exists with fee revenue

\[
R^*_p = \frac{(2-c_1-c_2)^2}{4(1+b)(3-b)} \tag{A78}
\]

if \(c_2 \leq \frac{2(1-b)+(5-b)c_1}{7-3b}\).

Case 2a-2: Suppose \(\lambda^*_3 > 0\). Then (A76) and (A77) suggest \(r^*_p = \frac{[(1-b)+c_1-(2-b)c_2]}{1-b}\) and \(\delta^*_p = \delta_{p2} = \frac{(c_2-c_1)}{(1-b)} > 0\). Moreover, (A74) implies \(\lambda^*_2 = \frac{[(1-b)(2+b)+(1+b)c_1-(6-b^2)c_2]}{2-b^2}\), and (A73) implies \(\lambda^*_3 = \frac{[-2(1-b)-(5-b)c_1+(7-b)c_2]}{(2-b)(1-b)}\). By some calculations, we get \(\delta^*_p = \delta_{p2} > \delta_{p1}\), \(r^* \geq 0\) iff \(c_2 \leq \frac{(1-b)+c_1}{2-b}\), \(\lambda^*_2 > 0\) iff \(c_2 < \frac{(1-b)(2+b)+(4-b)c_1}{(6-b^2)}\), and \(\lambda^*_3 > 0\) iff \(c_2 > \frac{2(1-b)+(5-b)c_1}{7-3b}\) with \(\frac{2(1-b)+(5-b)c_1}{7-3b} < \frac{(1-b)(2+b)+(4-b)c_1}{(6-b^2)} < \frac{(1-b)+c_1}{2-b}\). Thus, under condition \(\frac{2(1-b)+(5-b)c_1}{7-3b} < c_2 < \frac{(1-b)(2+b)+(4-b)c_1}{(6-b^2)}\), the equilibrium exists with fee revenue

\[
R^*_p = \frac{2(c_2-c_1)(1-b) + c_1 - (2-b)c_2}{(1-b)(1-b^2)} \tag{A79}
\]

Case 2b: Suppose \(r > \frac{[(1-b)+c_1-(2-b)c_2]}{1-b}\). Then the problem in (A65) becomes

\[
\max_{r, f, \delta} 2f + r[q^*_p + q^*_r] \]

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s.t. $\delta_{p1} < \delta \leq \frac{(1-b)(2+b)(1-r) + bc_1 - (2-b^2)c_2}{2(1-b^2)}$ and $\frac{[(1-b) + c_1 - (2-b)c_2]}{1-b} < r < \bar{r}_p$.

Then, the associated Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial \delta} = 1 - \frac{2r - c_1}{2 - b^2} + \frac{\lambda_1}{(1+b)(2-b)} - \frac{(1-b)(2+b)\lambda_2}{2(1-b^2)} + \lambda_3 - \lambda_4 \leq 0,$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_{p1} \geq 0,$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{(1-b)(2+b)(1-r) + bc_1 - (2-b^2)c_2}{2(1-b^2)} - \delta \geq 0,$$

$$\frac{\partial L}{\partial \lambda_3} = r - \frac{[(1-b) + c_1 - (2-b)c_2]}{1-b} \geq 0,$$

$$\frac{\partial L}{\partial \lambda_4} = \bar{r}_p - r \geq 0.$$
Solving these equations yields \( r^*_p = \frac{[(3 + b) - c_1 - (2 + b)c_2]}{(3 + b)} \), \( \delta^*_p = \frac{(1 - b)(2 + b)(1 - r^*_p) + bc_1 - (2 - b^2)c_2}{2(1 - b^2)} \), and \( \lambda^*_2 = \frac{2(1 + b)(c_2 - c_1)}{(3 + b)(2 - b^2)} > 0 \). Since \( r^*_p > \frac{[(1 - b) + c_1 - (2 - b^2)c_2]}{1 - b} \), we have \( r^*_p > \frac{[(1 - b) + c_1 - (2 - b^2)c_2]}{1 - b} \) if \( c_2 > \frac{(3 - 2b - b^2) + (7 + b)c_1}{(10 - b^2)} \). Moreover, we have \( \delta^* > \delta_{p1} \) by \( r^*_p \leq \bar{r}_p \). Thus, under condition \( \frac{(3 - 2b - b^2) + (7 + b)c_1}{(10 - b^2)} < c_2 \leq \frac{(1 - b)(2 + b)(3 + b) + (2 + 5b + b^2)c_1}{(8 + 4b - 3b^2 - b^3)} \), the equilibrium exists with fee revenue

\[
R^*_p = \frac{[(3 + b) - c_1 - (2 + b)c_2]^2}{8(1 + b)(3 + b)}. \tag{A87}
\]

Case 2b-2: Suppose \( \lambda^*_2 > 0 \) and \( \lambda^*_4 > 0 \). Then, \( r^*_p = \bar{r}_p \) is implied by (A86), and \( \delta^*_p = \frac{(1 - b)(2 + b)(1 - r^*_p) + bc_1 - (2 - b^2)c_2}{2(1 - b^2)} = 0 \) by (A84). However, since \( \delta_{p1} = 0 \) at \( r^*_p = \bar{r}_p \), we have \( \delta^*_p = \delta_{p1} = 0 \), which contradicts \( \delta > \delta_{p1} \). Thus, no solution exists in this case.

Case 2b-3: Suppose \( \lambda^*_2 = 0 \) and \( \lambda^*_4 = 0 \). Then, (A81) and (A82) suggest \( r^*_p = \frac{1 - c_1}{2} > 0 \) and \( \delta^*_p = \frac{(1 - b)(2 + b)(1 - r^*_p) + bc_1 - (2 - b^2)c_2}{4(1 - b^2)} = \frac{-(2 - b^2)(c_2 - c_1)}{4(1 - b^2)} < 0 \), which contradicts \( \delta \leq \frac{(1 - b)(2 + b)(1 - r) + bc_1 - (2 - b^2)c_2}{2(1 - b^2)} \) required by (A80). Thus, no solution exists in this case.

Case 2b-4: Suppose \( \lambda^*_2 = 0 \) and \( \lambda^*_4 > 0 \). Then, \( r^*_p = \bar{r}_p \) is implied by (A86), and \( \delta^*_p = \frac{(1 - b)(2 + b) + bc_1 - (2 - b^2)c_2}{4(1 - b^4)} \) by (A82). Since \( \frac{(1 - b)(2 + b)(1 - r^*_p) + bc_1 - (2 - b^2)c_2}{2(1 - b^2)} - \delta^*_p = \frac{(1 - b)(2 + b) - bc_1 + (2 - b^2)c_2}{4(1 - b^2)} < 0 \) due to \( c_2 < \bar{c}_p \), which contradicts \( \delta \leq \frac{(1 - b)(2 + b)(1 - r) + bc_1 - (2 - b^2)c_2}{2(1 - b^2)} \) required by (A80). Thus, no solution exists in this case.

Case 3: Suppose \( \delta \in (\delta_{p2}, \frac{1}{1 + b}] \). Then, Lemma 8(iii) implies \( \pi^*_{p1} > \pi^*_{p2} \), and \( f^*_p = \pi^*_{p2} = \frac{[1 - (1 + b)\delta - c_2 - r] \delta}{2(1 + b)} \) with \( f^*_p \geq 0 \) iff \( \delta \leq \frac{1 - c_2 - r}{(1 + b)} \) and \( r < (1 - c_2) \). Note that \( r < (1 - c_2) \) is implied by \( r \leq \bar{r}_p \equiv \frac{[(1 - b)(2 + b) + bc_1 - (2 - b^2)c_2]}{(1 - b)(2 + b)} \). Accordingly, the problem in (A65) becomes

\[
\max_{r, \delta} \left[ \delta (1 - (1 + b)\delta - c_2 - r) + 2r \delta \right] \text{ s.t. } \delta_{p2} < \delta \leq \frac{1 - c_2 - r}{(1 + b)} \text{ and } 0 \leq r \leq \bar{r}_p. \tag{A88}
\]
Its Lagrange function associated with the problem in (A88) is

\[
L = \delta[1 - (1 + b)\delta - c_2 - r] + 2r\delta + \lambda_1(\delta - \delta_p) + \lambda_2\left(\frac{1 - c_2 - r}{1 + b}\right) - \delta + \lambda_3(r_p - r).
\]

Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = \delta + \frac{\lambda_1}{(1 + b)(2 - b)} - \frac{\lambda_2}{1 + b} - \lambda_3 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \quad (A89)
\]

\[
\frac{\partial L}{\partial \delta} = 1 - 2(1 + b)\delta - c_2 + r + \lambda_1 - \lambda_2 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad (A90)
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta - \delta_p \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A91)
\]

\[
\frac{\partial L}{\partial \lambda_2} = \frac{1 - c_2 - r}{(1 + b)} - \delta \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad \text{and} \quad (A92)
\]

\[
\frac{\partial L}{\partial \lambda_3} = r_p - r \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0. \quad (A93)
\]

Since \( \delta > \delta_p \), we have \( \lambda_1^* = 0 \) by (A91). According to the values of \( \lambda_2^* \) and \( \lambda_3^* \), there are four sub-cases.

**Case 3a:** Suppose \( \lambda_2^* > 0 \) and \( \lambda_3^* = 0 \). Then, (A89), (A90) and (A92) suggest \( \delta - \frac{\lambda_2}{1 + b} = 0, \quad 1 - 2(1 + b)\delta - c_2 + r - \lambda_2 = 0, \quad \text{and} \quad \frac{(1 - c_2 - r)}{(1 + b)} - \delta = 0 \). Solving these equations yields \( r_p^* = \frac{(1 - c_2)}{2} > 0, \quad \delta_p^* = \frac{1 - c_2}{2(1 + b)} > 0, \quad \text{and} \quad \lambda_2^* = (1 + b)\delta_p^* > 0 \). Some calculations yield \( (\bar{r}_p - r_p^*) = \frac{(1 - c_2)}{(1 + b)(2 + b)} \) and \( (\delta_p^* - \delta_p) = \frac{1 - c_2 - r_p^*}{2(1 + b)} - \frac{1 - c_2}{2(1 + b)(2 + b)} = \frac{(1 - b)(2 + b) + 2c_1}{2(1 + b)(2 + b)} \). Thus, we have \( r_p^* \leq \bar{r}_p \) iff \( c_2 \leq \frac{(1 - b)(2 + b) + 2c_1}{(1 + b)(2 + b)} \), and \( \delta_p < \delta_p^* \) iff \( c_2 < \frac{(1 - b)(2 + b) + 2c_1}{(1 + b)(2 + b)} \). Under condition \( c_2 < \frac{(1 - b)(2 + b) + 2c_1}{(3 - b)} \), the equilibrium exists with fee revenue

\[
R_p^* = \frac{(1 - c_2)^2}{2(1 + b)}. \quad (A94)
\]

**Case 3b:** Suppose \( \lambda_2^* > 0 \) and \( \lambda_3^* > 0 \). Then we have \( r_p^* = \bar{r}_p \) by (A93), and \( \delta_p = \frac{b(c_2 - c_1)}{(1 - b^2)(2 + b)} > 0 \) by \( r_p^* = \bar{r}_p \) and (A92). However, we have \( (\delta_p^* - \delta_p) = \frac{-2(c_2 - c_1)}{(1 + b)(4 - b^2)} < 0 \), which contradicts \( \delta > \delta_p \). Thus, no solution exists in this case.
Case 3c: Suppose $\lambda_2^* = 0$ and $\lambda_3^* = 0$. Then we have $\delta_p^* \leq 0$ by (A89), which contradicts $\delta > \delta_p^2 > 0$. Thus, no solution exists in this case.

Case 3d: Suppose $\lambda_2^* = 0$ and $\lambda_3^* > 0$. Then (A90) and (A93) suggest $r_p^* = \bar{r}_p$ and $\delta_p^* = \frac{1+\hat{r}_p}{2(1+b)}$. Some calculations yield $\delta_p^* - \delta_{p2}^* = \frac{2(1-b)(4-b^2)+(4+2b-3b_2)\bar{c}1-(2-b^2)(4-b)c_2}{(1-b^2)(4-b^2)} > 0$ iff $c_2 < \frac{2(1-b)(4-b^2)+(4+2b-3b_2)c_1}{(2-b^2)(4-b)}$, and $\delta_p^* - \delta_{p2}^* = \frac{1+c_2-\hat{r}_p}{(1+b)} - \frac{1+\hat{r}_p}{2(1+b)} = \frac{1}{2(1-b^2)}(-2(2-b-b^2)-3bc_1+(2+2b-b^2)c_2) \geq 0$ iff $c_2 \geq \frac{2(2-b-b^2)+3bc_1}{(2+2b-b^2)}$, which contradicts $c_2 < \frac{2(1-b)(4-b^2)+(4+2b-3b_2)c_1}{(2-b^2)(4-b)}$ derived above. Thus, no solution exists in this case.

Finally, by comparing port authority’s equilibrium fee revenues in Cases 1-3, we can derive the best concession contracts. Before doing this, we need to know relative sizes of the critical points in Cases 1-3, including $\hat{c}_{p2} \equiv \frac{2(1-b)(2+b)(+2+3b-2b^2)c_1}{(6+b-3b^2)}$, and $\bar{c}_{p2} \equiv \frac{(1-b)(2+b)(+2+3b-2b^2)c_1}{(6+b-3b^2)}$ in Case 1, $\hat{c}_{p2} \equiv \frac{2(1-b)(5+b+b_1)}{7-3b}$, and $\bar{c}_{p2} \equiv \frac{(1-b)(b)(+2+3b-2b^2)c_1}{(6+b-3b^2)}$ in Case 2a, $c_{p2}^P = \frac{(1-b)(2+b)(+2+3b-2b^2)c_1}{(6+b-3b^2)}$ in Case 2b-1, and $c_{p2}^P = \frac{(1-b)(2+b)(+2+3b-2b^2)c_1}{(6+b-3b^2)}$ in Case 3a. Since $c_{p2}^P - c_{p2}^P = \frac{(1-b)(1-\bar{c}^2(1-c))}{(6+b-3b^2)} > 0$, $(c_{p2}^P - c_{p2}^P) = \frac{(1-b)(1-\bar{c}^2(1-c))}{(6+b-3b^2)} > 0$, $(\bar{c}_{p2} - \bar{c}_{p2}) = \frac{(1-b)(2+b)(+2+3b-2b^2)c_1}{(6+b-3b^2)} > 0$, and $(\bar{c}_{p2} - \bar{c}_{p2}) = \frac{(1-b)(2+b)(+2+3b-2b^2)c_1}{(6+b-3b^2)} > 0$, we have $\hat{c}_{p2} < c_{p2}^P < c_{p2}^P < \bar{c}_{p2} < \bar{c}_{p2}$, and seven cases as follows.

Case A: For $c_2 \in (c_1, \hat{c}_{p2})$, an equilibrium exists in Case 1a with $R_p^* = 2f_p^* + r_p^*[\frac{2(1-r_p^*)(-c_1+c_2)}{(1+b)(2-b)}]$ in (A70), where $r_p^* = \frac{1}{3-b} - \frac{4+2b-3b^2(1+b)(2-b)}{2(1+b)(3-b)}$ and $f_p^* = \frac{1}{2}(1-b^2)$. $\frac{2(1-b)(2+b)+2+3b-2b^2)c_1}{(2+b)(3-b)(1-b^2)}$, an equilibrium in Case 2a-1 with $R_p^* = \frac{2-c_2}{2(1+b)}$ in (A78), and an equilibrium in Case 3a with $R_p^* = \frac{1-c_2}{2(1+b)}$ in (A94). Define $M_1 = \frac{(1-c_2)^2}{2(1+b)} - 2f_p^* - r_p^*[\frac{2(1-r_p^*)(-c_1+c_2)}{(1+b)(2-b)}] = \frac{1}{4(1-b^2)(3-b)(2-b)}[8b - 6b^2 + 4b^3 + 2b^4] + (8 - 4b - 4b^2)c_1 + (-24 + 20b + 16b^2 - 8b^3 - 4b^4)c_2 + (12b + 6b^2 - b^3)c_1c_2 - (4 + 4b + b^2 - b^3)c_1^2 + (12 - 16b - 11b^2 + 5b^3 + 2b^4)c_2^2$ with $\frac{\partial M_1}{\partial c_2} = \frac{-12 + 10b + b^2 - 2b^3}{2(1-b^2)(3-b)(2-b)(1-b^2)}c_1 - (12 - 16b - 11b^2 + 5b^3 + 2b^4)c_2^2$. By some calculations, we get $\frac{\partial M_1}{\partial c_2} |_{c_2 = c_1} = \frac{-3(3-b-b^2)(1-c_1)}{12(b)(2-b)(3-b)(1-b^2)} < 0$, $\frac{\partial M_1}{\partial c_2} |_{c_2 = \hat{c}_{p2}} = \frac{1}{(3-b)(1+b)(1-b^2)(3-b)(2-b)(1-b^2)}c_1^2 > 0$, and $M_1 |_{c_2 = \hat{c}_{p2}} = \frac{(1-b)(1-c_1)^2}{2(1+b)(3-b)(2-b)(1-b^2)}c_1 > 0$. Thus, we have $M_1 \geq \max\{M_1 |_{c_2 = c_1}, M_1 |_{c_2 = \hat{c}_{p2}}\} > 0$, which implies $\frac{(1-c_2)^2}{2(1+b)} > 2f_p^* + r_p^*[\frac{2(1-r_p^*)(-c_1+c_2)}{(1+b)(2-b)}]$.
Next, define \( M_2 = \frac{(1-c_2)^2}{2(1+b)} - \frac{2c_2-2c_1-c_2}{4(1+b)(3-b)} \). Since 
\[
\frac{\partial M_2}{\partial c_2} = \frac{-2(2b-c_1+3b-c_2)}{2(1+b)(3-b)} \
\frac{\partial^2 M_2}{\partial c_2^2} = \frac{2(1-b)}{2(1+b)(3-b)} > 0,
\]
we have \( \frac{\partial M_2}{\partial c_2} < 0 \) and \( \frac{\partial^2 M_2}{\partial c_2^2} < 0 \). Moreover, since \( M_2 > \frac{(1-c_2)^2}{2(1+b)} - \frac{2(1-c_2)^2}{4(1+b)(3-b)} = \frac{2}{4(1+b)(3-b)} > 0 \), we have \( \frac{\partial M_2}{\partial c_2} < \). Thus, the port authority will choose the optimal unit-fee contract in Case 3a with 
\[
r_p^* = \frac{(1-c_2)^2}{2}, \quad \delta_p = \frac{1-c_2}{2}, \quad f_p^* = 0, \quad \text{and} \quad R_p^* = \frac{(1-c_2)^2}{2}.
\]

Case B: For \( c_2 \in (\tilde{c}_p, c'_p) \), an equilibrium exists in Case 1a with \( R_p^* = 2f_p^* + r_p^* \). 
\[
\left[\frac{2(1-b)(2+b)^2(3+b)^2}{2(1+b)(3-b)}\right]^2, \quad \text{an equilibrium in Case 2a-2 with} \quad R_p^* = \frac{1}{2} \left( 1 - b^2 \right).
\]

Next, define \( M_2 = \frac{(1-c_2)^2}{2(1+b)} - \frac{2c_2-2c_1-c_2}{4(1+b)(3-b)} \). Since 
\[
\frac{\partial M_2}{\partial c_2} = \frac{-2(2b-c_1+3b-c_2)}{2(1+b)(3-b)} \
\frac{\partial^2 M_2}{\partial c_2^2} = \frac{2(1-b)}{2(1+b)(3-b)} > 0,
\]
we have \( \frac{\partial M_2}{\partial c_2} < 0 \) and \( \frac{\partial^2 M_2}{\partial c_2^2} < 0 \). Moreover, since \( M_2 > \frac{(1-c_2)^2}{2(1+b)} - \frac{2(1-c_2)^2}{4(1+b)(3-b)} = \frac{2}{4(1+b)(3-b)} > 0 \), we have \( \frac{\partial M_2}{\partial c_2} < \). Thus, the port authority will choose the optimal unit-fee contract in Case 3a with 
\[
r_p^* = \frac{(1-c_2)^2}{2}, \quad \delta_p = \frac{1-c_2}{2}, \quad f_p^* = 0, \quad \text{and} \quad R_p^* = \frac{(1-c_2)^2}{2}.
\]

Case C: For \( c_2 \in (c'_p, c''_p) \), an equilibrium exists in Case 1a with \( R_p^* = 2f_p^* + r_p^* \). 
\[
\left[\frac{2(1-b)(2+b)^2(3+b)^2}{2(1+b)(3-b)}\right]^2, \quad \text{an equilibrium in Case 2b-1 with} \quad R_p^* = \frac{(3+b-c_1-(2+b)c_2)^2}{8(1+b)(3-b)} \text{ in (A87), and an equilibrium in Case 3a with} \quad R_p^* = \frac{(1-c_2)^2}{2(1+b)} \text{ in (A94). As in Case A, some calculations show} \frac{\partial M_2}{\partial c_2} |_{c_2 = c''_p} = \frac{-2(1-b)(12+20b+8b^2+3b)^2}{4(1+b)(3-b)^2(2+b)(3-b)} > 0 \text{ and} \quad M_1 |_{c_2 = c''_p} > \text{max} \{ M_1 |_{c_2 = c_2'}, M_1 |_{c_2 = c''_p} \} \text{ > 0, and hence} \frac{(1-c_2)^2}{2(1+b)} > \frac{2f_p^* + r_p^*}{2(1+b)(3-b)} > 0 \text{. Moreover, we have} \frac{(1-c_2)^2}{2(1+b)} > \frac{1}{(1+b)(b-2)} > \frac{1}{(1+b)(1-b)} > 0 \text{. Accordingly, the port authority will choose the optimal unit-fee contract in Case 3a with} \quad r_p^* = \frac{(1-c_2)^2}{2}, \quad \delta_p = \frac{1-c_2}{2}, \quad f_p^* = 0, \quad \text{and} \quad R_p^* = \frac{(1-c_2)^2}{2}.
\]
Thus, the port authority will choose the unit-fee contract in Case 2b-1 with \(c'_{p2} < \hat{c}_p < c''_{p2}\) so that \(M_3 = 0\) at \(c'_{p2} = \frac{1}{8(1+b)} \{(16-b-b^2) + (2+b)c_1 - 2\sqrt{(3+b)(1-2c_1 + c_1^2)}\}\). Accordingly, we have \(\frac{(1-c_2)^2}{2(1+b)} > \frac{(3+b)-c_1-(2+b)c_2}{8(1+b)(3+b)}\) for \(c_1 \in (c'_{p2}, \hat{c}_p)\), and \(\frac{(1-c_2)^2}{2(1+b)} < \frac{(3+b)-c_1-(2+b)c_2}{8(1+b)(3+b)}\) for \(c_1 \in (\hat{c}_p, c''_{p2})\).

Thus, if \(c_2 \in (c'_{p2}, c''_{p2})\), there are two sub-cases. For \(c_2 \in (\hat{c}_p, c''_{p2})\), the port authority will choose the unit-fee contract in Case 3a with \(r_p^* = \frac{(1-c_2)}{2}, f_p^* = 0\), \(\delta_p^* = \frac{(1-c_2)}{2(1+b)}\), and \(R_p^* = \frac{(1-c_2)^2}{2(1+b)}\) in (A94). For \(c_2 \in (\hat{c}_p, c''_{p2})\), the port authority will choose the unit-fee contract in Case 2b-1 with \(r_p^* = \frac{(3+b)-c_1-(2+b)c_2}{2(3+b)}, \delta_p^* = \frac{(1-b)(2+b)(1-r_p^*)+bc_1-(2-b^2)c_2}{2(1-b^2)}, f_p^* = 0\), and \(R_p^* = \frac{(3+b)-c_1-(2+b)c_2}{8(1+b)(3+b)}\) in (A87).

**Case D:** For \(c_2 \in [c''_{p2}, c''_{p2}]\), an equilibrium exists in Case 1a with \(R_p^* = \frac{(1-c_2)}{2(1+b)}\) in (A70), where \(r_p^* = \frac{1}{3-b} - \frac{(4+2b-b^2)c_1+b^2c_2}{2(2+b)(3-b)}\) and \(f_p^* = \frac{1}{2}(1-b^2)\).

\[
\frac{(2(1-b)(2+b)+2(3-2b^2)c_1-6(3+2b^2)c_2)^2}{2(2+b)(3-b)(1-b^2)} < 0
\]

22. \(\frac{(3+b)-c_1-(2+b)c_2}{8(1+b)(3+b)} \in (A79), and an equilibrium in Case 2b-1 with \(R_p^* = \frac{(3+b)-c_1-(2+b)c_2}{8(1+b)(3+b)}\) in (A87). Define \(M_4 = \frac{(3+b)-c_1-(2+b)c_2}{8(1+b)(3+b)} - 2f_p^* - r_p^* \left(\frac{(1-r_p^*)}{(1+b)(2-b)}\right)\). Then \(M_4 = \frac{(3+b)-c_1-(2+b)c_2}{8(1+b)(3+b)}\) and \(\frac{\partial M_4}{\partial c_2} = \frac{-2(24+16b-2b^2-3b^4+b^5)(4+16b-8b^2-8b^3+b^4)(12-8b-13b^2+3b^3+5b^4+b^5)+(24+16b-8b^2-8b^3+b^4)(12-8b-13b^2+3b^3+5b^4+b^5)}{4(1-b)(2+b)^2(9-b^2)} < 0\), we have \(\frac{\partial M_4}{\partial c_2} < \frac{1}{4(1-b)(2+b)^2(9-b^2)} \left(6b^2-2b^3-4b^4-b^5\right)\). Thus, the port authority will choose the unit-fee contract in Case 2b-1 with \(r_p^* = \frac{(3+b)-c_1-(2+b)c_2}{2(3+b)}, f_p^* = 0\), \(\delta_p^* = \frac{(1-b)(2+b)(1-r_p^*)+bc_1-(2-b^2)c_2}{2(1-b^2)}\), and \(R_p^* = \frac{(3+b)-c_1-(2+b)c_2}{8(1+b)(3+b)}\) in (A87).
(A87).

Case E: For \( c_2 \in [c''_{p2}, \hat{c}_{p2}] \), an equilibrium exists in Case 1a with \( R^*_p = 2f^*_p + r^*_p\left[\frac{2(1-r^*_p)-(c_1+c_2)}{(1+b)(2-b)}\right] \) in (A70), where \( r^* = \frac{1}{3b} - \frac{(4+2b-b^2)c_1+b^2c_2}{2(2+b)(3-b)} \) and \( f^*_p = \frac{1}{2} (1-b^2) \). An equilibrium exists in Case 2b-1 with \( R^*_p = \frac{1}{8(1+b)(3+b)} \) \((3+b)-c_1-(2+b)c_2\)^2 in (A87). Since \( \frac{\partial M_4}{\partial c_2} < 0 \) by the results of Case D, we have

\[
M_4 > \frac{1}{8(1-b)(2+b)^2(9-b^2)} \left[(24+12b^2+b^4+b^6)+(24+16b+14b^2-4b^3-2b^4)c_1+(12b-2b^3-8b^4-2b^5)c_{p2}+(48+32b-16b^2-2b^3+2b^4)c_1\hat{c}_{p2}+(12-24b+b^2+3b^3)c_2^2+(-24-16b+2b^2+2b^3+3b^4+b^5)c_2^2\right] = \frac{1}{8(3+b)(6+b-3b^2)^2} > 0.\]

Thus, the port authority will choose the unit-fee contract in Case 2b-1 with \( r^*_p = \frac{[3+b]-c_1-(2+b)c_2}{2(3+b)} \), \( f^*_p = 0 \), \( \delta^*_p = \frac{(b-1)(b+1)^2+b+c_1-(2-b^2)c_2}{2(1-b^2)} \), and \( R^*_p = \frac{[3+b]-c_1-(2+b)c_2}{8(1+b)(3+b)} \) in (A87).

Case F: For \( c_2 \in [\hat{c}_{p2}, \bar{c}_{p2}] \), an equilibrium exists in Case 1b with \( R^*_p = \bar{r}_p q_{p1} = \frac{(c_2-c_1)(1-(b-2+b^2)\bar{c}_{p1}-(2-b^2)c_2)}{(1-b)^2(2+b)^2} \) in (A71), and an equilibrium in Case 2b-1 with \( R^*_p = \frac{[3+b]-c_1-(2+b)c_2}{8(1+b)(3+b)} \) in (A87). By some calculations, we have

\[
\{ (c_2-c_1)[(1-b)(2+b)+bc_1-(2-b^2)c_2] \}^2 = \frac{[6-b-4b^2+b^3](2+b^2)+b^3+c_2(8+4b-3b^2-b^3)c_2^2}{8(1+b)(3+b)(1-b)^2(2+b)^2} > 0.\]

Thus, the best choice for the port authority is the unit-fee contract with \( r^*_p = \frac{[3+b]-c_1-(2+b)c_2}{2(3+b)} \), \( \delta^*_p = \frac{(1-b)(2+b)(1-r^*_p)+bc_1-(2-b^2)c_2}{2(1-b^2)} \), \( f^*_p = 0 \), and \( R^*_p = \frac{[3+b]-c_1-(2+b)c_2}{8(1+b)(3+b)} \) in (A87).

Case G: For \( c_2 \in (\bar{c}_{p2}, \hat{c}_{p2}) \), only the equilibrium in Case 1b exists with \( R^*_p = \bar{r}_p q_{p1} = \frac{(c_2-c_1)(1-(b-2+b^2)\bar{c}_{p1}-(2-b^2)c_2)}{(1-b)^2(2+b)^2} \) in (A71). Thus, the best choice for the port authority is the unit-fee contract with \( r^*_p = \bar{r}_p \equiv \frac{(1-b)(2+b)+bc_1-(2-b^2)c_2}{(1-b)(2+b)^2} \), \( \delta^*_p = 0 \), \( f^*_p = 0 \), and \( R^*_p = \bar{r}_p q_{p1} = \frac{[c_2-c_1][1-(b-2+b^2)\bar{c}_{p1}-(2-b^2)c_2]}{(1-b)^2(2+b)^2} \) in (A71).

Proof of Corollary 4: By some calculations, we have \( \bar{c}_{p2} < \hat{c}_{p2} \geq \hat{c}_{p2} \) if \( b < (\gtrless) 0.830504 \), \( \hat{c}_{p2} < \bar{c}_{p2} < \hat{c}_{p2} \), \( \bar{c}_{p2} \geq \hat{c}_{p2} \) if \( b < (\gtrless) 0.830504 \). According to relative sizes of \( c_1, \bar{c}_{p2}, \hat{c}_{p2}, \tilde{c}_2, \hat{c}_{p2}, \tilde{c}_2, \) and \( \hat{c}_2 \), we can prove Corollary 4 by the ensuing Lemmas A, B, C and D.

**Lemma A.** Suppose \( b < 0.80734 \). Then we have the following.
(i) For $c_2 \in (c_1, \hat{c}_{p2}]$, terminal operators’ best choices and port authority’s optimal contracts under both competition modes are the same.

(ii) For $c_2 \in (\hat{c}_{p2}, \bar{c}_2]$, operators will rent terminals under both competition modes, but operator 2’s equilibrium profit equals zero. Moreover, we have $R_p^u > R_u^u$, $\pi_{p2}^u = \pi_2^u = 0$, $\pi_{p1}^u \leq \pi_1^u$ for $c_2 \in (\hat{c}_{p2}, v]$, $\pi_{p1}^u > \pi_1^u$ for $c_2 \in (v, \bar{c}_2]$ if $b > 0.226985$, and $\pi_{p1}^u < \pi_1^u$ for $c_2 \in (\hat{c}_{p2}, \bar{c}_2]$ if $b < 0.226985$, where $v$ is defined in the proofs below.

(iii) For $c_2 \in (\bar{c}_2, \hat{c}_{p2}]$, we have $R_p^u > R_u^u$, $\pi_{p2}^u = \pi_2^u = 0$, $\pi_{p1}^u \geq \pi_1^u$ for $c_2 \in (\bar{c}_2, t]$ and $\pi_{p1}^u < \pi_1^u$ for $c_2 \in (t, \hat{c}_{p2}]$ if $0.391041 < b < 0.45193$, $\pi_{p1}^u > \pi_1^u$ for $c_2 \in (\bar{c}_2, \hat{c}_{p2}]$ if $b > 0.45193$ and $\pi_{p1}^u < \pi_1^u$ for $c_2 \in (\bar{c}_2, \hat{c}_{p2}]$ if $b < 0.391041$, where $t$ is defined in the proofs below.

(iv) For $c_2 \in (\hat{c}_{p2}, \bar{c}_2]$, operator 2 will not rent terminals under price competition, but operator 1 will always rent them. Accordingly, we have $R_p^u > R_u^u$ for $c_2 \in (\hat{c}_{p2}, \bar{c}_2]$ if $b < 0.643333$, $R_p^u \geq R_u^u$ for $c_2 \in (\hat{c}_{p2}, y]$ and $R_p^u < R_u^u$ for $c_2 \in (y, \bar{c}_2]$ if $b > 0.643333$, $\pi_{p2}^u = \pi_2^u = 0$, $\pi_{p1}^u > \pi_1^u$ for $c_2 \in (\hat{c}_{p2}, \bar{c}_2]$ if $b > 0.45193$, $\pi_{p1}^u \leq \pi_1^u$ for $c_2 \in (\hat{c}_{p2}, k]$ and $\pi_{p1}^u > \pi_1^u$ for $c_2 \in (k, \bar{c}_2]$ if $b < 0.45193$, where $y$ and $k$ are defined in the proofs below.

(v) For $c_2 \in [\bar{c}_2, \hat{c}_{p2}]$, operator 2 will not rent terminals under both competition modes, but operator 1 will always rent them. Accordingly, we have $R_p^u < R_u^u$ for $c_2 \in [\bar{c}_2, \hat{c}_{p2}]$ if $b > 0.643333$, $R_p^u > R_u^u$ for $c_2 \in [\bar{c}_2, w]$ and $R_p^u \leq R_u^u$ for $c_2 \in [w, \hat{c}_{p2}]$ if $b < 0.643333$, $\pi_{p2}^u = \pi_2^u = 0$, and $\pi_{p1}^u > \pi_1^u$, where $w$ is defined in the proofs below.

Proof. For $b < 0.807374$, we have $c_1 < \hat{c}_{p2} < \bar{c}_2 < \bar{c}_{p2} < \hat{c}_2$. Then, there are five sub-cases.

(i) For $c_2 \in (c_1, \hat{c}_{p2}]$, Lemma 2(iii), Lemma 5(i), Lemma 8(iii), and Lemma 10(i) imply $r_p^u = r_u^u = \frac{1-c_2}{2}$, $\delta_p^u = \delta_u^u = \frac{1-c_1}{2(1+b)}$, $R_p^u = R_u^u = \frac{(1-c_2)^2}{2(1+b)}$, $q_{p2}^u = q_{u2}^u = \frac{1-c_2}{2(1+b)}$, $p_{p2}^u = p_{u2}^u = \frac{1+c_2}{2}$, $\pi_{p1}^u = \pi_1^u = \frac{(1-c_2)(c_2-c_1)}{2(1+b)}$, and $\pi_{p2}^u = \pi_2^u = 0$ for $i = 1, 2.$
(ii) For $c_2 \in (\bar{c}_p, \tilde{c}_2]$, by Lemma 2(iii) and Lemma 5(i), we have $R^u = \frac{(1-c_2)^2}{2(1+b)}$, $\tau_1^u = \frac{(1-c_2)(2-c_1)}{2(1+b)}$, and $\pi_2^u = 0$. Moreover, Lemma 8(ii) and Lemma 10(ii) suggest $R^u_p = \frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)}$, $\pi_{p1}^u = \frac{[-(3-2b-c_2)+(5+3b)c_1-(2+5b+b^2)c_2]}{16(1-b^2)(3+b)^2}$, and $\pi_{p2}^u = 0$. Thus, we can get $\frac{[(3+b)-c_1-(2+b)c_2]^2}{8(1+b)(3+b)} > \frac{(1-c_2)^2}{2(1+b)}$ if $c_2 > \hat{c}_p$. This means $R^u_p > R^u$. On the other hand, $(\pi_{p1}^u - \pi_1^u) = \frac{[-(3-2b-c_2)+(5+3b)c_1-(2+5b+b^2)c_2]}{16(1-b^2)(3+b)^2} - \frac{(1-c_2)(2-c_1)}{2(1+b)}$ is a convex function of $c_2$ because $\frac{\partial^2(\pi_{p1}^u - \pi_1^u)}{\partial c_2^2} = \frac{76-4b-11b^2+2b^3+4b^4}{8(1-b^2)(3+b)^2} > 0$. By some calculations, we have $\pi_{p1}^u = \pi_1^u$ at $c_2 = u$ and $c_2 = v$, where

$$u = \frac{[(30-23b-11b^2+3b^3+b^4)+(46+19b-b^3)c_1] - 2(1-c_1)\sqrt{2(3+b)^3(1-b)^3}}{(76-4b-11b^2+2b^3+b^4)}$$

and

$$v = \frac{[(30-23b-11b^2+3b^3+b^4)+(46+19b-b^3)c_1] + 2(1-c_1)\sqrt{2(3+b)^3(1-b)^3}}{(76-4b-11b^2+2b^3+b^4)}$$

with $v > u$. Then, we can know relative sizes of $\pi_{p1}^u$ and $\pi_1^u$ by comparing $\hat{c}_p$, $\tilde{c}_2$, $u$, and $v$. Since $\frac{\partial^2(\pi_{p1}^u - \pi_1^u)}{\partial c_2^2} > 0$, $\frac{\partial(\pi_{p1}^u - \pi_1^u)}{\partial c_2} = \frac{-(1-b)(2+b)(7+b)(1-c_1)}{8(1+b)(3+b)(3+b)^2} < 0$ at $c_2 = c'_2$, and $(\pi_{p1}^u - \pi_1^u) = \frac{-(3+b)(1-b)(1-c_1)^2}{2(1+b)(10-b-b^2)^2} < 0$ at $c_2 = c'_2$ with $c'_2 < \hat{c}_p < c''_2$, where $c'_2 = \frac{(3-2b-b^2)+(7+b)c_1}{(10-b-b^2)}$ and $c''_2 = \frac{(1-b)+2c_1}{(3-b)}$; we have $(\pi_{p1}^u - \pi_1^u) < 0$ at $c_2 = \hat{c}_p$ and hence $u < \hat{c}_p < v$. Then, we can show $(v - \tilde{c}_2) = \frac{(1-c_1)}{(8-3b^2)(76-4b-11b^2+2b^3+b^4)}[-(216 + 84b - 36b^2 - 62b^3 + 15b^4 + 4b^5 + b^6) + 2(8-3b^2)\sqrt{2(3+b)^3(1-b)^3} + (76-4b-11b^2+2b^3+b^4)\sqrt{2(6+2b-7b^2-b^3+2b^4)} \geq (\leq) 0$ iff $b \leq (\geq) 0.226985$. Accordingly, $u < \hat{c}_p < \tilde{c}_2 < v$ if $b < 0.226985$, and $u < \hat{c}_p < v < \tilde{c}_2$ if $b > 0.226985$. These suggest $\pi_{p1}^u < \pi_1^u$ if $b < 0.226985$, and $\pi_{p1}^u \leq \pi_1^u$ for $c_2 \in (\hat{c}_p, v]$ and $\pi_{p1}^u > \pi_1^u$ for $c_2 \in (v, \tilde{c}_2]$ if $b > 0.226985$.

(iii) For $c_2 \in (\tilde{c}_2, \hat{c}_p]$, Lemma 2(ii) and Lemma 5(ii) imply $R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2)^2}{4(3-2b)(2-b^2)}$, $\pi_1^u = \frac{[(3-5b+2b^2)-(5-2b-b^2)c_1+(2+3b-3b^2)c_2)^2}{4(3-2b)^2(2-b^2)^2}$, and $\pi_2^u = 0$. Moreover, Lemma 8(ii) and Lemma 10(ii) suggest $R^u_p = \frac{[(3+b)-c_1-(2+b)c_2)^2}{8(1+b)(3+b)}$, $\pi_{p1}^u = \frac{[-(3-2b-b^2)+(5+3b)c_1-(2+5b+b^2)c_2]}{16(1-b^2)(3+b)^2}$, and $\pi_{p2}^u = 0$. By some calculations, we have $(R^u_p - R^u) = \frac{[(3+b)-(1-b)c_1-(2-b)c_2)^2}{8(1+b)(3+b)} - \frac{[(3-2b)-(1-b)c_1-(2-b)c_2)^2}{4(3-2b)(2-b^2)^2}$ with $\frac{\partial^2(R^u_p - R^u)}{\partial c_2^2} = \frac{-b^2(4+8b-3b^2-2b^3)}{4(1+b)(3+b)(3-2b)(2-b^2)} < 0$. That is, $(R^u_p - R^u)$ is a concave function of $c_2$. In addition, we can get $(R^u_p - R^u) = \frac{b^2(1-b)(72+48b-44b^2-16b^3+3b^4+b^5)-(1-c_1)^2}{4(2-b^2)(3-2b)(8+4b-3b^2-b^4)^2} > 0$ at $c_2 = \tilde{c}_p$, and $R^u_p > R^u$ at $c_2 = \tilde{c}_p$ because $\frac{[(3+b)-(1-b)c_1-(2-b)c_2)^2}{8(1+b)(3+b)} > \frac{(1-c_1)^2}{2(1+b)}$ for $c_2 > \hat{c}_p$.
and \( \frac{(1-c_2)^2}{2(1+b)} = \frac{[3(3-2b)-(1-b)c_1-(2-b)c_2]}{4(3-2b)(2-b^2)} \) at \( c_2 = \tilde{c}_2 \). Thus, \( R_p^u > R^u \) for \( c_2 \in (\tilde{c}_2, \tilde{c}_{p2}] \) by \\
\( \frac{\partial^2(R_p^u-R^u)}{\partial c_2^2} < 0 \), and \( R_p^u > R^u \) at \( c_2 = \tilde{c}_{p2} \) and \( c_2 = \tilde{c}_2 \).

On the other hand, we have \( (\pi_p^u-\pi_1^u) = \frac{b^2 H(b, c_1, c_2)}{16(1-b^2)(3+b)(3-2b)(2-b^2)} \), where \( H(b, c_1, c_2) = (81b^2-216b^3+162b^4+12b^5+47b^6+4b^7+4b^8) \) \( (72-168b-238b^2+554b^3-66b^4-210b^5+32b^6+24b^7) c_1 \) \( (-72+168b+76b^2-122b^3-258b^4+186b^5+62b^6-32b^7-8b^8) c_2 \) \( (168-88b-524b^2-158b^3+504b^4+106b^5+112b^6-24b^7) c_1 c_2 \) \( (-120+128b+381b^2-198b^3-219b^4+52b^5+40b^6) c_1^2 \) \( (-48-40b+224b^2+140b^3-123b^4-146b^5+26b^6+28b^7+4b^8) c_2^2 \) with \\
\( \frac{\partial^2 H(b, c_1, c_2)}{\partial c_2^2} = 2b^2 (-48-40b+224b^2+140b^3-123b^4-146b^5+26b^6+28b^7+4b^8) \leq (\geq 0) \) iff \( b \leq (\geq) 0.53173 \). This implies that \( H(b, c_1, c_2) \) is a concave (convex) function of \( c_2 \) if \( b < (>) 0.53173 \). In addition, \( H(b, c_1, c_2) = 0 \) at \( c_2 = s \) and \( c_2 = t \), where

\[
s = \frac{1}{(-48-40b+224b^2+140b^3-123b^4-146b^5+26b^6+28b^7+4b^8)} \times [(36-84b-38b^2+61b^3+129b^4-93b^5-31b^6+16b^7+4b^8) + (-84+44b+262b^2+79b^3-252b^4-53b^5+56b^6+12b^7) c_1 + 2(1-c_1) \sqrt{(1+b)(1-b)^5(18-6b-13b^2+3b^3+2b^4)^2}],
\]

and

\[
t = \frac{1}{(-48-40b+224b^2+140b^3-123b^4-146b^5+26b^6+28b^7+4b^8)} \times [(36-84b-38b^2+61b^3+129b^4-93b^5-31b^6+16b^7+4b^8) + (-84+44b+262b^2+79b^3-252b^4-53b^5+56b^6+12b^7) c_1 + 2(1-c_1) \sqrt{(1+b)(1-b)^5(18-6b-13b^2+3b^3+2b^4)^2}],
\]

and \( s \leq (\geq) t \) iff \( b \leq (\geq) 0.53173 \). Under the circumstance, \( s - \tilde{c}_2 = (1-c_1)(s_1 + s_2 + s_3) \times \frac{1}{(8-3b^2)(-48-40b+224b^2+140b^3-123b^4-146b^5+26b^6+28b^7+4b^8)} \), where \( s_1 = -(3-2b)(2-b^2)(2b^7-3b^6-90b^5-133b^4+218b^3+278b^2+16b-96) \leq (\geq 0) \) iff \( b \geq (\leq) 0.506603 \), \( s_2 = 2(8-3b^2) \sqrt{(1+b)(1-b)^5(18-6b-13b^2+3b^3+2b^4)^2} > 0 \), and \( s_3 = (-48-40b+224b^2+140b^3-123b^4-146b^5+26b^6+28b^7+4b^8) \sqrt{2(6+2b-7b^2-b^3+2b^4)} \leq (\geq 0) \) iff \( b \leq (\geq) 0.53173 \). By some calculations, we can show \( s_1 + s_2 + s_3 > 0 \) if \( b < 0.53173 \), and \( s_1 + s_2 + s_3 < 0 \) if \( b > 0.53173 \). Thus, \( \tilde{c}_2 > s \). On the other hand, we have \( (\tilde{c}_{p2} - t) = \frac{(8+4b-3b^2-b^3) \times (8+4b+224b^2+140b^3-123b^4-146b^5+26b^6+28b^7+4b^8)}{t_1 + t_2} \), where \( t_1 = 2(8+4b-3b^2-b^3) \sqrt{(1+b)(1-b)^5(18-6b-13b^2+3b^3+2b^4)^2} > 0 \), and \( t_2 = \)
4(1 - b^2)(3 + b)^2(4b^6 + 11b^5 - 38b^4 - 10b^3 + 37b^2 + 20b - 16) \leq (\geq) 0 \text{ if } b \leq (\geq) 0.4991.

Some calculations can show \((t_1 + t_2) < 0\) if \(b < 0.45193\) and \((t_1 + t_2) > 0\) if \(b > 0.45193\). That is, \(\tilde{c}_{p_2} > t\) if \(b < 0.45193\), \(\tilde{c}_{p_2} < t\) if \(0.45193 < b < 0.53173\), and \(\tilde{c}_{p_2} > t\) if \(b > 0.53173\). Accordingly, if \(b < 0.45193\), then \(s < t\), \(s < \tilde{c}_2\), \(t < \tilde{c}_{p_2}\), and \(H(b, c_1, c_2)\) is a concave function of \(c_2\). If \(0.45193 < b < 0.53173\), we have \(s < \tilde{c}_2 < \tilde{c}_{p_2} < t\); and we have \(t < s < \tilde{c}_2 < \tilde{c}_{p_2}\) and \(H(b, c_1, c_2)\) is a convex function of \(c_2\) if \(b > 0.53173\).

Next, we compare \(\tilde{c}_2\) and \(t\) as \(b < 0.45193\). By some calculations, we have \((t - \tilde{c}_2) = \frac{(1-c_1)(s_1-s_2+s_3)}{(8-3b^2)(-48-40b+22b^2+14b^3-12b^4-14b^5+25b^6+2b^7+b^8)}\) with \(s_1 > 0\), \(s_2 > 0\), and \(s_3 < 0\) if \(b < 0.45193\). We can show \(t < \tilde{c}_2\) if \(b < 0.391041\) and \(t > \tilde{c}_2\) if \(0.391041 < b < 0.45193\). Thus, if \(b < 0.53173\), then \(H(b, c_1, c_2)\) is a convex function of \(c_2\). Moreover, \(s < t < \tilde{c}_2 < \tilde{c}_{p_2}\), \(\pi^{u_1}_{p_1} < \pi^{u_1}_1\) for \(c_2 \in (\tilde{c}_2, \tilde{c}_{p_2}]\) if \(b < 0.391041\); \(s < \tilde{c}_2 < t < \tilde{c}_{p_2}\), \(\pi^{u_1}_{p_1} \geq \pi^{u_1}_1\) for \(c_2 \in (\tilde{c}_2, \tilde{c}_{p_2}]\) if \(0.391041 < b < 0.45193\); \(s < \tilde{c}_2 < \tilde{c}_{p_2} < t\), \(\pi^{u_1}_{p_1} > \pi^{u_1}_1\) for \(c_2 \in (\tilde{c}_2, \tilde{c}_{p_2}]\) if \(0.45193 < b < 0.53173\). As \(b > 0.53173\), \(H(b, c_1, c_2)\) is a convex function of \(c_2\), \(t < s < \tilde{c}_2 < \tilde{c}_{p_2}\), and \(\pi^{u_1}_{p_1} > \pi^{u_1}_1\) for \(c_2 \in (\tilde{c}_2, \tilde{c}_{p_2}]\).

(iv) For \(c_2 \in (\tilde{c}_{p_2}, \tilde{c}_2)\), Lemma 2(ii) and Lemma 5(ii) suggest \(R^u = \frac{[(3-2b)-(1-b)(c_1-(2-b)c_2)]^2}{4(3-2b)(2-b^2)}\), \(\pi^{u_1}_1 = \frac{[(3-5b+2b^2)-(5-2b-b^2)c_1+(2+3b-3b^2)c_2]^2}{4(3-2b)^2(2-b^2)^2}\), and \(\pi^u_2 = 0\). On the other hand, Lemma 8(i) and Lemma 10(iii) imply \(R^u_p = \frac{(c_2-c_1)(1-b)(2+b)}{(1-b)(2-b)^2}\), \(\pi^{u_1}_{p_1} = \frac{(1+b)(c_2-c_1)^2}{(1-b)(2+b)^2}\), and \(\pi^u_{p_2} = 0\). Thus, we have \((R^u_p - R^u) = \frac{(c_2-c_1)(1-b)(2+b)}{(1-b)(2+b)^2}\) and \(\frac{\partial^2(R^u_p - R^u)}{\partial c_2^2} = \frac{[(48-80b+2b^2+47b^3-12b^4-7b^5+2b^6)+(16+16b-42b^2+b^3+17b^4-3b^5-b^6)c_1-2b(1-b)(6-b-2b^2)}{\sqrt{(1-b)(2-b^2)(1-c_1)^2}}\), where

\[x = \frac{1}{(64-64b-40b^2+48b^3+5b^4-10b^5+b^6) \times [(48-80b+2b^2+47b^3-12b^4-7b^5+2b^6)+(16+16b-42b^2+b^3+17b^4-3b^5-b^6)c_1-2b(1-b)(6-b-2b^2)}}{\sqrt{(1-b)(2-b^2)(1-c_1)^2}},\]
and

\[
y = \frac{1}{(64 - 64b - 40b^2 + 48b^3 + 5b^4 - 10b^5 + b^6)[(48 - 80b + 2b^2 + 47b^3 - 12b^4 - 7b^5 + 2b^6) + (16 + 16b - 42b^2 + b^3 + 17b^4 - 3b^5 - b^6c_1 + 2b(1 - b)(6 - b - 2b^2)]} \\
\sqrt{(1 - b)(2 - b)(1 - c_1)^2}
\]

with \((y - x) = \frac{4b(1-b)(6-b-2b^2)}{(64 - 64b - 40b^2 + 48b^3 + 5b^4 - 10b^5 + b^6)(1-b)(2-b)(1-c_1)^2} > 0\). Thus, \(R_p^u > R_u^u\) if \(x < c_2 < y\), \(R_p^u < R_u^u\) if \(c_2 < x\), and \(R_p^u < R_u^u\) if \(c_2 > y\). By some calculations, we can obtain \((R_p^u - R_u^u) = \frac{b^2(1-b)(72 + 48b + 44b^3 + 167b^4 + 30b^5 + 6b^6)(1-c_1)^2}{(1-b)^2(2-b)^2(8+4b-2b^2-b^3)^2} > 0\) at \(c_2 = \tilde{c}_p^2\), \(\frac{\partial(R_p^u - R_u^u)}{\partial c_2} = \frac{-b^2(4+2b-5b^2+b^3)(1-c_1)}{(1-b)^2(2+b)^2(8-4b-b^2)^2} < 0\), and \((R_p^u - R_u^u) = \frac{b^2(3-2b)(4-8b+3b^2-b^3-b^4)(1-c_1)^2}{(1-b)^2(2+b)^2(8-4b-b^2)^2} \leq (\geq) 0\) if \(b \geq (\leq) 0.643333\) at \(c_2 = \tilde{c}_2\). These suggest \(x < \tilde{c}_p^2 < y\), \(\tilde{c}_2 < y\) if \(b < 0.643333\), and \(\tilde{c}_2 > y\) if \(b > 0.643333\). Accordingly, \(R_p^u > R_u^u\) for \(c_2 \in (\tilde{c}_p^2, \tilde{c}_2)\) if \(b < 0.643333\), \(R_p^u \geq R_u^u\) for \(c_2 \in (\tilde{c}_p^2, y]\), and \(R_p^u < R_u^u\) for \(c_2 \in (y, \tilde{c}_2)\) if \(b > 0.643333\).

On the other hand, we have \((\pi^u_{p1} - \pi^u_{1}) = \frac{(1+b)(c_2-c_1)^2}{(1-b)(2+b)^2} - \frac{[(1-3b+2b^2) - (5-2b-b^2)c_1 + (2+3b-3b^2)c_2]^2}{4(3-2b)^2(2-b^2)^2} > 0\). That is, \((\pi^u_{p1} - \pi^u_{1})\) is a strictly convex function of \(c_2\). Moreover, \(\pi^u_{p1} = \pi^u_{1}\) at \(c_2 = j\) and \(c_2 = k\), where

\[
j = \frac{1}{(128 - 96b - 248b^2 + 224b^3 + 131b^4 - 133b^5 - 23b^6 + 25b^7)} \times [(24 - 4b - 98b^2 + 81b^3 + 37b^4 - 43b^5 - 3b^6 + 6b^7) + (104 - 92b - 150b^2 + 143b^3 + 94b^4 - 90b^5 - 20b^6 + 19b^7)]c_1 - 2(1-b)(2+b)(2-b^2)(3-2b^2)\sqrt{(1-b^2)(1-c_1)^2}
\]

and

\[
k = \frac{1}{(128 - 96b - 248b^2 + 224b^3 + 131b^4 - 133b^5 - 23b^6 + 25b^7)} \times [(24 - 4b - 98b^2 + 81b^3 + 37b^4 - 43b^5 - 3b^6 + 6b^7) + (104 - 92b - 150b^2 + 143b^3 + 94b^4 - 90b^5 - 20b^6 + 19b^7)]c_1 + 2(1-b)(2+b)(2-b^2)(3-2b^2)\sqrt{(1-b^2)(1-c_1)^2}
\]

with \(k > j\). Then, \(\pi^u_{p1} < \pi^u_{1}\) if \(j < c_2 < k\), and \(\pi^u_{p1} > \pi^u_{1}\) if \(c_2 < j\) or \(c_2 > k\). Since \(\frac{\partial(\pi^u_{p1} - \pi^u_{1})}{\partial c_2} = \frac{b^2(1-b)(1-44+120b+544b^2-44b^3-42b^4+b^5+b^6+17b^7)(1-c_1)}{4(3-2b)^2(2-b^2)^2(8+4b-2b^2-b^3)^2} \leq (\geq) 0\) if \(b \leq (\geq) 0.45193\) at \(c_2 = \tilde{c}_p^2\), and \((\pi^u_{p1} - \pi^u_{1}) = \frac{b^2(3-2b)^2(1-c_1)^2}{(1-b)(2+b)^2(8-4b-b^2)^2} > 0\) at \(c_2 = \tilde{c}_2\); we can infer \(\tilde{c}_2 > k\), \(\tilde{c}_p^2 < k\).
if $b < 0.45193$, and $\tilde{c}_{p2} > k$ if $b > 0.45193$. Thus, $\pi^u_{p1} > \pi^u_1$ for $c_2 \in (\tilde{c}_{p2}, \tilde{c}_2)$ if $b > 0.45193$, $\pi^u_{p1} \leq \pi^u_1$ for $c_2 \in (\bar{c}_{p2}, k]$, and $\pi^u_{p1} > \pi^u_1$ for $c_2 \in (k, \tilde{c}_2)$ if $b < 0.45193$.

(v) For $c_2 \in [\tilde{c}_2, \bar{c}_{p2}]$, Lemma 2(i) and Lemma 5(iii) suggest $R^u = \frac{(c_2-c_1)(2-b)+bc_1-2c_2}{(2-b)^2}$, $\pi^u_1 = \frac{(c_2-c_1)^2}{2-b}$, and $\pi^u_2 = 0$. On the other hand, Lemma 8(i) and Lemma 10(iii) imply $R^u_p = \frac{(c_2-c_1)(1-b)(2+b)+bc_1(2-b^2)c_2}{(1-b)(2+b)^2}$, $\pi^u_{p1} = \frac{1+b)(c_2-c_1)^2}{(1-b)(2+b)^2}$, and $\pi^u_{p2} = 0$. Thus, we have

$$R^u_p - R^u = \frac{b^2[(4-4b-b^3)+(b(4-2b-b^2)c_1-(4-3b^2)c_2)(c_2-c_1)]}{(1-b)^2(4-b^2)}$$

with $\frac{\partial^2 (R^u_p - R^u)}{\partial c_2^2} = -\frac{2b^2(4-3b^2)}{(1-b)^2(4-b^2)} < 0$. These mean that $(R^u_p - R^u)$ is a strictly concave function of $c_2$. In addition, $R^u_p = R^u$ at $c_2 = c_1$ and

$$c_2 = \frac{(4-4b-b^2+b^3)+b(4-2b-b^2)c_1}{(4-3b^2)} \equiv w,$$

with $(w - c_1) = \frac{(4-4b-b^2+b^3)(1-c_1)}{(4-3b^2)} > 0$. Then, we can infer $c_1 < \tilde{c}_2 < w < \bar{c}_{p2}$ if $b < 0.643333$, and $w < \tilde{c}_2 < \bar{c}_{p2}$ if $b > 0.643333$. Thus, $R^u_p < R^u$ for $c_2 \in [\tilde{c}_2, \bar{c}_{p2})$ if $b > 0.643333$, $R^u_p > R^u$ for $c_2 \in (\bar{c}_{p2}, w]$, and $R^u_p \leq R^u$ for $c_2 \in [w, \bar{c}_{p2})$ if $b < 0.643333$.

We also have $(\pi^u_{p1} - \pi^u_1) = \frac{2b^2(c_2-c_1)^2}{(1-b)(4-b^2)^2} > 0$. □

**Lemma B.** Suppose $0.807374 < b < 0.830504$. Then we have the following.

(i) For $c_2 \in (c_1, \tilde{c}_{p2}]$, the results are the same as those in Lemma A(i).

(ii) For $c_2 \in (\tilde{c}_{p2}, \tilde{c}_2]$, we have $R^u_p > R^u$, $\pi^u_{p2} = \pi^u_2 = 0$, $\pi^u_{p1} \leq \pi^u_1$ for $c_2 \in (\tilde{c}_{p2}, v]$, and $\pi^u_{p1} > \pi^u_1$ for $c_2 \in (v, \tilde{c}_2]$.

(iii) For $c_2 \in (\tilde{c}_2, \bar{c}_{p2}]$, we have $R^u_p > R^u$, $\pi^u_{p2} = \pi^u_2 = 0$, and $\pi^u_{p1} > \pi^u_1$ for $c_2 \in (\tilde{c}_2, \bar{c}_{p2}]$.

(iv) For $c_2 \in (\tilde{c}_{p2}, \bar{c}_{p2})$, operator 2 will not rent terminals under both competition modes, but operator 1 will always rent them. Accordingly, we have $R^u_p \geq R^u$ for $c_2 \in (\tilde{c}_{p2}, y]$ and $R^u_p < R^u$ for $c_2 \in (y, \tilde{c}_2)$, $\pi^u_{p2} = \pi^u_2 = 0$, and $\pi^u_{p1} > \pi^u_1$ for $c_2 \in (\tilde{c}_{p2}, \tilde{c}_{p2})$.

**Proof.** For $0.807374 < b < 0.830504$, we have $c_1 < \tilde{c}_{p2} < \tilde{c}_2 < \bar{c}_{p2} < \tilde{c}_{p2} < \bar{c}_2 < \tilde{c}_2$.  

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Then, there are four sub-cases.

(i) For \( c_2 \in (c_1, \tilde{c}_2) \), we have \( r_p^u = r^u = \frac{1-c_2}{2}, \delta^u = \delta^u = \frac{1-c_2}{2(1+b)^3}, R_p^u = R^u = \frac{(1-c_2)^2}{2(1+b)}, \pi_{p1}^u = \pi_1^u = \frac{(1-c_2)(c_2-c_1)}{2(1+b)}, \) and \( \pi_{p2}^u = \pi_2^u = 0. \)

(ii) For \( c_2 \in (\tilde{c}_2, \bar{c}_2) \), the proofs are the same as those in Lemma A(ii).

(iii) For \( c_2 \in (\bar{c}_2, \bar{c}_2) \), the proofs are the same as those in Lemma A(iii).

(iv) For \( c_2 \in (\bar{c}_2, \bar{c}_2) \), Lemma 2(ii) and Lemma 5(ii) imply \( R_p^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]^2}{4(3-2b)(2-b)^2}, \pi_1^u = \frac{[(3-5b+2b^2)-(5-2b-b^2)c_1+(2+3b-3b^2)c_2]^2}{4(3-2b)^2(2-b)^2}, \) and \( \pi_2^u = 0. \) On the other hand, Lemma 8(i) and Lemma 10(iii) suggest \( R_p^u = \frac{(c_2-c_1)(1-b)(2+b)c_1-(2-b)c_2}{(1-b)^2(2+b)^2}, \pi_{p1}^u = \frac{(1+b)(c_2-c_1)^2}{(1-b)(2+b)^2}, \) and \( \pi_{p2}^u = 0. \) We have \( R_p^u \geq R_p^u \) for \( c_2 \in (\tilde{c}_2, y) \) and \( R_p^u < R_p^u \) for \( c_2 \in (y, \bar{c}_2) \), because

\[
\frac{\partial^2(R_p^u - R^u)}{\partial c_2^2} = \frac{(-64-64b-40b^2+48b^3+5b^4-10b^5+b^6)}{2(2-b)^2(3-2b)(1-b)^2(2+b)^2} < 0, (R_p^u - R^u) = \frac{1}{4(2-b)^2(3-2b)(8+4b-3b^2-b^3)^2}b^2(1-b)(72 + 48b - 44b^2 - 16b^3 + 3b^4 + b^5)(1-c_1)^2 > 0 \]

at \( c_2 = \tilde{c}_2, \) and \( (R_p^u - R_p^u) = \frac{(-2-2b^2+b^3)^3(1-c_1)^2}{4(3-2b)(1-b)^2(2+b)^2} < 0 \) at \( c_2 = \bar{c}_2. \) As shown in the proof of Lemma A(iv), we have \( \pi_{p1}^u > \pi_1^u \) by \( \frac{\partial^2(\pi_{p1}^u - \pi_1^u)}{\partial c_2^2} > 0, \frac{\partial(\pi_{p1}^u - \pi_1^u)}{\partial c_2} > 0, \) and \( (\pi_{p1}^u - \pi_1^u) = \frac{1}{4(3-2b)^2(2-b)^2(8+4b-3b^2-b^3)^2}b^2(1-b)(-144 + 120b + 544b^2 - 44b^3 - 421b^4 - 25b^5 + 81b^6 + 17b^7)(1-c_1)^2 > 0 \) at \( c_2 = \tilde{c}_2 \) when \( 0.807374 < b < 0.830504. \)

**Lemma C.** Suppose \( 0.830504 < b < 0.918708. \) Then we have the following.

(i) For \( c_2 \in (c_1, \tilde{c}_2) \), the results are the same as those in Lemma A(i).

(ii) For \( c_2 \in (\tilde{c}_2, \bar{c}_2) \), we have \( R_p^u > R^u, \pi_{p2}^u = \pi_2^u = 0, \pi_{p1}^u \leq \pi_1^u \) for \( c_2 \in (\tilde{c}_2, v) \), and \( \pi_{p1}^u > \pi_1^u \) for \( c_2 \in (v, \bar{c}_2) \).

(iii) For \( c_2 \in (\bar{c}_2, \bar{c}_2) \), operator 2 will not rent terminals under both competition modes, but operator 1 will always rent them. Accordingly, we have \( R_p^u > R^u \) for \( c_2 \in (\bar{c}_2, \bar{c}_2) \) if \( b < 0.872981 \), \( R_p^u \geq R^u \) for \( c_2 \in (\bar{c}_2, g) \) and \( R_p^u < R^u \) for \( c_2 \in (g, \bar{c}_2) \) if \( b > 0.872981 \), \( \pi_{p1}^u > \pi_1^u, \) and \( \pi_{p2}^u = \pi_2^u = 0, \) where \( g \) is defined in the proofs below.

(iv) For \( c_2 \in (\bar{c}_2, \bar{c}_2) \), we have \( R_p^u < R^u \) for \( c_2 \in (\bar{c}_2, \bar{c}_2) \) if \( b > 0.872981 \), \( R_p^u \geq R^u \)
for \( c_2 \in (\tilde{c}_2, y) \) and \( R_p^u < R^u \) for \( c_2 \in (y, \tilde{c}_p2) \) if \( b < 0.872981 \), \( \pi_{p1}^u > \pi_1^u \) for \( c_2 \in (\tilde{c}_2, \tilde{c}_p2) \) and \( \pi_{p2}^u = \pi_2^u = 0 \).

**Proof.** For \( 0.830504 < b < 0.918708 \), we have \( c_1 < \tilde{c}_p2 < \tilde{c}_p2 < \tilde{c}_2 < \tilde{c}_2 < \tilde{c}_2 \).

Then, there are four sub-cases:

(i) For \( c_2 \in (c_1, \tilde{c}_p2) \), the proofs are the same as those in Lemma A(i).

(ii) For \( c_2 \in (\tilde{c}_p2, \tilde{c}_p2) \), Lemma 2(iii) and Lemma 5(i) suggest \( R^u = \frac{(1-c_2)^2}{2(1+b)}, \pi_1^u = \frac{(1-c_2)(c_2-c_1)}{2(1+b)}, \) and \( \pi_2^u = 0 \). On the other hand, Lemma 8(ii) and Lemma 10(ii) imply

\[
R_p^u = \frac{[(3+b)-c_2^2(3+b)]^2}{8(1+b)(3+b)}, \quad \pi_{p1}^u = \frac{[-(3-2b-b^2)+(5+3b)c_2-(2+5b+b^2)c_2]}{16(1-b^2)(3+b)^2}, \quad \text{and} \quad \pi_{p2}^u = 0. \]

Thus, we can get \( (R_p^u - R^u) = \frac{[(3+b)-c_2^2(3+b)]^2}{8(1+b)(3+b)} - \frac{(1-c_2)^2}{2(1+b)} > 0 \) from Lemma B(i). On the other hand, \( (\pi_{p1}^u - \pi_1^u) = \frac{[-(3-2b-b^2)+(5+3b)c_2-(2+5b+b^2)c_2]}{16(1-b^2)(3+b)^2} - \frac{(1-c_2)(c_2-c_1)}{2(1+b)} \) is a convex function of \( c_2 \) because

\[
\frac{\partial^2(\pi_{p1}^u - \pi_1^u)}{\partial c_2^2} = \frac{76-4b-11b^2+2b^3+b^4}{8(1-b^2)(3+b)^2} > 0. \]

By some calculations, we have \( \pi_{p1}^u = \pi_1^u \) at \( c_2 = u \) and \( c_2 = v \), where

\[
u = \frac{[(30 - 23b - 11b^2 + 3b^3 + b^4) + (46 + 19b - b^3)c_1] - 2(1 - c_1)\sqrt{2(3 + b)^3(1 - b)^3}}{(76 - 4b - 11b^2 + 2b^3 + b^4)} \]

and

\[
u = \frac{[(30 - 23b - 11b^2 + 3b^3 + b^4) + (46 + 19b - b^3)c_1] + 2(1 - c_1)\sqrt{2(3 + b)^3(1 - b)^3}}{(76 - 4b - 11b^2 + 2b^3 + b^4)} \]

with \( v > u \). Then, it remains to compare \( \tilde{c}_2, \tilde{c}_p2, u, \) and \( v \), which will decide relative sizes of \( \pi_{p1}^u \) and \( \pi_1^u \). Since

\[
\frac{\partial^2(\pi_{p1}^u - \pi_1^u)}{\partial c_2^2} > 0, \quad \frac{\partial(\pi_{p1}^u - \pi_1^u)}{\partial c_2} = \frac{-(1-b)(2+b)(7+b)(1-c_1)}{8(1+b)(3-b)(3+b)^2} < 0 \]

at \( c_2 = c_2' \), \( c_2'' \), and \( (\pi_{p1}^u - \pi_1^u) = \frac{-(3b+b)(1-c_1)^2}{2(1+b)(3-b)(3+b)^2} < 0 \) at \( c_2 = c_2' \) with \( c_2' < \tilde{c}_2 < c_2'' \), where

\[
c_2' = \frac{(3-2b-b^2)+(7+b)c_1}{(10-b-b^2)} \quad \text{and} \quad c_2'' = \frac{(1-b)+2c_1}{(3-b)}; \quad \text{we have} \quad (\pi_{p1}^u - \pi_1^u) < 0 \] at \( c_2 = c_2' \) it means \( u < \tilde{c}_p2 < v \). Thus, we have

\[
\frac{\partial(\pi_{p1}^u - \pi_1^u)}{\partial c_2} = \frac{(6+b+b^2)(1-c_1)}{2(1+b)(8+4b-3b^2-b^4)} > 0 \]

at \( c_2 = \tilde{c}_p2 \). These suggest \( \tilde{c}_p2 > v \), and we can obtain \( u < \tilde{c}_p2 < v < \tilde{c}_p2 \). Accordingly, \( \pi_{p1}^u < \pi_1^u \) for \( c_2 \in (\tilde{c}_p2, v) \) and \( \pi_{p1}^u > \pi_1^u \) for \( c_2 \in (v, \tilde{c}_p2) \).

(iii) For \( c_2 \in (\tilde{c}_p2, \tilde{c}_2) \), Lemma 2(iii) and Lemma 5(i) suggest \( R^u = \frac{(1-c_2)^2}{2(1+b)}, \pi_1^u = \frac{(1-c_2)(c_2-c_1)}{2(1+b)}, \) and \( \pi_2^u = 0 \). While Lemma 8(i) and Lemma 10(iii) imply \( R_p^u = \frac{1}{(1-b)^2(2+b)^2}[(c_2-c_1)(1-b)(2+b)+bc_1-(2-b^2)c_2]], \pi_{p1}^u = \frac{(1+b)(c_2-c_1)^2}{(1-b)(2+b)^2}, \) and \( \pi_{p2}^u = 0 \). Thus, \( (R_p^u -
\[ R^u = \frac{(c_2-c_1)((1-b)(2b+6b+bc_1)-(2-b^2)c_2)}{(1-b)^2(2b+6b+bc_1)^2} - \frac{(1-c_2)(c_2-c_1)}{2(1-b)} \] is a strictly concave function of \( c_2 \) by
\[ \frac{\partial^2 (R^u - R^u)}{\partial c_2^2} = \frac{-8(8-5b^2+b^4)}{(1-b)(1-b)^2(2-b)^2} < 0. \] In addition, \( R^u_p = R^u \) at \( c_2 = e \) and \( c_2 = g \), where
\[ e = \frac{[(6-3b-5b^2+b^3+b^4) + (2+3b-b^3)c_1 - \sqrt{(1+b)(2+b)^2(1-b)^3(1-c_1)}]}{(8-5b^2+b^4)} \] and
\[ g = \frac{[(6-3b-5b^2+b^3+b^4) + (2+3b-b^3)c_1 + \sqrt{(1+b)(2+b)^2(1-b)^3(1-c_1)}]}{(8-5b^2+b^4)} \] with \( g > e \). By some calculations, we have \( e < \tilde{c}_p < g \) by \( \frac{\partial^2 (R^u_p - R^u)}{\partial c_2^2} < 0 \) and
\( (R^u_p - R^u) = \frac{(1-b)(8+10b+7b^2+b^3)(1-c_1)^2}{2(1-b)^2(8+4b-3b^2-b^3)^2} > 0 \) at \( c_2 = \tilde{c}_p \), and \( (g - \tilde{c}_p) = \frac{1}{(8-5b^2)(2-b)^2} \). If \( b \leq \sqrt{2} \),
\[ \sqrt{2(6+2b-7b^2-b^3+2b^4)} \geq (\leq) 0 \text{ if } b \leq (\geq) 0.872981. \] Then, we can get \( e < \tilde{c}_p < c_2 < g \) if \( b < 0.872981 \) and \( e < \tilde{c}_p < g < \tilde{c}_p \) if \( b > 0.872981 \). Thus,
\( R^u_p > R^u \) for \( c_2 \in (\tilde{c}_p, c_2) \) if \( b < 0.872981 \), \( R^u_p \geq R^u \) for \( c_2 \in (c_2, g) \), and
\( R^u_p < R^u \) for \( c_2 \in (g, \tilde{c}_p) \) if \( b > 0.872981 \). On the other hand, we have \( \pi^u_1 - \pi^u_1 = \frac{-(4-3b^2-b^3)-(2+4b+2b^2)c_1+(6+4b-b^2-b^3)c_2}{2(1-b)^2(2b+6b+bc_1)(1-c_1)^2} > 0 \).

(iii) For \( c_2 \in (\tilde{c}_p, \tilde{c}_p) \), Lemma 2(ii) and Lemma 5(ii) imply
\[ R^u = \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]}{4(3-2b)(2-b^2)}, \] \( \pi^u_1 = \frac{[(3-5b+2b^2)-(5-2b-b^2)c_1+(2+3b-3b^2)c_2]}{4(3-2b)(2-b^2)}, \) and \( \pi^u_2 = 0 \), while Lemma 8(i) and Lemma 10(iii) suggest
\( (R^u_p - R^u) = \frac{(c_2-c_1)((1-b)(2b+bc_1)-(2-b^2)c_2)}{(1-b)^2(2-b^2)^2} - \frac{[(3-2b)-(1-b)c_1-(2-b)c_2]}{4(3-2b)(2-b^2)} \) is a strictly concave function of \( c_2 \) by
\[ \frac{\partial^2 (R^u_p - R^u)}{\partial c_2^2} = \frac{-8(64-64b-40b^2+48b^3+5b^4-10b^5+b^6)}{2(b-2)(1-b)^2(2-b)^2(3-2b)} < 0. \] In addition, \( R^u_p = R^u \) at \( c_2 = x \) and \( c_2 = y \). Since \( (R^u_p - R^u) = \frac{b^2(1-b)(72+4b-4b^2+16b^3+3b^4+6b^5+b^6)(1-c_1)^2}{4(2-b^2)(3-2b)(2-b^2)^2} > 0 \) at \( c_2 = \tilde{c}_p \), and \( (R^u_p - R^u) = \frac{b^2(2b-2b^2)^2(1-c_1)^2}{4(3-2b)(2-b^2)^2} < 0 \) at \( c_2 = \tilde{c}_p \); we have \( x < \tilde{c}_p < \bar{c}_2 \) and \( \bar{c}_p > y \). Moreover, we can obtain \( (y - \bar{c}_p) = \frac{b^2(1-b)(72+4b-4b^2+16b^3+3b^4+6b^5+b^6)(1-c_1)^2}{4(2-b^2)(3-2b)(2-b^2)^2} \) \( \geq (\leq) 0 \) iff \( b \leq (\geq) 0.872981 \), where \( y_1 = 2b(3-2b)(2-b^2)(4-b^2)(4-b^2) > 0 \), \( y_2 = 2b(1-b)(8-3b^2)(6-2b^2)(1-b)(2-b^2)^2 \), and \( y_3 = (64-64b-40b^2+48b^3+5b^4-10b^5+b^6) \sqrt{2(6+2b-7b^2-b^3+2b^4)} \). Some calculations can show \( (y_2+y_3)^2 - (y_1)^2 \geq (\leq) 0 \) iff \( b \leq (\geq) 0.872981 \). Thus, \( x < \bar{c}_p < y < \tilde{c}_p \) if \( b < 0.872981 \) and \( x < y < \bar{c}_p \) if \( b > 0.872981 \).
if \( b > 0.872981 \). We have \( R^u_p < R^u \) for \( c_2 \in (\bar{c}_2, \bar{c}_p) \) if \( b > 0.872981 \), \( R^u_p \geq R^u \) for \( c_2 \in (\bar{c}_2, y] \), and \( R^u_p < R^u \) for \( c_2 \in (y, \bar{c}_p) \) if \( b < 0.872981 \). As in the proofs of Lemma A(iv) and Lemma B(iv), we also have \( \pi^u_{p1} > \pi^u_1 \) by \( \bar{c}_2 > \bar{c}_p2 \) for \( c_2 \in (\bar{c}_2, \bar{c}_p) \). □

**Lemma D.** Suppose \( b > 0.918708 \). Then we have the following.

(i) For \( c_2 \in (c_1, \bar{c}_p) \), the results are the same as those in Lemma A(i).

(ii) For \( c_2 \in (\bar{c}_p, \bar{c}_p) \), the results are the same as those in Lemma C(ii).

(iii) For \( c_2 \in (\bar{c}_p, \bar{c}_p) \), the results are the same as those in Lemma B(iv).

**Proof.** For \( b > 0.918708 \), we have \( c_1 < \bar{c}_p < \bar{c}_p < \bar{c}_2 < \bar{c}_2 \). Then, there are three sub-cases.

(i) For \( c_2 \in (c_1, \bar{c}_p) \), the proofs are the same as those in Lemma A(i).

(ii) For \( c_2 \in (\bar{c}_p, \bar{c}_p) \), the proofs are the same as those in Lemma C(ii).

(iii) For \( c_2 \in (\bar{c}_p, \bar{c}_p) \), the proofs are the same as those in Lemma B(iv). □

In sum, Lemmas A-D imply that operator 2 will always earn zero profit. However, we can have either \( R^u_p \geq R^u \) or \( R^u_p < R^u \), and similarly either \( \pi^u_{p1} \geq \pi^u_1 \) or \( \pi^u_{p1} < \pi^u_1 \). □

**Lemma 12.** Given two-part tariff scheme \( (r, f) \) and minimum throughput guarantee \( \delta \), terminal operators’ optimal behaviors are given below. Define \( \alpha_1 \equiv (1 + \frac{d_1}{K_1}) \) and \( \alpha_2 \equiv (1 + \frac{d_2}{K_2}) \).

(i) Suppose \( \alpha_1 \leq \alpha_2 \).

(ia) For \( \delta \in [0, \delta_{c_1}] \) with \( \delta_{c_1} = \frac{(2\alpha_1-b)(1-r)+bc_1-2\alpha_1 c_2}{4\alpha_1 \alpha_2-b^2} \), both operators’ equilibrium cargo-handling amounts are

\[
q^*_c = \frac{(2\alpha_1-c_1)(1-r)-2\alpha_2 c_1 + bc_2}{(4\alpha_1 \alpha_2 - b^2)} > \delta_{c_1}
\]

and

\[
q^*_c = \frac{(2\alpha_1-c_2)(1-r)+bc_1-2\alpha_1 c_2}{(4\alpha_1 \alpha_2 - b^2)} = \delta_{c_1},
\]

the equilibrium service prices are

\[
p^*_c = c_i + r + (1 + \frac{2d_i}{K_i})q^*_c > 0,
\]

and their equilibrium profits are

\[
\pi^*_c = \alpha_i (q^*_c)^2 - f \quad \text{for} \quad i = 1, 2.
\]
(ii) For \( \delta \in (\delta_{c1}, \delta_{c2}] \) with \( \delta_{c2} = \frac{1-c_1-r}{2\alpha_1+b} \), both operators’ equilibrium cargo-handling amounts are
\[
q^*_c = \frac{(1 - b\delta - c_1 - r)}{2\alpha_1} \quad \text{and} \quad q^*_r = \delta,
\]
the equilibrium prices are \( p^*_c = \frac{[(2\alpha_1-1)(1-b\delta)+c_1+r]}{2\alpha_1} > 0 \) and \( p^*_r = \frac{[(2\alpha_1-b)(2\alpha_1-b^2)\delta+bc_1+br]}{2\alpha_1} > 0 \), and their equilibrium profits are \( \pi^*_c = \alpha_1(q^*_c)^2 - f \) and \( \pi^*_r = \frac{\delta}{2\alpha_1}[(2\alpha_1 - b)(1 - r) - (2\alpha_1\alpha_2 - b^2)\delta + bc_1 - 2\alpha_1c_2] - f. \)

(iii) For \( \delta \in (\delta_{c2}, \frac{1}{1+b}] \), both operators’ equilibrium cargo-handling amounts are
\[
q^*_c = \delta \quad \text{and} \quad q^*_r = \delta,
\]
the equilibrium service prices are \( p^*_c = p^*_r = 1 - (1 + b)\delta > 0 \), and their equilibrium profits are \( \pi^*_c = \delta[1 - (\alpha_i + b)\delta - r - c_i] - f \) for \( i = 1, 2. \)

(ii) Suppose \( \alpha_1 > \alpha_2 \) and \( r \geq r_{12} \equiv \frac{2(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1-(2\alpha_1+b)c_2}{2(\alpha_1-\alpha_2)} \). Then operators’ optimal behaviors are the same as those in part (i).

(iii) Suppose \( \alpha_1 > \alpha_2 \) and \( r < r_{12}. \)

(iiiia) For \( \delta \in [0, \delta'_{c1}] \) with \( \delta'_{c1} = \frac{(2\alpha_2-b)(1-r)+bc_2-2\alpha_2c_1}{4\alpha_1\alpha_2-b^2} \), both operators’ equilibrium cargo-handling amounts are
\[
q^*_c = \frac{(2\alpha_2-b)(1-r)-2\alpha_2c_1+bc_2}{(4\alpha_1\alpha_2-b^2)} = \delta'_{c1} \quad \text{and} \quad q^*_r = \frac{(2\alpha_1-b)(1-r)+bc_1-2\alpha_1c_2}{(4\alpha_1\alpha_2-b^2)} > \delta'_{c1},
\]
the equilibrium service prices are \( p^*_c = c_i + r + (1 + \frac{2\delta}{K_i})q^*_c > 0 \), and their equilibrium profits are \( \pi^*_c = \alpha_i(q^*_c)^2 - f \) for \( i = 1, 2. \)

(iiiib) For \( \delta \in (\delta'_{c1}, \delta'_{c2}] \) with \( \delta'_{c2} = \frac{1-c_2-r}{2\alpha_2+b} \), both operators’ equilibrium cargo-handling amounts are
\[
q^*_c = \delta \quad \text{and} \quad q^*_r = \frac{(1 - b\delta - c_2 - r)}{2\alpha_2},
\]
the equilibrium prices are \( p^*_c = \frac{[(2\alpha_2-b)(2\alpha_2-b^2)\delta+bc_2+br]}{2\alpha_2} > 0 \) and \( p^*_r = \frac{[(2\alpha_2-1)(1-b\delta)+c_2+r]}{2\alpha_2} > 0 \), and their equilibrium profits are \( \pi^*_c = \frac{1}{2\alpha_2}(\delta[(2\alpha_2 - b)(1 - r) - (2\alpha_1\alpha_2 - b^2)\delta + bc_2 - 2\alpha_2c_1]) - f \) and \( \pi^*_r = \alpha_2(q^*_c)^2 - f. \)
For \( \delta \in (\delta_2', \frac{1}{1+b}) \), both operators’ equilibrium cargo-handling amounts are
\[
q_{c1}^* = \delta \text{ and } q_{c2}^* = \delta,
\]
the equilibrium service prices are \( p_{c1}^* = p_{c2}^* = 1 - (1 + b)\delta > 0 \), and their equilibrium profits are \( \pi_{ci}^* = \delta [1 - (\alpha_i + b)\delta - r - c_i] - f \) for \( i = 1, 2 \).

**Proof of Lemma 12:** Denote \( L_1 \) and \( L_2 \) the Lagrange functions of operators 1 and 2, respectively, in problem (39) with
\[
L_1 = [1 - q_1 - bq_2 - c_1 - r - \frac{d_1}{K_1} q_1] q_1 - f + \lambda_1 (q_1 - \delta) \text{and}
L_2 = [1 - q_2 - bq_1 - c_2 - r - \frac{d_2}{K_2} q_2] q_2 - f + \lambda_2 (q_2 - \delta),
\]
where \( \lambda_1 \) and \( \lambda_2 \) are their associated Lagrange multipliers. Then, the Kuhn-Tucker conditions for operator 1 are
\[
\frac{\partial L_1}{\partial q_1} = 1 - 2(1 + \frac{d_1}{K_1}) q_1 - bq_2 - c_1 - r + \lambda_1 \leq 0, \quad q_1 \cdot \frac{\partial L_1}{\partial q_1} = 0 \quad \text{and} \quad (A95)
\]
\[
\frac{\partial L_1}{\partial \lambda_1} = q_1 - \delta \geq 0, \quad \lambda_1 \cdot \frac{\partial L_1}{\partial \lambda_1} = 0, \quad (A96)
\]
and for operator 2 are
\[
\frac{\partial L_2}{\partial q_2} = 1 - 2(1 + \frac{d_2}{K_2}) q_2 - bq_1 - c_2 - r + \lambda_2 \leq 0, \quad q_2 \cdot \frac{\partial L_2}{\partial q_2} = 0 \quad \text{and} \quad (A97)
\]
\[
\frac{\partial L_2}{\partial \lambda_2} = q_2 - \delta \geq 0, \quad \lambda_2 \cdot \frac{\partial L_2}{\partial \lambda_2} = 0. \quad (A98)
\]
Based on the values of \( \lambda_1 \) and \( \lambda_2 \), there are four cases as follows.

**Case 1:** Suppose \( \lambda_1^* = 0 \) and \( \lambda_2^* = 0 \). Then (A95) and (A97) become
\[
1 - 2\alpha_1 q_1 - bq_2 - c_1 - r = 0 \quad \text{and} \quad 1 - 2\alpha_2 q_2 - bq_1 - c_2 - r = 0
\]
Solving these equations yields \( q_{c1}^* = \frac{(2\alpha_2-b)(1-r)-2\alpha_2 c_1+c_2}{4\alpha_1\alpha_2-b^2} \) and \( q_{c2}^* = \frac{(2\alpha_1-b)(1-r)+bc_1-2\alpha_1 c_2}{4\alpha_1\alpha_2-b^2} \).
Then we have \( q_{c1}^* - q_{c2}^* = \frac{(2\alpha_2-b)(1-r)-2\alpha_2 c_1+c_2}{(4\alpha_1\alpha_2-b^2)} \geq 0 \) iff \( \alpha_1 \leq \alpha_2 \), or \( \alpha_1 > \alpha_2 \).
and \( r \geq \frac{2(\alpha_1 - \alpha_2) + (2\alpha_2 + b)c_1 - (2\alpha_1 + b)c_2}{2(\alpha_1 - \alpha_2)} \equiv r_{12} \), and \( (q_{c1}^* - q_{c2}^*) < 0 \) iff \( \alpha_1 > \alpha_2 \) and \( r < r_{12} \).

Based on relative sizes of \( q_{c1}^* \) and \( q_{c2}^* \), there are three sub-cases as follows.

**Case 1a:** Suppose \( \alpha_1 \leq \alpha_2 \). To guarantee \( q_{c1}^* \geq \delta \) and \( q_{c2}^* \geq \delta \), condition \( 0 \leq \delta \leq \delta_{c1} \equiv \frac{(2\alpha_1 - b)(1-r) + bc_1 - 2\alpha_1 c_2}{4\alpha_1 \alpha_2 - b^2} = q_{c2}^* \) is needed because \( c_1 < c_2 \) implies \( q_{c1}^* > q_{c2}^* \), and \( \alpha_2 \geq \delta \) implies \( q_{c2}^* \geq \delta \). Substituting \( q_{c1}^* \) and \( q_{c2}^* \) into (1)-(2) yields \( p_{ci}^* = c_i + r + (1 + \frac{2\delta}{K_i})q_{c1}^* > 0 \), and into (38) yields \( \pi_{ci}^* = \alpha_i(q_{c1}^*)^2 - f \) for \( i = 1, 2 \). To guarantee \( q_{c1}^* \geq 0 \) and \( q_{c2}^* \geq 0 \), conditions \( r < r_c \equiv \frac{[(2\alpha_1 - b) + bc_1 - 2\alpha_1 c_2]}{2\alpha_1 - b} \) and \( c_2 < c_{c2} \equiv \frac{(2\alpha_1 - b) + bc_1}{2\alpha_1 \alpha_2 - b^2} \) are needed. That is because \( r < r_c \) and \( c_2 < c_{c2} \) imply \( q_{c2}^* \geq 0 \), and \( r < \frac{[(2\alpha_2 - b) + bc_2 - 2\alpha_2 c_1]}{2\alpha_2 - b} \) implies \( q_{c1}^* \geq 0 \) with \( \frac{[(2\alpha_2 - b) + bc_2 - 2\alpha_2 c_1]}{2\alpha_2 - b} - r_c = \frac{(4\alpha_1 \alpha_2 - b^2)(c_2 - c_1)}{(2\alpha_1)(2\alpha_2 - b)} > 0 \). These prove Lemma 12(iia).

**Case 1b:** Suppose \( \alpha_1 > \alpha_2 \) and \( r \geq r_{12} \). Then, \( q_{c1}^* > q_{c2}^* \), and the analyses are the same as those in Case 1a. To guarantee \( q_{c1}^* \geq \delta \) and \( q_{c2}^* \geq \delta \), condition \( 0 \leq \delta \leq \delta_{c1} \) is needed. Under the circumstance, we have \( p_{ci}^* = c_i + r + (1 + \frac{2\delta}{K_i})q_{c1}^* > 0 \) and \( \pi_{ci}^* = \alpha_i(q_{c1}^*)^2 - f \) for \( i = 1, 2 \). These prove Lemma 12(iia).

**Case 1c:** Suppose \( \alpha_1 > \alpha_2 \) and \( r < r_{12} \). Then, \( q_{c1}^* > q_{c2}^* \). To guarantee \( q_{c1}^* \geq \delta \) and \( q_{c2}^* \geq \delta \), condition \( 0 \leq \delta \leq \delta_{c1} \equiv \frac{(2\alpha_2 - b)(1-r) - 2\alpha_2 c_1 + bc_2}{4\alpha_1 \alpha_2 - b^2} = q_{c1}^* \) is needed. That is because \( \alpha_1 > \alpha_2 \) and \( r < r_{12} \) imply \( q_{c2}^* > q_{c1}^* \), and \( c_1 \geq \delta \) implies \( q_{c1}^* \geq \delta \). Substituting \( q_{c1}^* \) and \( q_{c2}^* \) into (1)-(2) yields \( p_{ci}^* = c_i + r + (1 + \frac{2\delta}{K_i})q_{c1}^* > 0 \), and into (38) yields \( \pi_{ci}^* = \alpha_i(q_{c1}^*)^2 - f \) for \( i = 1, 2 \). These prove Lemma 12(iia).

**Case 2:** Suppose \( \lambda_1^* = 0 \) and \( \lambda_2^* > 0 \). Then (A95), (A97) and (A98) suggest

\[
1 - 2\alpha_1 q_1 - bq_2 - c_1 - r = 0, \quad 1 - 2\alpha_2 q_2 - bq_1 - c_2 - r + \lambda_2 = 0, \quad \text{and} \quad q_2 - \delta = 0.
\]

Solving these equations yields \( q_{c1}^* = \frac{(1-b\delta-c_1-r)}{2\alpha_1} \), \( q_{c2}^* = \delta \), and \( \lambda_2^* = \frac{(4\alpha_1 \alpha_2 - b^2)(\delta - \delta_1)}{2\alpha_1} \). To guarantee \( \lambda_2^* > 0 \) and \( \delta_{c1} \geq 0 \), conditions \( \delta > \delta_{c1} \), \( r \leq r_c \), and \( c_2 \leq c_{c2} \) are needed. On the other hand, to have \( q_{c1}^* \geq \delta \), condition \( \delta \leq \delta_{c2} \equiv \frac{1-{c_1-c_2}}{2\alpha_1+b} \) should be imposed, where \( (\delta_{c2} - \delta_{c1}) = \frac{2\alpha_1[2(\alpha_1 - \alpha_2) - c_1 - (2\alpha_1 + b)c_1 - (2\alpha_1 - b)c_2]}{(2\alpha_1 + b)(4\alpha_1 \alpha_2 - b^2)} \). If \( \alpha_1 \leq \alpha_2 \), or \( \alpha_1 > \alpha_2 \) and \( r > r_{12} \), the plausible range for \( \delta \) is \( (\delta_{c1}, \delta_{c2}] \). Accordingly, substituting \( q_{c1}^* \) and \( q_{c2}^* \) into (1)-(2) gives \( p_{c1}^* = \frac{[(2\alpha_1 - 1)(1-b\delta + c_1 + r]}{2\alpha_1} > 0 \) and
Suppose $12(ii)$. Solving these equations yields $\pi^*_c = \alpha_1(q^*_c)^2 - f$ and $\pi^*_c = \frac{\delta((2\alpha_1-b)(1-r)-(2\alpha_1-b^2)\delta + bc_1 - 2\alpha_1c_2)}{2\alpha_1} - f$. These prove Lemma 12(ib) and Lemma 12(ii).

Case 3: Suppose $\lambda^*_1 > 0$ and $\lambda^*_2 = 0$. Then (A95)-(A97) suggest

$$q_1 - \delta = 0, \quad (1 - 2\alpha_1q_1 - bq_2 - c_1 - r + \lambda_1) = 0, \quad (1 - 2\alpha_2q_2 - bq_1 - c_2 - r) = 0.$$ Solving these equations yields $q^*_c = \delta$, $q^*_c = \frac{(1 - \delta - c_2 - r)}{2\alpha_2}$, and $\lambda^*_1 = \frac{(4\alpha_1\alpha_2-b^2)\delta - \delta'_c}{2\alpha_2}$. To guarantee $\lambda^*_1 > 0$, condition $\delta > \delta'_c$ is needed. On the other hand, $q^*_c \geq \delta$ is implied by assuming $\delta \leq \delta'_c \equiv \frac{1-c_2-r}{2\alpha_2+b}$. However, we have $(\delta'_c - \delta'_c) = \frac{2\alpha_2[2(\alpha_1-q_1)(1-r)-(2\alpha_1+b)c_2+(2\alpha_2+b)c_1]}{(2\alpha_2+b)(4\alpha_1\alpha_2-b^2)} > 0$ iff $\alpha_1 > \alpha_2$ and $r < r_{12}$. Thus, if $\alpha_1 > \alpha_2$ and $r < r_{12}$, the plausible range for $\delta$ is $[\delta'_c, \delta'_c]$. Substituting $q^*_c$ and $q^*_c$ into (1)-(2) yields $p^*_c = \frac{[2\alpha_2-1(1-b)-c_2-r]}{2\alpha_2} > 0$ and $p^*_c = \frac{[(2\alpha_2-b^2)\delta + bc_2 + br]}{2\alpha_2} > 0$ if $\delta \leq \delta'_c$, and into (38) gives $\pi^*_c = \frac{1}{2\alpha_2}\delta[(2\alpha_2 - b)(1-r) - (2\alpha_1\alpha_2 - b^2)\delta + bc_2 - 2\alpha_2c_1] - f$ and $\pi^*_c = \alpha_2(q^*_c)^2 - f$. These prove Lemma 12(iiib).

Case 4: Suppose $\lambda^*_1 > 0$ and $\lambda^*_2 > 0$. Then (A95)-(A98) suggest

$$q^*_c = q^*_c = \delta, \quad \lambda^*_1 = -1 + (2\alpha_1 + b)\delta + c_1 + r, \quad \lambda^*_2 = -1 + (2\alpha_2 + b)\delta + c_2 + r.$$ To have $\lambda^*_1 > 0$ and $\lambda^*_2 > 0$, conditions $\delta > \delta'_c \equiv \frac{1-c_2-r}{2\alpha_2+b}$, $\delta > \delta'_c \equiv \frac{1-c_2-r}{2\alpha_2+b}$, and $r < (1 - c_2)$ are needed. Note that $r < (1 - c_2)$ is implied by $r < r$. In addition, we have $(\delta_c - \delta'_c) = \frac{2(\alpha_2-q_2)(1-r)-(2\alpha_1+b)c_2+(2\alpha_2+b)c_1}{(2\alpha_1+b)(2\alpha_2+b)} \geq 0$ iff $\alpha_1 \leq \alpha_2$, or $\alpha_1 > \alpha_2$ and $r \geq r_{12}$, and $(\delta_c - \delta'_c) > 0$ if $\alpha_1 > \alpha_2$ and $r < r_{12}$. Thus, if $\alpha_1 \leq \alpha_2$, or $\alpha_1 > \alpha_2$ and $r \geq r_{12}$, the conditions needed are $\delta > \delta_c \equiv \frac{1-c_1-r}{2\alpha_1+b}$ and $r < r$. Then, substituting $q^*_c = q^*_c = \delta$ into (1)-(2) gives $p^*_c = p^*_c = 1 - (1+b)\delta > 0$ if $\delta < \frac{1}{1+b}$, and into (38) gives $\pi^*_c = \delta[1 - (\alpha_i + b)\delta - r - c_i] - f$ for $i = 1, 2$. These prove Lemma 12(ic) and 12(ii). By contrast, if $\alpha_1 > \alpha_2$ and $r < r_{12}$, condition $\delta > \delta'_c$ is needed. Substituting $q^*_c = q^*_c = \delta$ into (1)-(2) generates $p^*_c = p^*_c = 1 - (1+b)\delta > 0$ if $\delta < \frac{1}{1+b}$, and into (38) gives $\pi^*_c = \delta[1 - (\alpha_i + b)\delta - r - c_i] - f$ for $i = 1, 2$. These prove Lemma 12(iic).

□
Lemma 13. Suppose the conditions in (40) hold. Then we have the following.

(i) Suppose \( \alpha_1 \leq \frac{\alpha_1}{2} \). Then, for \( c_2 \in (c_1, \hat{c}_2) \) with \( \hat{c}_2 = \frac{1}{(4\alpha_1^2+8\alpha_1\alpha_2-4\alpha_1\alpha_2^2+b^2)}[(4\alpha_1^2+4\alpha_1\alpha_2-6\alpha_1-2b_2+2b^2)+(4\alpha_1\alpha_2+2b_1+2b_2-3b^2)c_1] \), port authority’s optimal two-part tariff contract and minimum throughput requirement are \((r_c^*, f_c^*, \delta_c^*)\) with

\[
r_c^* = \frac{[(2a_2-b)(4a_1a_2-2b_2)-4b_2^2a_2+3b_1^2c_1^2+b^2(2a_1-b)c_2]}{2(8\alpha_1a_2^2-4\alpha_1a_2^2-3b_2^2a_2-2b_1^2a_2^2+b_2^2)} \quad \text{and} \quad \delta_c^* = \left[ 0, \frac{2a_1-b(1-r_c^*)+bc_1-2a_1c_2}{4\alpha_1a_2^2-b^2} \right].
\]

At the equilibrium, operators’ cargo-handling amounts are

\[
q_{c1}^* = \frac{(2a_2-b)(1-r_c^*)-2a_1c_1+bc_2}{4\alpha_1a_2-b^2} \quad \text{and} \quad q_{c2}^* = \frac{1}{(4\alpha_1a_2^2-b^2)}[(2a_1-b)(1-r_c^*)+bc_1-2a_1c_2]
\]
as in Lemma 12(iia), and port authority’s equilibrium fee revenue equals

\[
R_c^* = 2f_c^* + r_c^* \left[ (2a_1+2b_1-2a_1c_1-2a_1-b_2) \right].
\]

(ii) Suppose \( \frac{\alpha_1}{2} < \alpha_1 \leq \alpha_2 \).

(iia) If \( c_2 \in (c_1, \hat{c}_2) \) with \( \hat{c}_2 = \frac{2(2a_1-a_2)+(2a_1+3a_2+4b)c_1}{(6\alpha_1+\alpha_2+4b)} \), then port authority’s optimal two-part tariff contract and minimum throughput requirement are \((r_c^*, f_c^*, \delta_c^*)\) with

\[
r_c^* = \frac{2(\alpha_1-b)-(2a_1+2a_2+3b)c_1+(2a_1+b)c_2}{2(2a_1+2a_2+2b)} \quad \text{and} \quad f_c^* = \frac{1}{8(2a_1+2a_2+2b)} \{ (2-c_1-c_2)[2(2\alpha_1-\alpha_2)+(2\alpha_1+3a_2+4b)c_1-(6a_1+\alpha_2+4b)c_2] \} \quad \text{and} \quad \delta_c^* = \frac{2-c_1-c_2}{2(2a_1+2a_2+2b)}.
\]
At the equilibrium, operators’ cargo-handling amounts are \( q_{ci}^* = \delta_c^* \) for \( i = 1, 2 \), as in Lemma 12(iiia), and port authority’s fee revenue equals

\[
R_c^* = \frac{(2-c_1-c_2)^2}{4(2a_1+2a_2+2b)}
\]

(iii) Suppose \( \alpha_2 < \alpha_1 \leq 2\alpha_2 \).

(iiiia) If \( c_2 \in (c_1, \hat{c}_2) \) with \( \hat{c}_2 = \frac{2(\alpha_1-a_2)+(3a_2+2b)c_1}{(2a_1+2a_2+2b)} \), then port authority’s optimal two-part tariff contract and minimum throughput requirement are \((r_c^*, f_c^*, \delta_c^*)\) with

\[
r_c^* = \frac{2(\alpha_1+b)+(2a_1+b)c_1-(2a_1+2a_2+3b)c_2}{2(2a_1+2a_2+2b)} \quad \text{and} \quad f_c^* = \frac{(2-c_1-c_2)[2(2\alpha_1-a_2)+(2a_1+3a_2+4b)c_1-(6a_1+a_2+4b)c_2]}{4(2a_1+2a_2+2b)}
\]
and

\[
\delta_c^* = \frac{2-c_1-c_2}{2(2a_1+2a_2+2b)}.
\]
At the equilibrium, operators’ cargo-handling amounts are \( q_{ci}^* = \delta_c^* \) for \( i = 1, 2 \), as in Lemma 12(iiia), and port authority’s fee revenue equals

\[
R_c^* = \frac{(2-c_1-c_2)^2}{4(2a_1+2a_2+2b)}
\]
(iib) If $c_2 \in [\hat{c}, \bar{c})$, then port authority’s optimal two-part tariff contract and minimum throughput requirement are the same as those in Lemma 13(iiia).

(iic) If $c_2 \in [\hat{c}, \check{c})$, then port authority’s optimal two-part tariff contract and minimum throughput requirement are the same as those in Lemma 13(i).

(iv) Suppose $\alpha_1 > 2\alpha_2$.

(iva) If $c_2 \in (c_1, c''_2]$ with $c''_2 = \frac{(\alpha_1 - 2\alpha_2) + (2\alpha_2 + b) c_1}{\alpha_1 + b}$, then port authority’s optimal two-part tariff contract and minimum throughput requirement are $(r^*_c, f^*_c, \delta^*_c)$ with $r^*_c = \frac{1 - c_2}{2}$, $f^*_c = \frac{2\alpha_2 - b - 2\alpha_2 c_1 + bc_2}{8(2\alpha_1 \alpha_2 - b^2)} (c_2 - c_1)$, and $\delta^*_c = \frac{2\alpha_2 - b + bc_2 - 2\alpha_2 c_1}{2(2\alpha_1 \alpha_2 - b^2)}$. At the equilibrium, operators’ cargo-handling amounts are $q^*_{c_1} = \delta^*_c$ and $q^*_{c_2} = \frac{1 - b \delta^*_c - r^*_c}{2\alpha_2}$ as in Lemma 12(iiib), and port authority’s fee revenue equals $R^*_c = \frac{1}{4(2\alpha_1 \alpha_2 - b^2)} [(\alpha_1 + 2\alpha_2 - 2b) - 2(2\alpha_2 - b)c_1 - 2(\alpha_1 - b)c_2 - 2bc_1c_2 + 2\alpha_2 c_2^2 + \alpha_1 c_2^2]$.

(ivb) If $c_2 \in (c''_2, \check{c})$, then port authority’s optimal two-part tariff contract and minimum throughput requirement are the same as those in Lemma 13(iiia).

(ivc) Suppose $\check{c} < \hat{c}$ and $c_2 \in [\hat{c}, \check{c})$. Then port authority’s optimal two-part tariff contract and minimum throughput requirement are the same as those in Lemma 13(iiia).

(ivd) If $\max \{\check{c}, \hat{c}\} \leq c_2 < \hat{c}$, then port authority’s optimal two-part tariff contract and minimum throughput requirement are the same as those in Lemma 13(i).

Proof of Lemma 13: Based on the values of $\alpha_1$ and $\alpha_2$, we have three cases as follows.

Case 1: Suppose $\alpha_1 \leq \alpha_2$. Then, there are two sub-cases.

Case 1(1): Suppose $\delta \in [0, \delta_{c_1}]$. Lemma 12(ia) implies $\pi^*_c > \pi^*_2$ and $f^*_c = \pi^*_c = \frac{1}{2} \alpha_2 (q^*_c)^2 > 0$. Thus, the problem in (43) becomes

$$\max_{r, f, \delta} \quad 2f + r(q^*_c + q^*_2)$$

s.t. $0 \leq \delta \leq \delta_{c_1}$ and $0 < r < \bar{r}_c$.  

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Denote $L$ its Lagrange function with $L = \alpha_2(q_{c2}^*)^2 + r(q_{c1}^* + q_{c2}^*) + \lambda_1(\delta_{c1} - \delta) + \lambda_2(\bar{r}_c - r)$, where $\lambda_1$ and $\lambda_2$ are the Lagrange multipliers associated with the constraints. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = 2\alpha_2 q_{c2}^* \frac{\partial q_{c2}^*}{\partial r} + r(\frac{\partial q_{c1}^*}{\partial r} + \frac{\partial q_{c2}^*}{\partial r}) + (q_{c1}^* + q_{c2}^*) + \lambda_1 \frac{\partial \delta_1}{\partial r} - \lambda_2 \leq 0, \ r = 0, \ \frac{\partial L}{\partial \delta} = 0, \ \frac{\partial L}{\partial \lambda_1} = 0, \ \frac{\partial L}{\partial \lambda_2} = 0.$$  

(A99)

(A100)

(A101)

(A102)

Constraint $r < \bar{r}_c$ suggests $\lambda_2^* = 0$ by (A102). Based on the values of $\lambda_1$, we have two situations.

**Case 1(1):** Suppose $\lambda_1^* = 0$. Then (A99) becomes $\frac{\partial L}{\partial \delta} = \frac{1}{2}\left((2\alpha_2 - b)(4\alpha_1\alpha_2 + 2b\alpha_1 - 2b^2) - (8\alpha_1\alpha_2 - 4b^2\alpha_1 + 3b^3)c_1 + b^2(2\alpha_1 - b)c_2 - [(16\alpha_1\alpha_2 + (8\alpha_1^2 - 8b\alpha_1 - 6b^2)\alpha_2 - 4b^2)(\alpha_1 - b)]\right) = 0$. Solving this equation yields $r_c^* = \frac{1}{2}(\alpha_1^2 + 4\alpha_1\alpha_2 - 6b\alpha_1 - 2b^2 + 2b^3)c_1 + b^2(2\alpha_1 - b)c_2$. It remains to check whether $r_c^* < \bar{r}_c$ holds. By some calculations, we have $r_c^* < \bar{r}_c$ iff $c_2 < \tilde{c}_2 = \frac{(2\alpha_1^2 - b)(1-r_c^*) + bc_1 - 2\alpha_1 c_2}{4\alpha_1^2 - b^2}$. Therefore, (A101) implies both $\delta_c^* \in [0, \delta_{c1}]$ with $\delta_{c1} = \frac{(2\alpha_1 - b)(1-r_c^*) + bc_1 - 2\alpha_1 c_2}{4\alpha_1^2 - b^2}$ and $f_c^* = \frac{\alpha_2^2}{2}(\frac{2\alpha_1 - b)(1-r_c^*) + bc_1 - 2\alpha_1 c_2}{4\alpha_1^2 - b^2} > 0$. Thus, at the equilibrium, port authority’s fee revenue equals

$$R_c^* = \frac{\alpha_2^2}{4\alpha_1^2 - b^2} \left[\frac{(2\alpha_1 - b)(1-r_c^*) + bc_1 - 2\alpha_1 c_2}{4\alpha_1^2 - b^2} \right]^2 + r_c^* \left[\frac{2(\alpha_1 + \alpha_2 - b)(1-r_c^*) - (2\alpha_2 - b)c_2 - (2\alpha_1 - b)c_2}{4\alpha_1^2 - b^2} \right] = R_1^*.$$  

(A103)

**Case 1(1):** Suppose $\lambda_1^* > 0$. Then, (A101) suggests $\delta_c^* = \delta_{c1} > 0$. This in turn implies $\lambda_1^* = 0$ by (A100). It is a contradiction. Thus, no solution exists in this case.

**Case 1(2):** Suppose $\delta \in (\delta_{c1}, \delta_{c2}]$. Then, Lemma 12(1b) implies $\pi_{c1}^* > \pi_{c2}^*$ and $f_c^* = \pi_{c2}^* = \frac{\alpha_2^2}{4\alpha_1^2 - b^2} \left[\frac{(2\alpha_1 - b)(1-r_c^*) + bc_1 - 2\alpha_1 c_2}{4\alpha_1^2 - b^2} \right]^2 + r_c^* \left[\frac{2(\alpha_1 + \alpha_2 - b)(1-r_c^*) - (2\alpha_2 - b)c_2 - (2\alpha_1 - b)c_2}{4\alpha_1^2 - b^2} \right] = R_1^*$. Moreover, if $(2\alpha_1 - \alpha_2) > 0$, we have $\tilde{\delta}_c > \delta_{c2}$ if $r < \bar{r}_c$ and $\tilde{\delta}_c < \delta_{c2}$ if $r > \bar{r}_c$. Therefore, if $(2\alpha_1 - \alpha_2) > 0$, we have $\tilde{\delta}_c > \delta_{c2}$ and if $(2\alpha_1 - \alpha_2) \leq 0$, we have $\tilde{\delta}_c < \delta_{c2}$.  

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Also, \((\bar{r}_c - \tilde{r}_c) = \frac{(2\alpha_1\alpha_2 - b^2)(c_2 - c_1)}{(2\alpha_1 - b)(2\alpha_1 - \alpha_2)} \leq 0\) iff \((2\alpha_1 - \alpha_2) > (\leq) 0\). Thus, we have two sub-cases below.

**Case 1(2)-1:** Suppose \((2\alpha_1 - \alpha_2) > 0\). Then, there are two situations.

**Case 1(2)-1a:** Suppose \(r < \tilde{r}_c\). Then the problem in (43) becomes

\[
\begin{align*}
\max_{r, f, \delta} & \quad 2f + r(q_c^* + \tilde{q}_c^*) \\
\text{s.t.} & \quad \delta_1 < \delta \leq \delta_2 \text{ and } 0 < r < \tilde{r}_c.
\end{align*}
\]

(A104)

Denote \(L\) its Lagrange function with \(L = \frac{\delta}{2\alpha_1} [(2\alpha_1 - b)(1 - r) - (2\alpha_1\alpha_2 - b^2) \delta + bc_1 - 2\alpha_1c_2] + \frac{\delta}{2\alpha_1} [1 + (2\alpha_1 - b)\delta - c_1 - r] + \lambda_1(\delta - \delta_1) + \lambda_2(\delta_2 - \delta) + \lambda_3[\tilde{r}_c - r]\). Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = \frac{(1 - 2r - c_1)}{2\alpha_1} + \frac{(2\alpha_1 - b)\lambda_1}{4\alpha_1\alpha_2 - b^2} - \frac{\lambda_2}{2\alpha_1 + b} - \lambda_3 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \quad (A105)
\]

\[
\frac{\partial L}{\partial \delta} = \frac{1}{2\alpha_1} [(2\alpha_1 - b) - 2(2\alpha_1\alpha_2 - b^2)\delta + bc_1 - 2\alpha_1c_2] + \lambda_1 - \lambda_2 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad (A106)
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta - \delta_1 \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A107)
\]

\[
\frac{\partial L}{\partial \lambda_2} = \delta_2 - \delta \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad \text{and} \quad (A108)
\]

\[
\frac{\partial L}{\partial \lambda_3} = \tilde{r}_c - r \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \quad (A109)
\]

where \(\lambda_1, \lambda_2, \lambda_3\) are the Lagrange multipliers associated with the constraints in (A104). Constraints \(\delta_1 < \delta\) and \(r < \tilde{r}_c\) suggest \(\lambda_1^* = \lambda_3^* = 0\) by (A107) and (A109). If \(\lambda_2^* = 0\), we have \(r_c^* = \frac{1-c_1}{2}\) and \(\delta_c^* = \frac{(2\alpha_1 - b + bc_1 - 2\alpha_1c_2)}{2(2\alpha_1\alpha_2 - b^2)}\) by (A106) and (A107). We have \((\delta_c - \delta_c^*) = \frac{\alpha_1(2\alpha_1\alpha_2 - b^2)c_1 + (2\alpha_1 + b)c_2}{(2\alpha_1 + b)(2\alpha_1\alpha_2 - b^2)} \geq 0\) iff \(c_2 \geq \frac{(2\alpha_1 - a_2 + (\alpha_2 + b)c_1}{2\alpha_1 + b}\), and \(\tilde{r}_c \leq 0 < r_c^*\) iff \(c_2 \geq \frac{(2\alpha_1 - a_2 + (\alpha_2 + b)c_1}{2\alpha_1 + b}\). This is a contradiction. Thus, no solution exists in this case.

If \(\lambda_2^* > 0\), (A105), (A106), and (A108) suggest

\[
\frac{1}{2\alpha_1} - \frac{1}{2\alpha_1 + b} \lambda_2 = 0, \quad \frac{2\alpha_1 - b}{2\alpha_1} - 2(2\alpha_1\alpha_2 - b^2)\delta + bc_1 - 2\alpha_1c_2 - \lambda_2 = 0, \quad \text{and} \quad (\delta_c - \delta) = 0.
\]

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Solving these equations yields $r^*_c = \frac{(2(α_1+a) - (2α_1+2α_2+3b)c_1+(2α_1+b)c_2)}{2(2α_1+α_2+2b)} > 0$, $δ^*_c = \frac{-c_1-c_2}{2(2α_1+α_2+2b)}$, and $λ^*_2 = \frac{(2α_1+b)[(2α_1-a)2+(a_2+b)c_1-(2α_1+b)c_2]}{2α_12(2α_1+α_2+2b)}$. By some calculations, we have $(δ^*_c - δ_{c_1}) = \frac{2α_12(2α_2-a)-2(2α_1+2α_2+2b)c_1+c_2(2α_1+b)}{(2α_1+α_2+2b)(4α_1α_2-b^2)} > 0$, and $λ^*_2 > 0$ iff $c_2 < \frac{(2α_1-a)+(a_2+b)c_1}{2α_1+b}$. Since $\left(r^*_c - \tilde{r}_c\right) = \frac{(2α_1+b)[-2(2α_1-a)-(2α_1+2α_2+b)c_1+(6α_1+α_2+2b)c_2]}{2(2α_1-a)(2α_1+α_2+2b)}$, we have $r^*_c < \tilde{r}_c$ iff $c_2 < \frac{-2(2α_1-a)+(a_2+b)c_1}{(2α_1+b)(6α_1+α_2+2b)} \equiv \tilde{c}_2$. In addition, we have $\tilde{c}_2 - \frac{(2α_1-a)+(a_2+b)c_1}{(2α_1+b)} = \frac{1}{(2α_1+b)(6α_1+α_2+2b)}[-(2α_1-a)(2α_1+α_2+2b)(1-c_1)] < 0$. Thus, an equilibrium exists when $c_2 < \tilde{c}_2$ with fee revenue

$$R^*_c = \frac{(2-c_1-c_2)^2}{4(2α_1+α_2+2b)} = R^*_3. \quad (A110)$$

**Case 1(2)-1b:** Suppose $r \geq \tilde{r}_c \equiv \frac{[2(2α_1-a)+(a_2+b)c_1-(2α_1+b)c_2]}{(2α_1-a)}$. Then the problem in (43) becomes

$$\max_{r, f, \delta} \quad 2f + r(q^*_c + q^*_b)$$

s.t. $δ_{c_1} < δ < \tilde{δ}_c$ and $\tilde{r}_c \leq r < \tilde{r}_c$. \quad (A111)

Denote $L$ its Lagrange function with $L = \frac{δ}{2α_1}[(2α_1-a)(1-r)-(2α_1α_2-b^2)δ + bc_1-2α_1c_2] + \frac{r}{2α_1}[1+(2α_1-a)δ-c_1-r] + λ_1(δ - δ_{c_1}) + λ_2(\tilde{δ}_c - δ) + λ_3(r - \tilde{r}_c) + λ_4(\tilde{r}_c - r)$. Then, the Kuhn-Tucker conditions are

$$\frac{∂L}{∂r} = \frac{(1-2r-c_1)}{2α_1} + \frac{(2α_1-b)λ_1}{4α_1α_2-b^2} - \frac{(2α_1-b)λ_2}{2α_1α_2-b^2} + λ_3 - λ_4 \leq 0, \quad r \cdot \frac{∂L}{∂r} = 0, \quad (A112)$$

$$\frac{∂L}{∂δ} = \frac{1}{2α_1}[(2α_1-b)-2(2α_1α_2-b^2)δ + bc_1-2α_1c_2] + λ_1 - λ_2 \leq 0, \quad \frac{∂L}{∂δ} = 0, \quad (A113)$$

$$\frac{∂L}{∂λ_1} = δ - δ_{c_1} \geq 0, \quad λ_1 \cdot \frac{∂L}{∂λ_1} = 0,$$

$$\frac{∂L}{∂λ_2} = \tilde{δ}_c - δ \geq 0, \quad λ_2 \cdot \frac{∂L}{∂λ_2} = 0, \quad (A114)$$

$$\frac{∂L}{∂λ_3} = r - \tilde{r}_c \geq 0, \quad λ_3 \cdot \frac{∂L}{∂λ_3} = 0, \quad (A115)$$

$$\frac{∂L}{∂λ_4} = \tilde{r}_c - r \geq 0, \quad λ_4 \cdot \frac{∂L}{∂λ_4} = 0.$$

We have $λ^*_1 = λ^*_2 = λ^*_3 = 0$ by the three strict inequalities in (A111). If $λ^*_4 = 0$, we have $r^*_c = 1-c_1$ and $δ^*_c = \frac{(2α_1-b)+bc_1-2α_1c_2}{2(2α_1+α_2-b^2)}$ by (A112) and (A113). Some calculations show
\[(\delta^*_c - \tilde{\delta}_c) = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)} - \frac{(2\alpha_1-b)(1-r^*_c)+bc_1-2\alpha_1c_2}{(2\alpha_1\alpha_2-b^2)} = \frac{\alpha_1(c_2-c_1)}{2(2\alpha_1\alpha_2-b^2)} > 0.\] It is impossible to meet the requirement of \(\delta^*_c \leq \tilde{\delta}_c\). Thus, no solution exists in this case.

By contrast, if \(\lambda^*_3 > 0\), we have \(r^*_c = \tilde{r}_c = \frac{[(2\alpha_1-a_2)+(a_2+b)c_1-(2\alpha_1+b)c_2]{(2\alpha_1-a_2)}}{2\alpha_1} \delta^*_c = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)}, \) and \(\lambda^*_3 = \frac{(2\alpha_1-a_2)+(a_2+b)c_1-2(2\alpha_1+b)c_2}{2\alpha_1(2\alpha_1-a_2)}\) by (A112), (A113), and (A115).

Note that \(\lambda^*_3 > 0\) if \(c_2 < \frac{(2\alpha_1-a_2)+(2\alpha_1+a_2+2b)c_1}{2(2\alpha_1+b)}\). On the other hand, (A114) requires \((\tilde{\delta}_c - \delta^*_c) > 0\). Some calculations show \((\tilde{\delta}_c - \delta^*_c) = \frac{1}{2(2\alpha_1-a_2)(2\alpha_1\alpha_2-b^2)}[-(2\alpha_1-a_2)(2\alpha_1-b) - (4\alpha_1a_2 + 2ba_1 - ba_2 - 2b^2)c_1 + 2(\alpha_1a_2 + 2a_1^2 - b^2)c_2] > 0\) iff \(c_2 > \frac{(2\alpha_1-a_2)(2\alpha_1-b) + (4\alpha_1a_2 + 2ba_1 - ba_2 - 2b^2)c_1}{2(2\alpha_1\alpha_2 + 2\alpha_1^2 - b^2)c_1} - \frac{(2\alpha_1-a_2) + (2\alpha_1+a_2+2b)c_1}{2(2\alpha_1+b)} = \frac{(2\alpha_1-a_2)^2(1-c_1)}{2(\alpha_1\alpha_2 + 2\alpha_1^2 - b^2)} > 0\). Thus, no solution exists in this case.

Case 1(2)-2: Suppose \((2\alpha_1-a_2) \leq 0\), we have \(\tilde{\delta}_c < \delta^*_c\). Then the problem in (43) becomes

\[\max_{r, f, \delta} 2f + r(q^*_c + q^*_c)\]
\[\text{s.t. } \delta_{c1} < \delta < \tilde{\delta}_c \text{ and } 0 < r < \tilde{r}_c.\]

Solving this problem yields \(r^*_c = \frac{1-c_1}{2}\) and \(\delta^*_c = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)}, \) the same as those in Case 1(2)-1b. Some calculations yield \((\tilde{\delta}_c - \delta^*_c) = \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2(2\alpha_1\alpha_2-b^2)} - \frac{(2\alpha_1-b)(1-r^*_c)+bc_1-2\alpha_1c_2}{(2\alpha_1\alpha_2-b^2)} = \frac{\alpha_1(c_2-c_1)}{2(2\alpha_1\alpha_2-b^2)} > 0\), which contradicts \(\delta^*_c < \tilde{\delta}_c\). Thus, no solution exists in this case.

Case 1(3): Suppose \(\delta \in (\delta_{c2}, \frac{1}{1+b}]\). Then, Lemma 12(ic) implies \(\pi^*_1 > \pi^*_c\) and \(f^*_c = \pi^*_c = \frac{1}{2}(1 - (\alpha_2 + b)\delta - c_2 - r)\delta\) with \(f^*_c > 0\) iff \(\delta < \frac{1-c_2-r}{(\alpha_2+b)}\) and \(r < 1 - c_2\). Note that \(r < 1 - c_2\) is implied by \(r < \tilde{r}_c\) \(\equiv \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]{(2\alpha_1-b)}}{2\alpha_1}\). Accordingly, the problem in (43) becomes

\[\max_{r, \delta} \delta[1 - (\alpha_2 + b)\delta - c_2 - r] + 2r\delta\]
\[\text{s.t. } \delta_{c2} < \delta < \frac{1-c_2-r}{(\alpha_2+b)} \text{ and } 0 < r < \tilde{r}_c. \quad (A116)\]

Denote \(L\) its Lagrange function with \(L = \delta[1 - (\alpha_2 + b)\delta - c_2 - r] + 2r\delta + \lambda_1(\delta - \delta_c) + \ldots\)
\[ \lambda_2 \left[ \frac{1-c_2-r}{\alpha_2+b} \right] - \delta \right] + \lambda_3 (\bar{r}_c - r). \] Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = \delta + \frac{\lambda_1}{2\alpha_1 + b} - \frac{\lambda_2}{\alpha_2 + b} - \lambda_3 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \tag{A117}
\]

\[
\frac{\partial L}{\partial \delta} = 1 - 2(\alpha_2 + b)\delta - c_2 + r + \lambda_1 - \lambda_2 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0,
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta - \delta_c \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0,
\]

\[
\frac{\partial L}{\partial \lambda_2} = \frac{1-c_2-r}{(\alpha_2+b)} - \delta \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad \text{and}
\]

\[
\frac{\partial L}{\partial \lambda_3} = \bar{r}_c - r \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0.
\]

Since all constraints in (A116) are strict inequalities, we must have \( \lambda_1^* = \lambda_2^* = \lambda_3^* = 0. \)

However, some calculations show \( r_c^* = 0 \) by \( \delta > 0 \) and (A117), which contradicts \( r_c^* > 0. \)

Thus, no solution exists in this case.

**Case 2:** Suppose \( \alpha_1 > \alpha_2 \) and \( r \geq r_{12} \equiv \frac{2(\alpha_1-\alpha_2)+(2\alpha_2+b)c_1-(2\alpha_1+b)c_2}{2(\alpha_1-\alpha_2)}. \) Then, there are three sub-cases as follows.

**Case 2(1):** Suppose \( \delta \in [0, \delta_{c1}] \). Lemma 12(iia) implies \( \pi_{c1}^* > \pi_{c2}^* \) and \( f_c^* = \pi_{c2}^* = \frac{\alpha_2^2 (q_{c2}^*)^2}{2} > 0. \) Then, the problem in (43) becomes

\[
\max_{r, f, \delta} \quad 2f + r(q_{c1}^* + q_{c2}^*)
\]

s.t. \( 0 \leq \delta \leq \delta_{c1} \) and \( r_{12} \leq r < \bar{r}_c. \) \( \tag{A118} \)

Its Lagrange function is \( L = \alpha_2(q_{c2}^*)^2 + r(q_{c1}^* + q_{c2}^*) + \lambda_1(\delta_{c1} - \delta) + \lambda_2(r-r_{12}) + \lambda_3(\bar{r}_c-r), \)

where \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are the Lagrange multipliers for the inequality constraints in problem (A118). Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = 2\alpha_2 q_{c2}^* \frac{\partial q_{c2}^*}{\partial r} + r(\frac{\partial q_{c1}^*}{\partial r} + \frac{\partial q_{c2}^*}{\partial r}) + (q_{c1}^* + q_{c2}^*) + \lambda_1 \frac{\partial \delta}{\partial r} + \lambda_2 - \lambda_3 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \tag{A119}
\]

\[
\frac{\partial L}{\partial \delta} = -\lambda_1 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \tag{A120}
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta_{c1} - \delta \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \tag{A121}
\]

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\[
\frac{\partial L}{\partial \lambda_2} = r - r_{12} \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad \text{and} \quad \frac{\partial L}{\partial \lambda_3} = \bar{r}_c - r \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0.
\] (A122)

Constraint \(r < \bar{r}_c\) suggests \(\lambda_3^* = 0\) by (A123). If \(\lambda_1^* > 0\), then (A121) suggests \(\delta_c^* = \delta_{c1} > 0\). This in turn implies \(\lambda_1^* = 0\) by (A120). It is a contradiction. Thus, we must have \(\lambda_1^* = 0\). According to the values of \(\lambda_2\), there are two situations.

Case 2(1)a: Suppose \(\lambda_2^* = 0\). Then (A119) becomes

\[
\frac{1}{(4\alpha_1\alpha_2-b^2)r}(2(2a_2-b)(4\alpha_1\alpha_2 + 2ba_1 - 2b^2) - (8\alpha_1\alpha_2 - 4b^2\alpha_2 + b^3)c_1 + b^2(2\alpha_1 - b)c_2 - [16\alpha_1\alpha_2^2 + (8\alpha_1^2 - 8\alpha_1 - 6b^2)\alpha_2 - 4b^2(\alpha_1 - b)]r) = 0.
\]
Solving this equation yields \(r_c^* = \frac{1}{2(8\alpha_1\alpha_2 - 4\alpha_1^2\alpha_2 - 8\alpha_1 - 6b^2 + 2b^3)}[(2a_2-b)(4\alpha_1\alpha_2 + 2ba_1 - 2b^2) - (8\alpha_1\alpha_2 - 4b^2\alpha_2 + b^3)c_1 + b^2(2\alpha_1 - b)c_2]\) It remains to check whether \(r_{12} \leq r_c < \bar{r}_c\) holds. By some calculations, we have \(r_c^* < \bar{r}_c\) iff \(c_2 < \hat{c}_2 \equiv \frac{2(\alpha_1 - \alpha_2)(\alpha_1 + \alpha_2 - b) + (4\alpha_1\alpha_2 + 2b\alpha_1 + 2\alpha_2^2 - 2b^2)c_1}{(4\alpha_1\alpha_2 + 4\alpha_1\alpha_2^2 - 8\alpha_1^2\alpha_2 - 8\alpha_1 - 6b^2 + 2b^3)c_1} < \hat{c}_2\). In addition, (A121) implies both \(\delta_c^* \in [0, \delta_{c1}]\) with \(\delta_{c1} = \frac{(2a_2-b)(1-r_c^*) + bc_1 - 2a_1c_2}{4\alpha_1\alpha_2 - b^2}\) and \(f_c^* = \frac{\alpha_2 (2a_1-b)(1-r_c^*) + bc_1 - 2a_1c_2}{4\alpha_1\alpha_2 - b^2} > 0\).

Thus, under condition \(c_2 < \hat{c}_2\), port authority’s equilibrium fee revenue equals

\[
R_c^* = \alpha_2 \frac{2(\alpha_1 - \alpha_2)(\alpha_1 + \alpha_2 - b) + (4\alpha_1\alpha_2 + 2b\alpha_1 + 2\alpha_2^2 - 2b^2)c_1}{(4\alpha_1\alpha_2 + 4\alpha_1\alpha_2^2 - 8\alpha_1^2\alpha_2 - 8\alpha_1 - 6b^2 + 2b^3)c_1} \frac{(4\alpha_1\alpha_2 - b^2)}{4\alpha_1\alpha_2 - b^2} = R_c^*.
\] (A124)

Case 2(1)b: Suppose \(\lambda_2^* > 0\). Then, (A122) suggests \(r_c^* = r_{12}\) and \(\lambda_2^* = \frac{1}{(\alpha_1 - \alpha_2)(4\alpha_1\alpha_2 - b^2)}[2(\alpha_1 - \alpha_2)(\alpha_1 + \alpha_2 - b) + (4\alpha_1\alpha_2 + 2b\alpha_1 + 2\alpha_2^2 - 2b^2)c_1 - (4\alpha_1\alpha_2 + 2\alpha_2^2 - b\alpha_1 + 2b\alpha_2 - 2b^2)c_2]\) by (A119). Note that \(\lambda_2^* > 0\) iff \(c_2 < \frac{2(\alpha_1 - \alpha_2)(\alpha_1 + \alpha_2 - b) + (4\alpha_1\alpha_2 + 2b\alpha_1 + 2\alpha_2^2 - 2b^2)c_1}{(4\alpha_1\alpha_2 + 4\alpha_1\alpha_2^2 - 8\alpha_1^2\alpha_2 - 8\alpha_1 - 6b^2 + 2b^3)c_1}.\)

On the other hand, \((\bar{r}_c - r_{12}) = \frac{2(\alpha_1 + \alpha_2 - b)(c_2 - c_1)}{2(\alpha_1 - \alpha_2)(2\alpha_1 - b)} > 0\). In addition, (A121) implies both \(\delta_c^* \in [0, \delta_{c1}]\) with \(\delta_{c1} = \frac{c_2 - c_1}{2(\alpha_1 - \alpha_2)}\) and \(f_c^* = \frac{\alpha_2 (c_2 - c_1)^2}{8(\alpha_1 - \alpha_2)^2} > 0\). Thus, if \(c_2 < \frac{2(\alpha_1 - \alpha_2)(\alpha_1 + \alpha_2 - b) + (4\alpha_1\alpha_2 + 2b\alpha_1 + 2\alpha_2^2 - 2b^2)c_1}{(4\alpha_1\alpha_2 + 4\alpha_1\alpha_2^2 - 8\alpha_1^2\alpha_2 - 8\alpha_1 - 6b^2 + 2b^3)c_1}\), port authority’s equilibrium fee revenue equals

\[
R_c^* = \frac{(c_2 - c_1)(4(\alpha_1 - \alpha_2) + (3a_2 + 2b)c_1 - (4a_1 - \alpha_2 + 2b)c_2)}{4(\alpha_1 - \alpha_2)^2} = R_c^*.
\] (A125)
Case 2(2): Suppose $\delta \in (\delta_{c1}, \delta_{c2}]$. Then, Lemma 12(ii) implies $\pi_{c1}^{\ast} > \pi_{c2}^{\ast}$ and $f_c^{\ast} = \frac{\pi_{c2}^{\ast} \equiv \frac{\delta(2\alpha_1-b)(1-r)-(2\alpha_1-b^2)b+bc_1-2\alpha_1c_2}{4\alpha_1}}{4\alpha_1}$. We have $f_c^{\ast} > 0$ iff $\delta < \frac{(2\alpha_1-b)(1-r)+bc_1-2\alpha_1c_2}{(2\alpha_1-b^2)} \equiv \tilde{\delta}_c$ and $r \leq \tilde{r}_c \equiv \frac{(2\alpha_1-b)+bc_1-2\alpha_1c_2}{2\alpha_1-b}$. In addition, $\delta > (\leq) \delta_{c2}$ iff $r < (\geq) \frac{1}{(2\alpha_1-\alpha_2)}[(2\alpha_1-\alpha_2) - (\alpha_2 + b)c_1 - (2\alpha_1 + b)c_2] \equiv \tilde{r}_c$. By some calculations, we have $(\tilde{r}_c - r_{12}) = \frac{(4\alpha_1-b^2)(c_2-c_1)}{2(\alpha_1-b)} > 0$, $(\tilde{r}_c - \tilde{r}_c) = \frac{(2\alpha_1-b^2)(c_2-c_1)}{(2\alpha_1-b)(2\alpha_1-\alpha_2)} > 0$, $(\tilde{r}_c - r_{12}) = \frac{(2\alpha_1-\alpha_2+b)c_1}{2(\alpha_1-\alpha_2)(2\alpha_1-\alpha_2)} > 0$ iff $\alpha_1 > \alpha_2$, and $(\delta_{c2}-\delta_{c1}) = \frac{2(\alpha_2-1)(1-r)-(2\alpha_2+b)c_1+(2\alpha_1+b)c_2}{2(\alpha_1-b)(4\alpha_1-b^2)} > 0$ iff $r > r_{12}$. Accordingly, there are another two sub-cases as follows.

Case 2(2)-1a: Suppose $r_{12} < r < \tilde{r}_c$. Then the problem in (43) becomes

$$\max_{r, f, \delta} \quad 2f + r(q_{c1}^{\ast} + q_{c2}^{\ast})$$
$$\text{s.t.} \quad \delta_{c1} < \delta \leq \delta_{c2} \text{ and } r_{12} < r < \tilde{r}_c.$$ 

Since this problem is similar to that in (A104), we just need to check whether $r_{12} < r$ holds at the equilibria of (A104). Since $(r_c^{\ast} - r_{12}) = \frac{-2(\alpha_1-\alpha_2)-(\alpha_1+2\alpha_2+2b)c_1+(3\alpha_1+2b)c_2}{2(\alpha_1-\alpha_2)(2\alpha_1-\alpha_2)b} > 0$ iff $c_2 > \frac{2(\alpha_1-\alpha_2)+(\alpha_1+2\alpha_2+2b)c_1}{3\alpha_1+2b}$, and $2(\alpha_1-\alpha_2)+(\alpha_1+2\alpha_2+2b)c_1 < \frac{2(2\alpha_1-\alpha_2)+2(2\alpha_2+4b)c_1}{(6\alpha_1+\alpha_2+4b)} = \tilde{c}_c$, an equilibrium exists when $\frac{2(\alpha_1-\alpha_2)+(\alpha_1+2\alpha_2+2b)c_1}{(3\alpha_1+2b)} < c_2 < \tilde{c}_c$ with $r_c^{\ast} = \frac{1}{2(\alpha_1+b+2b)}[(\alpha_2+b) - (2\alpha_1+2\alpha_2+3b)c_1 + (2\alpha_1+b)c_2]$ > 0, $\delta_c^{\ast} = \frac{2-c_1-c_2}{2(\alpha_1+b+2b)}$, and port authority’s fee revenue

$$R_c^{\ast} = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)} = R_3^{\ast}. \quad (A126)$$

Case 2(2)-1b: Suppose $r \geq \tilde{r}_c \equiv \frac{[(2\alpha_1-\alpha_2)+(\alpha_1+2\alpha_2+2b)c_1-(2\alpha_1+b)c_2]}{(2\alpha_1-\alpha_2)}$. Then the problem in (43) becomes

$$\max_{r, f, \delta} \quad 2f + r(q_{c1}^{\ast} + q_{c2}^{\ast})$$
$$\text{s.t.} \quad \delta_{c1} < \delta \leq \tilde{\delta}_c \text{ and } \tilde{r}_c \leq r < \tilde{r}_c.$$ 

This problem is the same as that in (A111). Thus, no solution exists in this case.

Case 2(3): Suppose $\delta \in (\delta_{c2}, \frac{1}{1+b})$. Then, Lemma 12(iic) implies $\pi_{c1}^{\ast} > (\leq) \pi_{c2}^{\ast}$ iff $\delta < (>) \frac{\alpha_1-c_1}{\alpha_1-\alpha_2}$. Thus, there are two situations.

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Case 2(3)-1: Suppose $\delta < \frac{c_2-c_1}{a_1-a_2}$. Then we have $f^*_c = \pi^*_c = \frac{\delta[1-(\alpha_2+b)\delta-c_2-r]}{2}$ with $f^* > 0$ iff $\delta < \frac{1-(c_2-r)}{(a_2+b)}$ and $r < (1-c_2)$. Note that $r < (1-c_2)$ is implied by $r < \bar{r}_c = \frac{[(2a_1-b)+b(1-2a_1c_2)]}{2a_1-b}$. Accordingly, the problem in (43) becomes

$$\max_{r, \delta} \delta[1-(\alpha_2+b)\delta-c_2-r] + 2r\delta$$

s.t. $\delta, 2 < \delta < \left(\frac{1-c_2-r}{(a_2+b)}\right)$ and $r_{12} \leq r < \bar{r}_c$. (A127)

Its Lagrange function is $L = \delta[1-(\alpha_2+b)\delta-c_2-r] + 2r\delta + \lambda_1(\delta - \delta_2) + \lambda_2\left[\frac{1-c_2-r}{(a_2+b)}-\delta\right] + \lambda_3(r-r_{12}) + \lambda_4(\bar{r}_c-r)$. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \delta + \frac{\lambda_1}{2a_1+b} - \frac{\lambda_2}{\alpha_2+b} + \lambda_3 - \lambda_4 \leq 0, \text{ and } r \cdot \frac{\partial L}{\partial r} = 0,$$

$$\frac{\partial L}{\partial \delta} = 1 - 2(\alpha_2+b)\delta - c_2 + r + \lambda_1 - \lambda_2 \leq 0, \text{ and } \delta \cdot \frac{\partial L}{\partial \delta} = 0,$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_2 \geq 0, \text{ and } \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0,$$

$$\frac{\partial L}{\partial \lambda_2} = 1 - \frac{c_2-r}{(a_2+b)} - \delta \geq 0, \text{ and } \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0,$$

$$\frac{\partial L}{\partial \lambda_3} = r - r_{12} \geq 0, \text{ and } \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0,$$

$$\frac{\partial L}{\partial \lambda_4} = \bar{r}_c - r \geq 0, \text{ and } \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0.$$ (A129)

Since the three constraints in (A127) are strict inequalities, we must have $\lambda^*_1 = \lambda^*_2 = \lambda^*_4 = 0$. Suppose $\lambda^*_3 = 0$. Then, we obtain $r^*_c = 0$ by $\delta > 0$ and (A128), which contradicts the requirement of $r^*_c > 0$. Thus, we must have $\lambda^*_3 > 0$. Under the circumstance, $r^*_c = r_{12} = \frac{2(a_1-a_2)+(2a_2+b)(1-2a_1c_2)}{2(a_1-a_2)}$ by (A129) and $\frac{\partial L}{\partial r} = \delta + \lambda_3 = 0$ due to (A128).

In addition, $\delta^*_c < 0$ implied by $\frac{\partial L}{\partial r} = \delta + \lambda_3 = 0$, which contradicts $\delta^*_c \geq 0$. Thus, no solution exists in this case.

Case 2(3)-2: Suppose $\delta > \frac{c_2-c_1}{a_1-a_2}$. Then we have $f^*_c = \pi^*_c = \frac{\delta[1-(a_1+b)\delta-c_1-r]}{2}$ with $f^*_c > 0$ iff $\delta < \frac{(1+c_1-r)}{(a_1+b)}$ and $r < (1-c_1)$. As in Case 1(3), there exists no solution because (A128) does not hold.
Case 3: Suppose \( \alpha_1 > \alpha_2 \) and \( r < r_{12} \). Since \( (\bar{r}_c - r_{12}) = \frac{(4\alpha_1\alpha_2 - b^2)(c_2 - c_1)}{2(\alpha_1 - \alpha_2)(\alpha_1 - b)} > 0 \), \( r < r_{12} \) implies \( r < \bar{r}_c \). Accordingly, there are three subcases as follows.

Case 3(1): Suppose \( \delta \in [0, \delta_{c1}'] \). Lemma 12(iiiia) implies \( \pi_{c2}^* > \pi_{c1}^* \) and \( f_c^* = \pi_{c1}^* = \frac{1}{2}\alpha_1(q_{c1}^*)^2 > 0 \). Then the problem in (43) becomes

\[
\max_{r, f, \delta} 2f + r(q_{c1}^* + q_{c2}^*) \\
\text{s.t. } 0 \leq \delta \leq \delta_{c1}' \text{ and } r < r_{12}. \tag{A130}
\]

Its Lagrange function is \( L = \alpha_1(q_{c1}^*)^2 + r(q_{c1}^* + q_{c2}^*) + \lambda_1(\delta_{c1}' - \delta) + \lambda_2(r_{12} - r) \) with the Lagrange multipliers \( \lambda_1 \) and \( \lambda_2 \). Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = 2\alpha_2q_{c2}^* \frac{\partial q_{c2}^*}{\partial r} + r(\frac{\partial q_{c1}^*}{\partial r} + \frac{\partial q_{c2}^*}{\partial r}) + (q_{c1}^* + q_{c2}^*) + \lambda_1 \frac{\partial \delta_{c1}^*}{\partial r} = -\lambda_1 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \tag{A131}
\]

\[
\frac{\partial L}{\partial \delta} = -\lambda_1 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \tag{A132}
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta_{c1}' - \delta \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad \text{and} \tag{A133}
\]

\[
\frac{\partial L}{\partial \lambda_2} = r_{12} - r \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0. \tag{A134}
\]

Constraint \( r < r_{12} \) in (A130) suggests \( \lambda_2^* = 0 \) by (A134). Based on the values of \( \lambda_1 \), there are two situations below.

Case 3(1)a: Suppose \( \lambda_1^* = 0 \). Then (A131) becomes

\[
(4\alpha_1\alpha_2 - b^2)(2\alpha_1 - b) + b^2(2\alpha_1 - b)c_1 - (8\alpha_1^2\alpha_2 - 4\alpha_2^2 + b^3)c_2 - 2(8\alpha_1^2\alpha_2 + 4\alpha_1\alpha_2^2 - 4b\alpha_1\alpha_2 - 3b^2\alpha_1 - 2b^2\alpha_2 + b^2)r = 0. \]

Solving this equation yields

\[
r_c^* = \frac{1}{2(8\alpha_1^2\alpha_2 + 4\alpha_1\alpha_2^2 - 4b\alpha_1\alpha_2 - 3b^2\alpha_1 - 2b^2\alpha_2 + b^2)}[(2\alpha_1 - b)(4\alpha_1\alpha_2 + 2b\alpha_2 - b^2) + b^2(2\alpha_1 - b)c_1 - (8\alpha_1^2\alpha_2 - 4\alpha_2^2 + b^3)c_2 - 2(8\alpha_1^2\alpha_2 + 4\alpha_1\alpha_2^2 - 4b\alpha_1\alpha_2 - 3b^2\alpha_1 - 2b^2\alpha_2 + b^2)r]. \]

It remains to check whether \( r_c^* < r_{12} \) holds. By some calculations, we have \( r_c^* < r_{12} \) iff \( c_2 < \frac{[2(\alpha_1 - \alpha_2)(\alpha_1 + \alpha_2) + (4\alpha_1\alpha_2 + 2b\alpha_2 - b^2 - 2b^2) - (4\alpha_1\alpha_2 + 2b\alpha_2 - b^2 - 2b^2)]c_1}{(4\alpha_1\alpha_2 - b^2)(4\alpha_1\alpha_2 - b^2)} \). Moreover, (A133) implies both \( \delta_c^* \in [0, \delta_{c1}'] \) with \( \delta_{c1}' = \frac{(2\alpha_2 - b)(1 - r_c^*) - 2b\alpha_2 c_1 + bc_2}{4\alpha_1\alpha_2 - b^2} \) and \( f_c^* = \frac{\alpha_1}{2}[(2\alpha_2 - b)(1 - r_c^*) - 2b\alpha_2 c_1 + bc_2]^2 > 0 \). Thus, at the equilibrium, port authority’s fee revenue equals

\[
R_c^* = \alpha_1 \left( \frac{2(\alpha_2 - b)(1 - r_c^*) - 2b\alpha_2 c_1 + bc_2}{4\alpha_1\alpha_2 - b^2} \right)^2 + r_c^* \left( \frac{2(\alpha_1 + \alpha_2 - b)(1 - r_c^*) - (2\alpha_2 - b)c_1 - (2\alpha_1 - b)c_2}{4\alpha_1\alpha_2 - b^2} \right) = R_c^*. \tag{A135}
\]
Case 3(1): Suppose \( \lambda_1^* > 0 \). Then, (A133) suggests \( \delta_c^* = \delta_{c1}^* > 0 \). This in turn implies \( \lambda_1^* = 0 \) by (A132). It is a contradiction. Thus, no solution exists in this case.

Case 3(2): Suppose \( \delta \in (\delta_{c1}', \delta_{c2}'] \). Then, Lemma 12(iiiib) implies \( \pi_{c2}^* > \pi_{c1}^* \) and \( f_c^* = \pi_{c1}^* - \delta_{c1}' \) and \( r < (\geq) \). We have \( f_c^* > 0 \) if \( \delta \in (\pi_{c2}^* - \delta_{c2}') \) with \( \delta_{c1}' \). Moreover, if \( (2\alpha_2 - \alpha_1)^* > 0 \), we have \( \delta_{c1}' > (\geq) \) iff \( \delta_{c2}^- < (\geq) \). Then, (A133) suggests \( \tilde{\delta}_{c}^- \). By contrast, if \( (2\alpha_1 - \alpha_2)^* < 0 \), we have \( \tilde{\delta}_{c}^- \leq (>) \) iff \( \delta_{c2}^- \leq (>) \). Since \( \tilde{\delta}_{c}^- \) are the Lagrange multipliers for the three constraints in (A136). Then, the Kuhn-Tucker conditions are

\[
\begin{align*}
\frac{\partial L}{\partial r} &= \frac{(2\alpha_2 - b)(1 - r) - (2\alpha_1 \alpha_2 - b^2)\delta + bc_2 - 2\alpha_2 c_1}{(4\alpha_1 \alpha_2 - b^2)} - \frac{\lambda_2}{(2\alpha_2 + b)} - \lambda_3 r \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \quad (A137) \\
\frac{\partial L}{\partial \delta} &= \frac{1}{2\alpha_2} [(2\alpha_2 - b) - 2(2\alpha_1 \alpha_2 - b^2)\delta + bc_2 - 2\alpha_2 c_1] + \lambda_1 - \lambda_2 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad (A138) \\
\frac{\partial L}{\partial \lambda_1} &= \delta - \delta_{c1}' \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A139) \\
\frac{\partial L}{\partial \lambda_2} &= \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad (A140) \\
\frac{\partial L}{\partial \lambda_3} &= \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0. \quad (A141)
\end{align*}
\]

Constraints \( \delta_{c1}' < \delta \) and \( r < r_{c1} \) suggest \( \lambda_1^* = \lambda_3^* = 0 \) by (A139) and (A141). If \( \lambda_2^* = 0 \), we have \( r_{c}^* = \frac{(1-c_2)}{2} \) and \( \delta_{c}^* = \frac{(2\alpha_2 - b) + bc_2 - 2\alpha_2 c_1}{2(2\alpha_1 \alpha_2 - b^2)} \) by (A137) and (A138). Since
(\delta_c^* - \delta_c^{'*}) = \frac{\alpha_2\left[2(2\alpha_2 - \alpha_1) - (\alpha_1 + b)c_1 + (2\alpha_2 + b)c_1\right]}{(2\alpha_2 + b)(2\alpha_2 + b)} < 0$, it is impossible to meet the requirement of $\delta_c^* \leq \delta_c^{'*}$. Thus, no solution exists in this case.

By contrast, if $\lambda^*_c > 0$, then (A137), (A138), and (A140) suggest
\[
\frac{1 - 2r - c_2}{2\alpha_2} - \frac{\lambda_2}{2\alpha_2 + b} = 0, \quad \frac{[(2\alpha_2 - b) - 2(\alpha_1 + b - \delta_c^*)]c_2 + 2(\alpha_2 + b)(\alpha_1 - \delta_c^*)}{2\alpha_2} - \lambda_2 = 0,
\]
and $(\delta_c^* - \delta) = 0$.

Solving these equations yields $r_c^* = \frac{2(\alpha_1 + b)(2\alpha_2 - \alpha_1)(2\alpha_2 + b)c_2}{(2\alpha_2 + b)(2\alpha_2 + b)(\alpha_1 + b)c_2} - (\alpha_1 + b)c_2 > 0$. By some calculations, we have $(r_{12} - r_c^*) = \frac{2(\alpha_2 + b)(2(\alpha_1 - \alpha_2) + 2(\alpha_2 + b)c_1 - (\alpha_1 + 2\alpha_2 + b)c_2)}{2(\alpha_1 + \alpha_2 + 2b)} > 0$ iff $c_2 < \frac{2(\alpha_2 + b)(2(\alpha_1 - \alpha_2) + 3(\alpha_2 + b)c_1 - (\alpha_1 + 2\alpha_2 + b)c_2)}{2(\alpha_1 + 2\alpha_2 + b)}$. Thus, an equilibrium exists when $c_2 < \frac{2(\alpha_1 - \alpha_2) + 3(\alpha_2 + b)c_1}{(2\alpha_1 + 2\alpha_2 + b)}$, and $(\delta_c^* - \delta_c^{'*}) > 0$ iff $r_c^* < r_{12}$.

In addition, we have $2(\alpha_1 + b - \alpha_2)c_2 < \frac{2(\alpha_1 + b)(2\alpha_2 + b)c_2}{(2\alpha_1 + 2\alpha_2 + b)}$. Thus, an equilibrium exists when $c_2 < \frac{2(\alpha_1 - \alpha_2) + 3(\alpha_2 + b)c_1}{(2\alpha_1 + 2\alpha_2 + b)}$ with port authority’s fee revenue
\[
R_c^* = \frac{(2 - c_1 - c_2)^2}{4(\alpha_1 + 2\alpha_2 + 2b)} = R_{c'}^*. \quad (A142)
\]

Case 3(2)-2: Suppose $(2\alpha_2 - \alpha_1) < 0$, we have $\tilde{r}_c < r_{12}$. Then there are two situations below.

Case 3(2)-2a: Suppose $r \leq \frac{[(2\alpha_2 - \alpha_1) - (\alpha_1 + b)c_1 + (2\alpha_2 + b)c_1]}{(2\alpha_2 - \alpha_1)} = \tilde{r}_c$. Then, the problem in (43) becomes
\[
\max_{r, f, \delta} \quad 2f + r(q_1^* + q_2^*)
\]
s.t. $\delta_c^{'*} < \delta < \tilde{\delta}_c$ and $r \leq \tilde{r}_c$. \quad (A143)

Its Lagrange function is $L = \frac{\delta}{2\alpha_2}[(2\alpha_2 - b)(1 - r) - (\alpha_1 + b - \delta)c_2 - 2\alpha_2c_1] + \frac{r[1 + (2\alpha_2 - b)c_2 - \tilde{r}_c]}{2\alpha_2} + \lambda_1(\delta - \delta_c^{'*}) + \lambda_2(\tilde{r}_c - \delta) + \lambda_3(\tilde{r}_c - r)$. Then, the Kuhn-Tucker conditions are
\[
\frac{\partial L}{\partial r} = \frac{(1 - 2r - c_2)}{2\alpha_2} + \frac{(2\alpha_2 - b)}{(4\alpha_1 \alpha_2 - b^2)} \lambda_1 - \frac{(2\alpha_2 - b)}{(2\alpha_1 \alpha_2 - b^2)} \lambda_2 - \lambda_3 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \quad (A144)
\]
\[
\frac{\partial L}{\partial \delta} = \frac{1}{2\alpha_2}[(2\alpha_2 - b) - 2(\alpha_1 \alpha_2 - b^2)\delta + bc_2 - 2\alpha_2c_1] + \lambda_1 - \lambda_2 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad (A145)
\]
\[
\frac{\partial L}{\partial \lambda_1} = \delta - \delta'_c \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0,
\]
\[
\frac{\partial L}{\partial \lambda_2} = \tilde{\delta}_c - \delta \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad \text{and}
\]
\[
\frac{\partial L}{\partial \lambda_3} = \tilde{r}_c - r \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0,
\]

where \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are the Lagrange multipliers for the three constraints in (A143).

We have \( \lambda^*_1 = \lambda^*_2 = 0 \) by the three strict inequalities in (A143). According to the values of \( \lambda^*_3 \), there are two sub-cases as follows.

Case 3(2)-2a-1: Suppose \( \lambda^*_3 = 0 \). Then \( r^*_c = \frac{(1-c_2)}{2} \) and \( \delta^*_c = \frac{(2a_2-b)+bc_2-2a_2c_1}{2(2a_1a_2-b^2)} \) by (A144) and (A145). Some calculations yield \( (\tilde{\delta}_c - \delta^*_c) = \frac{a^2(c_2-c_1)}{2(2a_1a_2-b^2)} > 0 \) if \( c_2 < \frac{a^2(2a_2-b)-b^2c_1}{2a_1a_2} \). Moreover, we have \( (\tilde{r}_c - r^*_c) = \frac{(2a_2-b) \alpha_1 \alpha_2 + 2(2a_2-b) \alpha_1 b_1 + 2(2a_2-b) \alpha_2 c_1}{2(2a_2-a_1)} \geq 0 \) if \( c_2 < \frac{(a^2(2a_2-b)-b^2c_1)}{2(2a_1a_2-b^2)} \) due to \( (2a_2 - a_1) \).

Thus, an equilibrium exists when \( c_2 \leq \frac{(a^2(2a_2-b)+b_2-2a_2c_1)}{(2a_1a_2+b^2)} \) with port authority’s fee revenue

\[
R^*_c = \frac{(a_1 + 2a_2 - 2b) - 2(2a_2 - b)c_1 - 2(a_1 - b)c_2 - 2b_1c_2 + 2a_2c_1^2 + a_1c_2^2}{4(2a_1a_2 - b^2)}
\]

Case 3(2)-2a-2: Suppose \( \lambda^*_3 > 0 \). Then, \( r^*_c = \tilde{r}_c = \frac{[2(a_2-a_1)-(2a_2+b)c_1+(a_1+b)c_2]}{(2a_1-a_2)} \), \( \delta^*_c = \frac{(a_1-2a_2)(2a_2-b)-b^2c_1}{2(a_1-a_2)(2a_1a_2-b^2)} \), and \( \lambda^*_3 = \frac{-(a_1-2a_2)-(2a_2+b)c_1+(a_1+2a_2+b)c_2}{2(a_1-a_2)} \) by (A144)-(A146). Note that \( \lambda^*_3 > 0 \) iff \( c_2 > \frac{(a_1-2a_2)+2(2a_2+b)c_1+(a_1+2a_2+b)c_2}{a_1+2a_2+b} \), and \( r^*_c > 0 \) iff \( c_2 < \frac{(a_1-2a_2)+(2a_2+b)c_1}{a_1+b} \). On the other hand, condition \( \delta'_c < \delta^*_c < \tilde{\delta}_c \) is needed. Some calculations yield \( (\delta^*_c - \delta'_c) = \frac{(2a_2-b)+b_2-2a_2c_1}{2(2a_1a_2-b^2)}, \quad (\delta^*_c - \tilde{\delta}_c) = \frac{(a_1-2a_2)(2a_2-b)-2a_1a_2-2a_2b_1+2a_1b_2-2b^2c_1}{2(2a_1a_2-b^2)} > 0 \) iff \( c_2 < \frac{A+Bc_1}{H} \), where \( A = (2a_2-b)(4a_1a_2-b^2)(a_1-a_2) > 0, \quad B = (2a_2-b) \quad 2a_2(4a_1a_2-b^2)(a_1-a_2) > 0, \quad H = 2(2a_1a_2-b^2)(a_1-a_2) > 0. \)

Moreover, we have \( \frac{(a_1-2a_2)+2(2a_2+b)c_1}{a_1+b} < \frac{A+Bc_1}{H} \), and \( \frac{(a_1-2a_2)(2a_2-b)+2(a_1a_2+2a_2b)^2c_1}{4(a_1-a_2)(2a_1a_2-b^2)} < \frac{(a_1-2a_2)(2a_2-b)+2(a_1a_2+2a_2b)^2c_1}{4(a_1-a_2)(2a_1a_2-b^2)} \) because \( \frac{(a_1-2a_2)(2a_2-b)+2(a_1a_2+2a_2b)^2c_1}{4(a_1-a_2)(2a_1a_2-b^2)} < \frac{(a_1-2a_2)(2a_2-b)+2(a_1a_2+2a_2b)^2c_1}{4(a_1-a_2)(2a_1a_2-b^2)} \).

\[
R^*_c = \frac{2a_2(a_1-a_2)(2a_2-a_1)^2}{(a_1+2a_2+b)(4a_1a_2-b_1+2a_2b_2-b^2)} > 0, \quad \frac{a_1+b}{H(a_1+b)} > 0.
\]
Thus, if \(\frac{(a_1-2a_2)+2(2a_2+b)c_1}{a_1+2a_2+2b} < c_2 < \frac{(a_1-2a_2)+(2a_2+b)c_1}{a_1+b}\), an equilibrium exists with port authority’s fee revenue

\[
R^*_c = \frac{\delta^*_c}{2a_2} - [(2a_2-b)(1-r^*_c) - (2a_1a_2-b^2)\delta^*_c + bc_2 - 2a_2c_1] + \frac{r^*_c}{2a_2}[(1+(2a_2-b)\delta^*_c - c_2 - r^*_c) = R^*_c.
\]

(A148)

Case 3(2)-2b: Suppose \(r > \tilde{r}_c\). Then the problem in (43) becomes

\[
\max_{r,f, \delta} 2f + r(q^*_c + 1)\]

s.t. \(\delta'_{c1} < \delta < \delta'_{c2}\) and \(\tilde{r}_c < r < r_{12}\).

(A149)

Its Lagrange function is \(L = \frac{\delta}{2a_2} - [(2a_2-b)(1-r) - (2a_1a_2-b^2)\delta + bc_2 - 2a_2c_1] + \frac{r}{2a_2}[1+(2a_2-b)\delta - c_2 - r] + \lambda_1(\delta - \delta'_{c1}) + \lambda_2(\delta'_{c2} - \delta) + \lambda_3(r - \tilde{r}_c) + \lambda_4(r_{12} - r).\) Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = \frac{(1-2r-c_2)}{2a_2} + \frac{(2a_2-b)\lambda_1}{(4a_1a_2-b^2)} - \frac{\lambda_2}{(2a_2+b)} + \lambda_3 - \lambda_4 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \quad (A150)
\]

\[
\frac{\partial L}{\partial \delta} = \frac{[(2a_2-b) - 2(2a_1a_2-b^2)\delta + bc_2 - 2a_2c_1]}{2a_2} + \lambda_1 - \lambda_2 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad (A151)
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta - \delta'_{c1} \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A152)
\]

\[
\frac{\partial L}{\partial \lambda_2} = \delta'_{c2} - \delta \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad (A153)
\]

\[
\frac{\partial L}{\partial \lambda_3} = r - \tilde{r}_c \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \quad \text{and} \quad (A154)
\]

\[
\frac{\partial L}{\partial \lambda_4} = r_{12} - r \geq 0, \quad \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0, \quad (A155)
\]

where \(\lambda_1, \lambda_2, \lambda_3,\) and \(\lambda_4\) are the Lagrange multipliers for the problem in (A149). Constraints \(\delta'_{c1} < \delta < \delta'_{c2}\) and \(\tilde{r}_c < r < r_{12}\) suggest \(\lambda_1^* = \lambda_3^* = \lambda_4^* = 0\) by (A152), (A154), and (A155). According to the sign of \(\lambda_2\), we have two situations below.

Case 3(2)-2b-1: Suppose \(\lambda_2^* = 0\). Then, we have \(r^*_c = \frac{1-c_2}{2}\) and \(\delta^*_c = \frac{(2a_2-b)+2a_2c_1}{2(2a_1a_2-b^2)}\) by (A150) and (A151). It remains to find the conditions under which \(\delta'_{c1} < \delta^*_c \leq \delta'_{c2}\) and \(\tilde{r}_c < r < r_{12}\) hold. By some calculations, we have \(\delta'_{c2} - \delta^*_c = \frac{a_2[(a_1-b)+(a_2+b)c_1-(a_1+b)c_2]}{(2a_2+b)(2a_1a_2-b^2)}\)
Suppose 

\[ \alpha \in (0, 1) \text{ and } \beta \in (0, 1) \]

and \((\alpha, \beta) \in \mathcal{C} \cap \mathcal{D} \text{ due to } \alpha_2^2 \geq \lambda_2 > 0 \) if \( \alpha \geq \lambda_2 \) and \( \alpha < \lambda_2 \). Thus, if \( \alpha > \lambda_2 \), there exists an equilibrium with port authority’s fee revenue

\[ R^* = \frac{(\alpha + 2\alpha - 2b - 2(2\alpha - b)c_1 - 2(\alpha - b)c_2 - 2bc_1c_2 + 2\alpha c_1^2 + \alpha_1 c_2^2)}{4(\alpha_1 + 2\alpha_2 - b^2)} \]

(A156)

Case 3(2)-2b-2: Suppose \( \lambda_2^* > 0 \). Then (A150), (A151), and (A153) suggest \( \frac{1 - 2r - c_2}{2\alpha_2} - \frac{\lambda_2}{2\alpha_2 + b} = 0 \), \( \frac{1}{2\alpha_2} [(2\alpha - b) - 2(2\alpha - b^2)\delta + bc_2 - 2\alpha c_1] - \lambda_2 = 0 \), and \( (\delta_c^* - \delta) = 0 \). Solving these equations yields

\[ \frac{\lambda_2}{2\alpha_2 + b} = \frac{2(2\alpha - b) - 2\alpha_2 - 2(2\alpha - b^2)\delta + bc_2 - 2\alpha c_1}{\alpha_1 + 2\alpha_2 + 2b} \]

(A157)

Case 3(3): Suppose \( \delta \in (\delta_c^*, \frac{1}{\alpha_1 + 2\alpha_2 + 2b}) \). Then, Lemma 12(iiic) implies \( \pi^*_c > (\pi^*_c \text{ iff } \delta < (>) \frac{(c_2 - c_1)}{\alpha_1 - \alpha_2} \). Thus, we have two sub-cases below.

### Case 3(3): \( \delta \in (\delta_c^*, \frac{1}{\alpha_1 + 2\alpha_2 + 2b}) \)

\[ R^* = \frac{2 - c_1 - c_2}{4(\alpha_1 + 2\alpha_2 + 2b)} = R^* \]

Thus, an equilibrium exists when

\[ \frac{(\alpha - 2\alpha_2 + 2(2\alpha + 2b)c_1}{(\alpha + b)} \]

(A157)

with port authority’s fee revenue

\[ R_c^* = \frac{(2 - c_1 - c_2)^2}{4(\alpha_1 + 2\alpha_2 + 2b)} = R^* \]

Thus, an equilibrium exists when

\[ \frac{(\alpha - 2\alpha_2 + 2(2\alpha + 2b)c_1}{(\alpha + b)} \]

(A157)

with port authority’s fee revenue

\[ R_c^* = \frac{(2 - c_1 - c_2)^2}{4(\alpha_1 + 2\alpha_2 + 2b)} = R^* \]

Thus, an equilibrium exists when

\[ \frac{(\alpha - 2\alpha_2 + 2(2\alpha + 2b)c_1}{(\alpha + b)} \]

(A157)

with port authority’s fee revenue

\[ R_c^* = \frac{(2 - c_1 - c_2)^2}{4(\alpha_1 + 2\alpha_2 + 2b)} = R^* \]

Thus, an equilibrium exists when

\[ \frac{(\alpha - 2\alpha_2 + 2(2\alpha + 2b)c_1}{(\alpha + b)} \]

(A157)

with port authority’s fee revenue

\[ R_c^* = \frac{(2 - c_1 - c_2)^2}{4(\alpha_1 + 2\alpha_2 + 2b)} = R^* \]
Case 3(3)-1: Suppose $\delta > \frac{(c_2-c_1)}{\alpha_1-\alpha_2}$ and $f^*_c = \pi^*_c = \frac{\delta[1-(\alpha_1+b)\delta-c_1-r]}{2}$ with $f^*_c > 0$ iff $\delta < \frac{(1-c_1-r)}{(\alpha_1+b)}$ and $r < (1-c_1)$. Note that $r < (1-c_1)$ is implied by $r < r_{12}$. Accordingly, the problem in (43) becomes

$$\max_{r, \delta} \delta[1 - (\alpha_1 + b)\delta - c_1 - r] + 2r\delta$$

s.t. $\delta'_{c_2} < \delta < \frac{(1-c_1-r)}{(\alpha_1 + b)}$ and $0 < r < r_{12}$. \hspace{1cm} (A158)

Its Lagrange function is $L = \delta[1 - (\alpha_1 + b)\delta - c_1 - r] + 2r\delta + \lambda_1(\delta - \delta'_{c_2}) + \lambda_2\frac{[1-c_1-r]}{(\alpha_1+b)} - \delta] + \lambda_3(r_{12} - r)$. Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \delta + \frac{\lambda_1}{2\alpha_2 + b} - \frac{\lambda_2}{\alpha_1 + b} - \lambda_3 \leq 0, \hspace{0.5cm} r \cdot \frac{\partial L}{\partial r} = 0, \hspace{1cm} (A159)$$

$$\frac{\partial L}{\partial \delta} = 1 - 2(\alpha_1 + b)\delta - c_1 + r + \lambda_1 - \lambda_2 \leq 0, \hspace{0.5cm} \delta \cdot \frac{\partial L}{\partial \delta} = 0,$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta'_{c_2} \geq 0, \hspace{0.5cm} \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0,$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{1-c_1-r}{(\alpha_1 + b)} - \delta \geq 0, \hspace{0.5cm} \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \hspace{0.5cm} \text{and}$$

$$\frac{\partial L}{\partial \lambda_3} = r_{12} - r \geq 0, \hspace{0.5cm} \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0,$$

where $\lambda_1$, $\lambda_2$, and $\lambda_3$ are the Lagrange multipliers. Since all constraints in (A158) are strict inequalities, we must have $\lambda_1^* = \lambda_2^* = \lambda_3^* = 0$. However, by some calculations, we obtain $r^*_c = 0$ by $\delta > 0$ and (A159), which contradicts the requirement of $r^*_c > 0$. Thus, no solution exists in this case.

Case 3(3)-2: Suppose $\delta \leq \frac{(c_2-c_1)}{\alpha_1-\alpha_2}$ and $f^*_c = \pi^*_c = \frac{\delta[1-(\alpha_2+b)\delta-c_2-r]}{2}$ with $f^*_c > 0$ iff $\delta < \frac{(1-c_2-r)}{(\alpha_2+b)}$ and $r < (1-c_2)$. As in Case 1(3), no solution exists here because (A159) does not hold.

Based on the above and the values of $\alpha_1$ and $\alpha_2$, we can obtain optimal two-part tariff contracts as follows.

Case (i): Suppose $\alpha_1 \leq \frac{\alpha_2}{2}$. The proofs are the same as those of Case (ii) below. They verify Lemma 13(i).

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Case (ii): Suppose $\frac{\alpha_2^2}{2} < \alpha_1 \leq \alpha_2$. Since $\hat{c}_2 \equiv \frac{2(2\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1}{(6\alpha_1+\alpha_2+4b)} < \hat{c}_2$, we have two cases as follows.

First, if $c_2 \in (c_1, \hat{c}_2)$, then equilibria of $R^*_2$ in (A103) of Case 1(1)a and $R^*_3$ in (A110) of Case 1(2)-1a exist. Define $M_1 = (R^*_3 - R^*_1)$. Some calculations show

$$\frac{\partial^2 M_1}{\partial c^2} = \frac{-(4\alpha_1^2+8\alpha_2\alpha_1+8b\alpha_1\alpha_2+2b^2\alpha_2-2b\alpha_1^2)}{(2\alpha_1+\alpha_2+2b)(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4\alpha_1\alpha_2^2-3\alpha_2^2-2\alpha_1^2+2b^2)} < 0,$$

$$M_1 = \frac{-(4\alpha_1^2+12\alpha_2\alpha_1+11\alpha_2^2+2\alpha_1\alpha_2^2+4b\alpha_1\alpha_2^2-8b^2\alpha_1\alpha_2-2b\alpha_2^2-\alpha_2^2)}{(6\alpha_1+\alpha_2+4b)^2(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4\alpha_1\alpha_2^2-3\alpha_2^2-2\alpha_1^2+2b^2)} > 0$$

at $c_2 = c_1$ due to $(2\alpha_1^2 + 3\alpha_1\alpha_2 - \alpha_2^2 - 2b\alpha_1) > 0$ by $\alpha_1 \leq \alpha_2 < 2\alpha_1$, and

$$M_1 = \frac{-(4\alpha_1^2+12\alpha_2\alpha_1+11\alpha_2^2+2\alpha_1\alpha_2^2+4b\alpha_1\alpha_2^2-8b^2\alpha_1\alpha_2-2b\alpha_2^2-\alpha_2^2)}{(6\alpha_1+\alpha_2+4b)^2(8\alpha_1\alpha_2^2+4\alpha_1^2\alpha_2-4\alpha_1\alpha_2^2-3\alpha_2^2-2\alpha_1^2+2b^2)} > 0$$

at $c_2 = \hat{c}_2$ due to $(4\alpha_1^2+12\alpha_2\alpha_1+11\alpha_2^2+2\alpha_1\alpha_2^2+4b\alpha_1\alpha_2^2-8b^2\alpha_1\alpha_2-2b\alpha_2^2-\alpha_2^2) > (4\alpha_1^2+12\alpha_2\alpha_1+11\alpha_2^2+2\alpha_1\alpha_2^2+4b\alpha_1\alpha_2^2-8b^2\alpha_1\alpha_2-2b\alpha_2^2-\alpha_2^2) = (4\alpha_1^2+12\alpha_2\alpha_1+11\alpha_2^2+2\alpha_1\alpha_2^2+4b\alpha_1\alpha_2^2-8b^2\alpha_1\alpha_2-2b\alpha_2^2-\alpha_2^2) > 0$. These imply $R^*_3 > R^*_1$.

Thus, for $c_2 \in (c_1, \hat{c}_2)$, port authority’s optimal two-part tariff contract and minimum throughput guarantee are those in Case 1(2)-1a with $r^*_c = \frac{2(2\alpha_1+b)-(2\alpha_1+2\alpha_2+3b)c_1+(2\alpha_1+b)c_2}{(6\alpha_1+\alpha_2+2b)}$, $\delta^*_c = \frac{(2\alpha_1-c_2)}{2(2\alpha_1+\alpha_2+2b)}$, $f^*_c = \frac{(2\alpha_1-c_2)(2(2\alpha_1-c_2)+(2\alpha_1+3\alpha_2+4b)c_1-(6\alpha_1+\alpha_2+4b)c_2)}{8(2\alpha_1+\alpha_2+2b)^2}$, and $R^*_c = \frac{(2\alpha_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$.

For $c_2 \in [\hat{c}_2, \hat{c}_2]$, a unique solution in Case 1(1)a exists. Thus, port authority’s optimal two-part tariff contract and minimum throughput guarantee are those in Case 1(1)a with $r^*_c = \frac{2(2\alpha_1-b)-(2\alpha_1+2\alpha_2+3b)c_1+(2\alpha_1+b)c_2}{(6\alpha_1+\alpha_2+2b)}$, $\delta^*_c \in [0, \frac{2(2\alpha_1-b)(1-r^*_c)+(2\alpha_1+2\alpha_2+3b)c_1}{4\alpha_1+\alpha_2-b^2}]$, $f^*_c = \frac{(2\alpha_1-b)(1-r^*_c)\alpha_2}{4\alpha_1+\alpha_2-b^2}$, and $R^*_c = 2f^*_c + r^*_c \left[\frac{2(2\alpha_1-b)-(2\alpha_1+2\alpha_2+3b)c_1+(2\alpha_1+b)c_2}{(6\alpha_1+\alpha_2+2b)}\right]$.

Then, Lemma 13(ii) is proved.

Case (iii): Suppose $\alpha_2 < \alpha_1 \leq 2\alpha_2$. We need to know relative sizes of critical points

$$c^*_2 \equiv \frac{2(\alpha_1-\alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1^2+2\alpha_1\alpha_2+2\alpha_2^2-2\alpha_1^2)c_1}{(4\alpha_1+2\alpha_1+2\alpha_2^2-2\alpha_1^2+2b^2)}$$

and $\hat{c}_2 \equiv \frac{1}{(4\alpha_1^2+8\alpha_1\alpha_2-4\alpha_1^2-2b^2)} \left[(4\alpha_2^2+4\alpha_1\alpha_2-6b\alpha_1-2b\alpha_2+2b^2)+(4\alpha_1\alpha_2+2b\alpha_1+2b\alpha_2-2b^2)c_1\right]$ in Case 2(1)a, $c^*_2 \equiv \frac{2(\alpha_1-\alpha_2)+(2\alpha_1+3\alpha_2+4b)c_1}{(6\alpha_1+\alpha_2+4b)}$ in Case 2(2)-1a, and $c^*_3 \equiv \frac{1}{(4\alpha_1+2\alpha_1+2\alpha_2^2-2\alpha_1^2+2b^2)} \left[2(\alpha_1-\alpha_2)(\alpha_1+\alpha_2-b)+(4\alpha_1\alpha_2+2b\alpha_1+2b\alpha_2-2b^2)c_1\right]$ in Case 3(1)a, and $c^*_2 \equiv \frac{2(\alpha_1-\alpha_2)+(3\alpha_2+2b)c_1}{(2\alpha_1+2\alpha_2+4b)}$ in Case 3(2)-1. Some calculations show

$$(\hat{c}_2-c^*_2) = \frac{2(\alpha_1-\alpha_2)(\alpha_1-b)}{(6\alpha_1+\alpha_2+2b)^2(2\alpha_1+2\alpha_2+4b)} > 0,$$

$$(c^*_2-c^*_3) = \frac{2(\alpha_1-\alpha_2)^2(\alpha_1-b)(1-c_1)}{(4\alpha_1+2\alpha_1+2\alpha_2^2-2\alpha_1^2+2b^2)(2\alpha_1+2\alpha_2+4b)} > 0.$$
\( \hat{c}_2 < \check{c}_2 < \hat{c}_2 \), and optimal two-part tariff contracts can be derived by the following steps.

First, for \( c_2 < c_2^3 \), we need to comparing \( R_4^* \) in (A135) of Case 3(1)a and \( R_5^* \) in (A142) of Case 3(2)-1. Define \( M_2 = (R_5^* - R_4^*) \). Since \( \frac{\partial^2 M_2}{\partial c_2^2} = \frac{-2(2\alpha_1^2 + 2\alpha_2)(\alpha_1 + 2\alpha_2 + 2b\alpha_2 + b\alpha_2)(b\alpha_2 + b\alpha_2 + b\alpha_2) + \partial^2 M_2}{\partial c_2} = \frac{0}{(a_1 + 2a_2 + 2b)(4\alpha_1 + 2a_1^2 + 2b\alpha_2 + 2b^2\alpha_2)} < 0 \) and \( \frac{\partial M_2}{\partial c_2} = \frac{0}{(a_1 + 2a_2 + 2b)(4\alpha_1 + 2a_1^2 + 2b\alpha_2 + 2b^2\alpha_2)} < 0 \) at \( c_2 = c_1 \), we have \( \frac{\partial M_2}{\partial c_2} < 0 \). Moreover, since \( M_2 = \frac{\alpha_2(1 - 1)^2 - (1 - 1)^2}{(a_1 + 2a_2 + 2b)(4\alpha_1 + 2a_1^2 + 2b\alpha_2 + 2b^2\alpha_2)} > 0 \) at \( c_2 = c_1 \), we get \( R_5^* > R_4^* \) when \( c_2 < c_2^3 \).

Second, for \( c_2 < c_2^1 \), we need to comparing \( R_2^* \) in (A125) of Case 2(1)b and \( R_5^* \) in (A142) of Case 3(2)-1. Define \( M_3 = (R_5^* - R_2^*) \). Since \( \frac{\partial^2 M_3}{\partial c_2^2} = \frac{5(2\alpha_1^2 + 2\alpha_2)(\alpha_1 + 2\alpha_2 + 2b\alpha_2 + b\alpha_2 + 2b^2\alpha_2)}{2(\alpha_1 + 2\alpha_2 + 2b)(4\alpha_1 + 2a_1^2 + 2b\alpha_2 + 2b^2\alpha_2)} > 0 \), and \( M_3 = \frac{0}{(a_1 + 2a_2 + 2b)(4\alpha_1 + 2a_1^2 + 2b\alpha_2 + 2b^2\alpha_2)} > 0 \) at \( c_2 = c_1 \), which is the solution of \( \frac{\partial M_3}{\partial c_2} = 0 \). Accordingly, the minimum value of \( M_3 \) is \( \frac{0}{(a_1 + 2a_2 + 2b)(4\alpha_1 + 2a_1^2 + 2b\alpha_2 + 2b^2\alpha_2)} > 0 \), and \( R_5^* > R_2^* \) for all \( c_2 \).

Third, we always have \( (R_5^* - R_3^* \frac{4(\alpha_1 + 2a_2 + 2b)}{4(\alpha_1 + 2a_2 + 2b)} = \frac{4(\alpha_1 + 2a_2 + 2b)}{4(\alpha_1 + 2a_2 + 2b)} = \frac{4(\alpha_1 + 2a_2 + 2b)}{4(\alpha_1 + 2a_2 + 2b)} > 0 \).

Fourth, we have \( R_3^* > R_1^* \) for \( c_2 \in (c_1, \check{c}_2) \) from Case (i). Moreover, \( \frac{2(2a_1 - c_2)^2}{4(2a_1 + 2a_2 + 2b)} \) and \( \check{c}_2 < \check{c}_2 \). Thus, for \( c_2 < \check{c}_2 \), we have \( R_5^* > R_1^* \).

In sum, for \( c_2 \in (c_1, \check{c}_2) \), the above results imply that the optimal two-part tariff contract and the minimum throughput guarantee are those in Case 3(2)-1 with \( r_c^* = \frac{2(2a_1 + 2a_2 + 2b)c_1}{2(\alpha_1 + 2a_2 + 2b)} \), \( \delta_c^* = \frac{2(2a_1 + 2a_2 + 2b)c_1}{2(\alpha_1 + 2a_2 + 2b)} \), \( \gamma_c^* = \frac{2(2a_1 + 2a_2 + 2b)c_1}{2(\alpha_1 + 2a_2 + 2b)} \), \( \nu_c^* = \frac{2(2a_1 + 2a_2 + 2b)c_1}{2(\alpha_1 + 2a_2 + 2b)} \), and \( R_c^* = \frac{2(2a_1 + 2a_2 + 2b)c_1}{2(\alpha_1 + 2a_2 + 2b)} \). For \( c_2 \in (c_2, \check{c}_2) \), the fourth result implies that the optimal two-part tariff contract and the minimum throughput guarantee are those in Case 2(2)-1a with \( r_c^* = \frac{2(2a_1 + 2a_2 + 2b)c_1}{2(\alpha_1 + 2a_2 + 2b)} \), \( \delta_c^* = \frac{2(2a_1 + 2a_2 + 2b)c_1}{2(\alpha_1 + 2a_2 + 2b)} \), \( \gamma_c^* = \frac{2(2a_1 + 2a_2 + 2b)c_1}{2(\alpha_1 + 2a_2 + 2b)} \), \( \nu_c^* = \frac{2(2a_1 + 2a_2 + 2b)c_1}{2(\alpha_1 + 2a_2 + 2b)} \), and \( R_c^* = \frac{2(2a_1 + 2a_2 + 2b)c_1}{2(\alpha_1 + 2a_2 + 2b)} \). For \( c_2 \in (\check{c}_2, \check{c}_2) \), a unique equilibrium exists in Case 2(1)a with \( r_c^* = \frac{1}{2(8a_1^2 + 4a_1^2 + 4b^2 \alpha_2 - 2b^2 \alpha_2 + 2b^2 \alpha_1 + 2b^2)} \).
[(2α_2 - b)(4α_1α_2 + 2bα_1 - 2b^2) - (8α_1α_2^2 - 4b^2α_2 + b^3)c_1 + b^2(2α_1 - b)c_2], \ δ_c^* \in [0, \ (2α_1 - b)(1 - r_2') + bα_1 - 2α_2], \ f_c^* = \frac{α_1}{2} \left[ \frac{(2α_1 - b)(1 - r_2') + bα_1 - 2α_2}{4α_1α_2 - b^2} \right]^2, \ \text{and} \ R_c^* = 2f_c^* + r_c^*.

These prove Lemma 13(iii).

Case (iv): Suppose α_1 > 2α_2. We need to know relative sizes of critical points c_{21}^* \equiv \frac{2(α_1 - α_2)(α_1 + α_2) - (4α_1α_2 + bα_1 + 2b^2 - 2b^2)c_1}{(4α_1α_2 + 2α_1^2 + bα_1 + 2b^2 - 2b^2)} and c_2^* \equiv \frac{1}{(4α_1^2 + 8α_1α_2 - 4bα_1 - 2α_2 + 2b^2)} \left[ (4α_1^2 + 4α_1α_2 - 6bα_1 - 2b^2) + (4α_1α_2 + 2bα_1 + 2bα_2 - 3b^2)c_1 \right] in Case 2(1)a, \ c_{22}^* \equiv \frac{2(α_1 - α_2) + (α_1 + 2bα_1 + 2bα_2 - 2b^2)c_2}{(3α_1 + 2b)} \text{ and } \ c_{22}'' \equiv \frac{(α_1 - 2α_2)(α_1 + α_2)}{(α_1 + b_2)} \text{ in Case 2(2)-a-2. Some calculations yield } c_{22}' > c_{22}^*, \ (c_2^* - c_{21}^*) = \frac{2(α_1 - α_2)(α_2 - b)(1 - c_1)}{(2α_1 + α_2 + 2b)(4α_1α_2 + 2α_1^2 + bα_1 + 2b^2 - 2b^2)} > 0, \ (c_{22}^* - c_{23}^*) = \frac{2(α_1 - α_2)(α_1 - b)(1 - c_1)}{(4α_1α_2 + 2α_1^2 + bα_1 + 2b^2)(3α_1 + 2b)} > 0, \ (c_{23}^* - c_{22}^*) = \frac{2(α_1 - α_2)^2(α_1 - b)(1 - c_1)}{(4α_1α_2 + 2α_1^2 + bα_1 + 2b^2)(3α_1 + 2b)} > 0, \ (c_{23}^* - c_{24}^*) = \frac{2(α_1 - α_2)^2(α_1 - b)(1 - c_1)}{(4α_1α_2 + 2α_1^2 + bα_1 + 2b^2)(3α_1 + 2b)} > 0. \ \text{However, the relative size of } c_{22}'' \text{ and } c_{22}'' \text{ is uncertain, and we are not sure whether } c_{22}'' \text{ is larger than the values of } c_{22}^*, \ c_{22}^*, \ c_{21}^*, \text{ and } c_{22}^*. \ \text{Thus, optimal two-part tariff contracts can be derived by the following steps.}

First, if c_2 \in (c_1, \ c_{21}^*], then equilibria of R_3^* in (A126) of Case 2(2)-a-1 for c_{22}^* < c_{22}^* \text{ in (A135) of Case 3}(1)a, R_3^* \text{ in (A125) of Case 2}(1)b, R_6^* \text{ in (A147) of Case 3}(2)-a-1 and (A156) of Case 3(2)-b-1, and R_7^* \text{ in (A148) of Case 3}(2)-a-2 may exist.

Define M_4 = (R_3^* - R_4^*). Since \ \frac{∂^2 M_4}{∂c_2^2} = \frac{4α_1^2α_2 + 4α_1α_2^2 - 8α_1α_2 - 8bα_1 - 5b^2α_1 - 3b^2α_2}{2(2α_1 + α_2 + 2b)(8α_1^2 + 4α_1α_2 + 16α_2 + 4b^2 - 3b^2α_1 + 2b^2α_2 + 2b^2)} < 0, \ M_4 \text{ is a strictly concave function of } c_2, \ \text{and has the maximum value of } -\frac{(α_1^2 - α_1α_2 - 2α_2)(1 - c_1)^2}{8α_1^2 - 4α_1α_2 - 4α_1α_2 + 8α_1 + 4bα_2 + 3b^2α_1 + 3b^2α_2} < 0 \ \text{if } (2α_2 - α_1) < 0. \ \text{Thus, we have } R_3^* < R_4^* \ \text{for all } c_2.

Next, defining M_5 = (R_6^* - R_4^*). Since \ \frac{∂^2 M_5}{∂c_2^2} = \frac{(2α_1α_2 - bα_1 - b^2c_1)^2}{2(2α_1 + α_2 + 2b)(8α_1^2 + 4α_1α_2 + 16α_2 + 4b^2 - 3b^2α_1 - 2b^2α_2 + 2b^2)} > 0, \ M_5 \text{ is a strictly convex function of } c_2, \ \text{and has the minimum value 0 at } c_2 = \frac{(2α_1α_2 - bα_1 - b^2c_1)}{(2α_1 + α_2 + 2b)} \text{ with } \frac{∂^2 M_5}{∂c_2^2} = \frac{(2α_1α_2 - bα_1 - b^2c_1)}{(2α_1 + α_2 + 2b)} > c_{2c}^*. \ \text{Thus, } R_6^* > R_4^* \ \text{for } c_2 \in (c_1, \ c_{2c}^*]. \ \text{Then, we define } M_6 = (R_6^* - R_2^*). Since \ \frac{∂^2 M_6}{∂c_2^2} = \frac{4α_1α_2 + 2bα_1 + 2bα_2}{2(α_1 - α_2)^2} + \frac{α_1}{2(2α_1 + α_2)} > 0, \ M_6 \text{ is a strictly convex function of } c_2, \ \text{and has the minimum value } \frac{(α_1 + α_2) - 2bα_1 + 2bα_2(1 - c_1)^2}{4α_1^2 + 6α_1α_2 - 4α_1α_2 + 3b^2α_1 + 2b^2α_2 - 2b^2} > 0. \ \text{Thus, we get } R_6^* > R_2^* \ \text{for all } c_2. \ \text{In addition, } \frac{R_6^* - R_7^*}{R_6^*} = \frac{(α_1 + 2α_2 + 2bα_1 + 2bα_2)(1 - c_1)^2}{8α_1 + 2(α_1 - α_2)^2}.
> 0. Therefore, for \( c_2 \in (c_1, c''_2) \), port authority’s optimal two-part tariff contract and the minimum throughput guarantee are those in Case 3(2)-2a-1 and in Case 3(2)-2b-1 with \( r^*_c = \frac{(1-c_2)}{2}, \delta^*_c = \frac{(2a_2-b+c_2-2a_2c_2)}{2(2a_1a_2-b^2)}, f^*_c = \frac{(2a_2-b-2a_2c_1+c_2)(c_2-c_1)}{2(2a_1a_2-b^2)}, \) and \( R^*_c = \frac{(a_1+2a_2-2b)-2(2a_2-b)c_1-2(a_2-b)c_2-2bc_1c_2+2a_2c_1^2+a_1c_2^2}{4(2a_1a_2-b^2)} \).

Second, for \( c_2 \in (c''_2, \bar{c}_c) \), equilibria of \( R^*_c \) in (A124) of Case 2(1)a for \( c_2 \in [c''_2, \bar{c}_c) \), \( R^*_c \) in (A125) of Case 2(1)b for \( c_2 < c''_2 \), \( R^*_c \) in (A126) of Case 2(1)-1a for \( c_2 \in [c''_2, \bar{c}_c) \), \( R^*_c \) in (A135) of Case 3(1)a for \( c_2 < c''_2 \), and \( R^*_c \) in (A157) of Case 3(2)-2b-2 for \( c_2 \in (c''_2, \bar{c}_c) \) may exist. Since \( R^*_c > R^*_1, R^*_c > R^*_2, R^*_c > R^*_3, \) and \( R^*_c > R^*_4 \) are shown by Case (iii), \( R^*_c \) is the best. Thus, port authority’s optimal two-part tariff contract and the minimum throughput guarantee are those in Case 3(2)-2b-2 with \( r^*_c = \frac{[2(a_2+b)+(2a_2+b)c_1-(2a_1+2a_2+bc_2)]}{2(a_1+2a_2+2b)}, f^*_c = \frac{(2-c_1-c_2)[(2a_2-b-1)(a_1+2a_2+4b)c_1+(3a_1+2a_2+4b)c_2]}{8(a_1+2a_2+2b)^2}, \delta^*_c = \frac{(2-c_1-c_2)}{2(a_1+2a_2+2b)}, \) and \( R^*_c = \frac{(2-c_1-c_2)^2}{4(a_1+2a_2+2b)} \).

Third, suppose \( \bar{c}_c < \bar{c}_2 \). Then, for \( c_2 \in [\bar{c}_c, \bar{c}_c) \), the outcomes in Case (iii) show that port authority’s optimal two-part tariff contract and minimum throughput guarantee are those in Case 2(2)-1a with \( r^*_c = \frac{2(a_2+b)-(2a_1+2a_2+3b)c_1+(2a_1+b)c_2}{2(a_1+2a_2+2b)}, \delta^*_c = \frac{(2-c_1-c_2)}{2(a_1+2a_2+2b)}, f^*_c = \frac{(2-c_1-c_2)[(2a_2-b)+2a_2+2b)-(2a_1+2a_2+4b)c_1-(6a_1+2a_2+4b)c_2]}{8(a_1+2a_2+2b)^2}, \) and \( R^*_c = \frac{(2-c_1-c_2)^2}{4(a_1+2a_2+2b)} \).

Fourth, if \( \bar{c}_c < \bar{c}_2 \) and \( c_2 \in [\bar{c}_c, \bar{c}_c) \), a unique equilibrium exists in Case 2(1)a. Thus, port authority’s optimal two-part tariff contract and minimum throughput guarantee are those in Case 2(1)a with \( r^*_c = \frac{1}{2(8a_1a_2^2+4a_1a_2^2-4b_1a_1-2b_1)(a_1+2a_2+2b)^2}(2a_2-b)(4a_1a_2+2b\alpha_1-2b^2)-(8a_1a_2^2-4b_2\alpha_2+b)(2a_1-b)c_2], \delta^*_c \in [0, \frac{(2a_1-a_2-1)(a_1+2a_2+2b)(a_1+2a_2+2b)}{4a_1a_2-2a_1+2a_2}] \), \( f^*_c = \frac{a_2}{2}(2a_2-1)(1-\bar{c}_2)(a_1+2a_2+2b)^2, \) and \( R^*_c = 2f^*_c + r^*_c[a_2(2a_2-b)(1-\bar{c}_2)(a_1+2a_2+2b)(a_1+2a_2+2b)])], \)

By contrast, if \( \bar{c}_c > \bar{c}_2 \), then the outcomes in the third and fourth parts above will change. For \( c_2 \in [\bar{c}_2, \bar{c}_c) \), the optimal contract is the unique solution in Case 2(1)a. These prove Lemma 13(iv). □

**Lemma 14.** Suppose the conditions in (40) hold. Then we have the following.
(i) Suppose \( \alpha_1 \leq \alpha_2 \).

(ii) Suppose \( \alpha_2 < \alpha_1 \leq 2\alpha_2 \).

(iii) If \( c_2 \in (c_1, c_{12}) \) with \( c_{12} = \frac{(2\alpha_1-b)(2\alpha_1+b)}{2(\alpha_1+b)} \), then we have two sub-cases as follows.

(iii-1) Suppose \( \alpha_1 > \frac{2\alpha_2 + 5\alpha_2 + 4b^2}{(2\alpha_2 + b)} \). For \( c_2 \in [c_{11}, c'_{2}] \) with \( c'_{2} = \frac{1}{(2\alpha_1+b)} [(2\alpha_1 + b) - (1-c_1)\sqrt{(2\alpha_1+b)(2\alpha_2+b)}] \), port authority’s optimal unit-fee contract and minimum throughput requirement are \( (r^u_c, \delta^u_c) \) with \( r^u_c = \frac{1-c_1}{2} \) and \( \delta^u_c = \frac{1-c_1}{2(\alpha_1+b)} \). At the equilibrium, operators’ cargo-handling amounts are \( q^u_{c1} = \delta^u_c \) for \( i = 1, 2 \), and port authority’s fee revenue equals \( R^u_c = \frac{(1-c_2)^2}{2(\alpha_2+b)} \).
librium, operators’ cargo-handling amounts are $q_{c1}^u = \delta_c^u$ and $q_{c2}^u = \frac{1-b\delta_c^u-c_2-r_c^u}{2\alpha_2}$, and port authority’s fee revenue equals $R_c^u = \frac{(1-c_2)^2}{2(2\alpha_2+b)}$. By contrast, if $c_2 \in [c'_{c2}, c_{c2}]$, then port authority’s optimal unit-fee contract and minimum throughput requirement are $(r_c^u, \delta_c^u)$ with $r_c^u = \frac{1-c_1}{2}$ and $\delta_c^u = \frac{1-c_1}{2(2\alpha_1+b)}$. At the equilibrium, operators’ cargo-handling amounts are $q_{c1}^u = \frac{1-b\delta_c^u-c_1-r_c^u}{2\alpha_1}$ and $q_{c2}^u = \delta_c^u$, and port authority’s fee revenue equals $R_c^u = \frac{(1-c_1)^2}{2(2\alpha_1+b)}$.

(iib-2) Suppose $\alpha_1 < \frac{2\alpha_2^3+5b\alpha_2+4\alpha_2+b}{(2\alpha_2+b)}$. If $c_2 \in [c'_{c2}, c_{c2}]$, then port authority’s optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(iib-1).

(ii) If $c_2 \in (c'_{c2}, \bar{c}_c]$, then port authority’s optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(ia).

(iid) If $c_2 \in (\bar{c}_c, \bar{c}_c]$, then port authority’s optimal unit-fee contract and minimum throughput requirement are the same as those in Lemma 14(ib).

(iii) If $c_2 \in (\bar{c}_c, \bar{c}_c)$, then port authority’s optimal unit-fee contract and minimum throughput requirement are the same as those in Lemma 14(ic).

(iii) Suppose $(2\alpha_2 - \alpha_1) < 0$.

(iii-a) If $c_2 \in (c_1, \bar{c}_c)$. Then we have two sub-cases.

(iii-a-1) Suppose $(\alpha_1^2 + 6b\alpha_1\alpha_2 - 9b\alpha_2^2 - 7\alpha_1\alpha_2^2 + 8b^2\alpha_2 - 6\alpha_1^2\alpha_2 - b\alpha_2^2) < 0$. For $c_2 \in (c_1, \bar{c}_c]$ with $c_{c2}' = \frac{1}{\alpha_1}[(\alpha_1^2 + 2\alpha_1\alpha_2 - b\alpha_1 + 2b\alpha_2 - 2b^2) - (2\alpha_1\alpha_2 - b\alpha_1 + 2b\alpha_2 - b^2)c_1 - (1 - c_1)\sqrt{2(\alpha_1 + b)(\alpha_1 + 2\alpha_2 - 2b)(2\alpha_1\alpha_2 - b^2)}]$, port authority’s optimal unit-fee contract and minimum throughput requirement are $(r_c^w, \delta_c^w)$ with $r_c^w = \frac{(\alpha_1 + 2\alpha_2 - 2b) - (2\alpha_2 - b)c_1 - (\alpha_1 - b)c_2}{2(\alpha_1 + 2\alpha_2 - 2b)}$ and $\delta_c^w = \frac{(2\alpha_2 - b)(1-r_c^w) + b\alpha_2 - 2c_2c_1}{(2\alpha_1\alpha_2 - b^2)}$. At the equilibrium, operators’ cargo-handling amounts are $q_{c1}^w = \delta_c^w$ and $q_{c2}^w = \frac{1-b\delta_c^w-c_2-r_c^w}{2\alpha_2}$, and port authority’s fee revenue equals $R_c^w = \frac{(\alpha_1 + 2\alpha_2 - 2b) - (2\alpha_2 - b)c_1 - (\alpha_1 - b)c_2}{4(\alpha_1 + 2\alpha_2 - 2b)(2\alpha_1\alpha_2 - b^2)}$. By contrast, for $c_2 \in (c_{c2}', c_{c2}]$, port authority’s optimal unit-fee contract and minimum throughput
requirement are the same as those in Lemma 14(iiia).

(iia-2) Suppose $(\alpha_1^3 + 6\alpha_1\alpha_2 - 9\alpha_2^2 - 7\alpha_1\alpha_2^2 + 8b\alpha_2 - 6\alpha_1^2\alpha_2 - b\alpha_1^2) > 0$. Then, for $c_2 \in (c_1, c_{21}^{(11)})$, port authority’s optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(iiia-1).

(iiib) If $c_2 \in [c_{c_2}^{(11)}, c_{c_2}^{(12)}]$, then the optimal unit-fee contract and minimum throughput requirement depend on relative values of $(1-c_2)^2$, $(1-c_1)^2$ and $\frac{(\alpha_1 + 2\alpha_2 - 2b)(\alpha_1 - b)c_1 - (\alpha_1 - b)c_2}{4(\alpha_1 + 2\alpha_2 - 2b)(\alpha_1\alpha_2 - b^2)}$.

If $(1-c_2)^2$ is the largest, then port authority’s optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(iib-1).

If $(1-c_1)^2$ is the largest, then port authority’s optimal unit-fee contract and minimum throughput requirement are the same as those in the second part of Lemma 14(iib-1).

Finally, if $\frac{(\alpha_1 + 2\alpha_2 - 2b)(\alpha_1 - b)c_1 - (\alpha_1 - b)c_2}{4(\alpha_1 + 2\alpha_2 - 2b)(\alpha_1\alpha_2 - b^2)}$ is the largest, then port authority’s optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(iia-1).

(iiic) If $c_2 \in (c_{c_2}^{(12)}, \tilde{c}_{c_2}]$, then port authority’s optimal unit-fee contract and minimum throughput requirement are the same as those in the first part of Lemma 14(ia).

(iid) If $c_2 \in (\tilde{c}_{c_2}, \bar{c}_{c_2}]$, then port authority’s optimal unit-fee contract and minimum throughput requirement are the same as those in Lemma 14(ib).

(iie) If $c_2 \in (\bar{c}_{c_2}, \bar{c}_{c_2})$, then port authority’s optimal unit-fee contract and minimum throughput requirement are the same as those in Lemma 14(ic).

Proof of Lemma 14: The proofs are similar to those of Lemma 13, and thus omitted.

□

Lemma 15. Suppose the conditions in (40) hold. Then we have the following.

(i) Suppose $\alpha_1 \leq \alpha_2 < 2\alpha_1$. Then we have the following.

(ia) Suppose $(4\alpha_1^2 - 4\alpha_1\alpha_2 + b^2) > 0$. Then there are two sub-cases.
(ia-1) If \( c_2 \in (c_1, c_{c2}^{f1}) \) with \( c_{c2}^{f1} = \frac{(4\alpha_1^2-4\alpha_1\alpha_2+b^2)+(4\alpha_1\alpha_2+2\alpha_1-b^2)c_1}{2a_1(2\alpha_1+b)} \), then port authority’s optimal fixed-fee contract and minimum throughput requirement are \( (f_c^I, \delta_c^I) \) with \( f_c^I = \frac{(1-c_1)(2a_1-a_2)+(a_2+b)c_1-(2a_1+b)c_2}{2(2a_1+b)^2} \) and \( \delta_c^I = \frac{1-c_1}{2a_1+b} \). At the equilibrium, operators’ cargo-handling amounts are \( q_c^I = \delta_c^I \) for \( i = 1, 2 \), and port authority’s fee revenue equals \( R_c^I = \frac{(1-c_1)(2a_1-a_2)+(a_2+b)c_1-(2a_1+b)c_2}{(2a_1+b)^2} \).

(ia-2) If \( c_2 \in (c_{c2}^{f1}, c_{c2}) \) with \( c_{c2} = \frac{(2a_1-b)+bc_1}{2a_1} \), then port authority’s optimal fixed-fee contract and minimum throughput requirement are \( (f_c^I, \delta_c^I) \) with \( f_c^I = \frac{[2(2a_1-b)+bc_1-2a_1c_2]^2}{16a_1(2a_1a_2-b^2)} \) and \( \delta_c^I = \frac{(2a_1-b)+bc_1-2a_1c_2}{2(2a_1a_2-b^2)} \). At the equilibrium, operators’ cargo-handling amounts are \( q_c^I = \frac{1-c_1-b\delta_c^I}{2a_1} \) and \( q_c^F = \delta_c^I \), and port authority’s fee revenue equals \( R_c^I = \frac{[(2a_1-b)+bc_1-2a_1c_2]^2}{8a_1(2a_1a_2-b^2)} \).

(ii) Suppose \( (2\alpha_1 - \alpha_2) \leq 0 \). Then, for \( c_2 \in (c_1, c_{c2}) \) with \( c_{c2} = \frac{(2a_1-b)+bc_1}{2a_1} \), port authority’s optimal fixed-fee contract and minimum throughput requirement are \( (f_c^I, \delta_c^I) \) with \( f_c^I = \frac{[2(2a_1-b)+bc_1-2a_1c_2]^2}{16a_1(2a_1a_2-b^2)} \) and \( \delta_c^I = \frac{(2a_1-b)+bc_1-2a_1c_2}{2(2a_1a_2-b^2)} \). At the equilibrium, operators’ cargo-handling amounts are \( q_c^I = \frac{1-c_1-b\delta_c^I}{2a_1} \) and \( q_c^F = \delta_c^I \), and port authority’s fee revenue equals \( R_c^I = \frac{[(2a_1-b)+bc_1-2a_1c_2]^2}{8a_1(2a_1a_2-b^2)} \).

(iii) Suppose \( \alpha_2 < \alpha_1 \leq 2\alpha_2 \). Then there are four sub-cases as follows.

(iiiia) If \( (4\alpha_1\alpha_2-4\alpha_2^2-b^2) > 0 \) and \( c_2 \in (c_1, c_{c2}^{f4}) \) with \( c_{c2}^{f4} = \frac{(4\alpha_1\alpha_2-4\alpha_2^2-b^2)+2a_2(2a_2+b)c_1}{(4\alpha_1\alpha_2+2a_2-b^2)} \), then port authority’s optimal fixed-fee contract and minimum throughput requirement are \( (f_c^I, \delta_c^I) \) with \( f_c^I = \frac{[2(2a_2-b)+2a_2c_1+bc_2]^2}{16a_2(2a_1a_2-b^2)} \) and \( \delta_c^I = \frac{(2a_2-b)+2a_2c_1+bc_2}{2(2a_1a_2-b^2)} \). At the equilibrium, operators’ cargo-handling amounts are \( q_c^I = \delta_c^I \) and \( q_c^F = \frac{1-c_2-b\delta_c^I}{2a_2} \), and port authority’s fee revenue equals \( R_c^I = \frac{[(2a_2-b)-2a_2c_1+bc_2]^2}{8a_2(2a_1a_2-b^2)} \).
(iiib) If $c_2 \in (\max\{c_1, c_{c2}^{f1}\}, c_{c2}^{f3})$ with $c_{c2}^{f3} = \frac{2(a_1-a_2)+(2a_2+b)c_1}{(2a_1+b)}$, then port authority’s optimal fixed-fee contract and minimum throughput requirement are $(f_c^f, \delta_c^f)$ with $f_c^f = (1-c_2)(2a_2-a_1)-(2a_2+b)c_1+(a_1+b)c_2 \left/ \frac{2(2a_1+b)^2}{2} \right.$ and $\delta_c^f = \frac{1-c_2}{2a_2+b}$. At the equilibrium, operators’ cargo-handling amounts are $q_{c1}^f = q_{c2}^f = \delta_c^f$, and port authority’s fee revenue equals $R_c^f = (1-c_2)(2a_2-a_1)-(2a_2+b)c_1+(a_1+b)c_2 \left/ \frac{2(2a_2+b)^2}{2} \right.$.

(iic) If $c_2 \in [c_{c2}^{f3}, c_{c2}^{f1})$ with $c_{c2}^{f1} = \frac{(4a_1^2-4a_1a_2+b^2)+(4a_1a_2+2a_1b^2)c_1}{2a_1(2a_1+b)}$, then port authority’s optimal fixed-fee contract and minimum throughput requirement are $(f_c^f, \delta_c^f)$ with $f_c^f = (1-c_2)(2a_2-a_1)+(a_2+b)c_1-(2a_1+b)c_2 \left/ \frac{2(2a_1+b)^2}{2} \right.$ and $\delta_c^f = \frac{1-c_2}{2a_2+b}$. At the equilibrium, operators’ cargo-handling amounts are $q_{c1}^f = q_{c2}^f = \delta_c^f$, and port authority’s fee revenue equals $R_c^f = \frac{[(2a_1-b)+bc_1-2a_1c_2]^2}{16a_1(2a_1a_2-b^2)}$.

(iid) If $c_2 \in [c_{c2}^{f1}, c_{c2}^{-})$, then port authority’s optimal fixed-fee contract and minimum throughput requirement are $(f_c^f, \delta_c^f)$ with $f_c^f = \frac{[(2a_1-b)+bc_1-2a_1c_2]^2}{16a_1(2a_1a_2-b^2)}$ and $\delta_c^f = \frac{(2a_1-b)+bc_1-2a_1c_2}{2(2a_1a_2-b^2)}$. At the equilibrium, operators’ cargo-handling amounts are $q_{c1}^f = \delta_c^f$ and $q_{c2}^f = \frac{1-c_2-b^2}{2a_2}$, and port authority’s fee revenue equals $R_c^f = \frac{[(2a_1-b)+bc_1-2a_1c_2]^2}{8a_1(2a_1a_2-b^2)}$.

(iv) Suppose $(2a_2-a_1) < 0$. Then there are four sub-cases as follows.

(iva) If $(4a_1a_2-4a_2^2-b^2) > 0$ and $c_2 \in (c_1, c_{c2}^{f4})$ with $c_{c2}^{f4} = \frac{(4a_1a_2-4a_2^2-b^2)+2a_2(2a_2+b)c_1}{(4a_1a_2+2ba_2-b^2)}$, then port authority’s optimal fixed-fee contract and minimum throughput requirement are $(f_c^f, \delta_c^f)$ with $f_c^f = \frac{[(2a_1-b)+bc_1-c_2]^2}{16a_1(2a_1a_2-b^2)}$ and $\delta_c^f = \frac{2a_1-b+c_1+bc_2}{2(2a_1a_2-b^2)}$. At the equilibrium, operators’ cargo-handling amounts are $q_{c1}^f = \delta_c^f$ and $q_{c2}^f = \frac{1-c_2-b^2}{2a_2}$, and port authority’s fee revenue equals $R_c^f = \frac{[(2a_1-b)+bc_1-2a_1c_2]^2}{8a_1(2a_1a_2-b^2)}$.

(ivb) If $c_2 \in (c_{c2}^{f4}, c_{c2}^{f3})$ with $c_{c2}^{f3} = \frac{2(a_1-a_2)+(2a_2+b)c_1}{(2a_1+b)}$, then port authority’s optimal fixed-fee contract and minimum throughput requirement are $(f_c^f, \delta_c^f)$ with $f_c^f = \frac{(1-c_2)(2a_2-a_1)-(2a_2+b)c_1+(a_1+b)c_2}{2(2a_2+b)^2}$ and $\delta_c^f = \frac{1-c_2}{2a_2+b}$. At the equilibrium, operators’ cargo-handling amounts are $q_{c1}^f = q_{c2}^f = \delta_c^f$, and port authority’s fee revenue equals $R_c^f = \frac{(1-c_2)(2a_2-a_1)-(2a_2+b)c_1+(a_1+b)c_2}{(2a_2+b)^2}$.

(ivc) If $c_2 \in [c_{c2}^{f3}, c_{c2}^{f1})$ with $c_{c2}^{f1} = \frac{(4a_1^2-4a_1a_2+b^2)+(4a_1a_2+2a_1b^2)c_1}{2a_1(2a_1+b)}$, then port authority’s
optimal fixed-fee contract and minimum throughput requirement are \( (f_c^f, \delta_c^f) \) with \( f_c^f = \frac{(1-c_1)(2a_1-a_2)+a_2a_1c_1-a_1b}{2a_1(a_2-b^2)} \) and \( \delta_c^f = \frac{1-c_1}{a_2} \). At the equilibrium, operators’ cargo-handling amounts are \( q_c^f = q_c^f = \delta_c^f \), and port authority’s fee revenue equals \( R_c^f = 2f_c^f \).

**(ivd)** If \( c_2 \in [\bar{c}_c^f, \tilde{c}_c^f] \), then port authority’s optimal fixed-fee contract and minimum throughput requirement are \( (f_c^f, \delta_c^f) \) with \( f_c^f = \frac{(2a_1-b)+bc_1-2a_1c_2}{16a_1(2a_1a_2-b^2)} \) and \( \delta_c^f = \frac{(2a_1-b)+bc_1-2a_1c_2}{2(2a_1a_2-b^2)} \). At the equilibrium, operators’ cargo-handling amounts are \( q_c^f = \frac{1-c_1-bc_2}{2a_1} \) and \( q_c^f = \delta_c^f \), and port authority’s fee revenue equals \( R_c^f = \frac{(2a_1-b)+bc_1-2a_1c_2}{8a_1(2a_1a_2-b^2)} \).

**Proof of Lemma 15:** The proofs are similar to those of Lemma 13, and thus omitted.

\[ \square \]

**Proof of Proposition 4:** Note that \( \max\{c_1, c_2^f\} < \bar{c}_c \) and \( \bar{c}_2 < \bar{c}_c < c_2 \) In each of the following four cases, we will first compare the two-part tariff scheme with the unit-fee scheme, and then compare the better of these two with the fixed-fee scheme.

**Case 1:** Suppose \( \alpha_1 \leq \alpha_2 < 2a_1 \).

**Comparing the two-part tariff scheme with the unit-fee scheme:**

First, for \( c_2 \in (c_1, \bar{c}_2) \), we have \( R_c^u = \frac{(2-c_1-c_2)^2}{4(2a_1+a_2+2b)} \) by Lemma 13 (ia), and \( R_c^u = \frac{(1-c_2)^2}{2(a_2+b)} \) by Lemma 14 (ia). Define \( H_1 = (R_c^u - R_c^u) \). Since \( \frac{\partial^2 H_1}{\partial c_2^2} = \frac{(4a_1+a_2+3b)}{2(a_2+b)(2a_1+a_2+2b)} > 0 \), we have \( \frac{\partial H_1}{\partial c_2} = \frac{-2(2a_1+b)-a_2+b_2c_1+a_2a_2+3b^2c_2}{2(a_2+b)(2a_1+a_2+2b)} < 0 \) and\( H_1 > \frac{(1-c_2)^2}{2(a_2+b)} - \frac{(1-c_2)^2}{4(2a_1+a_2+2b)} = \frac{(2a_1-a_2)^2(1-c_2)^2}{2(2a_1+a_2+2b)} > 0 \). They imply \( R_c^u > R_c^u \).

Second, for \( c_2 \in [\bar{c}_2, \tilde{c}_2) \), we have \( R_c^u = 2f_c^u + r_c^u \left[ \frac{(2a_1+a_2+b)(1-r_c^u)-(2a_1-b)\tilde{c}_1-(2a_1-b)\tilde{c}_2}{4a_1a_2-b^2} \right] \) by Lemma 13(ib), and \( R_c^u = \frac{(1-c_2)^2}{2(a_2+b)} \) by Lemma 14(ia), and \( R_c^u = \frac{(2a_1+a_2-2b)-(a_2-b)\tilde{c}_1-(2a_1-b)\tilde{c}_2}{4(2a_1+a_2-2b)(2a_1a_2-b^2)} \) by Lemma 14(ib) with \( \left( \frac{(1-c_2)^2}{2(a_2+b)} \right) \geq \left( \frac{(2a_1+a_2-2b)-(a_2-b)\tilde{c}_1-(2a_1-b)\tilde{c}_2}{4(2a_1+a_2-2b)(2a_1a_2-b^2)} \right) \) iff \( c_2 \leq (>) \bar{c}_2 \).

Define \( H_2 = \frac{(2a_1+a_2-2b)-(a_2-b)\tilde{c}_1-(2a_1-b)\tilde{c}_2}{4(2a_1+a_2-2b)(2a_1a_2-b^2)} - R_c^u \). Since \( \frac{\partial^2 H_2}{\partial c_2^2} = \frac{-4a_2a_1a_2-4a_2^2a_1-a_2b^2a_2+4b^2a_2^2+7b^2a_1a_2-4b^2a_1-b}{(2a_1+a_2-2b)(2a_1a_2-b^2)(8a_1a_2^2+4a_1^2a_2-4b^2a_1-3b^2a_2+2b^2)} > 0 \) and \( \frac{\partial H_2}{\partial c_2} = \frac{-b^2a_2(2a_1-2b)(1-c_2)}{2(2a_1a_2-b^2)(8a_1a_2^2+4a_1^2a_2-4b^2a_1-3b^2a_2+2b^2)} < 0 \) at \( c_2 = c_1 \), we have \( \frac{\partial H_2}{\partial c_2} < 0 \) for
\( c_2 \in [\hat{c}_2, \tilde{c}_2] \). Moreover, we have \( H_2 = \frac{\alpha_2^2(2\alpha_1-b)^4(1-c_1)^2}{4(2\alpha_1+2\alpha_2-2b)(2\alpha_1+\alpha_2-b)^2(2\alpha_1+\alpha_2-b)^2} > 0 \) at \( c_2 = \hat{c}_2 \). These imply \( \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1+\alpha_2-b)^2} > R^*_c \) for \( c_2 \in [\hat{c}_2, \tilde{c}_2] \). Thus, we get \( R^*_c = \frac{(1-c_2)^2}{2(\alpha_2+b)} > R^*_c \) for \( c_2 \in [\hat{c}_2, \tilde{c}_2] \), and \( R^*_c = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b)^2} > R^*_c \) for \( c_2 \in [\hat{c}_2, \tilde{c}_2] \).

Third, for \( c_2 \in [\hat{c}_2, \tilde{c}_2] \), no equilibrium two-part tariff scheme exists. Thus, the unit-fee scheme is the better one.

**Comparing the better scheme above with the fixed-fee scheme:**

First, for \( c_2 \in (c_1, f_2^{(1)}) \), we have \( R^u_c = \frac{(1-c_2)^2}{2(\alpha_2+b)} \) by Lemma 14(ia), and \( R^f_c = \frac{(1-c_2)^2}{2(\alpha_2+b)} \) by Lemma 15(ia). Define \( H_3 = (R^u_c - R^f_c) \). Since
\[
\frac{\partial H_3}{\partial c_2} = -\frac{2(\alpha_1+\alpha_2)(\alpha_2-b)c_1-c_2(2\alpha_1+\alpha_2+b)(2\alpha_1+\alpha_2-b)}{(2\alpha_1+\alpha_2+b)(2\alpha_1+\alpha_2-b)}
\]
we can obtain \( \frac{\partial H_3}{\partial c_2} < 0 \) for \( c_2 \in (c_1, f_2^{(1)}) \). Define \( H_4 = \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b)^2} \). Since \( \frac{\partial H_4}{\partial c_2} = \frac{-(c_2-c_1)}{2(\alpha_2+b)} \), we have \( H_4 > 0 \), and hence \( \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b)^2} > \frac{[(2\alpha_1-b)+b_1-(2\alpha_1-b)c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b)^2} \) for \( c_2 \in (c_1, f_2^{(1)}) \).

Moreover, \( \frac{(1-c_2)^2}{2(\alpha_2+b)} > \frac{(1-c_2)^2}{2(\alpha_2+b)} \) for \( c_2 \in (c_1, f_2^{(1)}) \) if \( c_2 \in (c_1, f_2^{(1)}) \). Thus, we have
\[
\frac{(1-c_2)^2}{2(\alpha_2+b)} > \frac{(1-c_2)^2}{2(\alpha_2+b)} \quad \text{for } c_2 \in [\hat{c}_2, \tilde{c}_2], \quad \text{and} \quad \frac{[(2\alpha_1+\alpha_2-2b)-(\alpha_2-b)c_1-(2\alpha_1-b)c_2]^2}{4(2\alpha_1+\alpha_2-2b)(2\alpha_1\alpha_2-b)^2} > \frac{[(2\alpha_1-b)+b_1-(2\alpha_1-b)c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b)^2} \quad \text{for } c_2 \in (\hat{c}_2, \tilde{c}_2).\]
These suggest that the unit-fee scheme is optimal for \( c_2 \in [\hat{c}_2, \tilde{c}_2] \).

Third, for \( c_2 \in (\hat{c}_2, \tilde{c}_2) \), we have \( R^u_c = \frac{(c_2-c_1)[2\alpha_1-b+b_1-2\alpha_1c_2]}{(2\alpha_1-b)^2} \) by Lemma 14(ic), and \( R^f_c = \frac{[(2\alpha_1-b)+b_1-(2\alpha_1-b)c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b)^2} \) by Lemma 15(ib). Define \( H_5 = \frac{(c_2-c_1)[2\alpha_1-b+b_1-2\alpha_1c_2]}{(2\alpha_1-b)^2} - \frac{[(2\alpha_1-b)+b_1-(2\alpha_1-b)c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b)^2} \). Since \( \frac{\partial^2 H_5}{\partial c_2^2} = -\frac{a_1(4a_1^2+4\alpha_1b_1-3b^2)}{(2\alpha_1-b)^4(2\alpha_1\alpha_2-b)^2} < 0 \) and \( \frac{\partial H_5}{\partial c_2} = \frac{[-(2\alpha_1-b)(1-c_1)]}{2(4a_1^2+4\alpha_1b_1-3b^2)} \) < 0 at \( c_2 = \hat{c}_2 \), we have \( \frac{\partial H_5}{\partial c_2} < 0 \) for \( c_2 \in (\hat{c}_2, \tilde{c}_2) \), and \( H_5 > \frac{(c_2-c_1)[2\alpha_1-b+b_1-2\alpha_1c_2]}{(2\alpha_1-b)^2} - \frac{[(2\alpha_1-b)+b_1-(2\alpha_1-b)c_2]^2}{8\alpha_1(2\alpha_1\alpha_2-b)^2} \).
Thus, there are six intervals to be discussed.

Case 2: Suppose \((2\alpha_1 - \alpha_2) \leq 0\). As in Case 1, we can show that the unit-fee scheme is port authority’s best choice.

Case 3: Suppose \(\alpha_2 < \alpha_1 \leq 2\alpha_2\).

Comparing the two-part tariff scheme with the unit-fee scheme:

Lemma 14(ii) shows that \(R'^u_c = \frac{(1-c_2^1)^2}{2(\alpha_1 + \beta)}\) for \(c_2 \in (c_1, c_{21})\), \(R'^u_c = \max\{\frac{(1-c_2^1)^2}{2(\alpha_2 + \beta)}, \frac{(1-c_1)^2}{2(\alpha_1 + \beta)}\}\) for \(c_2 \in [c_{21}, c_{22}]\), \(R'^u_c = \frac{(1-c_2^1)^2}{2(\alpha_2 + \beta)}\) for \(c_2 \in (c_{22}, \bar{c}_2]\), \(R'^u_c = \frac{[(2\alpha_1 + \alpha_2 - 2\alpha_2)(\alpha_1 - (2\alpha_2)(\alpha_2))^2]}{4(2\alpha_1 + \alpha_2 - 2\alpha_2)(\alpha_1 + \alpha_2 - 2\alpha_2)^2}\) for \(c_2 \in (\bar{c}_2, \bar{c}_2]\). On the other hand, Lemma 13(iii) shows that \(R'^s_c = \frac{(2-c_2-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)}\) for \(c_2 \in (c_1, \bar{c}_2]\), \(R'^s_c = \frac{(2-c_2-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)}\) for \(c_2 \in [\bar{c}_2, \bar{c}_2]\), and \(R'^s_c = 2f'^c + r'^s_c\).

By some calculations, we have \(c_{21}^{u1} < c_{22}^{u1} < c_{22} < \bar{c}_2 < \bar{c}_2 < \bar{c}_2 < \bar{c}_2\). Thus, there are six intervals to be discussed.

First, for \(c_2 \in (c_1, c_{21}]\), we have \(R'^s_c = \frac{(2-c_2-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)}\) and \(R'^u_c = \frac{(1-c_2^1)^2}{2(\alpha_1 + \beta)}\). Define \(H_6 = (R'^u_c - R'^s_c) = \frac{(1-c_2^1)^2}{2(\alpha_1 + \beta)} - \frac{(2-c_2-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)}\). Since \(\frac{\partial H_6}{\partial c_2} = \frac{(2-c_2-c_2^1)}{2(\alpha_1 + 2\alpha_2 + 2\beta)} > 0\), we have \(H_6 > \frac{(1-c_2^1)^2}{2(\alpha_1 + \beta)} - \frac{(2-c_2-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)} > 0\). Thus, the unit-fee scheme is better than the two-part tariff scheme.

Second, for \(c_2 \in [c_{21}, c_{22}]\), we have \(R'^u_c = \max\{\frac{(1-c_2^1)^2}{2(\alpha_1 + \beta)}, \frac{(1-c_1)^2}{2(\alpha_1 + \beta)}\}\) and \(R'^s_c = \frac{(2-c_2-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)}\). Define \(H_7 = \frac{(2-c_2-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)} - \frac{(1-c_2^1)^2}{2(\alpha_1 + \beta)}\). Since \(\frac{\partial^2 H_7}{\partial c_2^2} = \frac{-(2\alpha_2 + \alpha_1)}{2(\alpha_1 + 2\alpha_2 + 2\beta)} < 0\) and \(\frac{\partial H_7}{\partial c_2} = \frac{(2\alpha_1 + 2\alpha_2 + 3\alpha_2 + 3\alpha_2 - (1-c_2^1)\alpha_1 + 2\alpha_2 + 2\beta)}{2(\alpha_1 + 2\alpha_2 + 2\beta)(\alpha_1 + 2\alpha_2 + 2\beta)} > 0\) at \(c_2 = c_{22}^{u2}\), we have \(\frac{\partial^2 H_7}{\partial c_2^2} > 0\) for \(c_2 \in [c_{21}, c_{22}]\), and \(H_7 > \frac{(2-c_2-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)} - \frac{(1-c_2^1)^2}{2(\alpha_1 + \beta)} = \frac{1}{16(\alpha_1 + 2\alpha_2 + 2\beta)(\alpha_1 + 2\alpha_2 + 2\beta)}[(2\alpha_2^2 + 2\alpha_1^2 + 2\alpha_1^2 + 2\alpha_1^2 + 3\alpha_1^2 - 3\alpha_1^2 + 8\alpha_2^2\alpha_2)(1-c_2^1)] > 0\), which implies \(\frac{(2-c_2-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)} > \frac{(1-c_2^1)^2}{2(\alpha_1 + \beta)}\). On the other hand, define \(H_8 = \frac{(2-c_2-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)} - \frac{(1-c_1)^2}{2(\alpha_1 + \beta)}\). Since \(\frac{\partial H_8}{\partial c_2} = \frac{(2-c_2-c_2^1)}{2(\alpha_1 + 2\alpha_2 + 2\beta)} < 0\), we have \(H_8 > \frac{(2-c_2-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)} - \frac{(1-c_1)^2}{2(\alpha_1 + \beta)} = \frac{(4\alpha_1^2 + 16\alpha_1^2 + 5\alpha_1^2 - 8\alpha_2^2 - 16\alpha_1^2 + 8\alpha_1^2 + 22\alpha_2\alpha_2 - 3\alpha_1^2)(1-c_2^1)^2}{4(\alpha_1 + 2\alpha_2 + 2\beta)(\alpha_1 + 2\alpha_2 + 2\beta)^2} > 0\). Thus, the two-part tariff scheme
is better than the unit-fee scheme for $c_2 \in [c_{c_2}^{u11}, c_{c_2}^{u12}]$.

Third, for $c_2 \in (c_{c_2}^{u12}, \bar{c}_{c_2})$, we have $R^u_c = \frac{(1-c_2)^2}{2(\alpha_2+b)}$ and $R^*_c = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$. Define $H_9 = (R^u_c - R^*_c) = \frac{(1-c_2)^2}{2(\alpha_2+b)} - \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$. Since $\frac{\partial^2 H_9}{\partial c_2^2} = \frac{(2-b)(1-c_1)}{(\alpha_2+b)(2\alpha_1+\alpha_2+2b)}>0$ and $\frac{\partial H_9}{\partial c_2} = \frac{-2(2+b)(1-c_1)}{(\alpha_2+b)(2\alpha_1+\alpha_2+2b)} < 0$ at $c_2 = \bar{c}_{c_2}$, we have $\frac{\partial H_9}{\partial c_2} < 0$ for $c_2 \in (c_{c_2}^{u12}, \bar{c}_{c_2})$, and $H_9 > \frac{(1-\bar{c}_{c_2})^2}{2(\alpha_2+b)} - \frac{(2-c_{c_2}^{u12}-\bar{c}_{c_2})^2}{4(\alpha_1+2\alpha_2+2b)} = \frac{5(\alpha_2 - 2\alpha_1 \alpha_2 - 2\alpha_1 + 4b \alpha_2)}{2(\alpha_2+b)(2\alpha_1+\alpha_2+2b)^2} > 0$. Thus, the unit-fee scheme is better than the two-part tariff scheme for $c_2 \in (c_{c_2}^{u12}, \bar{c}_{c_2})$.

Fourth, for $c_2 \in [\bar{c}_{c_2}, \bar{c}_{c_2})$, we have $R^u_c = \frac{(1-c_2)^2}{2(\alpha_2+b)}$ and $R^*_c = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$. As in Case 1, we have $R^u_c = \frac{(1-c_2)^2}{2(\alpha_2+b)} > R^*_c = \frac{(2-c_1-c_2)^2}{4(2\alpha_1+\alpha_2+2b)}$.

Fifth, for $c_2 \in [\bar{c}_{c_2}, \bar{c}_{c_2})$, the unit-fee scheme is better than the two-part tariff scheme as shown in Case 1.

Sixth, for $c_2 \in [\bar{c}_{c_2}, \bar{c}_{c_2})$, there exists no equilibrium two-part tariff scheme. Thus, the unit-fee scheme is better.

The outcomes of the six parts above are summarized below. The unit-fee scheme is better than the two-part tariff scheme for $c_2 \in (c_1, c_{c_2}^{u11})$ with $R^u_c = \frac{(1-c_2)^2}{2(\alpha_2+b)}$. The two-part tariff scheme is better than the unit-fee scheme for $c_2 \in [c_{c_2}^{u11}, c_{c_2}^{u12}]$ with $R^*_c = \frac{(2-c_1-c_2)^2}{4(\alpha_1+2\alpha_2+2b)}$. The unit-fee scheme is better than the two-part tariff scheme for $c_2 \in (c_{c_2}^{u12}, \bar{c}_{c_2})$ with $R^u_c = \frac{(1-c_2)^2}{2(\alpha_2+b)}$. The unit-fee scheme is better than the two-part tariff scheme for $c_2 \in (\bar{c}_{c_2}, \bar{c}_{c_2})$ with $R^u_c = \frac{(2-c_{c_2}^{u12}-\bar{c}_{c_2})^2}{4(\alpha_1+2\alpha_2+2b)}$. Finally, the unit-fee scheme is better than the two-part tariff scheme for $c_2 \in (\bar{c}_{c_2}, \bar{c}_{c_2})$ with $R^u_c = \frac{(c_2-c_1)(2\alpha_1-b)+bc_1-2\alpha_1c_2}{(2\alpha_1-b)^2}$.

Comparing the better scheme above with the fixed-fee scheme:

Lemma 15(iii) shows $R^f_c = \frac{[(2\alpha_3-b)-2\alpha_2c_1+bc_2]^2}{8\alpha_2^2(2\alpha_1+\alpha_2-b^2)}$ for $c_2 \in (c_1, c_{c_2}^{f1})$ iff $c_{c_2}^{f4} > c_1$, $R^f_c = \frac{(1-c_2)(2\alpha_3-\alpha_1)-(2\alpha_3+b)c_1+(\alpha_1+b)c_2}{(2\alpha_3+b)^2}$ for $c_2 \in (c_{c_2}^{f1}, c_{c_2}^{f3})$, $R^f_c = \frac{(1-c_2)(2\alpha_3-\alpha_1)+(\alpha_1+b)c_1-(2\alpha_1+b)c_2}{(2\alpha_1+b)^2}$ for $c_2 \in [c_{c_2}^{f3}, c_{c_2}^{f1})$, and $R^f_c = \frac{[(2\alpha_1-b)+bc_1-2\alpha_1c_2]^2}{8\alpha_1(2\alpha_1+\alpha_2-b^2)}$ for $c_2 \in [c_{c_2}^{f1}, \bar{c}_{c_2})$. 

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Some calculations show $\min\{c_{c_2}^{u11}, c_{c_2}^{f1}\} < \max\{c_{c_2}^{f4}, c_{c_2}^{b12}\} < c_{c_2}^{f3} < c_{c_2}^{f1} < \ddot{c}_c < c_{c_2}$. Thus, we need to discuss the following five intervals.

First, for $c_2 \in (c_1, c_{c_2}^{u11})$, one equilibrium unit-fee scheme exists with $R_u^{c} = \frac{(1-c_2)^2}{2(a_1+b)}$ and two equilibrium fixed-fee schemes may exist with $R_f^{c} = \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)}$ or $R_f^{c} = \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)}$. Define $H_{10} = \frac{(1-c_2)^2}{2(a_1+b)} - \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)}$. Since

$$\frac{\partial H_{10}}{\partial c_2} = \frac{-b[(2a_2-b)-2a_2c_1+c_2b_1]}{4a_2(2a_1a_2-b^2)} < 0,$$

we have $H_{10} = \frac{(1-c_2)^2}{2(a_1+b)} - \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)} > \frac{(1-c_2)^2}{2(a_1+b)}$. Since

$$\frac{\partial H_{10}}{\partial c_2} = \frac{-b[(2a_2-b)-2a_2c_1+c_2b_1]}{4a_2(2a_1a_2-b^2)} > 0,$$

we have $H_{10} < \frac{(1-c_2)^2}{2(a_1+b)} - \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)} < (1-c_2) < 0$. Thus, for $c_2 \in (c_1, c_{c_2}^{u11})$, the unit-fee scheme is better than the fixed-fee scheme, and thus the optimal contract.

Second, for $c_2 \in [c_{c_2}^{u11}, c_{c_2}^{u12}]$, one equilibrium two-part tariff scheme exists with $R^*_c = \frac{(2-c_2)^2}{4(a_1+2a_2+2b)}$ and two equilibrium fixed-fee schemes may exist with $R_f^{c} = \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)}$. Since

$$\frac{\partial H_{12}}{\partial c_2} = \frac{1}{8a_2(2a_1a_2-b^2)} - \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)} \cdot \frac{1}{8a_2(2a_1a_2-b^2)}\left[-(2a_2+b)(6a_2^2\alpha_2 + 2a_1\alpha_2^2 + 11b_1\alpha_2 - b_2\alpha_2^2 + 4b_2^2\alpha_2 - 2b_1\alpha_2 - 4b^2)(1-c_2)\right] < 0$$

at $c_2 = c_{c_2}^{u12}$, and

$$\frac{\partial H_{12}}{\partial c_2} = \frac{-(2a_2+b)(6a_2^2\alpha_2 + 2a_1\alpha_2^2 + 11b_1\alpha_2 - b_2\alpha_2^2 + 4b_2^2\alpha_2 - 2b_1\alpha_2 - 4b^2)(1-c_2)}{8a_2(2a_1a_2-b^2)} < 0$$

at $c_2 = c_{c_2}^{u12}$, we have $H_{12} > \frac{(2-c_2)^2}{4(a_1+2a_2+2b)}$ or $R_f^{c} = \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)}$. Define $H_{12} = \frac{(2-c_2)^2}{4(a_1+2a_2+2b)} - \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)}$. Since

$$\frac{\partial H_{12}}{\partial c_2} = \frac{-b[(2a_2-b)-2a_2c_1+c_2b_1]}{4a_2(2a_1a_2-b^2)} > 0,$$

we have $H_{12} > \frac{(2-c_2)^2}{4(a_1+2a_2+2b)} - \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)}$ or $R_f^{c} = \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)}$. Since

$$\frac{\partial H_{12}}{\partial c_2} = \frac{-b[(2a_2-b)-2a_2c_1+c_2b_1]}{4a_2(2a_1a_2-b^2)} < 0,$$

we have $H_{12} < \frac{(2-c_2)^2}{4(a_1+2a_2+2b)} - \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)}$. Moreover, since

$$\frac{\partial H_{12}}{\partial c_2} = \frac{-b[(2a_2-b)-2a_2c_1+c_2b_1]}{4a_2(2a_1a_2-b^2)} < 0,$$

we have $H_{12} < \frac{(2-c_2)^2}{4(a_1+2a_2+2b)} - \frac{[(2a_2-b)-2a_2c_1+c_2b_1]^2}{8a_2(2a_1a_2-b^2)}$. Thus, for $c_2 \in [c_{c_2}^{u11}, c_{c_2}^{u12}]$, the two-part tariff scheme is optimal.

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Third, for \( c_2 \in (c_{c_2}^{12}, \bar{c}_{c_2}] \), one equilibrium unit-fee scheme exists with \( R_c^u = \frac{(1-c_2)^2}{2(\alpha_2+b)} \), and four equilibrium fixed-fee schemes may exist with \( R_c^f = \frac{[(2\alpha_2-b)-2\alpha_2c_2-c_1(2\alpha_1)\alpha_2]}{8(\alpha_2)+(\alpha_2+b)^2} \), \( R_c^f = \frac{(1-c_2)(2\alpha_2+c_1+c_2)-(2\alpha_2+c_2)}{(\alpha_2+b)^2} \), or \( R_c^f = \frac{(1-c_2)(2\alpha_2+c_1+c_2)-(2\alpha_2+c_2)}{(\alpha_2+b)^2} \).

Define \( H_{13} = \frac{(1-c_2)^2}{2(\alpha_2+b)} - \frac{[(2\alpha_2-b)-2\alpha_2c_1+c_2]}{(\alpha_2+b)^2} \). Since \( \frac{\partial^2 H_{13}}{\partial c_2^2} = \frac{8(\alpha_2)^4-5b^2(\alpha_2-b)^2}{4(\alpha_2+b)} > 0 \) and \( \frac{\partial H_{13}}{\partial c_2} = \frac{-(4\alpha_2^2+3b\alpha_2+b)(1-c_1)}{8(\alpha_2+b)(4(\alpha_2+b)^2)} < 0 \) at \( c_2 = c_{c_2}^f \), we have \( \frac{\partial H_{13}}{\partial c_2} < 0 \) for \( c_2 \leq c_{c_2}^f \); and \( H_{13} = \frac{(1-c_2)^2}{2(\alpha_2+b)} - \frac{[(2\alpha_2-b)-2\alpha_2c_1+c_2]}{(\alpha_2+b)^2} > 0 \). Moreover, we have \( \frac{(1-c_2)^2}{2(\alpha_2+b)} > 0 \), \( \frac{(1-c_2^1)^2}{2(\alpha_2+b)} > 0 \), \( \frac{(1-c_2^3)^2}{2(\alpha_2+b)} > 0 \), and \( \frac{(1-c_2)^2}{2(\alpha_2+b)} > 0 \) for \( c_2 \in (c_1, c_{c_2}^1) \), \( c_2 \in (c_{c_2}^1, c_{c_2}^3) \), and \( c_2 \in (c_{c_2}^3, \bar{c}_{c_2}] \), respectively, by the outcomes of Case 1. Thus, for \( c_2 \in (c_{c_2}^{12}, \bar{c}_{c_2}] \), the unit-fee scheme is optimal.

Fourth, for \( c_2 \in (\bar{c}_{c_2}, \bar{c}_{c_2}] \), one equilibrium unit-fee scheme exists with \( R_c^u = \frac{[(2\alpha_1+\alpha_2-2b)-(2\alpha_2-b)c_1(2\alpha_1-b)c_2]}{4(\alpha_2+b)^2} \), and one equilibrium fixed-fee scheme exists with \( R_c^f = \frac{[(2\alpha_1+\alpha_2-2b)-(2\alpha_2-b)c_1(2\alpha_1-b)c_2]}{8(\alpha_2+b)(2\alpha_1+\alpha_2-2b)} \). As in Case 1, we have \( \frac{[(2\alpha_1+\alpha_2-2b)-(2\alpha_2-b)c_1(2\alpha_1-b)c_2]}{8(\alpha_2+b)(2\alpha_1+\alpha_2-2b)} > \frac{[(2\alpha_1+\alpha_2-2b)-(2\alpha_2-b)c_1(2\alpha_1-b)c_2]}{8(\alpha_2+b)(2\alpha_1+\alpha_2-2b)} \) \( 8(\alpha_2+b)(2\alpha_1+\alpha_2-2b) \). These suggest that the unit-fee scheme is optimal.

Fifth, for \( c_2 \in (\bar{c}_{c_2}, \bar{c}_{c_2}] \), one equilibrium unit-fee scheme exists with \( R_c^u = \frac{(c_2-c_1)(2\alpha_1-b)+bc_1-2\alpha_1c_2}{(2\alpha_2+b)^2} \), and one equilibrium fixed-fee scheme exists with \( R_c^f = \frac{1}{8(\alpha_2+b)} \). Thus, the unit-fee scheme is optimal.

Case 4: Suppose \((2\alpha_2-\alpha_1) < 0\).
Comparing the two-part tariff scheme with the unit-fee scheme:

Lemma 14(iii) shows that
\[ R_c^u = \frac{[(\alpha_1 + 2\alpha_2 - 2b) - (2\alpha_2 - b)c_1 - (\alpha_1 - 1)c_2]^2}{4(\alpha_1 + 2\alpha_2 - 2b)(2\alpha_1\alpha_2 - b^2)} \]
for \( c_2 \in (c_1, c_2^{\text{ul}}) \) if \( (\alpha_1^3 + 6\alpha_1\alpha_2 - 9\alpha_2^2 - 7\alpha_1\alpha_2^2 + 8b^2\alpha_2 - 6\alpha_1^2\alpha_2 - 6\alpha_1\alpha_2^2 - 2\alpha_1\alpha_2^2) > 0 \),
\[ R_c^u = \frac{[(\alpha_1 + 2\alpha_2 - 2b) - (2\alpha_2 - b)c_1 - (\alpha_1 - 1)c_2]^2}{4(\alpha_1 + 2\alpha_2 - 2b)(2\alpha_1\alpha_2 - b^2)} \]
for \( c_2 \in (c_1, c_2^{u}) \),
\[ R_c^u = \frac{(1-c_2)^2}{2(\alpha_1 + b)} \]
for \( c_2 \in (c_2^u, c_2^{ul}) \) if \( (\alpha_1^3 + 6\alpha_1\alpha_2 - 9\alpha_2^2 - 7\alpha_1\alpha_2^2 + 8b^2\alpha_2 - 6\alpha_1^2\alpha_2 - 6\alpha_1\alpha_2^2 - 2\alpha_1\alpha_2^2) \) \(< 0 \),
\[ R_c^u = \max\left\{ \frac{(1-c_2)^2}{2(\alpha_1 + b)}, \frac{(1-c_2)^2}{2(\alpha_1 + b)} \right\} \]
for \( c_2 \in [c_2^{ul}, c_2^{ul}] \),
\[ R_c^u = \frac{(1-c_2)^2}{2(\alpha_1 + b)} \]
for \( c_2 \in (c_2^{ul2}, c_2^{ul}) \),
\[ R_c^u = \frac{(c_2 - c_1)[(2\alpha_1 - b)c_1 - 2\alpha_1\alpha_2^2 - 2\alpha_1\alpha_2^2]}{(2\alpha_1 - b)^2} \]
for \( c_2 \in (\tilde{c}_2, \tilde{c}_2) \), and
\[ R_c^u = \frac{(1-c_2)^2}{2(\alpha_1 + b)} \]
for \( c_2 \in (c_2^{ul2}, c_2^{ul}) \),
\[ R_c^u = \frac{(2\alpha_1 - b)c_1 - 2\alpha_1\alpha_2^2 - \alpha_1\alpha_2^2}{(2\alpha_1 - b)^2} \]
for \( c_2 \in (\tilde{c}_2, \tilde{c}_2) \).

On the other hand, Lemma 13(iii) shows that
\[ R_c^* = \frac{1}{4(\alpha_1 + 2\alpha_2 - b^2)} \frac{1}{4(\alpha_1 + 2\alpha_2 - b^2)} \]
for \( c_2 \in (c_2^u, \tilde{c}_2) \),
\[ R_c^* = \frac{(2\alpha_1 - b)c_1 - 2\alpha_1\alpha_2^2 - \alpha_1\alpha_2^2}{(2\alpha_1 - b)^2} \]
for \( c_2 \in [\tilde{c}_2, \tilde{c}_2] \), \( c_2 < \tilde{c}_2 \), and \( R_c^* = \frac{1}{4(\alpha_1 + 2\alpha_2 - b^2)} \frac{1}{4(\alpha_1 + 2\alpha_2 - b^2)} \).

By some calculations, we have \( c_2^{u10} < c_2^{u11} \) and \( c_2^{u11} < c_2^{u12} < \min\{\tilde{c}_2, \tilde{c}_2\} < \max\{\tilde{c}_2, \tilde{c}_2\} < \tilde{c}_2 < \tilde{c}_2 < \tilde{c}_2 < \tilde{c}_2 \) as shown in Case 3. However, relative sizes of \( c_2^u \) and \( c_2^{u2} \) and of \( c_2^u \) and \( c_2^{u11} \) are unknown. Thus, we need to discuss the following five intervals.

First, if \( c_2 \in (c_1, c_2^{u11}) \) and \( (\alpha_1^3 + 6\alpha_1\alpha_2 - 9\alpha_2^2 - 7\alpha_1\alpha_2^2 + 8b^2\alpha_2 - 6\alpha_1^2\alpha_2 - 6\alpha_1\alpha_2^2 - 2\alpha_1\alpha_2^2) > 0 \), then we have \( c_2^{ul1} < c_2^u \). Under the circumstance, one equilibrium unit-fee scheme exists with
\[ R_c^u = \frac{[(\alpha_1 + 2\alpha_2 - 2b) - (2\alpha_2 - b)c_1 - (\alpha_1 - 1)c_2]^2}{4(\alpha_1 + 2\alpha_2 - 2b)(2\alpha_1\alpha_2 - b^2)} \]
and one equilibrium two-part tariff scheme exists with
\[ R_c^u = \frac{[(\alpha_1 + 2\alpha_2 - 2b) - (2\alpha_2 - b)c_1 - (\alpha_1 - 1)c_2]^2}{4(\alpha_1 + 2\alpha_2 - 2b)(2\alpha_1\alpha_2 - b^2)} \]
for \( c_2 \in (c_2^u, \tilde{c}_2) \).

By contrast, if \( (\alpha_1^3 + 6\alpha_1\alpha_2 - 9\alpha_2^2 - 7\alpha_1\alpha_2^2 + 8b^2\alpha_2 - 6\alpha_1^2\alpha_2 - 6\alpha_1\alpha_2^2 - 2\alpha_1\alpha_2^2) < 0 \), then \( c_2^u \) can be less than \( c_2^{u11} \). Two equilibrium unit-fee schemes may exist with
\[ R_c^u = \frac{[(\alpha_1 + 2\alpha_2 - 2b) - (2\alpha_2 - b)c_1 - (\alpha_1 - 1)c_2]^2}{4(\alpha_1 + 2\alpha_2 - 2b)(2\alpha_1\alpha_2 - b^2)} \]
or
\[ R_c^u = \frac{(1-c_2)^2}{2(\alpha_1 + b)} \]
for \( c_2 \in (c_2^u, c_2^{u11}) \). On the other hand, two equilibrium two-part tariff schemes may exist with
\[ R_c^* = \frac{1}{4(\alpha_1 + 2\alpha_2 - b^2)} \frac{1}{4(\alpha_1 + 2\alpha_2 - b^2)} \]
for \( c_2 \in (c_2^u, \tilde{c}_2) \) iff \( c_2^u < c_2^{u11} \). As in Case 3, we can show \( \frac{(1-c_2)^2}{2(\alpha_1 + b)} > \frac{(2\alpha_1 - b)c_1 - 2\alpha_1\alpha_2^2 - \alpha_1\alpha_2^2}{(2\alpha_1 - b)^2} \) for \( c_2 < c_2^{u11} \), and
Define $H_{15} = \frac{(1-c_2)^2}{2(a_1+b)} - \frac{(a_1+2a_2-2b)-(2a_2-b)c_1-2(a_1-b)c_2-2bc_1c_2+2a_2c_1^2+\alpha_1}{4(2a_1a_2-b^2)}$. Since $\frac{\partial H_{15}}{\partial c_2} = \frac{(a_1-b)+bc_1c_2}{2(2a_1a_2-b^2)} > 0$ at $c_2 = c_1$, $H_{15} = \frac{(a_1-b)(1-c_1)^2}{4(a_1+b)} > 0$ at $c_2 = c''_2$, and $H_{15} = \frac{(a_1-b)(1-c_1)^2}{4(a_1+b)(2a_1a_2-b^2)}$ if $(a_1-b)(1-c_1)^2 > 0$, $H_{15} = \frac{(a_1-b)(1-c_1)^2}{4(a_1+b)(2a_1a_2-b^2)}$ if $(a_1-b)(1-c_1)^2 < 0$, $H_{15}$ is increasing in $c_2$ for $c_2 < c''_2$, and $H_{15}$ is decreasing in $c_2$ for $c_2 > c''_2$.

Thus, if $(a_1-b)(1-c_1)^2 = (a_1-b)(1-c_1)^2 - 4b^2\alpha_2)$ < 0, the two-part tariff scheme is better than the unit-fee scheme, and so is when $(a_1-b)(1-c_1)^2 - 4b^2\alpha_2)$ > 0 and $c_2 \leq c''_2$. However, the unit-fee scheme will be better if $c''_2 < c_2 < c''_2$.

Second, for $c_2 \in [c''_2, c''_2]$, one equilibrium unit-fee scheme exists with $R_e^u = \max\{\frac{(1-c_2)^2}{2(a_1+b)}, \frac{(a_1+2a_2-2b)-(2a_2-b)c_1-2(a_1-b)c_2-2bc_1c_2+2a_2c_1^2+\alpha_1}{4(2a_1a_2-b^2)}\}$, and two equilibrium two-part tariff schemes may exist with $R_e^u = \frac{(a_1+2a_2-2b)-(2a_2-b)c_1-2(a_1-b)c_2-2bc_1c_2+2a_2c_1^2+\alpha_1}{4(2a_1a_2-b^2)}$, or $R_e^u = \frac{(a_1+2a_2-2b)-(2a_2-b)c_1-2(a_1-b)c_2-2bc_1c_2+2a_2c_1^2+\alpha_1}{4(2a_1a_2-b^2)}\frac{2(2a_1a_2-b^2)}{(a_1+2a_2-2b)-(2a_2-b)c_1-2(a_1-b)c_2-2bc_1c_2+2a_2c_1^2+\alpha_1}{4(2a_1a_2-b^2)}$ if $c_2 < c''_2$, and $R_e^u = 2f_2^* + r_2^*$ if $c_2 = c''_2$.

Third, for $c_2 \in (c''_2, c''_2]$, one equilibrium unit-fee scheme exists with $R_e^u = \frac{(1-c_2)^2}{2(a_2+b)}$, and four equilibrium two-part tariff schemes may exist with $R_e^u = \frac{(a_1+2a_2-2b)-(2a_2-b)c_1-2(a_1-b)c_2-2bc_1c_2+2a_2c_1^2+\alpha_1}{4(2a_1a_2-b^2)}$ if $c_2 > c''_2$, and $R_e^u = \frac{(a_1+2a_2-2b)-(2a_2-b)c_1-2(a_1-b)c_2-2bc_1c_2+2a_2c_1^2+\alpha_1}{4(2a_1a_2-b^2)}$ if $c_2 < c''_2$, and $R_e^u = 2f_2^* + r_2^*$ if $c_2 = c''_2$.

Note that $(c''_2 - c''_2) = (\frac{b\alpha_1 - 2a_2^2 - 3b\alpha_2(1-c_1)}{(a_1+b)(a_1+2a_2+2b)} > 0$ if $(b\alpha_1 - 2a_2^2 - 3b\alpha_2) > 0$. Define $H_{16} = \frac{(a_1+2a_2-2b)-(2a_2-b)c_1-2(a_1-b)c_2-2bc_1c_2+2a_2c_1^2+\alpha_1}{4(2a_1a_2-b^2)}$. Since $\frac{\partial^2 H_{16}}{\partial c_2} = \frac{(a_1-b)+bc_1c_2}{2(2a_1a_2-b^2)} > 0$ at $c_2 = c_1$, $H_{16} = \frac{(a_1-b)(1-c_1)^2}{4(a_1+b)} > 0$ at $c_2 = c''_2$, and $H_{16} = \frac{(a_1-b)(1-c_1)^2}{4(a_1+b)(2a_1a_2-b^2)}$ if $(a_1-b)(1-c_1)^2 > 0$, $H_{16} = \frac{(a_1-b)(1-c_1)^2}{4(a_1+b)(2a_1a_2-b^2)}$ if $(a_1-b)(1-c_1)^2 < 0$, $H_{16}$ is increasing in $c_2$ for $c_2 < c''_2$, and $H_{16}$ is decreasing in $c_2$ for $c_2 > c''_2$.

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Lemma 16. Given fee contract and minimum throughput requirement \((r_i, f_i, \delta_i)\) offered to operator \(i, i = 1, 2\), operators’ optimal behaviors are as follows.

(i) For \(\delta_1 \in [0, \delta_{d1}]\) with \(\dot{\delta}_{d1} = \frac{(2-b-2(r_1+c_1)+b(r_2+c_2)}{4-b^2}\) and \(\delta_2 \in [0, \delta_{d2}]\) with \(\dot{\delta}_{d2} = \frac{(2-b-2(r_2+c_2)+b(r_1+c_1)}{4-b^2}\), both operators’ equilibrium cargo-handling amounts are \(q^*_d = \dot{\delta}_{d1}\) and \(q^*_d = \dot{\delta}_{d2}\), the equilibrium service prices are \(p^*_{di} = c_i + r_i + q^*_d > 0\), and their
equilibrium profits are \( \pi_{d1}^* = (q_{d1}^*)^2 - f_i \) for \( i = 1, 2 \).

(ii) For \( \delta_1 \in [0, \frac{-b - c - r_1}{2}] \) and \( \delta_2 \in (\delta_{d2}, \frac{-(2-b) + b(c_1 + r_1)}{2-b}) \), both operators’ equilibrium cargo-handling amounts are \( q_{d1}^* = \frac{1-b - c_1 - r_1}{2} \) and \( q_{d2}^* = \delta_2 \), the equilibrium prices are \( p_{d1}^* = \frac{(1-b - c_1 - r_1)}{2} > 0 \) and \( p_{d2}^* = \frac{[(2-b) - (2-b^2) \delta_2 + bc_1 + br_1]}{2} > 0 \), and their equilibrium profits are \( \pi_{d1}^* = (q_{d1}^*)^2 - f_1 \) and \( \pi_{d2}^* = \delta_2 [(2-b) - (2-b^2) \delta_2 - 2(r_2 + c_2) + b(r_1 + c_1)] - f_2 \).

(iii) For \( \delta_1 \in (\delta_{d1}, \frac{-(b - c_2 + r_2)}{2}) \) and \( \delta_2 \in [0, \frac{-b - c + r_2}{2}] \), both operators’ equilibrium cargo-handling amounts are \( q_{d1}^* = \delta_1 \) and \( q_{d2}^* = \frac{-b \delta_1 - c_2 - r_2}{2} \), the equilibrium prices are \( p_{d1}^* = \frac{[(2-b) - (2-b^2) \delta_1 + bc_2 + br_2]}{2} > 0 \) and \( p_{d2}^* = \frac{1-b \delta_1 + c_2 + r_2}{2} > 0 \), and their equilibrium profits are \( \pi_{d1}^* = \delta_1 [(2-b) - (2-b^2) \delta_1 - 2(r_1 + c_1) + b(r_2 + c_2)] - f_1 \) and \( \pi_{d2}^* = (q_{d2}^*)^2 - f_2 \).

(iv) For \( \delta_1 \in (\frac{1-b - c_1 - r_1}{2}, 1-b \delta_2) \) and \( \delta_2 \in (\frac{1-b - c_2 - r_2}{2}, 1-b \delta_1) \), both operators’ equilibrium cargo-handling amounts are \( q_{d1}^* = \delta_1 \) and \( q_{d2}^* = \delta_2 \), the equilibrium service prices are \( p_{d1}^* = (1 - \delta_1 - b \delta_2) > 0 \) and \( p_{d2}^* = (1 - b \delta_1 - \delta_2) > 0 \), and their equilibrium profits are \( \pi_{d1}^* = \delta_1 [1 - \delta_1 - b \delta_2 - c_i - r_i] - f_i \) for \( i, j \in \{1, 2 \mid i \neq j \} \).

**Proof of Lemma 16:** Denote \( L_1 \) and \( L_2 \) the Lagrange functions of operators 1 and 2 in problem (46) with \( L_1 = (1 - q_1 - bq_2 - c_1 - r_1)q_1 - f_1 + \lambda_1 (q_1 - \delta_1) \) and \( L_2 = (1 - q_2 - bq_1 - c_2 - r_2)q_2 - f_2 + \lambda_2 (q_2 - \delta_2) \), where \( \lambda_1 \) and \( \lambda_2 \) are the Lagrange multipliers for the operators. Then, the Kuhn-Tucker conditions for operator 1 are

\[
\frac{\partial L_1}{\partial q_1} = 1 - 2q_1 - bq_2 - c_1 - r_1 + \lambda_1 \leq 0, \quad q_1 \cdot \frac{\partial L_1}{\partial \lambda_1} = 0, \quad (A160)
\]

\[
\frac{\partial L_1}{\partial \lambda_1} = q_1 - \delta_1 \geq 0, \quad \lambda_1 \cdot \frac{\partial L_1}{\partial \lambda_1} = 0, \quad (A161)
\]

and for operator 2 are

\[
\frac{\partial L_2}{\partial q_2} = 1 - 2q_2 - bq_1 - c_2 - r_2 + \lambda_2 \leq 0, \quad q_2 \cdot \frac{\partial L_2}{\partial \lambda_2} = 0, \quad (A162)
\]

\[
\frac{\partial L_2}{\partial \lambda_2} = q_2 - \delta_2 \geq 0, \quad \lambda_2 \cdot \frac{\partial L_2}{\partial \lambda_2} = 0. \quad (A163)
\]

Based on the values of \( \lambda_1 \) and \( \lambda_2 \), we have four cases as follows.
Case 1: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then (A160) and (A162) suggest $(1 - 2q_1 - bq_2 - c_1 - r_1) = 0$ and $(1 - 2q_2 - bq_1 - c_2 - r_2) = 0$. Solving these equations yields $q_{d1}^* = \frac{(2-b)-(c_1+c_2)+b(r_2+c_2)}{4b^2}$ and $q_{d2}^* = \frac{(2-b)-(c_1+c_2)+b(r_2+c_2)}{4b^2}$. To guarantee $q_{d1}^* \geq \delta_1$ and $q_{d2}^* \geq \delta_2$, conditions $0 \leq \delta_1 \leq \delta_1 = \frac{(2-b)-(c_1+c_2)+b(r_2+c_2)}{4b^2}$ and $0 \leq \delta_2 \leq \delta_2 = \frac{(2-b)-(c_1+c_2)+b(r_2+c_2)}{4b^2}$ are imposed. Moreover, we have $q_{d1}^* \geq 0$ iff $r_1 \leq \bar{r}_d \equiv \frac{1}{2}(2 - b - 2c_1 + b(r_2 + c_2)]$, and $q_{d2}^* \geq 0$ iff $r_2 \leq \bar{r}_d \equiv \frac{1}{2}[2 - b - 2c_2 + b(r_1 + c_1)]$. Substituting $q_{d1}^*$ and $q_{d2}^*$ into (1)-(2) yields $p_{d1}^* = c_i + r_i + q_{d1}^* > 0$, and into (4) yields $\pi_{d1}^* = (q_{d1})^2 - f_i$ for $i = 1, 2$. These prove Lemma 16(i).

Case 2: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then (A160), (A162), and (A163) suggest $(1 - 2q_1 - bq_2 - c_1 - r_1) = 0$, $(1 - 2q_2 - bq_1 - c_2 - r_2 + \lambda_2) = 0$, and $(q_2 - \delta_2) = 0$. Solving these equations yields $q_{d1}^* = \frac{(1-b+\lambda_2-c_1-r_1)}{2}$, $q_{d2}^* = \delta_2$, and $\lambda_2^* = \frac{(1-b+\lambda_2-c_1-r_1)}{2}$. To have $\lambda_2^* > 0$, conditions $\delta_2 > \delta_{d2}$ and $r \leq \bar{r}_{d2}$ are needed. On the other hand, to have $q_{d1}^* \geq \delta_1$, condition $\delta_1 \leq \frac{(1-b+\lambda_2-c_1-r_1)}{2}$ is needed. Moreover, we have $q_{d1}^* \geq 0$ iff $r_1 \leq (1 - b\delta_1 - c_1)$. Substituting $q_{d1}^*$ and $q_{d2}^*$ into (1)-(2) produces $p_{d1}^* = \frac{(1-b+\lambda_2-c_1-r_1)}{2} > 0$ and $p_{d2}^* = \frac{(1-b+\lambda_2-c_1-r_1)}{2} > 0$ iff $\delta_2 < \frac{(2-b)(c_1+r_1)}{(2-b^2)}$, and into (4) gives $\pi_{d1}^* = (q_{d1}^*)^2 - f_1$ and $\pi_{d2}^* = \delta_2[(2-b)(2-b^2)(2b_2-2(r_2+c_2)+b(r_1+c_1)] - f_2$. Thus, the plausible range for $\delta_1$ is $\delta_1 \in [0, \frac{1-b+b(c_1+r_1)}{2}]$, and for $\delta_2$ is $\delta_2 \in (\delta_{d2}, \frac{(2-b)(c_1+r_1)}{(2-b^2)})$. These prove Lemma 16(ii).

Case 3: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then (A160)-(A162) suggest $(q_1 - \delta_1) = 0$, $(1 - 2q_1 - bq_2 - c_1 - r_1 + \lambda_1) = 0$, and $(1 - 2q_2 - bq_1 - c_2 - r_2) = 0$. Solving these equations yields $q_{d1}^* = \delta_1$, $q_{d2}^* = \frac{(1-b+\lambda_2-c_1-r_2)}{2}$, and $\lambda_1^* = \frac{1}{2}(4 - b^2)(\delta_1 - \delta_{d1})$. To guarantee $\lambda_1^* > 0$, conditions $\delta_1 > \delta_{d1}$ and $r \leq \bar{r}_{d1}$ are needed. On the other hand, to have $q_{d2}^* \geq \delta_2$, condition $\delta_2 \leq \frac{(1-b+\lambda_2-c_1-r_2)}{2}$ is needed. Moreover, we have $q_{d2}^* \geq 0$ iff $r_2 \leq (1 - b\delta_1 - c_2)$. Substituting $q_{d1}^*$ and $q_{d2}^*$ into (1)-(2) produces $p_{d1}^* = \frac{(1-b+\lambda_2-c_1-r_2)}{2} > 0$ and $p_{d2}^* = \frac{(1-b+\lambda_2-c_1-r_2)}{2} > 0$ iff $\delta_1 < \frac{(2-b)(c_1+r_2)}{(2-b^2)}$, and into (4) gives $\pi_{d1}^* = (q_{d1}^*)^2 - f_1$ and $\pi_{d2}^* = \delta_1[(2-b)(2-b^2)(2b_2-2(r_1+c_1)+b(r_2+c_2)] - f_1$. Thus, the plausible range for $\delta_2$ is $\delta_2 \in [0, \frac{1-b+b(c_1+r_2)}{2}]$, and for $\delta_1$ is $\delta_1 \in (\delta_{d1}, \frac{(2-b)(c_1+r_2)}{(2-b^2)})$. These prove Lemma 16(iii).
Case 4: Suppose \( \lambda_1^* > 0 \) and \( \lambda_2^* > 0 \). Then (A160)-(A163) suggest \( q_{d1}^* = \delta_1, \ q_{d2}^* = \delta_2 \), \( \lambda_1^* = -1 + 2\delta_1 + b\delta_2 + c_1 + r_1 \), and \( \lambda_2^* = -1 + 2\delta_2 + b\delta_1 + c_2 + r_2 \). To have \( \lambda_1^* > 0 \) and \( \lambda_2^* > 0 \), conditions \( \delta_1 > \frac{(1-b\delta_2-c_1-r_1)}{2} \) and \( \delta_2 > \frac{(1-b\delta_1-c_2-r_2)}{2} \) are needed. Substituting \( q_{d1}^* = \delta_1 \) and \( q_{d2}^* = \delta_2 \) into (1)-(2) yields \( p_{d1}^* = (1-\delta_1-b\delta_2) > 0 \) iff \( \delta_1 < (1-b\delta_2) \) and \( p_{d2}^* = (1-b\delta_1-\delta_2) > 0 \) iff \( \delta_2 < (1-b\delta_1) \), and into (4) gives \( \pi_{\delta_i}^* = \delta_i(1-\delta_i-b\delta_j-c_i-r_i) - f_i \) for \( i, j \in \{1, 2| i \neq j\} \). These prove Lemma 16(iv). \( \square \)

**Proof of Proposition 5:** Suppose \( \delta_1 \in [0, \delta_{d1}] \) with \( \dot{\delta}_{d1} = \frac{2-b-2(r_1+c_1)+b(r_2+c_2)}{4-b^2} \) and \( \delta_2 \in [0, \delta_{d2}] \) with \( \dot{\delta}_{d2} = \frac{2-b-2(r_2+c_2)+b(r_1+c_1)}{4-b^2} \). Lemma 16(i) implies \( f_{d1}^* = \pi_{d1}^* = \frac{1}{2}(q_{d1}^*)^2 \geq 0 \) and \( f_{d2}^* = \pi_{d2}^* = \frac{1}{2}(q_{d2}^*)^2 \geq 0 \). Thus, the problem in (49) becomes

\[
\max_{r_1, f_1, \delta_1, r_2, f_2, \delta_2} f_1 + f_2 + r_1q_{d1}^* + r_2q_{d2}^*
\]

s.t. \( 0 \leq \delta_1 \leq \delta_{d1}, 0 \leq \delta_2 \leq \delta_{d2}, 0 \leq r_1 \leq \bar{r}_{d1}, \) and \( 0 \leq r_2 \leq \bar{r}_{d2} \).

Its Lagrange function is

\[
L = \frac{1}{2}(q_{d1}^*)^2 + \frac{1}{2}(q_{d2}^*)^2 + r_1q_{d1}^* + r_2q_{d2}^* + \lambda_1(\dot{\delta}_{d1}-\delta_1) + \lambda_2(\dot{\delta}_{d2}-\delta_2) + \lambda_3(\bar{r}_{d1}-r_1) + \lambda_4(\bar{r}_{d2}-r_2),
\]

where \( \lambda_1, \lambda_2, \lambda_3, \) and \( \lambda_4 \) are the Lagrange multipliers for the four inequality constraints. Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r_1} = \frac{1}{(4-b^2)^2}[(1+b)(2-b)^2 - (4-3b^2)c_1 - b^3c_2 - (12-5b^2)r_1 + 2b(2-b^2)r_2] 
\]

\[-\frac{2\lambda_1}{(4-b^2)} + \frac{b\lambda_2}{(4-b^2)} - \lambda_3 + \frac{b\lambda_4}{2} \leq 0, \quad r_1 \cdot \frac{\partial L}{\partial r_1} = 0, \quad (A164)\]

\[
\frac{\partial L}{\partial r_2} = \frac{1}{(4-b^2)^2}[(1+b)(2-b)^2 - b^3c_1 - (4-3b^2)c_2 + 2b(2-b^2)r_1 - (12-5b^2)r_2] 
\]

\[+ \frac{b\lambda_1}{(4-b^2)} - \frac{2\lambda_2}{(4-b^2)} + \frac{b\lambda_3}{2} - \lambda_4 \leq 0, \quad r_2 \cdot \frac{\partial L}{\partial r_2} = 0, \quad (A165)\]

\[
\frac{\partial L}{\partial \delta_1} = -\lambda_1 \leq 0, \quad \delta_1 \cdot \frac{\partial L}{\partial \delta_1} = 0, \quad (A166)\]

\[
\frac{\partial L}{\partial \delta_2} = -\lambda_2 \leq 0, \quad \delta_2 \cdot \frac{\partial L}{\partial \delta_2} = 0, \quad (A167)\]

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\[ \frac{\partial L}{\partial \lambda_1} = \delta_{d1} - \delta_1 \geq 0, \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A168) \]
\[ \frac{\partial L}{\partial \lambda_2} = \delta_{d2} - \delta_2 \geq 0, \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad (A169) \]
\[ \frac{\partial L}{\partial \lambda_3} = \bar{r}_{d1} - r_1 \geq 0, \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \quad (A170) \]
\[ \frac{\partial L}{\partial \lambda_4} = \bar{r}_{d2} - r_2 \geq 0, \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0. \quad (A171) \]

If \( \lambda_i^* > 0 \), then \( \frac{\partial L}{\partial \lambda_i} = -\lambda_1 < 0 \) and \( \delta_i^* = 0 \) by (A166). They in turn suggest \( \frac{\partial L}{\partial \lambda_1} = \delta_{d1} > 0 \) and \( \lambda_1^* = 0 \) by (A168). It is a contradiction. Thus, we must have \( \lambda_i^* = 0 \). Similarly, we have \( \lambda_2^* = 0 \) by (A167). Then, based on the values of \( \lambda_3 \) and \( \lambda_4 \), there are four sub-cases as follows.

**Case 1a:** Suppose \( \lambda_3^* = 0 \) and \( \lambda_4^* = 0 \). Then (A164) and (A165) suggest
\[
\frac{1}{(4-b^2)^2} [(1+b)(2-b)^2 - (4-3b^2)c_1 - b^3c_2 - (12-5b^2)r_1 + 2b(2-b^2)r_2] = 0, \quad \text{and}
\frac{1}{(3-b^2)^2} [(1+b)(2-b)^2 - b^3c_1 - (4-3b^2)c_2 + 2b(2-b^2)r_1 - (12-5b^2)r_2] = 0.
\]
Solving these equations yields \( r_{d1}^* = \frac{(1+b)(3-2b)-(3-2b)c_1-bc_2}{9-4b^2} > 0 \) and \( r_{d2}^* = \frac{1}{9-4b^2}[(1+b)(3-2b)-bc_1-(3-2b^2)c_2] > 0 \). It remains to check whether \( r_{d1}^* < \bar{r}_{d1} \) and \( r_{d2}^* < \bar{r}_{d2} \) hold. By some calculations, we have \( \bar{r}_{d1} - r_{d1}^* = \frac{(4-b^2)[(3-2b)-3c_1+2bc_2]}{2(3-b^2)} > 0 \), and \( \bar{r}_{d2} - r_{d2}^* = \frac{(4-b^2)[(3-2b)+2bc_1-3c_2]}{2(3-b^2)} \geq 0 \) iff \( c_2 \leq \hat{c}_d \).

In addition, (A168) and (A169) imply \( \delta_{d1}^* \in [0, \delta_{d1}], \delta_{d2}^* \in [0, \delta_{d2}], f_{d1}^* = \frac{1}{2}\left[(3-2b-3c_1+2bc_2)^2\right] > 0 \), and \( f_{d2}^* = \frac{1}{2}\left[(3-2b+2bc_1-3c_2)^2\right] > 0 \) iff \( c_2 \leq \hat{c}_d \). Thus, for \( c_2 \leq \hat{c}_d \), port authority’s equilibrium fee revenue equals
\[ R_d^* = \frac{2(3-2b)(1-c_1-c_2) - 4bc_1c_2 + 3(c_1^2 + c_2^2)}{2(9-4b^2)} \equiv R_1^*. \quad (A172) \]

**Case 1b:** Suppose \( \lambda_3^* = 0 \) and \( \lambda_4^* > 0 \). Then (A164), (A165), and (A171) suggest
\[
\frac{1}{(4-b^2)^2} [(1+b)(2-b)^2 - (4-3b^2)c_1 - b^3c_2 - (12-5b^2)r_1 + 2b(2-b^2)r_2] + \frac{1}{2}\lambda_4^* = 0, \quad \text{and}
\frac{1}{(4-b^2)^2} [(1+b)(2-b)^2 - b^3c_1 - (4-3b^2)c_2 + 2b(2-b^2)r_1 - (12-5b^2)r_2] - \lambda_4^* = 0, \quad \text{and}
\]
\[ r_2 - \frac{1}{2}(2-b) - 2c_2 + b(r_1 + c_1) = 0. \]
Solving these equations yields \( r_{d1}^* = \frac{1-c_1}{3} > 0 \), \( r_{d2}^* = \frac{(3-b)+bc_1-3c_2}{3} \), and \( \lambda_4^* = \frac{2[-(3-2b)+2bc_1+3c_2]}{3(4-b^2)} \).

Note that \( r_{d2}^* \geq 0 \) iff \( c_2 \leq \frac{(3-b)+bc_1}{3} \), \( (\tilde{r}_{d1} - r_{d1}^* ) = \frac{(4-b^2)(1-c_1)}{6} > 0 \), \( \lambda_4^* > 0 \) iff \( c_2 > \hat{c}_d \) with \( \frac{(3-b)+bc_1}{3} - \hat{c}_d = \frac{2b(3-b^2)(1-c_1)}{3(6-b^2)} > 0 \), \( f_{d2}^* = 0 \), and \( \delta_{d2}^* = 0 \) due to \( \dot{\lambda}_{d2} = \frac{2-b-2(r_{d2}^* + c_2) + b(r_{d2}^* + c_1)}{4-b^2} = 0 \). In addition, (A168) implies \( \tilde{\delta}_{d1} \in [0, \hat{\delta}_{d1}] \) with \( \hat{\delta}_{d1} = \frac{1-c_1}{3} \) and \( f_{d1}^* = \frac{(1-c_1)^2}{18} > 0 \). Thus, for \( \hat{c}_d < c_2 \leq \frac{(3-b)+bc_1}{3} \), port authority’s equilibrium fee revenue equals

\[
R_d = f_{d1}^* + r_{d1}^* \cdot q_{d1}^* = \frac{(1-c_1)^2}{6} \equiv R_2^*.
\]  

**Case 1c:** Suppose \( \lambda_3^* > 0 \) and \( \lambda_4^* = 0 \). Then, (A164), (A165), and (A170) suggest \( r_{d1}^* = \tilde{r}_{d1} = \frac{(3-b)-3c_1+bc_2}{3} > 0 \), \( r_{d2}^* = \frac{1-c_2}{3} > 0 \), and \( \lambda_3^* = \frac{2[-(3-2b)+3c_1-2bc_2]}{3(4-b^2)} < 0 \). It is a contradiction. Thus, no solution exists in this case.

**Case 1d:** Suppose \( \lambda_3^* > 0 \) and \( \lambda_4^* > 0 \). Then, (A164), (A165), (A170), and (A171) suggest \( r_{d1}^* = \tilde{r}_{d1} = 1-c_1 \), \( r_{d2}^* = \tilde{r}_{d2} = 1-c_2 \), \( \lambda_3^* = \frac{-2(1-c_1)}{(4-b^2)} < 0 \), and \( \lambda_4^* = \frac{-2(1-c_2)}{(4-b^2)} < 0 \). However, \( \lambda_3^* < 0 \) is a contradiction. Thus, no solution exists in this case.

**Case 2:** Suppose \( \delta_1 \in [0, \frac{1-bd_2-c_1-r_1}{2}] \) and \( \delta_2 \in (\tilde{\delta}_{d2}, \frac{(2-b)+b(c_1+r_1)}{(2-b^2)}) \). Then, Lemma 16(ii) implies \( \pi_{d1}^* = \left( \frac{1-bd_2-c_1-r_1}{2} \right)^2 - f_{d1}^* \) and \( \pi_{d2}^* = \frac{\delta_2}{4} \left( (2-b) - (2-b^2) \delta_2 - 2(r_2 + c_2) + b(r_1 + c_1) \right) - f_{d2}^* \) with \( f_{d1}^* = \frac{1}{2} \left( \frac{1-bd_2-c_1-r_1}{2} \right)^2 \) and \( f_{d2}^* = \frac{\delta_2}{4} \left( (2-b) - (2-b^2) \delta_2 - 2(r_2 + c_2) + b(r_1 + c_1) \right) \).

We have \( f_{d1}^* \geq 0 \) iff \( r_1 \leq 1 - b(\delta_2 - c_1) \) with \( (1 - b\delta_2 - c_1) < \tilde{r}_{d1} \), \( f_{d2}^* \geq 0 \) iff \( \delta_2 \leq \frac{(2-b)-2(r_2+c_2)+b(r_1+c_1)}{(2-b^2)} \) \( \equiv \tilde{\delta}_{d2} \) and \( r_2 \leq \tilde{r}_{d2} \). In addition, we have \( (2-b)-2(r_2+c_2)+b(r_1+c_1) \) \( < \frac{(2-b)+b(c_1+r_1)}{(2-b^2)} \). Thus, the problem in (49) becomes

\[
\max_{r_1, f_1, \delta_1, r_2, f_2, \delta_2} f_{d1}^* + f_{d2}^* + r_1 q_{d1}^* + r_2 q_{d2}^*
\]

s.t. \( 0 \leq \delta_1 \leq \frac{1-b\delta_2-c_1-r_1}{2} \), \( \tilde{\delta}_{d2} < \delta_2 \leq \tilde{\delta}_{d2} \), \( 0 \leq r_1 \leq 1 - b(\delta_2 - c_1) \), and \( r_2 \leq \tilde{r}_{d2} \).

(A174)
Its Lagrange function is

\[
L = \frac{r_1(1 - b\delta_2 - c_1 - r_1)}{2} + r_2\delta_2 + \frac{1}{2} \left( \frac{1 - b\delta_2 - c_1 - r_1}{2} \right)^2 + \frac{\delta_2}{4} \left[ (2 - b) - (2 - b^2)\delta_2 - 2(r_2 + c_2) + b(r_1 + c_1) \right] + \lambda_1\left( \frac{1 - b\delta_2 - c_1 - r_1}{2} - \delta_1 \right) + \lambda_2(\delta_2 - \dot{\delta}_{d2}) + \lambda_3(\delta_{d2} - \delta_2) + \lambda_4[(1 - b\delta_2 - c_1) - r_1] + \lambda_5(r_{d2} - r_2).
\]

Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r_1} = \frac{1 - 3r_1 - c_1}{4} - \frac{\lambda_1}{2} - \frac{b\lambda_3}{(4 - b^2)} + \frac{b\lambda_5}{2} - \lambda_4 + \frac{b\lambda_5}{2} \leq 0, \quad r_1 \cdot \frac{\partial L}{\partial r_1} = 0, \quad (A175)
\]

\[
\frac{\partial L}{\partial r_2} = \frac{\delta_2}{2} + \frac{2\lambda_2}{(4 - b^2)} - \frac{2\lambda_3}{(2 - b^2)} - \lambda_5 \leq 0, \quad r_2 \cdot \frac{\partial L}{\partial r_2} = 0, \quad (A176)
\]

\[
\frac{\partial L}{\partial \delta_1} = -\lambda_1 \leq 0, \quad \delta_1 \cdot \frac{\partial L}{\partial \delta_1} = 0, \quad (A177)
\]

Constraint \( \delta_2 > \dot{\delta}_{d2} \) in (A174) suggests \( \lambda_2^* < 0 \) by (A180). If \( \lambda_1^* > 0 \), then \( \frac{\partial L}{\partial \delta_1} = -\lambda_1 < 0 \) and \( \delta_{d1}^* = 0 \) by (A177). They in turn suggest \( \frac{\partial L}{\partial \lambda_1} = \frac{1 - b\delta_2 - c_1}{2} > 0 \) and \( \lambda_1^* = 0 \) by (A179). This is a contradiction. Thus, we must have \( \lambda_1^* = 0 \). Similarly, if \( \lambda_3^* > 0 \), we have \( r_{d2}^* = \bar{r}_{d2} \) by (A183) and \( \dot{\delta}_{d2} = \tilde{\delta}_{d2} = 0 \), which contradicts \( \dot{\delta}_{d2} < \delta_2 \leq \tilde{\delta}_{d2} \). Thus, we must have \( \lambda_3^* = 0 \). Moreover, if \( \lambda_1^* = \lambda_3^* = 0 \), (A176) becomes \( \frac{\partial L}{\partial \delta_2} = \frac{\delta_2}{2} - \frac{2\lambda_5}{(2 - b^2)} \leq 0 \). Thus, we must have \( \lambda_5^* > 0 \).

Based on the values of \( \lambda_4 \), there are two sub-cases as follows.
Case 2a: Suppose $\lambda^*_4 = 0$. Then (A175), (A176), (A178), and (A181) suggest $[\frac{1}{4}(1 - 3r_1 - c_1) + \frac{b\lambda_3}{(2-b^2)}] = 0$, $[\frac{\dot{b}}{2} - \frac{2b\lambda_3}{(2-b^2)}] = 0$, $\frac{1}{4}[2(1 - b) + 2bc_1 - 2c_2 + 2r_2 - (4 - 3\delta_2)^2] - \lambda_3 = 0$, and $\delta_2 = \tilde{\delta}_{d2} = \frac{(2-b-2(r_2+c_2)+b(r_1+c_1)}{(2-b^2)}$. Solving these equations yields $r^*_d = \frac{2(2-b-2b^2-2(1-b^2)c_1-bc_2)}{2(3-2b^2)} > 0$, $\delta^*_d = \frac{1-c_2}{2} > 0$, $\delta^*_d = \frac{(3-2b)+2bc_1-3c_2}{2(3-2b^2)}$, and $\lambda^*_3 = \frac{2(1-b)-c_1+bc_2}{3(3-2b^2)} > 0$. Note that $f^*_d = 0$ due to $\delta^*_d = \frac{(3^2b)+2bc_1-3c_2}{2(3-2b^2)}$, $(1-b\delta^*_d - c_1) - r^*_d = \frac{2(1-b)-c_1+bc_2}{3(3-2b^2)} > 0$, $(\tilde{r}_{d2} - r^*_d) = \frac{(2-b^2)(2-2b+2bc_1-3c_2)}{4(3-2b^2)} > 0$ iff $c_2 < \tilde{c}_{d2} \equiv \frac{(3-2b)+2bc_1}{3}$, and $\lambda^*_3 > 0$ iff $c_2 < \tilde{c}_{d2}$. In addition, (A179) implies $\delta^*_d \in [0, \frac{(1-b)-c_1+bc_2}{(3-2b^2)}]$ and $f^*_d = \frac{1}{2}\frac{(1-b)-c_1+bc_2}{(3-2b^2)} > 0$. Thus, for $c_2 < \tilde{c}_{d2}$, port authority’s equilibrium fee revenue equals

$$R^*_d = \frac{5 - 4b - 4(1-b)c_1 - 2(3-2b)c_2 - 4bc_1c_2 + 2c^2_1 + 3c^2_2}{4(3-2b^2)} \equiv R^*_3. \quad (A184)$$

Case 2b: Suppose $\lambda^*_4 > 0$. Then (A181) and (A182) suggest $\delta^*_d = \tilde{\delta}_{d2}$ and $r^*_d = (1-b\delta^*_d - c_1) > 0$. Substituting $\delta^*_d$ and $r^*_d$ into equations $[\frac{1}{4}(1 - 3r_1 - c_1) + \frac{b\lambda_3}{(2-b^2)} - \lambda_4] = 0$, $[\frac{\dot{b}}{2} - \frac{2b\lambda_3}{(2-b^2)}] = 0$ and $\frac{1}{4}[2(1 - b) + 2bc_1 - 2c_2 + 2r_2 - (4 - 3\delta_2)^2] - \lambda_3 - b\lambda_4 = 0$ yields $\lambda^*_4 = \frac{1}{2}[-(1-b)+c_1-bc_2] < 0$, which contradicts $\lambda^*_4 > 0$. Thus, no solution exists in this case.

Case 3: Suppose $\delta_1 \in (\tilde{\delta}_{d1}, \frac{(2-b)+b(r_1+r_2)}{(2-b^2)})$ and $\delta_2 \in [0, \frac{1-b\delta_1 - c_2 - r_2}{2}]$. Then, Lemma 16(iii) implies $\pi^*_d = \delta_1(2-b)(2-b^2)\delta_1 - 2(r_1+c_1) + b(r_2+c_2)] - f^*_d$, and $\pi^*_d = (q^*_d)^2 - f^*_d$ with $f^*_d = \frac{\delta_1}{2}(2-b)(2-b^2)\delta_1 - 2(r_1+c_1) + b(r_2+c_2)]$ and $f^*_d = \frac{1}{2}[(1-b)-c_2-r_2]^2$. We have $f^*_d \geq 0$ if $\delta_1 \leq \frac{(2-b)-2(r_1+c_1)+b(r_2+c_2)}{(2-b^2)} \equiv \tilde{\delta}_{d1}$, and $r_1 \leq \tilde{r}_{d1}$ and $f^*_d \geq 0$ iff $r_2 \leq (1-b\delta_1 - c_2)$ with $(1-b\delta_1 - c_2) < \tilde{r}_{d2}$. In addition, we have $\frac{(2-b)-2(r_1+c_1)+b(r_2+c_2)}{(2-b^2)} < \frac{(2-b)+b(r_2+c_2)}{(2-b^2)}$. Thus, the problem in (49) becomes

$$\max_{r_1, f_1, \delta_1, r_2, f_2, \delta_2} f^*_d + f^*_d + r_1q^*_d + r_2q^*_d$$

s.t. $0 \leq \delta_2 \leq \frac{1-b\delta_1 - c_2 - r_2}{2}$, $\tilde{\delta}_{d1} < \delta_1 \leq \tilde{\delta}_{d1}$, $r_2 \leq (1-b\delta_1 - c_2)$, and $r_1 \leq \tilde{r}_{d1}$.

(185)
Its Lagrange function is

\[
L = \frac{r_2}{2} (1 - b \delta_1 - c_2 - r_2) + r_1 \delta_1 + \frac{1}{2} \left[ \frac{1 - b \delta_1 - c_2 - r_2}{2} \right] + \frac{\delta_1}{4} [(2 - b) - (2 - b^2)] \delta_1
\]

\[
-2(r_1 + c_1) + b(r_2 + c_2)] + \lambda_1 \left( \frac{1 - b \delta_1 - c_2 - r_2}{2} - \delta_2 \right) + \lambda_2 (\delta_1 - \delta_{d1})
\]

\[
+ \lambda_3 (\delta_{d1} - \delta_1) + \lambda_4 [(1 - b \delta_1 - c_2) - r_2] + \lambda_5 (\bar{r}_{d1} - r_1)
\]

Then, the Kuhn-Tucker conditions are

\[\frac{\partial L}{\partial r_1} = \frac{\delta_1}{2} + \frac{2 \lambda_2}{(4 - b^2)} - \frac{2 \lambda_3}{(2 - b^2)} - \lambda_5 \leq 0, \quad r_1 \cdot \frac{\partial L}{\partial r_1} = 0, \quad (A186)\]

\[\frac{\partial L}{\partial r_2} = \frac{(1 - 3r_2 - c_2)}{4} - \frac{\lambda_1}{2} - \frac{b \lambda_2}{(4 - b^2)} + \frac{b \lambda_3}{(2 - b^2)} - \lambda_4 + \frac{b \lambda_5}{2} \leq 0, \quad r_2 \cdot \frac{\partial L}{\partial r_2} = 0, \quad (A187)\]

\[\frac{\partial L}{\partial \delta_1} = \frac{1}{4} [2(1 - b) + 2bc - 2c_1 + 2r_1 - (4 - 3b^2) \delta_1] - \frac{b \lambda_1}{2} + \lambda_2 - \lambda_3 - b \lambda_4 \leq 0, \quad \delta_1 \cdot \frac{\partial L}{\partial \delta_1} = 0, \quad (A188)\]

\[\frac{\partial L}{\partial \delta_2} = -\lambda_1 \leq 0, \quad \delta_2 \cdot \frac{\partial L}{\partial \delta_2} = 0, \quad (A189)\]

\[\frac{\partial L}{\partial \lambda_1} = \frac{1 - b \delta_1 - c_2 - r_2}{2} - \delta_2 \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A190)\]

\[\frac{\partial L}{\partial \lambda_2} = \delta_1 - \delta_{d1} \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad (A191)\]

\[\frac{\partial L}{\partial \lambda_3} = \delta_{d1} - \delta_1 \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \quad (A192)\]

\[\frac{\partial L}{\partial \lambda_4} = (1 - b \delta_1 - c_2) - r_2 \geq 0, \quad \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0, \quad \text{and} \quad (A193)\]

\[\frac{\partial L}{\partial \lambda_5} = \bar{r}_{d1} - r_1 \geq 0, \quad \lambda_5 \cdot \frac{\partial L}{\partial \lambda_5} = 0. \quad (A194)\]

Constraint \(\delta_1 > \delta_{d1}\) in (A185) suggests \(\lambda_2^* = 0\) by (A191). If \(\lambda_1^* > 0\), then \(\frac{\partial L}{\partial \lambda_2} = -\lambda_1 < 0\) and \(\delta_{d1}^* = 0\) by (A189). They in turn suggest \(\frac{\partial L}{\partial \lambda_1} = \frac{1 - b \delta_1 - c_2 - r_2}{2} > 0\) and \(\lambda_1^* = 0\) by (A190). It is a contradiction. Thus, we must have \(\lambda_1^* = 0\). On the other hand, if \(\lambda_5^* > 0\), we have \(\bar{r}_{d1}^* = \bar{r}_{d1}\) by (A194) and \(\delta_{d1} = \delta_{d1}\), which contradicts \(\delta_{d1} < \delta_2 \leq \delta_{d1}\). Thus, we must have \(\lambda_5^* = 0\). Moreover, if \(\lambda_2^* = \lambda_5^* = 0\), (A186) becomes \(\frac{\partial L}{\partial r_1} = \frac{\delta_1}{2} - \frac{2 \lambda_3}{(2 - b^2)} \leq 0\), which implies \(\lambda_3^* > 0\).

Based on the values of \(\lambda_4\), we have two sub-cases as follows.
Case 3a: Suppose $\lambda^*_1 = 0$. Then (A186)-(A188) and (A192) suggest $[\frac{1}{4}(1-3r_2-c_2)] + \frac{b\lambda_3}{2-b^2} - \lambda_4 = 0$, $[\frac{1}{2}\delta_1 - \frac{2\lambda_4}{2-b^2}] = 0$, $\frac{1}{4}[2(1-b)+2bc_2-2c_1+2r_1-(4-3b^2)\delta_1] - \lambda_3 = 0$, and $\delta_1 = \frac{\bar{c}_1}{2(2-b^2)+b(r_2+c_2)}$. Solving these equations yields $r^*_{d_1} = \frac{(2-b^2)^2-2(1-b^2)c_2-bc_1}{2(2-b^2)} > 0$, $r^*_{d_2} = 0$, $\delta^*_{d_1} = \frac{3-2b^2+2bc_2-3c_1}{2(3-2b^2)} > 0$, and $\lambda^*_3 = \frac{(2-b^2)[(3-2b^2)+2bc_2-3c_1]}{8(3-2b^2)} > 0$. Note that $f_{d_1} = 0$ by $\delta^*_{d_1} = \frac{(3-2b^2)+2bc_2-3c_1}{2(3-2b^2)}$, $(1-b)c_1 - c_2 > 0$ if $c_2 \leq (1-b) + bc_1$, and $(r^*_{d_1} - r^*_{d_2}) = \frac{(2-b^2)(3-2b^2)+2bc_2-3c_1}{4(3-2b^2)} > 0$. Moreover, (A190) implies $\delta^*_2 \in [0, \frac{(1-b)-c_2+bc_1}{(3-2b^2)}]$ and $f^*_{d_2} = \frac{1}{2}[(1-b)-c_2+bc_1]^2 > 0$ if $c_2 \leq (1-b) + bc_1$. Thus, for $c_2 \leq (1-b) + bc_1$, port authority’s equilibrium fee revenue equals

$$R^*_d = \frac{5-4b) - 4(1-b)c_2 - 2(3-2b)c_1 - 4bc_1c_2 + 3c_1^2 + 2c_2^2}{4(3-2b^2)} \equiv R^*_4. \quad (A195)$$

Case 3b-1: Suppose $\lambda^*_1 > 0$, $r^*_{d_1} > 0$, and $r^*_{d_2} > 0$. Then (A186)-(A188) and (A192)-(A193) suggest $[\frac{1}{4}(1-3r_2-c_2)] + \frac{b\lambda_3}{2-b^2} - \lambda_4 = 0$, $[\frac{1}{2}\delta_1 - \frac{2\lambda_4}{2-b^2}] = 0$, $\frac{1}{4}[2(1-b)+2bc_2-2c_1+2r_1-(4-3b^2)\delta_1] - \lambda_3 = 0$, $\delta_1 = \frac{\bar{c}_1}{2(2-b^2)+b(r_2+c_2)}$, and $r_2 = (1-b) - c_2$. Solving these equations yields $r^*_{d_1} = \frac{1-c_1}{2} > 0$, $r^*_{d_2} = \frac{[(2-b)+bc_1-2c_2]}{2}$, $\delta^*_{d_1} = \frac{1-c_1}{2} > 0$, $\lambda^*_3 = \frac{1-c_1}{2}$, $\lambda^*_4 = \frac{1-c_1}{2}$. Note that $f^*_{d_1} = 0$ due to $\delta^*_{d_1} = \frac{1-c_1}{2}$, $f^*_{d_2} = 0$ due to $q^*_{d_2} = 0$, $\lambda^*_1 > 0$ if $c_2 > (1-b) + bc_1$, $r^*_{d_2} > 0$ if $c_2 < \frac{1}{2}[(2-b)+bc_1]$ with $\frac{[(2-b)+bc_1]}{2} > (1-b) + bc_1$, and $(r^*_{d_1} - r^*_{d_2}) = \frac{(2-b^2)(1-c_1)}{4} > 0$. Thus, for $[(1-b)+bc_1] < c_2 < \frac{1}{2}[(2-b)+bc_1]$, port authority’s equilibrium fee revenue equals

$$R^*_d = r^*_{d_1} \cdot \delta^*_{d_1} \cdot \lambda^*_3 \cdot \frac{1-c_1}{2} \equiv R^*_5. \quad (A196)$$

Obviously, we have $r^*_{d_2} < 0$ if $c_2 > \frac{[(2-b)+bc_1]}{2}$ from the above. Thus, $r^*_{d_2} = 0$ for large $c_2$, and we have the following sub-cases.

Case 3b-2: Suppose $\lambda^*_1 > 0$, $r^*_{d_1} > 0$, and $r^*_{d_2} = 0$. Then (A186)-(A188) and (A192)-(A193) suggest $[\frac{1}{4}(1-3r_2-c_2)] + \frac{b\lambda_3}{2-b^2} - \lambda_4 \leq 0$, $[\frac{1}{2}\delta_1 - \frac{2\lambda_4}{2-b^2}] \leq 0$, $\frac{1}{4}[2(1-b)+2bc_2-2c_1+2r_1-(4-3b^2)\delta_1] - \lambda_3 = 0$, $\delta_1 = \frac{\bar{c}_1}{2(2-b^2)+b(r_2+c_2)}$, and $r_2 = (1-b) - c_2 = 0$. Solving these equations yields $r^*_{d_1} = \frac{[(1-b)-bc_1+c_2]}{b}$, $r^*_{d_2} = 0$, $\delta^*_{d_1} = \frac{1-c_1}{b} > 0$, $\lambda^*_3 = \frac{(2-b^2)(1-c_2)}{4b} > 0$, and $\lambda^*_4 = \frac{(-4-2b^2)-(2bc_1+(4-b^2)c_2)}{2b^2}$. Note
that $f_{d1}^* = 0$ due to $\delta_{d1}^* = \tilde{\delta}_{d1} = \frac{1-c_2}{b}$, $f_{d2}^* = 0$ due to $\delta_{d2}^* = 0$, $\lambda_4^* > 0$ iff $c_2 > \frac{(4-2b-b^2)+2bc_1}{(4-b^2)}, \left(\frac{(1-3r_2-c_2)}{2} + \frac{b\lambda_3}{(2-b^2)} - \lambda_4 \right) = \frac{(2-b)+bc_1-2c_2}{b^2} \leq 0$ iff $c_2 \geq \frac{[2-b]+bc_1}{2}$. With $\frac{[2-b]+bc_1}{2} > \frac{(4-2b-b^2)+2bc_1}{(4-b^2)} > (1-b) + bc_1$, and $(\bar{r}_{d1} - \bar{r}_{d1}^*) = \frac{(2-b^2)(1-c_1)}{4} > 0$. Thus, for $c_2 \geq \frac{[2-b]+bc_1}{2}$, port authority’s equilibrium fee revenue equals

$$R_\delta^* = r_{d1}^* \cdot \delta_{d1}^* = \frac{1}{b^2}(1-c_2)[-1 - b - bc_1 + c_2] \equiv R_6^*. \quad (A197)$$

Case 4: Suppose $\delta_1 \in (\frac{1-b\delta_2-c_1-r_1}{2}, (1-b\delta_2))$ and $\delta_2 \in (\frac{1-b\delta_1-c_2-r_2}{2}, (1-b\delta_1))$. Then, Lemma 16(iv) implies $\pi_{d1} = \delta_1[1 - \delta_1 - b\delta_j - c_i - r_i] - f_i$ for $i, j \in \{1, 2 \mid i \neq j\}$. Accordingly, we have $f_{d1}^* = \delta_1[1-\delta_1-b\delta_2-(r_1+c_1)]$ and $f_{d2}^* = \delta_2[1-\delta_2-b\delta_1-(r_2+c_2)]$. Moreover, $f_{d1}^* \geq 0$ iff $\delta_1 \leq [1 - b\delta_2 - r_1 - c_1]$ and $r_1 \leq \bar{r}_{d1}$, and $f_{d2}^* \geq 0$ iff $\delta_2 \leq [1 - b\delta_1 - (r_2 + c_2)]$ and $r_2 \leq \bar{r}_{d2}$. Then, the problem in (49) becomes

$$\max_{r_1, f_1, \delta_1, r_2, f_2, \delta_2} \quad f_{d1}^* + f_{d2}^* + r_1q_{d1}^* + r_2q_{d2}^*$$

s.t. $\frac{(1-b\delta_2-c_1-r_1)}{2} < \delta_1 \leq [1 - b\delta_2 - r_1 - c_1], r_1 \leq \bar{r}_{d1},$

$$\frac{(1-b\delta_1-c_2-r_2)}{2} < \delta_2 \leq [1 - b\delta_1 - (r_2 + c_2)], \text{ and } r_2 \leq \bar{r}_{d2}. \quad (A198)$$

Its Lagrange function is

$$L = r_1\delta_1 + r_2\delta_2 + \delta_1[1 - \delta_1 - b\delta_2 - (r_1 + c_1)] + \delta_2[1 - \delta_2 - b\delta_1 - (r_2 + c_2)]$$

$$+ \lambda_1[\delta_1 - \frac{(1-b\delta_2-c_1-r_1)}{2}] + \lambda_2[(1-b\delta_2-r_1-c_1)-\delta_1] + \lambda_3[\delta_2 - \frac{(1-b\delta_1-c_2-r_2)}{2}] + \lambda_4[1-b\delta_1-(r_2+c_2)-\delta_2] + \lambda_5(\bar{r}_{d1}-r_1) + \lambda_6(\bar{r}_{d2}-r_2).$$

Then, the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r_1} = \frac{1}{2}\delta_1 + \frac{1}{2}\lambda_1 - \lambda_2 - \lambda_5 + \frac{b}{2}\lambda_6 \leq 0, \quad r_1 \cdot \frac{\partial L}{\partial r_1} = 0, \quad (A199)$$

$$\frac{\partial L}{\partial r_2} = \frac{1}{2}\delta_2 + \frac{1}{2}\lambda_3 - \lambda_4 + \frac{b}{2}\lambda_5 - \lambda_6 \leq 0, \quad r_2 \cdot \frac{\partial L}{\partial r_2} = 0, \quad (A200)$$

$$\frac{\partial L}{\partial \delta_1} = \frac{1}{2}[1 - 2\delta_1 - 2b\delta_2 - c_1 + r_1] + \lambda_1 - \lambda_2 + \frac{b}{2}\lambda_3 - b\lambda_4 \leq 0, \quad \delta_1 \frac{\partial L}{\partial \delta_1} = 0, \quad (A201)$$
\[
\frac{\partial L}{\partial \delta_2} = \frac{1}{2}[1 - 2\delta_2 - 2b\delta_1 - c_2 + r_2] + \frac{b}{2}\lambda_1 - b\lambda_2 + \lambda_3 - \lambda_4 \leq 0, \quad \delta_2 \cdot \frac{\partial L}{\partial \delta_2} = 0, \quad (A202)
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta_1 - \frac{(1 - b\delta_2 - c_1 - r_1)}{2} \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A203)
\]

\[
\frac{\partial L}{\partial \lambda_2} = (1 - b\delta_2 - r_1 - c_1) - \delta_1 \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad (A204)
\]

\[
\frac{\partial L}{\partial \lambda_3} = \delta_2 - \frac{(1 - b\delta_1 - c_2 - r_2)}{2} \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \quad (A205)
\]

\[
\frac{\partial L}{\partial \lambda_4} = 1 - b\delta_1 - (r_2 + c_2) - \delta_2 \geq 0, \quad \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0, \quad (A206)
\]

Constraints \((\frac{1 - b\delta_2 - c_1 - r_1}{2}) < \delta_1\) and \((\frac{1 - b\delta_1 - c_2 - r_2}{2}) < \delta_2\) in (A198) suggest \(\lambda_1^* = \lambda_3^* = 0\) by (A203) and (A205). If \(\lambda_5^* > 0\), we have \(r_{d1}^* = \bar{r}_{d1}\) by (A207) and \(\dot{\delta}_{d1} = 0\). Then, we get \(\delta_1 \leq (1 - b\delta_2 - \bar{r}_{d1} - c_1) < 1 - \frac{b(1 - b\delta_1 - c_2 - r_2)}{2} - \bar{r}_{d1} - c_1 = \frac{\delta_1^2}{2}\). This is a contraction. Thus, we must have \(\lambda_5^* = 0\). Similarly, \(\lambda_6^* = 0\) can be shown by (A208). Moreover, we have \(\frac{\partial L}{\partial \bar{r}_{d1}} = \frac{\delta_1}{2} - \lambda_2 \leq 0\) by (A199), \(\frac{\partial L}{\partial r_2} = \frac{\delta_2}{2} - \lambda_4 \leq 0\) by (A200), \(\delta_1 > 0\) and \(\delta_2 > 0\) due to \(r_{d1}^* < \bar{r}_{d1}\) and \(r_2 < \bar{r}_{d2}\). Thus, we must have \(\lambda_5^* > 0\) and \(\lambda_4^* > 0\).

Then, (A199)-(A202), (A204) and (A206) suggest \((\frac{\delta_1}{2} - \lambda_2) = 0, \quad (\frac{\delta_2}{2} - \lambda_4) = 0, \quad \frac{1}{2}[1 - 2\delta_1 - 2b\delta_2 - c_1 + r_1] - \lambda_2 - b\lambda_4 = 0, \quad \frac{1}{2}[1 - 2\delta_2 - 2b\delta_1 - c_2 + r_2] - b\lambda_2 - \lambda_4 = 0, \quad [(1 - b\delta_2 - r_1 - c_1) - \delta_1] = 0, \quad [1 - b\delta_1 - (r_2 + c_2) - \delta_2] = 0\). Solving these equations yields \(r_{d1}^* = \frac{1 - c_1}{2} > 0, \quad r_{d2}^* = \frac{1 - c_2}{2}, \quad \delta_{d1}^* = \frac{(1 - b) - c_1 + bc_2}{2(1 - b^2)} > 0, \quad \delta_{d2}^* = \frac{(1 - b) + bc_1 - c_2}{2(1 - b^2)} > 0, \quad \lambda_2^* = \frac{(1 - b) - c_1 + bc_2}{4(1 - b^2)} > 0, \quad \lambda_4^* = \frac{(1 - b) + bc_1 - c_2}{4(1 - b^2)} > 0\). Note that \(f_{d1}^* = 0\) due to \(\delta_{d1}^* = (1 - b\delta_{d2}^* - r_{d1}^* - c_1), \quad f_{d2}^* = 0\) due to \(\delta_{d2}^* = (1 - b\delta_{d1}^* - r_{d2}^* - c_2), \quad \lambda_4^* > 0\) if \(c_2 < (1 - b + bc_1), \quad (\bar{r}_{d1} - r_{d1}^*) = \frac{(2 - b) - 2c_1 + bc_2}{4} > 0, \quad (\bar{r}_{d2} - r_{d2}^*) = \frac{(2 - b) + bc_1 - 2c_2}{4} \geq 0\) if \(c_2 \leq \frac{1}{2}[(2 - b) + bc_1] \) with \(\frac{1}{2}[(2 - b) + bc_1] > (1 - b + bc_1)\). Thus, for \(c_2 < [(1 - b) + bc_1]\), port authority’s equilibrium fee revenue equals

\[
R_d^* = r_{d1}^* \cdot \delta_{d1}^* + r_{d2}^* \cdot \delta_{d2}^* = \frac{(1 - c_1)[(1 - b) - c_1 + bc_2] + (1 - c_2)[(1 - b) + bc_1 - c_2]}{4(1 - b^2)} = R_7^*.
\]
By comparing port authority’s equilibrium fee revenues derived in Cases 1-4, we can obtain optimal concession contracts. We first compare critical points \( \hat{c}_{d2} \equiv \frac{(3-2b)+2bc_1}{3} \) in Case 1a, \( \hat{c}_{d2} \equiv \frac{(3-b)+bc_1}{3} \) in Case 1b, \( \hat{c}_{d2} \equiv (1-b) + bc_1 \) in Case 3a, and \( \hat{c}_{d2} \equiv \frac{1}{2}[(2-b) + bc_1] \) in Case 3b. Since \( (\hat{c}_{d2} - \hat{c}_{d2}) = \frac{b(1-c_1)}{6} > 0 \), \( (\hat{c}_{d2} - \hat{c}_{d2}) = \frac{b(1-c_1)}{6} > 0 \), and \( (\hat{c}_{d2} - \hat{c}_{d2}) = \frac{b(1-c_1)}{3} > 0 \), we have \( \hat{c}_{d2} < \hat{c}_{d2} < \hat{c}_{d2} < \hat{c}_{d2} \). Thus, there are four situations below.

First, for \( c_2 < \hat{c}_{d2} \), equilibria of \( R_1^* \) in (A172) for \( c_2 \leq \hat{c}_{d2} \) of Case 1a, \( R_3^* \) in (A184) for \( c_2 < \hat{c}_{d2} \) of Case 2a, \( R_4^* \) in (A195) for \( c_2 \leq \hat{c}_{d2} \) of Case 3a, and \( R_5^* \) in (A209) for \( c_2 < \hat{c}_{d2} \) of Case 4 exist. Since \( (R_1^* - R_3^*) = \frac{((1-b)-c_1+bc_1)^2}{4(1-b)^2(3-2b^2)} > 0 \), \( (R_1^* - R_4^*) = \frac{((1-b)+bc_1-c_2)^2}{4(1-b)^2(3-2b^2)} > 0 \), and \( (R_3^* - R_1^*) = \frac{((3-2b)+2bc_1-3c_2)^2}{4(9-4b^2)(3-2b^2)} > 0 \), \( R_1^* \) is optimal. Thus, port authority’s best choices are \( r_{d1}^* = \frac{1-c_1}{2}, r_{d2}^* = \frac{1-c_2}{2}, \delta_{d1}^* = \frac{(1-b)-c_1+bc_1}{2(1-b^2)}, \delta_{d2}^* = \frac{(1-b)+bc_1-c_2}{2(1-b^2)}, f_{d1}^* = 0, f_{d2}^* = 0 \), and

\[
R_d^* = \frac{(1-c_1)((1-b)-c_1+bc_2)+(1-c_2)((1-b)+bc_1-c_2)}{4(1-b^2)}
\]

in (A209). These prove Proposition 5(i).

Second, for \( \hat{c}_{d2} \leq c_2 \leq \hat{c}_{d2} \), equilibria of \( R_1^* \) in (A172) for \( c_2 \leq \hat{c}_{d2} \) in Case 1a, \( R_4^* \) in (A184) for \( c_2 < \hat{c}_{d2} \) in Case 2a, and \( R_5^* \) in (A196) for \( c_2 \leq \hat{c}_{d2} \) of Case 3b-1 exist. Since \( (R_4^* - R_3^*) = \frac{(1-c_1)^2}{4} - \frac{(5-4b)-4(1-b)c_1-2(3-2b)c_2-4bc_1c_2+2c_1^2+3c_2^2}{4(3-2b^2)} \) and \( \frac{\partial(R_4^* - R_3^*)}{\partial c_2} = \frac{(3-2b)+2bc_1-3c_2}{4(3-2b^2)} \geq 0 \) for \( c_2 \leq c_2 \leq c_2 \), we have \( (R_4^* - R_3^*) > \frac{(1-c_1)^2}{4} - \frac{(5-4b)-4(1-b)c_1-2(3-2b)c_2-4bc_1c_2+2c_1^2+3c_2^2}{4(3-2b^2)} = \frac{(1-b)^2(1-c_1)^2}{4(1-b^2)} > 0 \). In addition, \( (R_3^* - R_4^*) = \frac{((3-2b)+2bc_1-3c_2)^2}{4(9-4b^2)(3-2b^2)} \geq 0 \). Thus, \( R_4^* \) is optimal, and port authority’s best choices are \( r_{d1}^* = \frac{1-c_1}{2}, r_{d2}^* = \frac{1}{2}[(2-b) + bc_1 - 2c_2], \delta_{d1}^* = \frac{1-c_1}{2}, f_{d1}^* = 0, f_{d2}^* = \delta_{d2}^* = 0 \), and

\[
R_d^* = r_{d1}^* \cdot \delta_{d1}^* = \frac{(1-c_1)^2}{4}
\]

in (A196). These prove Proposition 5(ii) with \( c_2 \in [\hat{c}_{d2}, \hat{c}_{d2}] \).

Third, for \( \hat{c}_{d2} < c_2 < \hat{c}_{d2} \), equilibria of \( R_2^* \) in (A173) for \( \hat{c}_{d2} < c_2 \leq \hat{c}_{d2} \) of Case 1b and \( R_5^* \) in (A196) for \( c_2 < c_2 < \hat{c}_{d2} \) of Case 3b-1 exist. Since \( (R_5^* - R_2^*) = \frac{(1-c_1)^2}{4} - \frac{(1-c_1)^2}{6} > 0 \), \( R_5^* \) is optimal, the same as the second situation. These prove Proposition 5(ii) with \( c_2 \in (\hat{c}_{d2}, \hat{c}_{d2}) \).

Fourth, for \( c_2 \geq \hat{c}_{d2} \), equilibria of \( R_2^* \) in (A173) for \( \hat{c}_{d2} < c_2 \leq \hat{c}_{d2} \) of Case 1b and \( R_6^* \) in (A197) for \( c_2 \geq \hat{c}_{d2} \) of Case 3b-2 exist. Since \( (R_6^* - R_2^*) = \frac{1}{12}((1-}
Lemma 17. Given two-part tariff contract and minimum throughput requirement $(r, f, \delta)$, operators’ optimal behaviors are as follows.

(i) If $\delta \leq \frac{(1-c-r)}{(1+n)}$, the equilibrium cargo-handling amount of operator $k$ is $q_{gk}^* = \frac{(1-c-r)}{(1+n)}$, its equilibrium service price is $p_{gk}^* = \frac{(1+nc+nr)}{(1+n)} < 0$, and its equilibrium profit is $\pi_{gk}^* = (q_{gk}^*)^2 - f$ for $k = 1, 2, \ldots, n$.

(ii) If $\delta > \frac{(1-c-r)}{(1+n)}$, the equilibrium cargo-handling amount of operator $k$ is $q_{gk}^* = \delta$, its equilibrium service price is $p_{gk}^* = (1-n\delta)$, and its equilibrium profit is $\pi_{gk}^* = (1-n\delta-c-r)\delta - f$ for $k = 1, 2, \ldots, n$.

Proof of Lemma 17: The proofs are similar to those of Lemma 1, and thus omitted. □

Lemma 18. Suppose the conditions in (53) hold. Then we have $r_g^* = \frac{n(1-c)}{1+2n}$, $f_g^* = \frac{(1-c)^2}{2(1+2n)}$, and $\delta_g^* \in [0, \frac{(1-c)}{1+2n})$. At the equilibrium, operator $k$ will handle cargo amount $q_{gk}^* = \frac{1-c}{1+2n} > 0$, charge price $p_{gk}^* = \frac{1+n+nc}{1+2n} > 0$, and obtain profit $\pi_{gk}^* = \frac{(1-c)^2}{2(1+2n)^2} > 0$ for $k = 1, 2, \ldots, n$. Moreover, port authority’s fee revenue is $R_g^* = \frac{n(1-c)^2}{2(1+2n)} > 0$.

Proof of Lemma 18: According to Lemma 17, we have two cases as follow.

Case 1: Suppose $\delta \leq \frac{1-c-r}{1+n}$. Since $\pi_{g1}^* = \pi_{g2}^* = \ldots = \pi_{gn}^* = (q_{gk}^*)^2 - f$, we have $f \leq (q_{gk}^*)^2 - f$, and hence $f_g^* = \frac{(q_{gk}^*)^2}{2}$ and $\pi_{gk}^* = \frac{(q_{gk}^*)^2}{2} > 0$. Then, the problem in (54) becomes

$$\max_r \quad \frac{n(q_{gk}^*)^2}{2} + rn[\frac{(1-c-r)}{(1+n)}]$$

s.t. $0 < r \leq \bar{r}_g \equiv (1-c)$.

Its Lagrange function is $L = \frac{n}{2}[(1-c-r)]^2 + rn[\frac{(1-c-r)}{(1+n)}] + \lambda(\bar{r}_g - r)$, and the Kuhn-Tucker
conditions are

\[
\frac{\partial L}{\partial r} = \frac{n(1 - c - r)}{1 + n} - \frac{n(1 - c - r)}{(1 + n)^2} - \frac{rn}{1 + n} - \lambda \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0 \quad \text{and} \quad (A210)
\]

\[
\frac{\partial L}{\partial \lambda} = (1 - c - r) \geq 0, \quad \lambda \cdot \frac{\partial L}{\partial \lambda} = 0. \quad (A211)
\]

According to the values of \( \lambda \), there are two sub-cases.

**Case 1a:** Suppose \( \lambda^* = 0 \). Then (A210) implies \( r^*_g = \frac{n(1-c)}{1+2n} \) with \( (1 - c - r^*) = \frac{(1+n)(1-c)}{1+2n} > 0 \). Since no constraint is imposed on \( \delta^*_g \), it can be any number in interval \([0, \frac{(1-c)}{1+2n}]\). Substituting \( r^*_g \) into Lemma 17(i) yields \( q^*_g = \frac{(1-c)}{1+2n} > 0 \), into (50) yields \( p^*_g = \frac{(1+n+cn)}{1+2n} > 0 \), and into (51) yields \( \pi^*_g = f^*_g = \frac{(1-c)^2}{2(1+2n)} > 0 \). At the equilibrium, port authority’s fee revenue is \( R^*_g = \frac{n(1-c)^2}{2(1+2n)} > 0 \).

**Case 1b:** Suppose \( \lambda^* > 0 \). Then (A211) implies \( r^*_g = (1-c) > 0 \) and \( \lambda^* = \frac{n(c-1)}{1+n} < 0 \), which contradicts \( \lambda^* > 0 \). Thus, no solution exists in this case.

**Case 2:** Suppose \( \delta > \frac{1-c-r}{1+n} \). Then Lemma 17(ii) implies \( q^*_g = \delta \) with \( \delta < \frac{1}{n} \) and \( \pi^*_g = (1-n\delta - c - r)\delta - f \). On the other hand, since \( \pi^*_g = \pi^*_g = \ldots = \pi^*_g = (1-n\delta - c - r)\delta - f \), we have \( f \leq (1-n\delta - c - r)\delta - f \), and thus \( f^*_g = \frac{(1-n\delta - c-r)\delta}{2} \) and \( \pi^*_g = \frac{(1-n\delta - c-r)\delta}{2} > 0 \). Accordingly, the problem in (54) becomes

\[
\max_{r, \delta} \quad \frac{n\delta(1-n\delta - c - r)}{2} + nr\delta \\
\text{s.t.} \quad 0 < r \leq (1-c) \quad \text{and} \quad \frac{1-c-r}{1+n} < \delta \leq \frac{1-c-r}{n}.
\]

Since \( \delta \leq \frac{1-c-r}{n} \) implies \( r \leq (1-c-n\delta) \) and \( \delta > \frac{1-c-r}{(1+n)} \) implies \( r > [1-c-(n+1)\delta] \), this problem can be reduced to

\[
\max_{r, \delta} \quad R = \frac{n\delta(1-n\delta - c - r)}{2} + nr\delta \\
\text{s.t.} \quad [1-c-(n+1)\delta] < r \leq [1-c-n\delta] \text{.}
\]

Due to \( \frac{\partial R}{\partial r} = \frac{n\delta}{2} > 0 \), we have \( r^*_g = (1-c-n\delta) \). Accordingly, at \( r^*_g \), we have \( f^*_g = \pi^*_g = \frac{1-c-r^*_g-n\delta}{2} = 0 \), which contradicts \( f > 0 \). Thus, no solution exists in this case.

The solutions in Case 1 prove Lemma 18. □
Lemma 19. Suppose the conditions in (53) hold. Then we have $r_u = \frac{(1-c)}{2}$ and $\delta_u = \frac{(1-c)}{2n}$. At the equilibrium, operator $k$ will handle cargo amount $q_u^k = \frac{(1-c)}{2n} > 0$, charge price $p_u^k = \frac{1+c}{2} > 0$, and obtain profit $\pi_u = 0$ for $k = 1, 2, \ldots, n$. Moreover, port authority's fee revenue is $R_u = \frac{(1-c)^2}{4} > 0$.

Proof of Lemma 19: The proofs are similar to those of Lemma 18. □

Lemma 20. Suppose the conditions in (53) hold. Then we have $f_f = \frac{(1-c)^2}{2(1+n)^2}$ and $\delta_f \in [0, \frac{(1-c)}{1+n}]$. At the equilibrium, operator $k$ will handle cargo amount $q_f^k = \frac{(1-c)}{1+n} > 0$, charge price $p_f^k = \frac{1+cn}{1+n} > 0$, and obtain profit $\pi_f = \frac{(1-c)^2}{2(1+n)^2} > 0$ for $k = 1, 2, \ldots, n$. Moreover, port authority's fee revenue is $R_f = \frac{n(1-c)^2}{2(1+n)^2} > 0$. □

Proof of Lemma 20: The proofs are similar to those of Lemma 18. □

Proof of Footnote 8: By letting $c_1 = c_2 = c$ and $b = 0$ in Lemma 1, we get optimal cargo-handling amount $q^m$ and equilibrium service price $p^m$ for a monopolistic operator under the two-part tariff scheme $(r, f)$ as follows.

Lemma 21. Given two-part tariff scheme $(r, f)$ and minimum throughput guarantee $\delta$, we obtain the following optimal behaviors for a monopolistic operator.

(i) For $\delta \in [0, \delta_m]$ with $\delta_m = \frac{1-r-c}{2}$, the operator’s equilibrium cargo-handling amount is $q^m = \frac{1-r-c}{2} = \delta_m$, equilibrium service price is $p^m = \frac{1+r+c}{2} > 0$, and equilibrium profit is $\pi^m = (q^m)^2 - f$.

(ii) For $\delta \in (\delta_m, 1)$, the operator’s equilibrium cargo-handling amount is $q^m = \delta$, equilibrium service price is $p^m = (1-\delta) > 0$, and equilibrium profit is $\pi^m = (1-\delta - c-r)\delta - f$.

Based on these outcomes, the port authority will choose $(r^m, f^m, \delta^m)$ to solve the problem of

$$\max_{r, f, \delta} \quad f + rq^m$$

subject to $0 \leq \delta < 1, 0 \leq r \leq (1-c), \pi^m \geq 0$, and $0 \leq f \leq \pi^m$. 

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The solutions are listed in Lemma 22.

**Lemma 22.** Suppose \( r \leq (1 - c) \). Given \((q^m, \pi^m)\) in Lemma 21, the port authority’s optimal concession contract is the unit-fee contract with \( r^m = \frac{(1-c)}{2} \) and \( \delta^m = \frac{(1-c)}{2} \).

At the equilibrium, port authority’s equilibrium fee revenue is \( R^m = \frac{(1-c)^2}{4} \).

**Proof.** According to the sizes of \( \delta \), we have two cases.

**Case 1:** Suppose \( \delta \in [0, \hat{\delta}_m] \). Lemma 21(i) implies \( f^m = \frac{1}{2}(q^m)^2 \). Thus, port authority’s problem becomes

\[
\max_{r, f, \delta} \quad \frac{1}{2}(q^m)^2 + rq^m \\
\text{s.t.} \quad 0 \leq \delta < \hat{\delta}_m \text{ and } 0 \leq r \leq (1 - c).
\]

Its Lagrange function is \( L = \frac{1}{2}(q^m)^2 + rq^m + \lambda_1(\hat{\delta}_m - \delta) + \lambda_2[(1 - c) - r] \), where \( \lambda_1 \) and \( \lambda_2 \) are the Lagrange multipliers associated with the two inequalities. Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = \frac{(1 - 3r - c)}{4} - \lambda_1 - \lambda_2 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \quad \text{(A212)}
\]

\[
\frac{\partial L}{\partial \delta} = -\lambda_1 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad \text{(A213)}
\]

\[
\frac{\partial L}{\partial \lambda_1} = \hat{\delta}_m - \delta \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad \text{and} \quad \text{(A214)}
\]

\[
\frac{\partial L}{\partial \lambda_2} = (1 - c) - r \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0. \quad \text{(A215)}
\]

Based on the values of \( \lambda_1 \) and \( \lambda_2 \), we have four sub-cases.

**Case 1a:** Suppose \( \lambda_1^* = \lambda_2^* = 0 \). Then, \( r^m = \frac{(1-c)}{3} > 0 \) by (A212). It remains to check whether \( r^m \leq (1 - c) \) holds. We can show \( r^m \leq (1 - c) \) by some calculations. Accordingly, \( \delta^m \in [0, \hat{\delta}_m] \) with \( \delta^m = \frac{(1-c)}{3} \) and \( f^m = \frac{(1-c)^2}{18} > 0 \), and port authority’s equilibrium fee revenue is

\[
R^m = f^m + r^m q^m = \frac{(1-c)^2}{6}. \quad \text{(A216)}
\]
Case 1b: Suppose \( \lambda_1^* = 0 \) and \( \lambda_2^* > 0 \). Then, \( r^m = (1-c) > 0 \) by (A215). However, we have \( \lambda_2^* = \frac{-(1-c)}{2} < 0 \) by (A212), which leads to a contradiction. Thus, no solution exists in this case.

Case 1c: Suppose \( \lambda_1^* > 0 \) and \( \lambda_2^* = 0 \). Then, \( \delta^m = \hat{\delta}_m > 0 \) by (A214). This in turn implies \( \lambda_1^* = 0 \) by (A213), which leads to a contradiction. Thus, no solution exists in this case.

Case 1d: Suppose \( \lambda_1^* > 0 \) and \( \lambda_2^* > 0 \). As in Case 1b, we have \( \lambda_1^* = 0 \) by (A212), and no solution exists in this case.

Case 2: Suppose \( \delta \in (\hat{\delta}_m, 1) \). Lemma 21(ii) implies \( f^m = \frac{\delta[1-\delta-r-c]}{2} \). Thus, port authority’s problem becomes

\[
\max_{r, f, \delta} \quad r\delta + \frac{\delta[1-\delta-r-c]}{2}
\]

s.t. \( \hat{\delta}_m < \delta < 1 \) and \( 0 \leq r \leq (1-c) \).

Its Lagrange function is \( L = r\delta + \frac{\delta[1-\delta-r-c]}{2} + \lambda_1(\delta-\hat{\delta}_m) + \lambda_2[(1-r-c)-\delta] + \lambda_3[(1-c)-r] \), where \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are the Lagrange multipliers associated with the three inequalities.

Then, the Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial r} = \frac{\delta}{2} + \frac{\lambda_1}{2} - \lambda_2 - \lambda_3 \leq 0, \quad r \cdot \frac{\partial L}{\partial r} = 0, \quad (A217)
\]

\[
\frac{\partial L}{\partial \delta} = \frac{1 - 2\delta - c + r}{2} + \lambda_1 - \lambda_2 \leq 0, \quad \delta \cdot \frac{\partial L}{\partial \delta} = 0, \quad (A218)
\]

\[
\frac{\partial L}{\partial \lambda_1} = \delta - \hat{\delta}_m \geq 0, \quad \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \quad (A219)
\]

\[
\frac{\partial L}{\partial \lambda_2} = (1-r-c) - \delta \geq 0, \quad \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \quad (A220)
\]

\[
\frac{\partial L}{\partial \lambda_3} = (1-c) - r \geq 0, \quad \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0. \quad (A221)
\]

Since \( \hat{\delta}_m < \delta \), we must have \( \lambda_1^* = 0 \) by (A219). If \( \lambda_3^* > 0 \), then \( r^m = (1-c) > 0 \) by (A221) and \( \delta^m = 0 \) by (A219) and (A220). However, (A217) suggests \( r^m = 0 \) due to \( \frac{\partial L}{\partial r} < 0 \), which leads to a contradiction. Thus, we must have \( \lambda_3^* = 0 \). On the other
hand, if $\lambda_2^* > 0$, then $\frac{\partial L}{\partial r} = \delta_2 \leq 0$, which contradicts the requirement of $\delta > \hat{\delta}_m > 0$. Thus, we must have $\lambda_2^* = 0$. Under the circumstance, (A217), (A218) and (A220) imply $\frac{\delta}{2} = \lambda_2$, $\lambda_2 = \frac{(1-2\delta-c+r)}{2}$, and $(1 - r - c) = \delta$. Solving the three equations yields $r^m = \frac{(1-c)}{2} < (1 - c)$, $\delta^m = \frac{(1-c)}{2}$, and $\lambda_2^* = \frac{(1-c)}{4} > 0$ with $(\delta^m - \hat{\delta}_m) = \frac{(1-c)}{4} > 0$ and $f^m = 0$. Thus, port authority’s equilibrium fee revenue is

$$R_m^* = f^m + q^m r^m = \frac{(1-c)^2}{4}. \quad (A222)$$

Comparing $R_m = \frac{(1-c)^2}{6}$ in (A216) and $R_m^* = \frac{(1-c)^2}{4}$ in (A222), we obtain that port authority’s best choice is the unit-fee contract defined in Case 2 with $R_m = \frac{(1-c)^2}{4}$. $\square$

Finally, since port authority’s equilibrium fee revenue in Proposition 1 with $c_1 = c_2 = c$ is $R_u = \frac{(1-c)^2}{2(1+b)}$, we have $(R_u - R_m^*) = \frac{(1-c)^2}{2(1+b)} - \frac{(1-c)^2}{4} = \frac{(1-b)(1-c)^2}{4(1+b)} > 0$. This implies that the port authority is better off when there are two terminal operators, instead of one, in the market.