Appendix of the Manuscript "International Environmental Agreements Under Different Evolutionary Imitation Mechanisms"

<u>*Proof of Proposition 1*</u>: After some calculations, u is the minimum of k satisfying

$$k(d+b-\frac{c}{2})+\alpha-bn-\frac{c}{2}\geq 0, \tag{1}$$

and v is the minimum of k satisfying

$$k(d+b+\frac{c}{2})-dn-\alpha+\frac{c(n+1)}{2}\geq 0. \tag{2}$$

According to relative sizes of $(d+b)$ and $\frac{c}{2}$, we have three cases below.

<u>Case 1</u>: Suppose $(d+b)<\frac{c}{2}$. We then have $bn+\frac{c}{2}-(1-n)d-b-\frac{c(n+2)}{2}=(n-1)(d+b)-\frac{c(n+1)}{2}<\frac{c(n-1)}{2}-\frac{c(n+1)}{2}=-c<0$, which implies $bn+\frac{c}{2}<l_0\equiv(1-n)d+b+\frac{c(n+2)}{2}$. Accordingly, there are three sub-cases.

First, if $\alpha>l_0$, then (22) fails at $k=1$, and hence $v=\lceil\frac{dn+\alpha-\frac{c(n+1)}{2}}{d+b+\frac{c}{2}}\rceil\geq 2$. On the other hand, $\alpha>l_0$ implies $\alpha>bn+\frac{c}{2}$, which suggests $u=\lceil\frac{\alpha-bn-\frac{c}{2}}{\frac{c}{2}-d-b}\rceil\geq 1$ by (21). Thus, $S_*=\{\vec{C}\}$ and $E(T_\epsilon)=\epsilon^{-u}$ if $u<v$, $S_*=\{\vec{C},\ \vec{D}\}$ and $E(T_\epsilon)=\epsilon^{0}$ if $u=v$, and $S_*=\{\vec{D}\}$ and $E(T_\epsilon)=\epsilon^{-v}$ if $u>v$. These prove Proposition 1(ia).

Second, if $\alpha\in(bn+\frac{c}{2},\ l_0]$, then $u=\lceil\frac{\alpha-bn-\frac{c}{2}}{\frac{c}{2}-d-b}\rceil\geq 1$ by (21), and $v=1$ by $\alpha<l_0$ and (22). Thus, $S_*=\{\vec{C},\ \vec{D}\}$ and $E(T_\epsilon)=\epsilon^{0}$ if $u=1$ and $S_*=\{\vec{D}\}$ and $E(T_\epsilon)=\epsilon^{-1}$ if $u>1$. These prove Proposition 1(ib).

Third, if $\alpha\leq bn+\frac{c}{2}$, then (21) fails for all k, and hence $u=n$. On the other hand, since $\alpha\leq bn+\frac{c}{2}$, we have $\alpha<l_0$, which implies that (22) holds at $k=1$ and $v=1$. Since $u=n>v=1$, $S_*=\{\vec{D}\}$ and $E(T_\epsilon)=\epsilon^{-1}$. These prove Proposition 1(ic).

<u>Case 2</u>: Suppose $(d+b)=\frac{c}{2}$. Under the circumstance, we have $bn+\frac{c}{2}-(1-n)d-b-\frac{c(n+2)}{2}<0$. Thus, there are two sub-cases.

First, if $\alpha>l_0$, then $v\geq 2$ because (22) fails at $k=1$. We have $\alpha>bn+\frac{c}{2}$ by $\alpha>l_0$, which implies $u=1$. Thus, $S_*=\{\vec{C}\}$ and $E(T_\epsilon)=\epsilon^{-1}$ as shown by Proposition

1(iia). Second, if $\alpha \in [bn + \frac{c}{2},\ l_0]$, then $v = 1$ by $\alpha < l_0$ and $u = 1$ by $\alpha > bn + \frac{c}{2}$. Thus, $S_* = \{\vec{C},\ \vec{D}\}$ and $E(T_\epsilon) = \epsilon^0$ as shown by Proposition 1(iib). Third, if $\alpha < bn + \frac{c}{2}$, then (21) fails for all k, and hence $u = n$. But $\alpha < l_0$ implies $v = 1$. Thus, $S_* = \{\vec{D}\}$ and $E(T_\epsilon) = \epsilon^{-1}$ as shown by Proposition 1(iic).

<u>Case 3</u>: Suppose $(d + b) > \frac{c}{2}$. Then, relative sizes of $bn + \frac{c}{2}$ and l_0 become unsure, and there are two sub-cases. First, suppose $\alpha \geq bn + \frac{c}{2}$. Then (21) holds at $k = 1$, and hence $u = 1$. However, if $\alpha > l_0$, then $v \geq 2$, and hence $S_* = \{\vec{C}\}$ and $E(T_\epsilon) = \epsilon^{-1}$. In contrast, if $\alpha \leq l_0$, then $v = 1$, and hence $S_* = \{\vec{C},\ \vec{D}\}$ and $E(T_\epsilon) = \epsilon^0$. These are the content of 1(iiia).

Second, if $\alpha < bn + \frac{c}{2}$, then (21) implies $u = \lceil \frac{bn + \frac{c}{2} - \alpha}{d + b - \frac{c}{2}} \rceil \geq 1$ due to $\frac{bn + \frac{c}{2} - \alpha}{d + b - \frac{c}{2}} > 0$. If $\alpha \leq l_0$, then (22) implies $v = 1$. Accordingly, $S_* = \{\vec{C},\ \vec{D}\}$ and $E(T_\epsilon) = \epsilon^0$ if $u = 1$, and $S_* = \{\vec{D}\}$ and $E(T_\epsilon) = \epsilon^{-1}$ if $u > 1$. These prove Proposition 1(iiia).

In contrast, if $\alpha > l_0$, then (22) fails at $k = 1$. Thus, $v = \lceil \frac{dn + \alpha - \frac{c(n+1)}{2}}{d + b + \frac{c}{2}} \rceil \geq 2$. Accordingly, $S_* = \{\vec{C}\}$ and $E(T_\epsilon) = \epsilon^{-u}$ if $u < v$, $S_* = \{\vec{C},\ \vec{D}\}$ and $E(T_\epsilon) = \epsilon^0$ if $u = v$, and $S_* = \{\vec{D}\}$ and $E(T_\epsilon) = \epsilon^{-v}$ if $u > v$. These prove Proposition 1(iiic).

<u>*Proof of Theorem 1*</u>: Before comparing Propositions 1 and C1, we need to know relative

sizes of the following variables:

$$2(\alpha - bn) - 2\alpha = -2bn < 0, \tag{3}$$

$$2(\alpha - bn) - 2(d - b) = 2(\alpha - d) + 2b(1 - n) < 0 \text{ by } d > \alpha \text{ and } n \geq 2, \tag{4}$$

$$\frac{2[\alpha - (1-n)d - b]}{n+2} - \frac{2[\alpha + (d-b)(n-1)]}{n+2} = \frac{2b(n-2)}{n+2} > 0 \text{ by } n \geq 3, \tag{5}$$

$$2\alpha - \frac{2[\alpha - (1-n)d - b]}{n+2} = \frac{2[\alpha(n+1) - d(n-1) + b]}{n+2}$$

$$\geq \ (\leq)\ 0 \text{ iff } d \leq \ (\geq)\ \frac{b + \alpha(n+1)}{n-1}, \tag{6}$$

$$2\alpha - \frac{2[\alpha + (d-b)(n-1)]}{n+2} = \frac{2[\alpha(n+1) + b(n-1) - d(n-1)]}{n+2}$$

$$\geq \ (\leq)\ 0 \text{ iff } d \leq \ (\geq)\ b + \frac{\alpha(n+1)}{n-1}, \tag{7}$$

$$2(d-b) - \frac{2[\alpha - (1-n)d - b]}{n+2} = \frac{2[3d - b(n+1) - \alpha]}{n+2}$$

$$\geq \ (\leq)\ 0 \text{ iff } d \geq \ (\leq)\ \frac{b(n+1) + \alpha}{3}, \tag{8}$$

$$2(d-b) - \frac{2[\alpha + (d-b)(n-1)]}{n+2} = \frac{2[3(d-b) - \alpha]}{n+2}$$

$$\geq \ (\leq)\ 0 \text{ iff } d \geq \ (\leq)\ b + \frac{\alpha}{3}, \tag{9}$$

$$2(d+b) - \frac{2[\alpha - (1-n)d - b]}{n+2} = \frac{2[3d + n + 3b - \alpha]}{n+2} > 0, \tag{10}$$

$$2(\alpha - bn) - \frac{2[\alpha + (d-b)(n-1)]}{n+2} = \frac{-2[d(n-1) + b(n^2 + 3n - 1) - \alpha(n+1)]}{n+2}$$

$$< 0 \text{ by } n \geq 2, \tag{11}$$

$$(d - b + \alpha) - \frac{2[\alpha - (1-n)d - b]}{n+2} = \frac{-d(n-4) - bn + n\alpha}{n+2} < 0 \text{ by } n \geq 4, \tag{12}$$

$$(d - b + \alpha) - \frac{2[\alpha + (d-b)(n-1)]}{n+2} = \frac{(b-d)(n-4) + \alpha n}{n+2} > 0 \text{ if } d < b + \frac{\alpha}{3}. \tag{13}$$

Next, we need to know relative sizes of the thresholds given in (23)-(33). Some calculations yield

$$b + \frac{\alpha}{3} < \frac{b + \alpha(n+1)}{n-1} < b + \alpha < b + \frac{\alpha(n+1)}{n-1} < \frac{b(n+1) + \alpha}{3}$$

for $n \geq 4$. These inequalities divide the values of d into six mutually exclusive ranges as discussed below.

Case 1: Suppose $d \geq \frac{b(n+1)+\alpha}{3}$. We then have

$$2(\alpha - bn) < 2\alpha < \frac{2[\alpha + (d-b)(n-1)]}{n+2} < \frac{2[\alpha - (1-n)d - b]}{n+2} < 2(d-b) < 2(d+b)$$

by (23), (25), (27), and (28). These inequalities divide the values of c into seven mutually exclusive intervals. At each interval, we can derive the LREs under the two dynamics by Propositions 1 and C1. Under the imitating-the-best-total dynamic; $\vec{D}$ is the LRE for $c > 2(d+b)$ by Proposition 1(ic), $c > 2(d+b)$, $c \geq 2(\alpha - bn)$; $\vec{D}$ is the LRE at $c = 2(d+b)$ by Proposition 1(iic), $c = 2(d+b)$, and $c > 2(\alpha - bn)$; $\vec{D}$ is the LRE for $c \in [\frac{2[\alpha-(1-n)d-b]}{n+2},\ 2(d+b))$ by Proposition 1(iiib), $c < 2(d+b)$, $\alpha \leq (1-n)d + b + \frac{c(n+2)}{2}$, and $c > d + \alpha - b(n-1)$; $\vec{D}$ is the LRE for $c \in (\hat{c},\ \frac{2[\alpha-(1-n)d-b]}{n+2})$ by Proposition 1(iiic), $c > 2(\alpha - bn)$, $c < \frac{2[\alpha-(1-n)d-b]}{n+2}$, and $c > \hat{c}$; $\{\vec{C},\ \vec{D}\}$ is the LRE at $c = \hat{c}$ by Proposition 1(iiic), $c > 2(\alpha - bn)$, and $c < \frac{2[\alpha-(1-n)d-b]}{n+2}$; $\vec{C}$ is the LRE for $c \in (2(\alpha - bn),\ \hat{c})$ by Proposition 1(iiic), $c > 2(\alpha - bn)$, $c < \frac{2[\alpha-(1-n)d-b]}{n+2}$, and $c > \hat{c}$; and $\vec{C}$ is the LRE for $c \in (0,\ 2(\alpha - bn)]$ by Proposition 1(iiia), $c \leq \frac{\alpha - bn}{2}$, and $\alpha > (1-n)d + b + \frac{c(n+2)}{2}$. Here $\hat{c} \in (2(\alpha - bn),\ \frac{2[\alpha-(1-n)d-b]}{n+2})$ satisfies condition

$$\frac{bn + \frac{\hat{c}}{2} - \alpha}{d + b - \frac{\hat{c}}{2}} = \frac{dn + \alpha - \frac{\hat{c}(n+1)}{2}}{d + b + \frac{\hat{c}}{2}}. \tag{14}$$

Under the imitating-the-best-average dynamic; $\vec{D}$ is the LRE for $c > 2(d-b)$ by Proposition C1(ib) and $c \geq 2\alpha$; $\vec{D}$ is the LRE at $c = 2(d-b)$ by Proposition C1(iic) and $c \geq 2\alpha$; $\vec{D}$ is the LRE for $c \in (\hat{c}_c,\ 2(d-b))$ by Proposition C1(iiib), $c > 2\alpha$, and $c > \hat{c}_c$; $\{\vec{C},\ \vec{D}\}$ is the LRE at $c = \hat{c}$ by Proposition C1(iiib), $c > 2\alpha$, and $c = \hat{c}_c$; $\vec{C}$ is the LRE for $c \in (2\alpha,\ \hat{c}_c)$ by Proposition C1(iiib), $c > 2\alpha$, and $c < \hat{c}_c$; $\vec{C}$ is the LRE at $c = 2\alpha$ by Proposition C1(iiia), $c < 2(d-b)$, and $c = 2\alpha$; and $\vec{C}$ is the LRE for $c \in (0,\ 2\alpha)$ by Proposition C1(iiia), $c < 2\alpha$, and $\alpha > (1-n)d + b + \frac{c(n+2)}{2}$. Here $\hat{c}_c \in (2\alpha,\ 2(d-b))$ satisfies condition

$$\frac{\frac{\hat{c}_c}{2} - \alpha}{d - b - \frac{\hat{c}_c}{2}} = \frac{(d - b - \frac{\hat{c}_c}{2})n + \alpha - \frac{\hat{c}_c}{2}}{d - b + \frac{\hat{c}_c}{2}}. \tag{15}$$

In summary, the c interval making $\vec{C}$ the LRE under the imitating-the-best-average dynamic is $(0,\ \hat{c}_c]$, and the associated interval under the imitating-the-best-total dynamic is $(0,\ \hat{c}]$. Thus, if $\hat{c}_c > \hat{c}$, then $\vec{C}$ is more likely to be the LRE under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic. The converse will hold if $\hat{c}_c < \hat{c}$, and both dynamics will make $\vec{C}$ emerge equally likely if $\hat{c}_c = \hat{c}$.

Case 2: Suppose $b + \frac{\alpha(n+1)}{n-1} \leq d < \frac{b(n-1)+\alpha}{3}$. We then have

$$2(\alpha - bn) < 2\alpha < \frac{2[\alpha + (d-b)(n-1)]}{n+2} < 2(d-b) < \frac{2[\alpha - (1-n)d - b]}{n+2} < 2(d+b)$$

by (23), (27), (28), (29), and (30). These inequalities divide the values of c into seven mutually exclusive intervals. Proposition 1 implies that the c interval making $\vec{C}$ the LRE under the imitating-the-best-average dynamic is $(0,\ \hat{c}_c]$, and the associated interval under the imitating-the-best-total dynamic is $(0,\ \hat{c}]$ by Proposition C1. Thus, the conclusions are same as Case 1's.

Case 3: Suppose $(b+\alpha) \leq d < b + \frac{\alpha(n+1)}{n-1}$. We then have

$$2(\alpha - bn) < \frac{2[\alpha + (d-b)(n-1)]}{n+2} < 2\alpha < 2(d-b) < \frac{2[\alpha - (1-n)d - b]}{n+2} < 2(d+b)$$

by (27), (28), (30), (31), and $d \geq b + \alpha$. These inequalities divide the values of c into seven mutually exclusive intervals. According to Propositions 1 and C1, the c interval making $\vec{C}$ the LRE under the imitating-the-best-average dynamic is $(0,\ \hat{c}_c]$, and the associated interval under the imitating-the-best-total dynamic is $(0,\ \hat{c}]$. Again, the results are the same as Case 1's.

Case 4: Suppose $\frac{b+\alpha(n-1)}{n-1} \leq d < (b+\alpha)$. We then have

$$2(\alpha - bn) < \frac{2[\alpha + (d-b)(n-1)]}{n+2} < 2(d-b) < (d-b+\alpha) < 2\alpha < \frac{2[\alpha - (1-n)d - b]}{n+2}$$
$$< 2(d+b)$$

by (26), (29), (30), (31), and $(d-b) < \alpha$. These inequalities divide the values of c into eight mutually exclusive intervals. At each interval, we can derive the LREs under both dynamics by Propositions 1 and C1. Under the imitating-the-best-total dynamic; $\vec{D}$ is the LRE for $c > 2(d+b)$ by Proposition 1(ic); $\vec{D}$ is the LRE at $c = 2(d+b)$ by Proposition 1(iic); $\vec{D}$ is the LRE for $c \in [\frac{2[\alpha-(1-n)d-b]}{n+2},\ 2(d+b))$ by Proposition 1(iiib), $c > 2(\alpha - bn)$, $\alpha \leq (1-n)d + b + \frac{c(n+2)}{2}$, and $\lceil \frac{bn+c/2-\alpha}{d+b-c/2} \rceil > 1$;[1] $\vec{D}$ is the LRE for $c \in (\hat{c},\ \frac{2[\alpha-(1-n)d-b]}{n+2})$ by Proposition 1(iiic), $c > 2(\alpha - bn)$, $c < \frac{2[\alpha-(1-n)d-b]}{n+2}$, and $c > \hat{c}$; $\{\vec{C},\ \vec{D}\}$ is the LRE at $c = \hat{c}$ by Proposition 1(iiic), $c > 2(\alpha - bn)$, and $c < \frac{2[\alpha-(1-n)d-b]}{n+2}$; $\vec{C}$ is the LRE for $c \in (2(\alpha - bn),\ \hat{c})$ by Proposition 1(iiic), $c > 2(\alpha - bn)$, $c < \frac{2[\alpha-(1-n)d-b]}{n+2}$, and $c < \hat{c}$; and $\vec{C}$ is the LRE for $c \in (0,\ 2(\alpha - bn)]$ by Proposition 1(iiia), $c \leq \frac{\alpha-bn}{2}$, and $\alpha > (1-n)d + b + \frac{c(n+2)}{2}$. Here $\hat{c} \in (2(\alpha - bn),\ \frac{2[\alpha-(1-n)d-b]}{n+2})$ satisfies (34).

Under the imitating-the-best-average dynamic; $\vec{D}$ is the LRE for $c \geq 2\alpha$ by Proposition C1(ib), $c > 2(d-b)$, and $c \geq 2\alpha$; $\vec{D}$ is the LRE for $c \in ((d-b+\alpha),\ 2\alpha)$ by Proposition C1(ia), $c > 2(d-b)$, $c < 2\alpha$, and $c > (d-b+\alpha)$; $\{\vec{C},\ \vec{D}\}$ is the LRE for $c \in (2(d-b),\ (d-b+\alpha)]$ by Proposition C1(ia), $c > 2(d-b)$, $c \leq (d-b+\alpha)$, and $\alpha < l_0^c$; $\vec{C}$ is the LRE at $c = 2(d-b)$ by Proposition C1(iia); $\vec{C}$ is the LRE for $c \in [\frac{2[\alpha+(d-b)(n-1]}{n+2},\ 2(d-b))$ by Proposition C1(iiia), $c < 2(d-b)$, and $c < 2\alpha$; and $\vec{C}$ is the LRE for $c \in (0,\ \frac{2[\alpha+(d-b)(n-1)]}{n+2})$ by Proposition C1(iiia), $c < 2(d-b)$, $c \leq 2\alpha$, and $\alpha > l_0^c$.

In summary, the c interval making $\vec{C}$ the LRE under the imitating-the-best-average dynamic is $(0,\ (d-b+\alpha)]$, and the associated interval under the imitating-the-best-total dynamic is $(0,\ \hat{c}]$. Thus, if $(d-b+\alpha) > \hat{c}$, then $\vec{C}$ is more likely to be the LRE under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic. The converse will hold if $(d-b+\alpha) < \hat{c}$, and both dynamics will make $\vec{C}$ emerge equally likely if $(d-b+\alpha) = \hat{c}$.

[1] That is because $bn + c/2 - \alpha - [d + b - c/2] = -d + b(n-1) + c - \alpha > b(n-2) > 0$ by $c > d - b + \alpha$ and $n \geq 4$.

Case 5: Suppose $b + \frac{\alpha}{3} \leq d < \frac{\alpha(n-1)+b}{n-1}$. We then have

$$2(\alpha - bn) < \frac{2[\alpha + (d-b)(n-1)]}{n+2} < 2(d-b) < (d-b+\alpha) < \frac{2[\alpha - (1-n)d - b]}{n+2}$$
$$< 2\alpha < 2(d+b)$$

by (26), (29), (31), and (32).[2] These inequalities divide the values of c into eight mutually exclusive intervals. As in Case 4, the c interval making $\vec{C}$ the LRE under the imitating-the-best-average dynamic is $(0,\ (d-b+\alpha)]$, and the associated interval under the imitating-the-best-total dynamic is $(0,\ \hat{c}]$. Thus, the conclusions are the same as Case 4's.

Case 6: Suppose $d \leq b + \frac{\alpha}{3}$. We then have

$$2(\alpha - bn) < 2(d-b) < \frac{2[\alpha + (d-b)(n-1)]}{n+2} < d - b + \alpha < \frac{2[\alpha - (1-n)d - b]}{n+2}$$
$$< 2\alpha < 2(d+b)$$

by (24), (28), (29), (32), and (33).[3] These inequalities divide the values of c into eight mutually exclusive intervals. As in Case 4, the c interval making $\vec{C}$ the LRE under the imitating-the-best-average dynamic is $(0,\ (d-b+\alpha)]$, and the associated interval under the imitating-the-best-total dynamic is $(0,\ \hat{c}]$. Again, the conclusions are the same as Case 4's.

In summary, Cases 1-3 show that for $d \geq b+\alpha$, $\vec{C}$ is more likely to emerge when the imitating-the-best-average rule is adopted if $\hat{c} < \hat{c}_c$. For $d < b+\alpha$, the same conclusions can be drawn if $\hat{c} < (d-b+\alpha)$. The two conditions will hold as displayed below, and hence Theorem 1 is proved.

Claim 1. *Suppose* $d \geq b+\alpha$. *We then have* $\hat{c} < \hat{c}_c$ *with* $\hat{c} \in (2(\alpha - bn),\ \frac{2[\alpha-(1-n)d-b]}{n+2})$ and $\hat{c}_c \in (2\alpha,\ 2(d-b))$.

Proof. To simplify the notations, we define $x = \frac{\hat{c}}{2}$ and $y = \frac{\hat{c}_c}{2}$. Solving (34) and (35)

[2] Although $2\alpha > 2(d+b)$ may occur in the inequalities, our results will not change.

[3] Although $2\alpha > 2(d+b)$ may occur in the inequalities, our results will not change.

yields[4]

$$\begin{aligned} x &= \frac{(d+b)(n+1)-\sqrt{(d+b)^2(n+1)^2-n(d+b)[2\alpha+n(d-b)]}}{n} \text{ and} \\ y &= \frac{(d-b)(n+1)-\sqrt{(d-b)^2(n+1)^2-n(d-b)[2\alpha+n(d-b)]}}{n} \end{aligned}$$

Accordingly, we have

$$(x-y) = \frac{[2b(n+1)-\sqrt{A}+\sqrt{B}]}{n},$$

where $A = (d+b)^2(n+1)^2 - n(d+b)[2\alpha + n(d-b)] > 0$ and $B = (d-b)^2(n+1)^2 - n(d-b)[2\alpha + n(d-b)] > 0$ with $A > B$ by $d > b$. To show $x < y$, it is enough to show $A > 4(n+1)^2b^2 + B + 4b(n+1)\sqrt{B}$. Note that $\sqrt{\alpha-\beta} \leq \sqrt{\alpha} - \sqrt{\beta}$ if $\alpha > \beta > 0$. Thus,

$$\begin{aligned} & A - 4(n+1)^2b^2 - B - 4b(n+1)\sqrt{B} \\ =\; & (n+1)^2(4bd-4b^2) - 2bn[(d-b)n+2\alpha] - 4(n+1)b\sqrt{B} \\ >\; & (n+1)^2 4b(d-b) - 2bn[(d-b)n+2\alpha] - 4b(n+1)\sqrt{(d-b)^2(n+1)^2} \\ & +4b(n+1)\sqrt{n(d-b)[(d-b)n+2\alpha]} \\ =\; & 2b\{2(n+1)\sqrt{n(d-b)[(d-b)n+2\alpha]} - n[(d-b)n+2\alpha]\} \\ >\; & 0, \end{aligned}$$

where the first inequality is implied by $\sqrt{B} < \sqrt{(d-b)^2(n+1)^2} - \sqrt{n(d-b)[(d-b)n+2\alpha]}$ and the second inequality is because

$$\begin{aligned} & 4(n+1)^2 n(d-b)[(d-b)n+2\alpha] - n^2[(d-b)n+2\alpha]^2 \\ =\; & n[(d-b)n+2\alpha][4(n+1)^2(d-b) - n^2(d-b) - 2\alpha n] \\ >\; & n[(d-b)n+2\alpha][4(n+1)^2(d-b) - n^2(d-b) - 2n(d-b)] \\ =\; & n(d-b)[(d-b)n+2\alpha][3n^2+6n+4] > 0 \end{aligned}$$

by $-\alpha > -(d-b)$. Thus, we have $x < y$ and $\hat{c} < \hat{c}_c$, which prove Claim 1.

[4]To meet the range requirements, we take the negative roots.

Claim 2. *Suppose* $d < b + \alpha$ *and* $\hat{c} \in (2(\alpha - bn), \frac{2[\alpha-(1-n)d-b]}{n+2})$ *with* $\alpha > bn$. *We then have* $\hat{c} < (d - b + \alpha)$.

Proof. Plugging $\hat{c} = d - b + \alpha$ into (34) yields

$$L \equiv \frac{bn + \frac{d-b+\alpha}{2} - \alpha}{d + b - \frac{d-b+\alpha}{2}} = \frac{d + b(2n-1) - \alpha}{d + 3b - \alpha} \geq 1 \text{ by } n \geq 2, \text{ and}$$

$$R \equiv \frac{dn + \alpha - \frac{(n+1)(d-b+\alpha)}{2}}{d + b + \frac{d-b+\alpha}{2}} = \frac{d(n-1) + b(n-1) - \alpha(n+1)}{3d + b + \alpha}.$$

Thus

$$L - R = \frac{-d^2(n-4) - b^2(n+4) - \alpha^2(n+2) + 2nbd + 2d\alpha(n-1) + 2b\alpha(3n+1)}{[d + 3b - \alpha][3d + b + \alpha]}.$$

Denote $N \equiv -d^2(n-4) - b^2(n+4) - \alpha^2(n+2) + 2nbd + 2d\alpha(n-1) + 2b\alpha(3n+1)$. We then have

$$\begin{aligned} N &> -(n-4)(b+\alpha)^2 - b^2(n+4) - \alpha^2(n+2) + 2nb^2 + 2b\alpha(n-1) + 2b\alpha(3n+1) \\ &= \alpha^2(-2n+2) + b\alpha(6n+8) \\ &> \alpha^2(-2n+2) + \alpha^2(6n+8) \\ &= \alpha^2(4n+10) \\ &> 0. \end{aligned}$$

The first inequality is due to $d < b + \alpha$ and the second inequality is by $\alpha > bn$. These imply $L > R$. Thus, to make $\hat{c}$ satisfy (34), we must have $\hat{c} < (d - b + \alpha)$ to lower L and raise R. Claim 2 is then proved.

Proof of Proposition 2: After some calculations, u is the minimum of k satisfying

$$(dk + \alpha) - b(n-k) \geq \frac{c[2k^2 + 3k + 1]}{6}, \tag{16}$$

and v is the minimum of k satisfying

$$(d+b)k - \alpha - dn \geq \frac{-c[2(n^2 + nk + k^2) + 3(n+k) + 1]}{6}. \tag{17}$$

Define $g(k) \equiv 2k^2 + 3k + 1 - \frac{6[(d+b)k+\alpha-bn]}{c}$ with $g'(k) = 4k + 3 - \frac{6(d+b)}{c}$ and $g''(k) = 4 > 0$ for all k. These imply that $g(k)$ is a strictly convex function of k with $g'(k) \geq \ (\leq)\ 0$

iff $k \geq\ (\leq)\ \underline{k} \equiv \frac{6(d+b)-3c}{4c}$, $g'(1) = 7 - \frac{6(d+b)}{c} \geq\ (\leq)\ 0$ iff $c \geq\ (\leq)\ \frac{6(d+b)}{7}$, $\underline{k} \geq\ (\leq)\ 1$ iff $c \leq\ (\geq)\ \frac{6(d+b)}{7}$, and $g(1) \geq\ (\leq)\ 0$ iff $c \geq\ (\leq)\ [d + b(1-n) + \alpha]$. Since u is the minimum of k satisfying $g(k) \leq 0$ by (36), it depends on the values of $g'(1)$ and $g(1)$. Thus, if $c < \frac{6(d+b)}{7}$, we have $g'(1) < 0$ and $\underline{k} > 1$, which suggest

$$u = \begin{cases} 1 & \text{if } g(1) \leq 0, \\ \lceil k_g \rceil & \text{if } g(1) > 0 \text{ and } g(\underline{k}) < 0, \\ n & \text{if } g(1) > 0 \text{ and } g(\underline{k}) > 0, \end{cases} \tag{18}$$

where k_g satisfies $g(k_g) = 0$. For $c = \frac{6(d+b)}{7}$, we have $g'(1) = 0$ and $\underline{k} = 1$, which imply

$$u = \begin{cases} 1 & \text{if } g(1) \leq 0, \\ n & \text{if } g(1) > 0. \end{cases} \tag{19}$$

In contrast, if $c > \frac{6(d+b)}{7}$, we have $g'(1) > 0$ and $\underline{k} < 1$, which suggest

$$u = \begin{cases} 1 & \text{if } g(1) \leq 0, \\ n & \text{if } g(1) > 0. \end{cases} \tag{20}$$

On the other hand, define $h(k) \equiv 2(n^2+nk+k^2)+3(n+k)+1+\frac{6}{c}[(d+b)k-\alpha-dn]$ based on $k \geq 1$ with $h'(k) = 2n+4k+3+\frac{6(d+b)}{c} > 0$ for all $k \geq 1$, $h''(k) = 4 > 0$ for all $k \geq 1$, $h'(1) = 2n+7+\frac{6(d+b)}{c} > 0$, and $h(1) = 2n^2+5n+6+\frac{6(d+b)}{c} - \frac{6(\alpha+dn)}{c} \geq\ (\leq)\ 0$ iff $c \geq\ (\leq)\ \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}$. These suggest that $h(k)$ is a strictly convex function with minimum value $h(1)$. Accordingly, by (37), we have

$$v = \begin{cases} 1 & \text{if } h(1) \geq 0, \\ \lceil k_h \rceil \geq 2 & \text{if } h(1) < 0, \end{cases} \tag{21}$$

where k_h satisfies $h(k_h) = 0$.

Now we are ready to get relative sizes of u and v by comparing $g(k)$, $h(k)$, and (38)-(41). First, we need to know relative sizes of $\frac{6(d+b)}{7}$, $\frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}$, and $[d+\alpha+(1-n)b]$.

Some calculations show

$$\tfrac{6(d+b)}{7} - [d+\alpha+(1-n)b] = \tfrac{-[d+7\alpha-b(7n-1)]}{7} \geq \ (\leq)\ 0 \text{ iff } d \leq \ (\geq)\ b(7n-1)-7\alpha, \tag{22}$$

$$\tfrac{6(d+b)}{7} - \tfrac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} = \tfrac{6[d(2n^2-2n+13)+b(2n^2+5n+13)-7\alpha]}{7(2n^2+5n+6)} > 0 \text{ by } d > b > \alpha, \tag{23}$$

$$[d+\alpha+(1-n)b] - \tfrac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} = \tfrac{[d(2n^2-n+12)-b(2n^3+3n^2+n-12)+\alpha(2n^2+5n)]}{2n^2+5n+6}$$
$$\geq \ (\leq)\ 0 \text{ iff } d \geq \ (\leq)\ \tfrac{b(2n^3+3n^2+n-12)-\alpha(2n^2+5n)}{2n^2-n+12}. \tag{24}$$

Moreover, since $[b(7n-1)-7\alpha] > \frac{b(2n^3+3n^2+n-12)-\alpha(2n^2+5n)}{2n^2-n+12}$, (42)-(44) divide the values of d into three mutually exclusive ranges as stated below.

Case 1: Suppose $d \geq [b(7n-1)-7\alpha]$. We then have

$$l_1 \equiv \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d+b)}{7} < [d+\alpha+(1-n)b].$$

These inequalities divide the values of c into four mutually exclusive intervals.

Case 1a: Suppose $c \geq [d+\alpha+(1-n)b]$. We have $h(1) > 0$ by $c > l_1$ and $v = 1$ by (41). Then $g(1) > 0$ by $c \geq d+\alpha+b(1-n)$ and $g'(1) > 0$ by $c > \frac{6(d+b)}{7}$, which imply $u = n$ by (40). Thus, $S_* = \{\vec{D}\}$ and $E(T_\epsilon) = \epsilon^{-1}$.

Case 1b: Suppose $\frac{6(d+b)}{7} \leq c < [d+\alpha+(1-n)b]$. We have $h(1) > 0$ by $c > l_1$ and $v = 1$ by (41). Then $g(1) < 0$ by $c < d+\alpha+b(1-n)$ and $g'(1) \geq 0$ by $c \geq \frac{6(d+b)}{7}$, which imply $u = 1$ by (39)-(40). Thus, $S_* = \{\vec{C},\ \vec{D}\}$ and $E(T_\epsilon) = \epsilon^0$.

Case 1c: Suppose $l_1 \leq c < \frac{6(d+b)}{7}$. We have $h(1) \geq 0$ by $c \geq l_1$ and $v = 1$ by (41). Then $g(1) < 0$ by $c < [d+\alpha+b(1-n)]$ and $g'(1) < 0$ by $c < \frac{6(d+b)}{7}$, which imply $u = 1$ by (38). Thus, $S_* = \{\vec{C},\ \vec{D}\}$ and $E(T_\epsilon) = \epsilon^0$.

Case 1d: Suppose $c < l_1$. We have $h(1) < 0$ by $c < l_1$ and $v = \lceil k_h \rceil \geq 2$ by (41). Then $g(1) < 0$ by $c < [d+\alpha+b(1-n)]$ and $g'(1) < 0$ by $c < \frac{6(d+b)}{7}$, which imply $u = 1$ by (38). Thus, $S_* = \{\vec{C}\}$ and $E(T_\epsilon) = \epsilon^{-1}$.

Propositions 2(ia), 2(ib) and 2(ic) are proved by the results of Case 1a, Cases 1b-1c and Case 1d, respectively.

Case 2: Suppose $\frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12} \leq d < [b(7n-1)-7\alpha]$. We then have

$$l_1 \equiv \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < d+\alpha+(1-n)b < \frac{6(d+b)}{7}.$$

These inequalities divide the values of c into four mutually exclusive intervals.

Case 2a: Suppose $c \geq \frac{6(d+b)}{7}$. The results here are the same as Case 1a's.

Case 2b: Suppose $[d+\alpha+(1-n)b] \leq c < \frac{6(d+b)}{7}$. We have $h(1) > 0$ by $c > l_1$ and $v = 1$ by (41). Then $g(1) > 0$ by $c > [d+\alpha+b(1-n)]$ and $g'(1) < 0$ by $c < \frac{6(d+b)}{7}$, which imply $u = n$ or $\lceil k_g \rceil$ by (38). Thus, $S_* = \{\vec{D}\}$ and $E(T_\epsilon) = \epsilon^{-1}$.

Case 2c: Suppose $l_1 \leq c < [d+\alpha+(1-n)b]$. The results here are the same as Case 1c's.

Case 2d: Suppose $c < l_1$. The results here are the same as Case 1d's.

Propositions 2(iia), 2(iib) and 2(iic) are proved by the results of Cases 2a-2b, Case 2c and Case 2d, respectively.

Case 3: Suppose $d \leq \frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12}$. We then have

$$[d+\alpha+(1-n)b] < l_1 \equiv \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d+b)}{7}.$$

These inequalities divide the values of c into four mutually exclusive intervals.

Case 3a: Suppose $c \geq \frac{6(d+b)}{7}$. The results here are the same as Case 1a's.

Case 3b: Suppose $l_1 \leq c < \frac{6(d+b)}{7}$. We have $h(1) \geq 0$ by $c \geq l_1$ and $v = 1$ by (41). Then $g(1) > 0$ by $c > [d+\alpha+b(1-n)]$ and $g'(1) < 0$ by $c < \frac{6(d+b)}{7}$, which imply $u = n$ or $\lceil k_g \rceil$ by (38). Thus, $S_* = \{\vec{D}\}$ and $E(T_\epsilon) = \epsilon^{-1}$.

Case 3c: Suppose $[d+\alpha+(1-n)b] \leq c < l_1$. We have $h(1) < 0$ by $c < l_1$ and $v = \lceil k_h \rceil \geq 2$ by (41). Then $g(1) > 0$ by $c > [d+\alpha+b(1-n)]$ and $g'(1) < 0$ by $c < \frac{6(d+b)}{7}$, which imply $u = n$ or $\lceil k_g \rceil \geq 2$ by (38). We can show $\lceil k_g \rceil < \lceil k_h \rceil$ below, and it implies $S_* = \{\vec{D}\}$ and $E(T_\epsilon) = \epsilon^{-\lceil k_h \rceil}$.

Claim 3. *We have* $\lceil k_g \rceil > \lceil k_h \rceil$.

Proof. Since k_g satisfies $g(k_g) = 0$, we have[5]

$$k_g = \frac{-3 + \frac{6(d+b)}{c} + \sqrt{[3 - \frac{6(d+b)}{c}]^2 - 8[1 - \frac{6(\alpha - bn)}{c}]}}{4}.$$

Similarly, since k_h satisfies $h(k_g) = 0$, we have

$$k_h = \frac{-[2n + 3 + \frac{6(d+b)}{c}] + \sqrt{[2n + 3 + \frac{6(d+b)}{c}]^2 - 8[2n^2 + 3n + 1 - \frac{6(\alpha + dn)}{c}]}}{4}.$$

Define $A = [3 - \frac{6(d+b)}{c}]^2 - 8[1 - \frac{6(\alpha - bn)}{c}]$ and $B = [2n + 3 + \frac{6(d+b)}{c}]^2 - 8[2n^2 + 3n + 1 - \frac{6(\alpha + dn)}{c}]$. Then,

$$(k_g - k_h) = \frac{2n + \frac{12(d+b)}{c} + \sqrt{A} - \sqrt{B}}{4}.$$

To show $k_g > k_h$, it is enough to prove $B < A + 2\sqrt{A}[2n + \frac{12(d+b)}{c}] + 4n^2 + \frac{48(d+b)}{c} + \frac{144(d+b)^2}{c^2}$. Some calculations reveal

$$\begin{aligned} & B - A - 2\sqrt{A}[2n + \frac{12(d+b)}{c}] - 4n^2 - \frac{48(d+b)}{c} - \frac{144(d+b)^2}{c^2} \\ = \;& -[\frac{12(d+b)}{c} - (3n+1)]^2 - 7n^2 - 6n + 1 - 4\sqrt{A}[n + \frac{d+b}{c}] \\ < \;& 0. \end{aligned}$$

These suggest $k_g > k_h$. Moreover, the above inequality remains true if we replace n with $(n - 4)$. It means $k_g > k_h + 1$. Thus, we will have Claim 3, $\lceil k_g \rceil > \lceil k_h \rceil$.

Case 3d: Suppose $c < [d + \alpha + (1 - n)b]$. The results here are the same as Case 1d's.

Propositions 2(iiia), 2(iiib) and 2(iiic) are proved by the results of Cases 3a-3b, Case 3c and Case 3d, respectively.

Proof of Theorem 2: Before comparing Propositions 2 and C2, we need to know relative

[5] To have $k_g > 0$ and $k_h > 0$, we take the positive roots.

sizes of the following variables:

$$\frac{6(d-b)}{7}-[d+\alpha+(1-n)b]=\frac{-1}{7}[d+7\alpha-b(7n-13)]$$
$$\geq\ (\leq)\ 0 \text{ iff } d\leq\ (\geq)\ (7bn-13b-7\alpha), \tag{25}$$
$$\alpha-\frac{(d-b)[2n^2-2n+13]}{7}\geq\ (\leq)\ 0 \text{ iff } d\leq\ (\geq)\ b+\frac{7\alpha}{2n^2-2n+13}, \tag{26}$$
$$\frac{6(d-b)}{7}-\frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}=\frac{6[d(2n^2+4n+7)-b(2n^2+5n+5)-\alpha]}{7(2n^2+5n+6)}$$
$$\geq\ (\leq)\ 0 \text{ iff } d\geq\ (\leq)\ \frac{b(2n^2+5n+5)+\alpha}{2n^2+4n+7}, \tag{27}$$
$$d-b+\alpha-[d+\alpha+(1-n)b]=b(n-2)>0 \text{ by } n\geq 3, \tag{28}$$
$$\frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}-\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}=\frac{6b(n-2)}{2n^2+5n+6}>0 \text{ by } n\geq 3, \tag{29}$$
$$\frac{6(d+b)}{7}-[d-b+\alpha]=\frac{-(d-13b+7\alpha)}{7}\geq\ (\leq)\ 0 \text{ iff } d\leq\ (\geq)\ (13b-7\alpha), \tag{30}$$
$$\frac{6(d-b)}{7}-\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}=\frac{6(2n^2-12n+13)[d-b-\frac{7\alpha}{2n^2-12n+13}]}{7(2n^2+5n+6)}$$
$$\geq\ (\leq)\ 0 \text{ iff } d\geq\ (\leq)\ b+\frac{7\alpha}{2n^2-2n+13}, \tag{31}$$
$$[d+\alpha+(1-n)b]-\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}$$
$$=\frac{d(2n^2-n+12)-b(2n^3+3n^2-5n)+\alpha(2n^2+5n)}{2n^2+5n+6}$$
$$\geq\ (\leq)\ 0 \text{ iff } d\geq\ (\leq)\ \frac{b(2n^3+3n^2-5n)-\alpha(2n^2+5n)}{2n^2-n+12}. \tag{32}$$

Next, according to relative sizes of the thresholds of d specified in Proposition 2, there are three cases below.

Case 1: Suppose $d\geq[b(7n-1)-7\alpha]$. We then have

$$\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}<\frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}<\frac{6(d-b)}{7}<\frac{6(d+b)}{7}$$
$$<[d+\alpha+(1-n)b]<(d-b+\alpha)$$

by (42), (47), (48), (49), and $d\geq[b(7n-1)-7\alpha]>\frac{b[2n^2+5n+5]+\alpha}{2n^2+4n+7}$. These inequalities divide the values of c into seven exclusive ranges. At each range, we can derive the LREs in both dynamics. Under the imitating-the-best-total dynamic; $\vec{D}$ is the LRE for

$c \geq [d+\alpha+(1-n)b]$ by Proposition 2(ia); $\{\vec{C},\ \vec{D}\}$ is the LRE for $c \in [\frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6},\ [d+\alpha+(1-n)b])$ by Proposition 2(ib); and $\vec{C}$ is the LRE for $c < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}$ by Proposition 2(ic).

Under the imitating-the-best-average dynamic; $\vec{D}$ is the LRE for $c > (d-b+\alpha)$ by Proposition C2(i); $\{\vec{C},\ \vec{D}\}$ is the LRE for $c \in [\frac{6(d-b)}{7},\ (d-b+\alpha)]$ by Proposition C2(iia) and $\alpha \leq \frac{(d-b)[2n^2-2n+13]}{7}$; $\{\vec{C},\ \vec{D}\}$ is the LRE for $c \in [\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6},\ \frac{6(d-b)}{7})$ by Proposition C2(iiib), $c < \frac{6(d-b)}{7}$, $\alpha \leq \frac{(d-b)[2n^2-2n+13]}{7}$ implied by (46) and $d \geq [b(7n-1)-7\alpha]$, and $c \in [\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6},\ \frac{6(d-b)}{7})$; and $\vec{C}$ is the LRE for $c \in (0,\ \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6})$ by Proposition C2(iiia), $c < \frac{6(d-b)}{7}$, $\alpha \leq \frac{(d-b)[2n^2-2n+13]}{7}$ implied by (46) and $d \geq [b(7n-1)-7\alpha]$, and $c < \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}$.

In summary, the c interval making $\vec{C}$ the LRE under the imitating-the-best-average dynamic is $(0,\ (d-b+\alpha)]$, and the associated interval under the imitating-the-best-total dynamic is $(0,\ (d+\alpha+(1-n)b))$. Since $(0,\ (d+\alpha+(1-n)b)) \subset (0,\ (d-b+\alpha)]$ by $(d-b+\alpha) > [d+\alpha+(1-n)b]$ due to $n \geq 3$, $\vec{C}$ is more likely to be the LRE under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic.

Case 2: Suppose $(7bn-7\alpha-13b) \leq d < [b(7n-1)-7\alpha]$. Under the circumstance, we need to know relative sizes of the thresholds of d specified in (45)-(51). Some calculations reveal

$$b+\frac{7\alpha}{2n^2-2n+13} < \frac{b[2n^2+5n+5]+\alpha}{2n^2+4n+7} < 13b-7\alpha < 7bn-7\alpha-13b < b(7n-1)-7\alpha.$$

These inequalities divide the values of d into five mutually exclusive intervals.

Case 2a: Suppose $(7bn-7\alpha-13b) \leq d < [b(7n-1)-7\alpha]$. We then have

$$\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d-b)}{7} < [d+\alpha+(1-n)b]$$
$$< \frac{6(d+b)}{7} < (d-b+\alpha)$$

by (42), (45), (47), (49), and (50). These inequalities divide the values of c into seven mutually exclusive intervals. The results here are the same as Case 1's by $(d-b+\alpha) > [d+\alpha+(1-n)b]$.

Case 2b: Suppose $(13b-7\alpha) \leq d < (7bn-7\alpha-13b)$. We then have

$$\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < [d+\alpha+(1-n)b] < \frac{6(d-b)}{7} < \frac{6(d+b)}{7} < (d-b+\alpha)$$

by (44), (45), (49), and (50). These inequalities divide the values of c into seven mutually exclusive intervals. Again, the conclusions here are the same as Case 1's due to $(d-b+\alpha) > [d+\alpha+(1-n)b]$.

Case 2c: Suppose $\frac{b[2n^2+5n+5]+\alpha}{2n^2+4n+7} \leq d < 13b-7\alpha$. We then have

$$\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < [d+\alpha+(1-n)b] < \frac{6(d-b)}{7} < (d-b+\alpha) < \frac{6(d+b)}{7}$$

by (44), (45), (49), and (50). These inequalities divide the values of c into seven mutually exclusive intervals. Similarly, Propositions 2 and C2 imply that $\vec{C}$ is more likely to emerge when countries adopt the imitating-the-best-average rule by $(d-b+\alpha) > [d+\alpha+(1-n)b]$.

Case 2d: Suppose $b + \frac{7\alpha}{2n^2-2n+13} \leq d < \frac{b[2n^2+5n+5]+\alpha}{2n^2+4n+7}$. We then have

$$\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6(d-b)}{7} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < [d+\alpha+(1-n)b] < (d-b+\alpha) < \frac{6(d+b)}{7}$$

by (44), (47), (48), (50), and (51). These inequalities divide the values of c into seven mutually exclusive intervals. The results obtained here are the same as Case 1's by Propositions 2 and C2 due to $(d-b+\alpha) > [d+\alpha+(1-n)b]$.

Case 2e: Suppose $d \le b + \frac{7\alpha}{2n^2-2n+13}$. We then have

$$\frac{6(d-b)}{7} < \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < [d+\alpha+(1-n)b]$$
$$< (d-b+\alpha) < \frac{6(d+b)}{7}$$

by (44), (48), (49), (50), and (51). These inequalities divide the values of c into seven mutually exclusive intervals. Because $(d-b+\alpha) > [d+\alpha+(1-n)b]$, $\vec{C}$ is more likely to emerge in the long run under the imitating-the-best-average rule by Propositions 2 and C2.

Case 3: Suppose $d < \frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12}$. We can show $\frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12} < [7bn-7\alpha-13b]$ by $n \ge 4$. On the other hand, we have $\frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12} - (13b-7\alpha) < 0$ if $n < 13$, and > 0 if $n \ge 13$. Thus, the situation of $d \in [7bn-7\alpha-13b,\ b(7n-1)-7\alpha)$ discussed in Case 2a does not exist here. Accordingly, we will start with the case of $d \in [13b-7\alpha,\ 7bn-7\alpha-13b)$ as follows. In addition, we have

$$\frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12} < \frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12}$$

by $n \ge 3$.

Case 3a: Suppose $(13b-7\alpha) \le d < (7bn-7\alpha-13b)$. We then have

$$\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < d+\alpha+(1-n)b < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d-b)}{7}$$
$$< \frac{6(d+b)}{7} < (d-b+\alpha)$$

by (44), (47), (50), and (52) with $d > \frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12}$ assumed.[6] These inequalities divide the values of c into seven mutually exclusive intervals. At each range, we can derive the LREs in both dynamics. Under the imitating-the-best-total dynamic; $\vec{D}$ is the LRE for $c \ge \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}$ by Proposition 2(iiia); $\vec{D}$ is the LRE for

[6] If $d \le \frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12}$, then we have $[d+\alpha+(1-n)b] < \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d-b)}{7} < \frac{6(d+b)}{7} < (d-b+\alpha)$. Under the circumstance, the results of Case 3a remain true.

$c \in [d+\alpha+(1-n)b,\ \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6})$ by Proposition 2(iiib); and $\vec{C}$ is the LRE for $c < [d+\alpha+(1-n)b]$ by Proposition 2(iiic).

Under the imitating-the-best-average dynamic; $\vec{D}$ is the LRE for $c > (d-b+\alpha)$ by Proposition C2(i); $\{\vec{C},\ \vec{D}\}$ is the LRE for $c \in [\frac{6(d-b)}{7},\ (d-b+\alpha)]$ by Proposition C2(iia) and $\alpha \leq \frac{(d-b)[2n^2-2n+13]}{7}$ implied by (46) and $d > b + \frac{7\alpha}{2n^2-2n+13}$; $\{\vec{C},\ \vec{D}\}$ is the LRE for $c \in [\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6},\ \frac{6(d-b)}{7})$ by Proposition C2(iiib), $c < \frac{6(d-b)}{7}$, $\alpha \leq \frac{(d-b)[2n^2-2n+13]}{7}$ implied by (46) and $d > b + \frac{7\alpha}{2n^2-2n+13}$, and $c \in [\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6},\ \frac{6(d-b)}{7})$; and $\vec{C}$ is the LRE for $c \in (0,\ \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6})$ by Proposition C2(iiia), $c < \frac{6(d-b)}{7}$, $\alpha \leq \frac{(d-b)[2n^2-2n+13]}{7}$ implied by (46) and $d > b + \frac{7\alpha}{2n^2-2n+13}$, and $c < \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}$.

In summary, the c interval making $\vec{C}$ the LRE under the imitating-the-best-average dynamic is $(0,\ (d-b+\alpha)]$, and the associated interval under the imitating-the-best-total dynamic is $(0,\ (d+(1-n)b+\alpha))$. Since $(0,\ (d+(1-n)b+\alpha)) \subset (0,\ (d-b+\alpha)]$ by $(d-b+\alpha) > (d+(1-n)b+\alpha)$, $\vec{C}$ is more likely to be the LRE under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic.

<u>Case 3b</u>: Suppose $\frac{b[2n^2+5n+5]+\alpha}{2n^2+4n+7} \leq d < (13b-7\alpha)$. Some calculations show

$$\frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12} > \frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12} > \frac{b[2n^2+5n+5]+\alpha}{2n^2+4n+7}.$$

We then have

$$\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < d+\alpha+(1-n)b < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d-b)}{7} < (d-b+\alpha) < \frac{6(d+b)}{7}$$

by (44), (47), (50), and (52) with $d > \frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12}$ assumed.[7] These inequalities divide the values of c into seven mutually exclusive intervals. As in Case 3b, we obtain that $\vec{C}$ is more likely to emerge under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic due to $(0,\ (d+(1-n)b+\alpha)) \subset (0,\ (d-b+\alpha)]$ by Propositions 2 and C2.

[7] As in Case 3a, our results remain true if $d \leq \frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12}$.

Case 3c: Suppose $b+\frac{7\alpha}{2n^2-2n+13} \leq d < \frac{b[2n^2+5n+5]+\alpha}{2n^2+4n+7}$. We will always have $d < \frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12} < \frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12}$. Then

$$d+\alpha+(1-n)b < \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6(d-b)}{7} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < (d-b+\alpha) < \frac{6(d+b)}{7}$$

by (47), (50), (51), and (53) by $d > b+\frac{7\alpha}{2n^2-2n+13} > \frac{(b-\alpha)(2n^2+5n)}{2n^2-2n+12}$. These inequalities divide the values of c into seven mutually exclusive intervals. Again, the results here are the same as Case 3b's due to $(d-b+\alpha) > [d+(1-n)b+\alpha]$.

Case 3d: Suppose $d < b+\frac{7\alpha}{2n^2-2n+13}$. Under the circumstance, we need to know relative sizes of $(d-b+\alpha)$, $\frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}$, and $\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}$. Some calculations show

$$(d-b+\alpha) - \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} = \frac{[d(2n^2-n+12)-(b-\alpha)(2n^2+5n)]}{2n^2+5n+6} \geq (\leq)\ 0 \text{ iff } d \geq (\leq)\ \frac{(b-\alpha)(2n^2+5n)}{2n^2-n+12}, \tag{33}$$

$$(d-b+\alpha) - \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} = \frac{(2n^2-n+12)[d-b+\frac{\alpha(2n^2+5n)}{2n^2-n+12}]}{2n^2+5n+6} \geq (\leq)\ 0 \text{ iff } d \geq (\leq)\ b-\frac{\alpha(2n^2+5n)}{2n^2-n+12}, \tag{34}$$

$$b-\frac{\alpha(2n^2+5n)}{2n^2-n+12} - \frac{(b-\alpha)(2n^2+5n)}{2n^2-n+12} = \frac{b(2n^2-n+11)}{2n^2-n+12} > 0 \text{ by } n \geq 2, \tag{35}$$

$$b-\frac{\alpha(2n^2+5n)}{2n^2-n+12} < b+\frac{7\alpha}{2n^2-n+12}. \tag{36}$$

These suggest

$$\frac{(b-\alpha)(2n^2+5n)}{2n^2-n+12} < b-\frac{\alpha(2n^2+5n)}{2n^2-n+12} < b+\frac{7\alpha}{2n^2-n+12},$$

and the inequalities imply that there are three sub-cases below.

Case 3d-1: Suppose $b-\frac{\alpha(2n^2+5n)}{2n^2-n+12} \leq d < b+\frac{7\alpha}{2n^2-n+12}$. We then have

$$d+\alpha+(1-n)b < \frac{6(d-b)}{7} < \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < (d-b+\alpha) < \frac{6(d+b)}{7}$$

by (45), (49), (50), (51), and (53). These inequalities divide the values of c into seven mutually exclusive intervals. As in Case 3b, $\vec{C}$ is more likely to emerge under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic due to $(d-b+\alpha) > \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}$ by Propositions 2 and C2.

<u>Case 3d-2</u>: Suppose $\frac{(b-\alpha)(2n^2+5n)}{2n^2-n+12} \leq d < b - \frac{\alpha(2n^2+5n)}{2n^2-n+12}$. We then have $(d-b+\alpha) > \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}$ by (53), and $(d-b+\alpha) < \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}$ by (54). However, (49) implies $\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}$, which leads to a contradiction. Thus, no solution exists in this case.

<u>Case 3d-3</u>: Suppose $d \leq \frac{(b-\alpha)(2n^2+5n)}{2n^2-n+12}$. We then have

$$[d+\alpha+(1-n)b] < \frac{6(d-b)}{7} < (d-b+\alpha) < \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}$$
$$< \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d+b)}{7}$$

by (43), (45), (49), and (54). These inequalities divide the values of c into seven mutually exclusive intervals. At each range, we can derive the LREs in both dynamics. Under the imitating-the-best-total dynamic; $\vec{D}$ is the LRE for $c \geq \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}$ by Proposition 2(iiia); $\vec{D}$ can be the LRE for $c \in [d+\alpha+(1-n)b, \ \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6})$ by Proposition 2(iiib); and $\vec{C}$ is the LRE for $c \in (0, \ [d+\alpha+(1-n)b])$ by Proposition 2(iiic).

Under the imitating-the-best-average dynamic; $\vec{D}$ is the LRE for $c > (d-b+\alpha)$ by Proposition C2(i); $\vec{C}$ is the LRE for $c \in [\frac{6(d-b)}{7}, \ (d-b+\alpha)]$ by Proposition C2(iib) and $\alpha > \frac{(d-b)[2n^2-2n+13]}{7}$ and $c \in [\frac{6(d-b)}{7}, \ \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6})$; and $\vec{C}$ is the LRE for $c \in (0, \ \frac{6(d-b)}{7})$ by Proposition C2(iiic), $c < \frac{6(d-b)}{7}$, and $\alpha > \frac{(d-b)[2n^2-2n+13]}{7}$.

Accordingly, the c interval making $\vec{C}$ the LRE under the imitating-the-best-average dynamic is $(0, \ (d-b+\alpha)]$, and the associated interval under the imitating-the-best-total dynamic is $(0, \ (d+(1-n)b+\alpha))$. Since $(0, \ (d+(1-n)b+\alpha)) \subset (0, \ (d-b+\alpha))$, $\vec{C}$ is more likely to be the LRE under the imitating-the-best-total dynamic than under the imitating-the-best-average dynamic.

In summary, the results of Cases 1-3 prove Theorem 2.

Proof of Proposition 3: If $[d+\alpha-b(n-1)] < c_1$, then $[d+\alpha-b(n-1)] < \frac{\sum_{i=1}^{k} c_i}{k}$. That is, (8) fails at $k=1$, and hence $u \geq 2$. On the other hand, $[d(n-1)+\alpha-b] \leq \frac{\sum_{i=2}^{n} c_i}{n-1}$ implies that (11) holds at $k=1$, and hence $v=1$. We have $S_* = \{\vec{D}\}$ and $E(T_\epsilon) = \epsilon^{-1}$ as shown by Proposition 3(i). In contrast, if $[d+\alpha-b(n-1)] \geq c_1$, then $u=1$ due to (8) holding at $k=1$, and $v=1$ by $[d(n-1)+\alpha-b] \leq \frac{\sum_{i=2}^{n} c_i}{n-1}$ due to (11) holding at $k=1$. Thus, $S_* = \{\vec{C},\ \vec{D}\}$ and $E(T_\epsilon) = \epsilon^{0}$ as shown by Proposition 3(ii). Finally, if $[d+\alpha-b(n-1)] > c_n$, then we have $[d+\alpha-b(n-1)] > c_n > \frac{\sum_{i=1}^{k} c_i}{k}$, which implies $u=1$ by (8). Moreover, we have $d(n-1)+\alpha-b-\frac{\sum_{i=2}^{n} c_i}{n-1} > d(n-1)+\alpha-b-c_n > (n-1)[c_n+b(n-1)-\alpha]+\alpha-b-c_n = (n-2)c_n+b(n-1)^2-\alpha(n-2) > 0$ by $c_n > \frac{\sum_{i=1}^{n} c_i}{(n-1)}$ and $d > c_n+b(n-1)-\alpha$. This suggests that (11) fails at $k=1$, and hence $v \geq 2$. Thus, $S_* = \{\vec{C}\}$ and $E(T_\epsilon) = \epsilon^{-1}$ as shown by Proposition 3(iii).