Proof of Proposition 7: Under (3), (4), (13) and (14), we re-derive all the Lemmas and Propositions in Section 4. First, the changed problems are stated. Then, the associated Lemmas and Propositions are provided.

Under the new set-up, problem (5) becomes

$$\max_{q_i \geq 0} \ \pi_i = \left(a - bq_i - c_i - r\right) q_i - f \qquad (5A)$$

$$\text{s.t. } q_i \geq \delta$$

for $i = l, h$. To have optimal non-negative cargo-handling amounts for terminal operators, we need

$$r_i \leq \overline{r_i} \equiv (a - c_i) \ \text{ for } i = l, h. \qquad (6A)$$

The solutions of problem (5A) are listed below.

Lemma 1. *Suppose the conditions in (6A) hold. Given concession contract* $\left(r, f, \delta\right)$*, operator i's optimal behaviors are as follows.*

(i) *If* $\delta \in [0, \overline{\delta_i}]$ *with* $\overline{\delta_i} = \frac{(a-c_i-r)}{2b}$*, then we have* $q_i^c = \frac{(a-c_i-r)}{2b} = \overline{\delta_i}$ *with equilibrium service prices* $p_i^c = \frac{(a+c_i+r)}{2} > 0$ *and equilibrium profits* $\pi_i^c = b(q_i^c)^2 - f$ *for* $i = l, h$.

(ii) *If* $\delta \in (\overline{\delta_i}, \frac{a}{b})$*, then we have* $q_i^c = \delta$ *with equilibrium service prices* $p_i^c = (a - b\delta) > 0$ *and equilibrium profits* $\pi_i^c = (a - b\delta - c_i - r)\delta - f$ *for* $i = l, h$.

Proof of Lemma 1: Denote L the terminal operator's Lagrange function in problem (5A),

$$L = \left(a - bq_i - c_i - r\right) q_i - f + \lambda\left(q_i - \delta\right),$$

where λ is the Lagrange multiplier associated with the constraint in problem (5A). Then, the corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial q_i} = a - 2bq_i - c_i - r + \lambda \leq 0, \ q_i \cdot \frac{\partial L}{\partial q_i} = 0, \text{ and} \qquad (A1)$$

$$\frac{\partial L}{\partial \lambda} = q_i - \delta \geq 0, \ \lambda \cdot \frac{\partial L}{\partial \lambda} = 0. \qquad (A2)$$

Based on the values of λ, there are two cases below.

Case 1: Suppose $\lambda^* = 0$. We have $q_i^c = \dfrac{\left(a - c_i - r\right)}{2b}$. To guarantee $q_i^c \geq \delta$, condition $0 \leq \delta \leq \overline{\delta_i} \equiv \dfrac{\left(a - c_i - r\right)}{2b} = q_i^c$ should be met. Substituting q_i^c into (13) yields $p_i^c = \dfrac{\left(a + c_i + r\right)}{2} > 0$, and into (14) yields $\pi_i^c = b\left(q_i^c\right)^2 - f$ for $i = l, h$. These prove Lemma 1(i).

Case 2: Suppose $\lambda^* > 0$ .We have $q_i^c = \delta$ and $\lambda^* = 2b\left[\delta - \frac{(a - c_i - r)}{2b}\right] = 2b\left(\delta - \overline{\delta}_i\right)$ by (A1) and (A2). To guarantee $\lambda^* > 0$, conditions $\delta > \overline{\delta}_i$ and $r_i \le \overline{r}_i$ are needed. Substituting q_i^c into (13) yields $p_i^c = (a - b\delta) > 0$ if $\delta < \frac{a}{b}$, and into (14) gives $\pi_i^c = \delta\left(a - b\delta - c_i - r\right) - f$ for $i = l, h$. These prove Lemma 1(ii). □

Under the new set-up, problem (7) becomes

$$\max_{r_i, f_i, \delta_i} \; r_i q_i^c + f_i \tag{7A}$$

$$\text{s.t. } 0 \le \delta_i < \frac{a}{b},\; 0 \le r_i \le \overline{r}_i,\; f_i \ge 0 \text{ and } \pi_i^c \ge 0$$

for $i = l, h$. Its solutions are as follows.

Lemma 2. *Suppose the conditions in (6A) hold. The optimal concession contract* $(r_i^c, f_i^c, \delta_i^c)$ *offered to the operator with marginal service cost* c_i, $i = l, h$, *can be the fixed-fee contract* $f_i^c = \frac{(a - c_i)^2}{4b}$ *with minimum throughput requirement* $\delta_i^c \in \left[0, \frac{a - c_i}{2b}\right]$, *the unit-fee contract* $r_i^c = \frac{a - c_i}{2}$ *with minimum throughput requirement* $\delta_i^c = \frac{a - c_i}{2b}$, *or the two-part tariff contract with* $r_i^c \in \left(0, \frac{a - c_i}{2}\right)$, $f_i^c = \frac{(a - c_i)(a - c_i - 2r_i^c)}{4b}$, *and minimum throughput requirement* $\delta_i^c = \frac{a - c_i}{2b}$. *However, the port authority's equilibrium fee revenue always equals* $R_i^c = \frac{(a - c_i)^2}{4b}$, $i = l, h$.

Proof of Lemma 2:

Case 1: Suppose $\delta_i \in \left[0, \overline{\delta}_i\right]$. Lemma 1(i) implies $f_i^c = b\left(q_i^c\right)^2 > 0$, because the port authority will set f as large as possible. However, since $\frac{\partial \pi_i^c}{\partial f} < 0$, the optimal value of f will satisfy $\pi_i^c = 0$.

Accordingly, problem (7A) becomes

$$\max_{r_i, f_i, \delta_i} r_i q_i^c + b\left(q_i^c\right)^2$$

$$\text{s.t. } 0 \le \delta_i \le \overline{\delta}_i \text{ and } 0 \le r_i \le \overline{r}_i\,. \tag{A3}$$

Its Lagrange function is

$$L = r_i q_i^c + b\left(q_i^c\right)^2 + \lambda_1\left(\overline{\delta}_i - \delta\right) + \lambda_2\left(\overline{r}_i - r_i\right),$$

where λ_1 and λ_2 are the respective Lagrange multipliers associated with the two inequality constraints in (A3). Then, the corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r_i} = -\frac{b}{2} r_i - \frac{1}{2b}\lambda_1 - \lambda_2 \leq 0,\ r_i \cdot \frac{\partial L}{\partial r_i} = 0, \tag{A4}$$

$$\frac{\partial L}{\partial \delta_i} = -\lambda_1 \leq 0,\ \delta_i \cdot \frac{\partial L}{\partial \delta_i} = 0, \tag{A5}$$

$$\frac{\partial L}{\partial \lambda_1} = \overline{\delta}_i - \delta_i \geq 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \text{ and} \tag{A6}$$

$$\frac{\partial L}{\partial \lambda_2} = \overline{r}_i - r_i \geq 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0. \tag{A7}$$

Based on the values of λ_1 and λ_2, there are four cases below.

<u>Case 1a</u>: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. We have $r_i^c = 0$ by (A4). In addition, (A6) implies both $\delta_i^c \in [0, \overline{\delta}_i]$ with $\overline{\delta}_i = \frac{a - c_i}{2b}$ and $f_i^c = \frac{(a - c_i)^2}{4b} > 0$ by Lemma 1(i). Thus, the port authority's equilibrium fee revenue equals

$$R_i^c = \frac{(a - c_i)^2}{4b}. \tag{A8}$$

<u>Case 1b</u>: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then (A7) implies $r_i^c = \overline{r}_i = (a - c_i)$ for $i = l, h$. This suggests $\lambda_2^* = -\frac{b(a - c_i)}{2} < 0$ by (A4), which contradicts $\lambda_2^* > 0$. Thus, no equilibrium exists.

<u>Case 1c</u>: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then (A6) suggests $\delta_i^c = \delta_i > 0$ and $\lambda_1^* = 0$ by (A5), which lead to a contradiction. Thus, there is no solution in this case.

<u>Case 1d</u>: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. As in Case 1c, we have $\lambda_1^* = 0$. Again, no solution exists.

<u>Case 2</u>: Suppose $\delta_i \in \left(\overline{\delta}_i, \frac{a}{b}\right)$. As in Case 1, we have $\pi_i^c = 0$ and $f_i^c = \delta_i(a - b\delta_i - r_i - c_i)$ by Lemma 1(ii). Hence $f_i^c \geq 0$ iff $\delta_i \leq \frac{a - r_i - c_i}{b}$ and $r_i \leq \overline{r}_i \equiv a - c_i$. Problem (7A) thus becomes

$$\max_{r_i, f_i, \delta_i} r_i \cdot \delta_i + \delta_i(a - b\delta_i - r_i - c_i)$$

$$\text{s.t. } \overline{\delta}_i < \delta_i \leq \frac{a - r_i - c_i}{b},\ 0 \leq r_i \leq \overline{r}_i \text{ and } f_i \geq 0. \tag{A9}$$

Its Lagrange function is

$$L = r_i \cdot \delta_i + \delta_i\left(a - b\delta_i - r_i - c_i\right) + \lambda_1\left(\delta_i - \overline{\delta}_i\right) + \lambda_2\left[\frac{a - r_i - c_i}{b} - \delta_i\right] + \lambda_3\left(\overline{r}_i - r_i\right),$$

where λ_1, λ_2 and λ_3 are the respective Lagrange multipliers associated with the inequality constraints in (A9). Then, the corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial \delta_i} = \left(a - 2b\delta_i - c_i\right) + \lambda_1 - \lambda_2 \le 0,\ \delta_i \cdot \frac{\partial L}{\partial \delta_i} = 0, \tag{A10}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta_i - \overline{\delta}_i \ge 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \tag{A11}$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{\left(a - r_i - c_i\right)}{b} - \delta_i \ge 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \text{ and} \tag{A12}$$

$$\frac{\partial L}{\partial \lambda_3} = \overline{r}_i - r_i \ge 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0. \tag{A13}$$

Constraint $\overline{\delta}_i < \delta_i$ suggests $\lambda_1^* = 0$ by (A11). According to the sign of λ_2, there are two sub-cases.

<u>Case 2a</u>: If $\lambda_2^* = 0$, then we have $\delta_i^c = \frac{a - c_i}{2b}$ by (A10) and $r_i^c \in \left[0, \frac{a - c_i}{2}\right]$ by (A12), and hence $f_i^c = \delta_i^c\left(a - b\delta_i^c - r_i^c - c_i\right) = \frac{\left(a - c_i\right)\left(a - c_i - 2r_i^c\right)}{4b}$. Moreover, $\delta_i^c = \frac{a - c_i}{2b}$ and $r_i^c \in \left[0, \frac{a - c_i}{2}\right]$ satisfy (A10)-(A13). Thus, the port authority's fee revenue equals

$$R_i^c = \frac{\left(a - c_i\right)^2}{4b}. \tag{A14}$$

<u>Case 2b</u>: If $\lambda_2^* > 0$, then we have $\delta_i^c = \frac{\left(a - r_i - c_i\right)}{b}$ by (A12) and $\lambda_2^* = 2r_i^c - \left(a - c_i\right)$ by (A10). In addition, we have $(\delta_i^c - \overline{\delta}_i) = \frac{a - r_i^c - c_i}{2b} > 0$ iff $r_i^c < (a - c_i)$, and $\lambda_2^* > 0$ iff $r_i^c > \frac{a - c_i}{2}$. Hence $\delta_i^c = \frac{\left(a - r_i - c_i\right)}{b}$, $r_i^c \in \left(\frac{a - c_i}{2}, a - c_i\right)$. The port authority's equilibrium fee revenue equals

$$R_i^c = \frac{r_i^c \cdot \left(a - r_i^c - c_i\right)}{b}. \tag{A15}$$

Since $\frac{\partial R_i^c}{\partial r_i^c} = \frac{a - 2r_i^c - c_i}{b} < 0$ for $r_i^c \in \left(\frac{a - c_i}{2}, a - c_i\right)$, we have $R_i^c = \frac{r_i^c \cdot \left(a - r_i^c - c_i\right)}{b} <$ $\frac{1}{b} \cdot \frac{a - c_i}{2} \cdot \left(a - \frac{a - c_i}{2} - c_i\right) = \frac{\left(a - c_i\right)^2}{4b}$.

Thus, we have three solutions: $R_i^c = \frac{(a-c_i)^2}{4b}$ in (A8) of Case 1a, $R_i^c = \frac{(a-c_i)^2}{4b}$ in (A14) of Case 2a, and $R_i^c = \frac{r_i^c \cdot (a - r_i^c - c_i)}{b}$ in (A15) of Case 2b. Because $\frac{r_i^c \cdot (a - r_i^c - c_i)}{b} < \frac{(a-c_i)^2}{4b}$, the port authority should choose: $r_i^c = 0$, $\delta^c \in \left[0, \frac{a-c_i}{2b}\right]$, and $f^c = \frac{(a-c_i)^2}{4b}$ as in Case 1a; or $r_i^c \in \left[0, \frac{a-c_i}{2}\right]$, $\delta_i^c = \frac{a-c_i}{2b}$, and $f_i^c = \frac{(a-c_i)(a-c_i-2r_i^c)}{4b}$ as in Case 2a. In summary, the optimal contact can be the fixed-fee scheme in Case 1a, the unit-fee scheme in Case 2a if $r_i^c = \frac{a-c_i}{2}$, or the two-part tariff scheme in Case 2a if $r_i^c \in \left(0, \frac{a-c_i}{2}\right)$. □

Under the new set-up, problem (8) becomes

$$\max_{q_i \ge 0} \quad \pi_i = (a - bq_i - c_i - r)q_i - f \tag{8A}$$

$$\text{s.t. } q_i \ge \delta, \; i = l, \; h.$$

Condition $r \le \bar{r}_h \equiv (a - c_h)$ can guarantee optimal non-negative cargo-handling amounts for both-type operators. Solving problem (8A) yields the results below.

Lemma 3. *Suppose the conditions in (6A) hold. Given contract (r, f, δ), operator i's optimal behaviors are as follows for $i = l, h$.*

(i) *For $\delta \in [0, \delta_1^p]$ with $\delta_1^p = \frac{(a-c_h-r)}{2b} > 0$, both-type operators' equilibrium cargo-handling amounts are $q_l^p = \frac{(a-c_l-r)}{2b} > \delta_1^p$ and $q_h^p = \delta_1^p$, their equilibrium service prices are $p_i^p = \frac{(a+c_i+r)}{2} > 0$, and their equilibrium profits are $\pi_i^p = b(q_i^p)^2 - f$ for $i = l, h$.*

(ii) *For $\delta \in \left(\delta_1^p, \delta_2^p\right]$ with $\delta_2^p = \frac{(a-c_l-r)}{2b} > \delta_1^p$, both-type operators' equilibrium cargo-handling amounts are $q_l^p = \frac{(a-c_l-r)}{2b}$ and $q_h^p = \delta$, their equilibrium service prices are $p_l^p = \frac{(a+c_l+r)}{2} > 0$ and $p_h^p = a - b\delta > 0$, and their equilibrium profits are $\pi_l^p = b\left(q_l^p\right)^2 - f$ and $\pi_h^p = \delta\left[a - b\delta - c_h - r\right] - f$.*

(iii) *For $\delta \in \left(\delta_2^p, \frac{a}{b}\right)$, both-type operators' equilibrium cargo-handling amounts are $q_l^p = \delta$ and $q_h^p = \delta$, their equilibrium service prices are $p_l^p = p_h^p = (a - b\delta) > 0$, and their equilibrium profits are $\pi_i^p = \delta\left[a - b\delta - c_i - r\right] - f$ for $i = l, h$.*

<u>**Proof of Lemma 3**</u>: Denote L_1 and L_2 the respective Lagrange functions for the *l*-type and the *h*-type operators in problem (8A),

$$L_1 = (a - bq_l)q_l - (c_l + r)q_l - f + \lambda_1(q_l - \delta) \text{ and}$$
$$L_2 = (a - bq_h)q_2 - (c_h + r)q_h - f + \lambda_2(q_h - \delta),$$

where λ_1 and λ_2 are Lagrange multipliers for the *l*-type and the *h*-type operators, respectively. Then, the Kuhn-Tucker conditions for the *l*-type operator are

$$\frac{\partial L_1}{\partial q_l} = a - 2bq_l - c_l - r + \lambda_1 \leq 0,\ q_l \cdot \frac{\partial L_1}{\partial q_l} = 0 \text{ and} \tag{A16}$$

$$\frac{\partial L_1}{\partial \lambda_1} = q_l - \delta \geq 0,\ \lambda_1 \cdot \frac{\partial L_1}{\partial \lambda_1} = 0, \tag{A17}$$

and for the *h*-type operator are

$$\frac{\partial L_2}{\partial q_h} = a - 2bq_h - c_h - r + \lambda_2 \leq 0,\ q_h \cdot \frac{\partial L_2}{\partial q_h} = 0 \text{ and} \tag{A18}$$

$$\frac{\partial L_2}{\partial \lambda_2} = q_h - \delta \geq 0,\ \lambda_2 \cdot \frac{\partial L_2}{\partial \lambda_2} = 0. \tag{A19}$$

Based on the values of λ_1 and λ_2, there are four cases below.

<u>Case 1</u>: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$.Then (A16) and (A18) become $(a - 2bq_l - c_l - r) = 0$ and $(a - 2bq_h - c_h - r) = 0$, respectively. Solving the two equations yields $q_l^p = \frac{(a - c_l - r)}{2b}$ and $q_h^p = \frac{(a - c_h - r)}{2b}$. To guarantee $q_l^p \geq \delta$ and $q_h^p \geq \delta$, condition $0 \leq \delta \leq \delta_1^p \equiv \frac{(a - c_h - r)}{2b} = q_h^p$ should be imposed, because $c_l < c_h$ suggests $q_l^p > q_h^p$ and $q_h^p \geq \delta$ suggests $q_l^p \geq \delta$. Substituting q_l^p and q_h^p into (13) yields $p_h^p = \frac{(a + c_h + r)}{2} > p_l^p = \frac{(a + c_l + r)}{2} > 0$, and into (14) yields $\pi_i^p = b\left(q_i^p\right)^2 - f$ for $i = l, h$. These prove Lemma 3(i).

<u>Case 2</u>: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then (A16), (A18) and (A19) imply $(a - 2bq_l - c_l - r) = 0$, $(a - 2bq_h - c_h - r + \lambda_2) = 0$ and $(q_h - \delta) = 0$. Solving these equations yields $q_l^p = \frac{(a - c_l - r)}{2b}$, $q_h^p = \delta$ and $\lambda_2^* = 2b\left(\delta - \delta_1^p\right)$. To guarantee $\lambda_2^* > 0$, conditions $\delta > \delta_1^p$ and $r \leq \overline{r_h}$ are needed. On the other hand, to have $q_l^p \geq \delta$, condition $\delta \leq \delta_2^p \equiv \frac{a - c_l - r}{2b}$ should be imposed. Accordingly, the plausible range for δ is $\left(\delta_1^p, \delta_2^p\right]$. Substituting q_l^p and q_h^p into (13) produces $p_h^p = a - b\delta \geq p_l^p = \frac{(a + c_l + r)}{2} > 0$ if $\delta \leq \delta_2^p$, and into (14) gives $\pi_l^p = b\left(q_l^p\right)^2 - f$ and $\pi_h^p = \delta\left[a - b\delta - c_h - r\right] - f$. These prove Lemma 3(ii).

Case 3: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$ .Then (A16)-(A18) suggest $(q_l - \delta) = 0$, $(a - 2bq_l - c_l - r + \lambda_1) = 0$ and $(a - 2bq_h - c_h - r) = 0$. Solving these equations yields $q_l^p = \delta$, $q_h^p = \frac{(a - c_h - r)}{2b}$ and $\lambda_1^* = 2b\left[\delta - \frac{a - c_l - r}{2b}\right]$. To guarantee $\lambda_1^* > 0$, condition $\delta > \frac{(a - c_l - r)}{2b}$ is needed; and $q_h^p \geq \delta$ is guaranteed if $\delta \leq \frac{(a - c_h - r)}{2b}$. However, the two conditions are incompatible with each other because $\frac{(a - c_h - r)}{2b} - \frac{(a - c_l - r)}{2b} = \frac{-(c_h - c_l)}{2b} < 0$. Thus, no solution exists in this case.

Case 4: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. Then (A16)-(A19) suggest $q_l^p = q_h^p = \delta$, $\lambda_1^* = -a + 2b\delta + c_l + r$ and $\lambda_2^* = -a + 2b\delta + c_h + r$. To have $\lambda_1^* > 0$ and $\lambda_2^* > 0$, conditions $\delta > \delta_2^p \equiv \frac{a - c_l - r}{2b}$ and $r < (a - c_l)$ are needed. Note that we have $r < (a - c_l)$ because of $r \leq \overline{r}_h \equiv a - c_h$ and $c_l < c_h$. Substituting $q_l^p = q_h^p = \delta$ into (13) produces $p_l^p = p_h^p = (a - b\delta) > 0$ if $\delta < \frac{a}{b}$, and into (14) gives $\pi_i^p = \delta\left[a - b\delta - c_i - r\right] - f$ for $i = l, h$. These prove Lemma 3(iii). □

Under the new set-up, problem (9) becomes

$$\max_{r, f, \delta} \; R = \theta\left(rq_l^p + f\right) + (1 - \theta)\left(rq_h^p + f\right) \tag{9A}$$

$$\text{s.t. } 0 \leq \delta < \frac{a}{b}, \; r \geq 0, \; f \geq 0, \; \pi_l^p \geq 0 \text{ and } \pi_h^p \geq 0.$$

Solving this problem yields the results below.

Proposition 1. *Suppose the conditions in (6A) hold. Then, we have the following.*

(i) *If* $c_h \in \left(c_l, \dot{c}_h\right]$ *with* $\dot{c}_h = \frac{a + c_l}{2}$, *then the two-part tariff contract is the port authority's best choice. The optimal scheme and minimum throughput guarantee are* $r^p = \frac{(c_h - c_l)}{2 - \theta}$, $f^p = \frac{(a + c_l - 2c_h)\left[(2 - \theta)a + \theta c_l - 2c_h\right]}{4b(2 - \theta)}$, *and* $\delta^p = \frac{(2 - \theta)a + \theta c_l - 2c_h}{2b(2 - \theta)}$. *At the equilibrium, the port authority's fee revenue equals* $R^p = \frac{(2 - \theta)a^2 - 2(2 - \theta)ac_h - 2\theta c_l c_h + \theta c_l^2 + 2c_h^2}{4b(2 - \theta)}$.

(ii) *If* $c_h \in \left(\dot{c}_h, \hat{c}_h\right)$ *with* $\hat{c}_h \equiv \frac{(2 - \theta)a + \theta c_l}{2}$, *then the unit-fee scheme is the port authority's best choice. The optimal scheme and minimum throughput guarantee are*

$r^p = \frac{(2-\theta)a - \theta c_l - 2(1-\theta)c_h}{2(2-\theta)}$ *and* $\delta^p = \frac{(2-\theta)a + \theta c_l - 2c_h}{2b(2-\theta)}$. *At the equilibrium, the port authority's fee revenue equals* $R^p = \frac{\left[(2-\theta)a - \theta c_l - 2(1-\theta)c_h\right]^2}{8b(2-\theta)}$.

(iii) *If* $c_h \in [\hat{c}_h, a)$, *then the unit-fee scheme is the port authority's best choice. The optimal scheme and minimum throughput guarantee are* $r^p = \overline{r}_h \equiv (a - c_h)$ *and* $\delta^p = 0$. *At the equilibrium, the port authority's fee revenue equals* $R^p = \frac{\theta(a-c_h)(c_h - c_l)}{2b}$.

Proof of Proposition 1: According to Lemma 3, we have three cases below.

Case1: Suppose $\delta \in [0, \delta_1^p]$. Lemma 3(i) implies $\pi_l^p > \pi_h^p$. Again, $\frac{\partial R}{\partial f} > 0$ and $\pi_h^p \geq 0$ suggest $f^p = b\left(q_h^p\right)^2 \geq 0$, and hence $\pi_h^p = 0$ and $\pi_l^p > 0$. Problem (9A) then becomes

$$\max_{r, f, \delta} \; R = \theta\left(rq_l^p + f\right) + (1-\theta)\left(rq_h^p + f\right)$$

$$\text{s.t. } 0 \leq \delta \leq \delta_1^p \text{ and } 0 < r \leq \overline{r}_h.$$

Its Lagrange function is

$$L = b\left(q_h^p\right)^2 + \theta\left(r \cdot q_l^p\right) + (1-\theta)\left(r \cdot q_h^p\right) + \lambda_1\left(\delta_1^p - \delta\right) + \lambda_2\left(\overline{r}_h - r\right),$$

where λ_1 and λ_2 are the respective Lagrange multipliers associated with the two inequality constraints of this problem. The corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{1}{2b}\left[\theta(c_h - c_l) - r\right] - \frac{1}{2b}\lambda_1 - \lambda_2 \leq 0, \; r \cdot \frac{\partial L}{\partial r} = 0, \tag{A20}$$

$$\frac{\partial L}{\partial \delta} = -\lambda_1 \leq 0, \; \delta \cdot \frac{\partial L}{\partial \delta} = 0, \tag{A21}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta_1^p - \delta \geq 0, \; \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \text{ and} \tag{A22}$$

$$\frac{\partial L}{\partial \lambda_2} = \overline{r}_h - r \geq 0, \; \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0. \tag{A23}$$

Based on the values of λ_1 and λ_2, we have four sub-cases.

Case 1a: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Constraint (A20) suggests $r^p = \theta(c_h - c_l) > 0$. It remains to check whether $r^p \le \overline{r}_h$ holds. By some calculations, we have $r^p \le \overline{r}_h$ iff $c_h \le c_{hp1} \equiv \frac{a+\theta c_l}{1+\theta}$.

Moreover, (A22) implies both $\delta^p \in \left[0, \delta_1^p\right]$ with $\delta_1^p = \frac{\left(a-c_h-r^p\right)}{2b} = \frac{a+\theta c_l-(1+\theta)c_h}{2b}$ and $f^p = b\left[\frac{a+\theta c_l-(1+\theta)c_h}{2b}\right]^2 > 0$. Thus, a solution exists when $c_h \le c_{hp1}$, and the port authority's equilibrium fee revenue equals

$$R_{11}^p = \frac{a-2ac_h-2\theta^2 c_l c_h+\theta^2 c_l^2+\left(1+\theta^2\right)c_h^2}{4b}. \tag{A24}$$

Case 1b: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Constraints (A20) and (A23) imply $r^p = \overline{r}_h = (a-c_h)$, and $\lambda_2^* = \frac{-a-\theta c_l+(1+\theta)c_h}{2b} > 0$ iff $c_h > c_{hp1}$. Moreover, (A22) suggests $\delta^p = 0$ by $\delta_1^p = \frac{\left(a-c_h-\overline{r}\right)}{2b} = 0$, and $f^p = 0$. Thus, for $c_h > c_{hp1}$, the port authority's equilibrium fee revenue equals

$$R_{12}^p = \frac{\theta\left(c_h-c_l\right)\left(a-c_h\right)}{2b}. \tag{A25}$$

Case 1c: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Constraint (A22) suggests $\delta^p = \delta_1^p \ge 0$. If $\delta_1^p > 0$, then $\lambda_1^* = 0$ by (A21). It is a contradiction. Thus, no solution exists in this case. If $\delta_1^p = 0$, then $r^p = \overline{r}_h$ and the solution is same as Case 1b's.

Case 1d: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. The solution is the same as Case 1b's for $c_h > c_{hp1}$.

Case 2: Suppose $\delta \in \left(\delta_1^p, \delta_2^p\right]$. Similar to Case 1, we have $f^p = \delta\left(a-b\delta-c_h-r\right)$, and $f^p \ge 0$ iff $\delta \le \tilde{\delta} \equiv \frac{a-c_h-r}{b}$ and $r \le \overline{r}_h \equiv a-c_h$. Moreover, $\tilde{\delta} \ge (<) \delta_2^p$ iff $r \le (>)\ \tilde{r} \equiv a+c_l-2c_h$. Thus, there are two sub-cases.

Case 2a: Suppose $r \le \tilde{r} \equiv (a+c_l-2c_h)$. Problem (9A) becomes

$$\max_{r,f,\delta}\ R = \theta\left(rq_l^p + f\right) + (1-\theta)\left(rq_h^p + f\right)$$

$$\text{s.t. } \delta_1^p < \delta \le \delta_2^p \text{ and } 0 \le r \le \tilde{r} \tag{A26}$$

Its Lagrange function is

$$L = \delta\left(a-b\delta-c_h-r\right) + \frac{r}{2b}\left[\theta\left(a-c_l-r\right)+2b(1-\theta)\delta\right] + \lambda_1\left(\delta-\delta_1^p\right) + \lambda_2\left(\delta_2^p-\delta\right) + \lambda_3\left(\tilde{r}-r\right).$$

The corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{\theta}{2b}\left(a-2r-c_l-2b\delta\right) + \frac{1}{2b}\lambda_1 - \frac{1}{2b}\lambda_2 - \lambda_3 \le 0,\ r\cdot\frac{\partial L}{\partial r} = 0, \tag{A27}$$

$$\frac{\partial L}{\partial \delta} = a - 2b\delta - c_h - \theta r + \lambda_1 - \lambda_2 \le 0,\ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \tag{A28}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_1^p \ge 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \tag{A29}$$

$$\frac{\partial L}{\partial \lambda_2} = \delta_2^p - \delta \ge 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \text{ and} \tag{A30}$$

$$\frac{\partial L}{\partial \lambda_3} = \tilde{r} - r \ge 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \tag{A31}$$

where λ_1, λ_2 and λ_3 are the respective Lagrange multipliers associated with the inequality constraints in (A26). Constraint $\delta_1^p < \delta$ suggests $\lambda_1^* = 0$ by (A29). Based on the values of λ_2 and λ_3, there are four sub-cases.

<u>Case 2a-1</u>: Suppose $\lambda_2^* = 0$ and $\lambda_3^* = 0$. Then, (A27) and (A28) suggest $\frac{\theta}{2b}\left(a - 2r - c_l - 2b\delta\right) = 0$ and $(a - 2b\delta - c_h - \theta r) = 0$. Solving the two equations yields $r^p = \frac{c_h - c_l}{2-\theta}$ and $\delta^p = \frac{(2-\theta)a + \theta c_l - 2c_h}{2b(2-\theta)}$. By some calculations, we have $(\delta^p - \delta_1^p) = \frac{(1-\theta)(c_h - c_l)}{2b(2-\theta)} > 0$, $(\delta_2^p - \delta^p) = \frac{(c_h - c_l)}{2b(2-\theta)} > 0$ and $(\tilde{r} - r^p) = \frac{(2-\theta)a + (3-\theta)c_l - (5-2\theta)c_h}{(2-\theta)} \ge 0$ iff $c_h \le c_{hp2} \equiv \frac{(2-\theta)a + (3-\theta)c_l}{(5-2\theta)}$. Thus, a solution exists when $c_h \le c_{hp2}$ with $r^p = \frac{c_h - c_l}{2-\theta}$, $\delta^p = \frac{(2-\theta)a + \theta c_l - 2c_h}{2b(2-\theta)}$ and $f^p = \frac{(a + c_l - 2c_h)\left[(2-\theta)a + \theta c_l - 2c_h\right]}{4b(2-\theta)}$, and the port authority's equilibrium fee revenue equals

$$R_{21}^p = \frac{(2-\theta)a^2 - 2(2-\theta)ac_h - 2\theta c_l c_h + \theta c_l^2 + 2c_h^2}{4b(2-\theta)}. \tag{A32}$$

<u>Case 2a-2</u>: Suppose $\lambda_2^* = 0$ and $\lambda_3^* > 0$. Then, (A27), (A28) and (A31) suggest

$\frac{\theta}{2b}\left(a - 2r - c_l - 2b\delta\right) - \lambda_3 = 0$, $(a - 2b\delta - c_h - \theta r) = 0$ and $(\tilde{r} - r) = 0$. Solving these equations yields $r^p = (a + c_l - 2c_h)$, $\delta^p = \frac{1}{2b}\left[(1-\theta)a - \theta c_l - (1-2\theta)c_h\right]$ and $\lambda_3^* = \frac{-\theta\left[(2-\theta)a + (3-\theta)c_l - (5-2\theta)c_h\right]}{2}$. It remains to check whether $\delta_1^p < \delta^p \le \delta_2^p$, $r^p \ge 0$ and $\lambda_3^* > 0$ hold. By some calculations, we have $(\delta^p - \delta_1^p) = \frac{1}{2b}(1-\theta)\left(a + c_l - 2c_h\right) > 0$ iff

$c_h < \dot{c}_h \equiv \frac{a+c_l}{2}$, $r^p \geq 0$ iff $c_h \leq \dot{c}_h$, $(\delta_2^p - \delta^p) = \frac{1}{2b}\left[-(1-\theta)a - (2-\theta)c_l + (3-2\theta)c_h\right] \geq 0$ iff $c_h \geq \frac{(1-\theta)a+(2-\theta)c_l}{3-2\theta}$, and $\lambda_3^* > 0$ iff $c_h > c_{hp2}$. Moreover, we have $c_{hp2} - \frac{(1-\theta)a+(2-\theta)c_l}{3-2\theta} = \frac{(a-c_l)}{(3-2\theta)(5-2\theta)} > 0$, and $(\dot{c}_h - c_{hp2}) = \frac{(a-c_l)}{2(5-2\theta)} > 0$. Thus, a solution exists for $c_{hp2} < c_h < \dot{c}_h$ and the port authority's equilibrium fee revenue equals

$$R_{22}^p = \frac{1}{4b}\begin{bmatrix}(1-\theta)^2 a^2 - 2\theta(3-\theta)ac_l - 2(1-5\theta+2\theta^2)ac_h \\ +2\theta(7-2\theta)c_lc_h - \theta(4-\theta)c_l^2 + (1-12\theta+4\theta^2)c_h^2\end{bmatrix}. \quad \text{(A33)}$$

Case 2a-3: Suppose $\lambda_2^* > 0$ and $\lambda_3^* = 0$. Then, (A27), (A28) and (A30) suggest $\frac{\theta}{2b}(a-2r-c_l-2b\delta) - \frac{1}{2b}\lambda_2 = 0$, $(a-2b\delta-c_h-\theta r-\lambda_2) = 0$ and $(\delta_2^p - \delta) = 0$. Solving these equations yields $r^p = (c_h - c_l) > 0$, $\delta^p = \frac{a-c_h}{2b}$ and $\lambda_2^* = -\theta(c_h - c_l) < 0$. Thus, no solution exists in this case.

Case 2a-4: Suppose $\lambda_2^* > 0$ and $\lambda_3^* > 0$. Then, (A27), (A28), (A30) and (A31) suggest $\frac{\theta}{2b}(a-2r-c_l-2b\delta) - \frac{1}{2b}\lambda_2 - \lambda_3 = 0$, $(a-2b\delta-c_h-\theta r-\lambda_2) = 0$, $(\tilde{r} - r) = 0$ and $(\delta_2^p - \delta) = 0$. Solving these equations yields $r^p = \tilde{r} = a + c_l - 2c_h$, $\delta^p = \delta_2^p = \frac{(c_h - c_l)}{b}$, $\lambda_2^* = (1-\theta)a + (2-\theta)c_l - (3-2\theta)c_h > 0$ iff $c_h < \frac{(1-\theta)a+(2-\theta)c_l}{(3-2\theta)}$, and $\lambda_3^* = \frac{1}{2b}\left[-a - 2c_l + 3c_h\right] > 0$ iff $c_h > \frac{a+2c_l}{3}$. Since $\frac{(1-\theta)a+(2-\theta)c_l}{(3-2\theta)} - \frac{a+2c_l}{3} = \frac{-\theta(a-c_l)}{(3-2\theta)} < 0$, we cannot have both $\lambda_2^* > 0$ and $\lambda_3^* > 0$ together. Thus, no solution exists in this case.

Case 2b: Suppose $r > \tilde{r} \equiv (a + c_l - 2c_h)$. Note that $f^p = \delta(a - b\delta - c_h - r) \geq 0$ if $\delta \leq \tilde{\delta} \equiv \frac{(a-c_h-r)}{b}$, and $\tilde{\delta} \geq (<) \delta_2^p$ iff $r \leq (>)\ \tilde{r}$. Assuming $r > \tilde{r}$, we reduce constraint $\delta_1^p < \delta \leq \delta_2^p$ to $\delta_1^p < \delta \leq \tilde{\delta}$. Thus, problem (9A) becomes

$$\max_{r,f,\delta}\ R = \theta\left(rq_l^p + f\right) + (1-\theta)\left(rq_h^p + f\right)$$

$$\text{s.t. } \delta_1^p < \delta \leq \tilde{\delta} \text{ and } \tilde{r} < r \leq \overline{r}_h. \quad \text{(A34)}$$

Its Lagrange function is

$$L = \delta(a - b\delta - c_h - r) + \frac{r}{2b}\left[\theta(a - c_l - r) + 2b(1-\theta)\delta\right] + \lambda_1\left(\delta - \delta_1^p\right) + \lambda_2\left(\tilde{\delta} - \delta\right) + \lambda_3\left(r - \tilde{r}\right) + \lambda_4\left(\overline{r}_h - r\right).$$

The corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r}=\frac{\theta}{2b}\left(a-2r-c_l-2b\delta\right)+\frac{1}{2b}\lambda_1-\frac{1}{b}\lambda_2+\lambda_3-\lambda_4\le 0,\ r\cdot\frac{\partial L}{\partial r}=0, \tag{A35}$$

$$\frac{\partial L}{\partial \delta}=a-2b\delta-c_h-\theta r+\lambda_1-\lambda_2\le 0,\ \delta\cdot\frac{\partial L}{\partial \delta}=0, \tag{A36}$$

$$\frac{\partial L}{\partial \lambda_1}=\delta-\delta_1^p\ge 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1}=0, \tag{A37}$$

$$\frac{\partial L}{\partial \lambda_2}=\tilde{\delta}-\delta\ge 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2}=0, \tag{A38}$$

$$\frac{\partial L}{\partial \lambda_3}=r-\tilde{r}\ge 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3}=0,\ \text{and} \tag{A39}$$

$$\frac{\partial L}{\partial \lambda_4}=\overline{r}_h-r\ge 0,\ \lambda_4\cdot\frac{\partial L}{\partial \lambda_4}=0, \tag{A40}$$

where λ_1, λ_2, λ_3, and λ_4 are the respective Lagrange multipliers associated with the inequality constraints in (A34). Constraints $\delta_1^p<\delta$ and $\tilde{r}<r$ suggest $\lambda_1^*=0$ and $\lambda_3^*=0$ by (A37) and (A39). If $\lambda_4^*>0$, then $r^p=\overline{r}_h=a-c_h$ by (A40). Moreover, we obtain $\delta_1^p=\tilde{\delta}=0$ by $r^p=\overline{r}_h$, which contradicts the requirement of $\delta_1^p<\delta\le\tilde{\delta}$. Thus, we must have $\lambda_4^*=0$. Based on the value of λ_2, there are two sub-cases below.

<u>Case 2b-1</u>: Suppose $\lambda_2^*=0$. Then, (A35) and (A36) suggest $\frac{\theta}{2b}\left(a-2r-c_l-2b\delta\right)=0$ and $(a-2b\delta-c_h-\theta r)=0$. Solving these equations yields $r^p=\frac{c_h-c_l}{2-\theta}$ and $\delta^p=\frac{(2-\theta)a+\theta c_l-2c_h}{2b(2-\theta)}$. It remains to check whether $\delta_1^p<\delta^p\le\tilde{\delta}$ and $\tilde{r}<r^p\le\overline{r}_h$ hold. By some calculations, we obtain $(\delta^p-\delta_1^p)=\frac{(1-\theta)(c_h-c_l)}{2b(2-\theta)}>0$, $(\tilde{\delta}-\delta^p)=\frac{a+c_l-2c_h}{2b}\ge 0$ iff $c_h\le\dot{c}_h\equiv\frac{a+c_l}{2}$,

$(r^p-\tilde{r})=\frac{-(2-\theta)a-(3-\theta)c_l+(5-2\theta)c_h}{(2-\theta)}>0$ iff $c_h>c_{hp2}$, and

$(\overline{r}_h-r^p)=\frac{(2-\theta)a+c_l-(3-\theta)c_h}{(2-\theta)}\ge 0$ iff $c_h\le\frac{(2-\theta)a+c_l}{(3-\theta)}$. In addition, we have $\frac{(2-\theta)a+c_l}{(3-\theta)}-\dot{c}_h=\frac{(1-\theta)(a-c_l)}{2(3-\theta)}>0$ and $(\dot{c}_h-c_{hp2})=\frac{(2-\theta)^2(a-c_l)}{(3-\theta)(5-2\theta)}>0$. Thus, a solution exists for $c_{hp2}<c_h\le\dot{c}_h$, and the port authority's equilibrium fee revenue equals

$$R_{21}^p=\frac{(2-\theta)a^2-2(2-\theta)ac_h-2\theta c_lc_h+\theta c_l^2+2c_h^2}{4b(2-\theta)}. \tag{A41}$$

Case 2b-2: Suppose $\lambda_2^* > 0$. Then, (A35), (A36) and (A38) suggest $\frac{\theta}{2b}\left(a-2r-c_l-2b\delta\right)-\frac{1}{b}\lambda_2=0$, $(a-2b\delta-c_h-\theta r-\lambda_2)=0$, and $(\tilde{\delta}-\delta)=0$. Solving these equations yields $r^p=\frac{(2-\theta)a-\theta c_l-2(1-\theta)c_h}{2(2-\theta)}$, $\delta^p=\frac{(2-\theta)a+\theta c_l-2c_h}{2b(2-\theta)}$ and $\lambda_2^*=\frac{-\theta\left(a+c_l-2c_h\right)}{2}$. It remains to check whether $\delta_1^p<\delta^p$, $\tilde{r}<r^p\le\overline{r}_h$ and $\lambda_2^*>0$ hold. By some calculations, we obtain $(\delta^p-\delta_1^p)=\frac{(2-\theta)a+\theta c_l-2c_h}{4b(2-\theta)}>0$ iff $c_h<\hat{c}_h\equiv\frac{(2-\theta)a+\theta c_l}{2}$, $(r^p-\tilde{r})=\frac{-(2-\theta)a-(4-\theta)c_l+2(3-\theta)c_h}{2(2-\theta)}>0$ iff $c_h>\frac{(2-\theta)a+(4-\theta)c_l}{2(3-\theta)}$, $(\overline{r}-r^p)=\frac{(2-\theta)a+\theta c_l-2c_h}{2(2-\theta)}>0$ iff $c_h<\frac{(2-\theta)a+\theta c_l}{2}$, and $\lambda_2^*>0$ iff $c_h>\frac{a+c_l}{2}=\dot{c}_h$. In addition, we have $\frac{(2-\theta)a+\theta c_l}{2}-\frac{a+c_l}{2}=\frac{(1-\theta)(a-c_l)}{2}>0$ and $\frac{a+c_l}{2}-\frac{(2-\theta)a+(4-\theta)c_l}{2(3-\theta)}=\frac{(a-c_l)}{2(3-\theta)}>0$. Thus, $\delta_1^p<\delta^p$, $\tilde{r}<r^p\le\overline{r}_h$ and $\lambda_2^*>0$ hold if $\dot{c}_h<c_h<\hat{c}_h$. Under the circumstance, the port authority's equilibrium fee revenue equals

$$R_{23}^p=\frac{\left[(2-\theta)a-\theta c_l-2(1-\theta)c_h\right]^2}{8b(2-\theta)}. \qquad \text{(A42)}$$

Case 3: Suppose $\delta\in\left(\delta_2^p,\frac{a}{b}\right)$. Lemma 3(iii) then implies $\pi_l^p>\pi_h^p$, and hence $f^p=\delta\left(a-b\delta-c_h-r\right)$ with $f^p\ge 0$ iff $\delta\le\tilde{\delta}\equiv\frac{(a-c_h-r)}{b}<\frac{a}{b}$ and $r\le\overline{r}_h\equiv(a-c_h)$. In addition, $\tilde{\delta}>(\le)\delta_2^p$ iff $r<(\ge)\tilde{r}\equiv(a+c_l-2c_h)$. If $r\ge\tilde{r}$, then $\tilde{\delta}\le\delta_2^p$. To have $f^p\ge 0$, we need $\delta\le\tilde{\delta}$. Since δ should belong to the interval of $\left(\delta_2^p,a\right)$, the two conditions contradict with each other. Thus, we must have $r<\tilde{r}$ and $\tilde{\delta}>\delta_2^p$. Combining $\delta\in\left(\delta_2^p,a\right)$ and $f^p\ge 0$, we have $\delta\in(\delta_2^p,\tilde{\delta}]$. Accordingly, problem (9A) becomes

$$\max_{r,f,\delta}\ R=\theta\left(rq_l^p+f\right)+(1-\theta)\left(rq_h^p+f\right)$$

$$\text{s.t. } \delta_2^p<\delta\le\tilde{\delta} \text{ and } 0\le r<\tilde{r}. \qquad \text{(A43)}$$

Its Lagrange function is

$$L=\delta\left(a-b\delta-c_h\right)+\lambda_1\left(\delta-\delta_2^p\right)+\lambda_2\left(\tilde{\delta}-\delta\right)+\lambda_3\left(\tilde{r}-r\right).$$

The corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r} = \frac{1}{2b}\lambda_1 - \frac{1}{b}\lambda_2 - \lambda_3 \le 0,\ r \cdot \frac{\partial L}{\partial r} = 0, \tag{A44}$$

$$\frac{\partial L}{\partial \delta} = \left(a - 2b\delta - c_h\right) + \lambda_1 - \lambda_2 \le 0,\ \delta \cdot \frac{\partial L}{\partial \delta} = 0, \tag{A45}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta - \delta_2^p \ge 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \tag{A46}$$

$$\frac{\partial L}{\partial \lambda_2} = \tilde{\delta} - \delta \ge 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0 \text{, and} \tag{A47}$$

$$\frac{\partial L}{\partial \lambda_3} = \tilde{r} - r \ge 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0\,, \tag{A48}$$

where λ_1, λ_2, and λ_3 are the respective Lagrange multipliers associated with the inequality constraints in (A43). Since constraints $\delta_2^p < \delta$ and $r < \tilde{r}$ are strict inequalities, we must have $\lambda_1^* = \lambda_3^* = 0$ by (A46) and (A48). If $\lambda_2^* > 0$, then we obtain $\frac{\partial L}{\partial r} = -\frac{1}{b}\lambda_2 < 0$ by (A44), $\delta^p = \tilde{\delta} = \frac{(a - c_h)}{b}$ by (A47), and $\lambda_2^* = -\left(a - c_h\right) < 0$ by (A45), which contradicts requirement $\lambda_2^* > 0$. Thus, we must have $\lambda_2^* = 0$, and hence (A45) suggests $\delta^p = \frac{a - c_h}{2b}$. It remains to check whether $\delta_2^p < \delta^p \le \tilde{\delta}$, $r^p < \tilde{r}$ and $\lambda_2^* > 0$ hold. By some calculations, we have

$(\delta^p - \delta_2^p) = \frac{a - c_h}{2b} - \frac{a - r - c_l}{2b} = \frac{r - \left(c_h - c_l\right)}{2b} > 0$ if $r > \left(c_h - c_l\right)$,

$(\tilde{\delta} - \delta^p) = \frac{a - r - c_h}{b} - \frac{a - c_h}{2b} = \frac{a - c_h - 2r}{2b} \ge 0$ if $r \le \frac{a - c_h}{2}$, and $r^p < \tilde{r} = (a + c_l - 2c_h)$. Since $\frac{a - c_h}{2} - \left(c_h - c_l\right) = \frac{a + 2c_l - 3c_h}{2} > 0$ iff $c_h < \frac{a + 2c_l}{3}$, we have

$\left(a + c_l - 2c_h\right) - \left(c_h - c_l\right) = a + 2c_l - 3c_h > 0$ iff $c_h < \frac{a + 2c_l}{3}$, and

$\left(a + c_l - 2c_h\right) - \frac{a - c_h}{2} = \frac{a + 2c_l - 3c_h}{2} > \left(<\right) 0$ iff $c_h < \left(>\right) \frac{a + 2c_l}{3}$. When $c_h > \frac{a + 2c_l}{3}$, constraint $\delta_2^p < \delta^p \le \tilde{\delta}$ cannot hold. Thus, for $c_h < \frac{a + 2c_l}{3}$, a solution exists with $r^p \in \left(c_h - c_l, \frac{a - c_h}{2}\right]$ and $\delta^p = \frac{a - c_h}{2b}$, and the port authority's equilibrium fee revenue equals

$$R_3^p = \frac{\left(a - c_h\right)^2}{4b}\,. \tag{A49}$$

Cases 1-3 imply that problem (9A) has seven solutions. By comparing them, we can derive the port authority's best choices. Because $(\hat{c}_h - c_{hp1}) = \frac{\theta(1-\theta)(a-c_l)}{2(1+\theta)} > 0$,

$(c_{hp1} - \dot{c}_h) = \frac{(1-\theta)(a-c_l)}{2(1+\theta)} > 0$, $(\dot{c}_h - c_{hp2}) = \frac{(2-\theta)^2(a-c_l)}{(3-\theta)(5-2\theta)} > 0$, and

$(c_{hp2} - \frac{a+2c_l}{3}) = \frac{(1-\theta)(a-c_l)}{3(5-2\theta)} > 0$, we have $\frac{a+2c_l}{3} < c_{hp2} < \dot{c}_h < c_{hp1} < \hat{c}_h$, and there are the following six cases.

<u>Case A</u>: Suppose $c_h < \frac{a+2c_l}{3}$. Three solutions appear: $R_{11}^p = \frac{a^2 - 2ac_h - 2\theta^2 c_l c_h + \theta^2 c_l^2 + (1+\theta^2)c_h^2}{4b}$ for $c_h \le c_{hp1}$ defined in (A24), $R_{21}^p = \frac{(2-\theta)a^2 - 2(2-\theta)ac_h - 2\theta c_l c_h + \theta c_l^2 + 2c_h^2}{4b(2-\theta)}$ for $c_h \le c_{hp2}$ defined in (A32), and $R_3^p = \frac{(a-c_h)^2}{4b}$ for $c_h < \frac{a+2c_l}{3}$ defined in (A49). Since $(R_{21}^p - R_{11}^p) = \frac{\theta(1-\theta)^2(c_h-c_l)^2}{4b(2-\theta)} > 0$ and $(R_{11}^p - R_3^p) = \frac{\theta^2(c_h-c_l)^2}{4b} > 0$, we have $R_{21}^p > R_{11}^p > R_3^p$.

<u>Case B</u>: Suppose $\frac{a+2c_l}{3} \le c_h \le c_{hp2}$. Two solutions appear:

$R_{11}^p = \frac{a^2 - 2ac_h - 2\theta^2 c_l c_h + \theta^2 c_l^2 + (1+\theta^2)c_h^2}{4b}$ defined in (A24) and

$R_{21}^p = \frac{(2-\theta)a^2 - 2(2-\theta)ac_h - 2\theta c_l c_h + \theta c_l^2 + 2c_h^2}{4b(2-\theta)}$ defined in (A32). Because

$(R_{21}^p - R_{11}^p) = \frac{\theta(1-\theta)^2(c_h-c_l)^2}{4b(2-\theta)} > 0$, we have $R_{21}^p > R_{11}^p$.

<u>Case C</u>: Suppose $c_{hp2} < c_h \le \dot{c}_h$. Three solutions appear:

$R_{11}^p = \frac{a^2 - 2ac_h - 2\theta^2 c_l c_h + \theta^2 c_l^2 + (1+\theta^2)c_h^2}{4b}$ defined in (A24),

$R_{22}^p = \frac{1}{4b}\begin{bmatrix} (1-\theta)^2 a^2 - 2\theta(3-\theta)ac_l - 2(1-5\theta+2\theta^2)ac_h \\ +2\theta(7-2\theta)c_l c_h - \theta(4-\theta)c_l^2 + (1-12\theta+4\theta^2)c_h^2 \end{bmatrix}$ defined in (A33), and

$R_{21}^p = \frac{(2-\theta)a^2 - 2(2-\theta)ac_h - 2\theta c_l c_h + \theta c_l^2 + 2c_h^2}{4b(2-\theta)}$ defined in (A41). Because

$(R_{21}^p - R_{11}^p) = \frac{\theta(1-\theta)^2(c_h-c_l)^2}{4b(2-\theta)} > 0$ and $(R_{21}^p - R_{22}^p) = \frac{\theta[(2-\theta)a + (3-\theta)c_l - (5-2\theta)c_h]^2}{4b(2-\theta)} > 0$, we have R_{21}^p the largest.

Case D: Suppose $\dot{c}_h < c_h \le c_{hp1}$. Two solutions appear:

$R_{11}^p = \dfrac{a^2 - 2ac_h - 2\theta^2 c_l c_h + \theta^2 c_l^2 + \left(1+\theta^2\right) c_h^2}{4b}$ defined in (A24) and

$R_{23}^p = \dfrac{\left[\left(2-\theta\right)a - \theta c_l - 2\left(1-\theta\right)c_h\right]^2}{8b\left(2-\theta\right)}$ defined in (A42). Since

$\dfrac{\partial\left(R_{23}^p - R_{11}^p\right)}{\partial c_h} = \dfrac{\theta\left[\left(2-\theta\right)a + \left(1+\theta-\theta^2\right)c_l - \left(3-\theta^2\right)c_h\right]}{2b\left(2-\theta\right)}$, $\dfrac{\partial^2\left(R_{23}^p - R_{11}^p\right)}{\partial c_h{}^2} = \dfrac{-\theta\left(3-\theta^2\right)}{2b\left(2-\theta\right)} < 0$,

$\dfrac{\partial\left(R_{23}^p - R_{11}^p\right)}{\partial c_h} = \dfrac{\theta\left(1-\theta\right)^2\left(a-c_l\right)}{4b\left(2-\theta\right)} > 0$, $(R_{23}^p - R_{11}^p) = \dfrac{\theta\left(1-\theta\right)^2\left(a-c_l\right)^2}{16b\left(2-\theta\right)} > 0$ at $c_h = \dot{c}_h$,

$\dfrac{\partial\left(R_{23}^p - R_{11}^p\right)}{\partial c_h} = \dfrac{-\theta\left(1-\theta\right)\left(a-c_l\right)}{2b\left(1+\theta\right)\left(2-\theta\right)} < 0$, and $(R_{23}^p - R_{11}^p) = \dfrac{\theta^2\left(1-\theta\right)^2\left(a-c_l\right)^2}{8b\left(2-\theta\right)\left(1+\theta\right)^2} > 0$ at $c_h = c_{hp1}$, we have

$R_{23}^p > R_{11}^p$ for $\dot{c}_h < c_h \le c_{hp1}$.

Case E: Suppose $c_{hp1} < c_h < \hat{c}_h$. Two solutions appear:

$R_{12}^p = \dfrac{\theta\left(c_h - c_l\right)\left(a-c_h\right)}{2b}$ defined in (A25) and $R_{23}^p = \dfrac{\left[\left(2-\theta\right)a - \theta c_l - 2\left(1-\theta\right)c_h\right]^2}{8b\left(2-\theta\right)}$ defined in (A42).

Since $(R_{23}^p - R_{12}^p) = \dfrac{\left[\left(2-\theta\right)a + \theta c_l - 2c_h\right]^2}{8b\left(2-\theta\right)} > 0$, we have $R_{23}^p > R_{12}^p$.

Case F: Suppose $c_h \ge \hat{c}_h$. The unique solution is $R_{12}^p = \dfrac{\theta\left(c_h - c_l\right)\left(a-c_h\right)}{2b}$ defined in (A25).

We find that R_{21}^p is optimal when $c_h \le \dot{c}_h$ by Cases A-C, R_{23}^p is optimal when $\dot{c}_h < c_h < \hat{c}_h$ by Cases D-E, and R_{12}^p is optimal when $c_h \ge \hat{c}_h$ by Case F.

In summary, for $c_h \le \dot{c}_h$, the port authority should adopt the two-part tariff scheme with $r^p = \dfrac{c_h - c_l}{2-\theta}$, $f^p = \dfrac{\left(a + c_l - 2c_h\right)\left[\left(2-\theta\right)a + \theta c_l - 2c_h\right]}{4b\left(2-\theta\right)}$, minimum throughput requirement $\delta^p = \dfrac{\left(2-\theta\right)a + \theta c_l - 2c_h}{2b\left(2-\theta\right)}$, and equilibrium fee revenue $R^p = \dfrac{\left(2-\theta\right)a^2 - 2\left(2-\theta\right)ac_h - 2\theta c_l c_h + \theta c_l^2 + 2c_h^2}{4b\left(2-\theta\right)}$ as in (A32). These prove Proposition 1(i). For $\dot{c}_h < c_h < \hat{c}_h$, the port authority ought to adopt the unit-fee scheme with $r^p = \dfrac{\left(2-\theta\right)a - \theta c_l - 2\left(1-\theta\right)c_h}{2\left(2-\theta\right)}$, minimum throughput requirement $\delta^p = \dfrac{\left(2-\theta\right)a + \theta c_l - 2c_h}{2b\left(2-\theta\right)}$, and equilibrium fee revenue $R^p = \frac{\left[(2-\theta)a-\theta c_l - 2(1-\theta)c_h\right]^2}{8b(2-\theta)}$ as in (A42). These prove Proposition 1(ii). Finally, for $c_h \geq \hat{c}_h$, the port authority should offer the unit-fee scheme with $r^p = \overline{r}_h$, $\delta^p = 0$, and equilibrium fee revenue $R^p = \frac{\theta(a-c_h)(c_h-c_l)}{2b}$ as in (A25). These prove Proposition 1(iii). □

Under the new set-up, problem (10) becomes

$$\max_{q_i \geq 0} \quad \pi_i = (a - bq_i - c_i - r_l) q_i - f_l \tag{10A}$$

$$\text{s.t. } q_i \geq \delta_l$$

for $i = l, h$. Its solutions are given below.

Lemma 4. *Suppose the conditions in (6A) hold. Given contract* (r_l, f_l, δ_l), *operator i's optimal behaviors are as follows.*

(i) *For* $\delta_l \in [0, \delta_l']$ *with* $\delta_l' = \frac{(a-c_h-r_l)}{2b}$, *both-type operators' equilibrium cargo-handling amounts are* $q_l^{sl} = \frac{(a-c_l-r_l)}{2b} > \delta_l'$ *and* $q_h^{sl} = \frac{(a-c_h-r_l)}{2b} = \delta_l'$, *their equilibrium service prices are* $p_i^{sl} = \frac{(a+c_i+r_l)}{2} > 0$, *and their equilibrium profits are* $\pi_i^{sl} = b(q_i^{sl})^2 - f_l$ *for* $i = l, h$.

(ii) *For* $\delta_l \in (\delta_l', \delta_l'']$ *with* $\delta_l'' = \frac{(a-c_l-r_l)}{2b} > \delta_l'$, *both-type operators' equilibrium cargo-handling amounts are* $q_l^{sl} = \frac{(a-c_l-r_l)}{2b}$ *and* $q_h^{sl} = \delta_l$, *their equilibrium service prices are* $p_l^{sl} = \frac{(a+c_l+r_l)}{2} > 0$ *and* $p_h^{sl} = (a - b\delta_l) > 0$, *and their equilibrium profits are* $\pi_l^{sl} = b\left(q_l^{sl}\right)^2 - f_l$ *and* $\pi_h^{sl} = \delta_l\left[a - b\delta_l - c_h - r_l\right] - f_l$.

(iii) *For* $\delta_l \in \left(\delta_l'', \frac{a}{b}\right)$, *both-type operators' equilibrium cargo-handling amounts are* $q_l^{sl} = \delta_l$ *and* $q_h^{sl} = \delta_l$, *their equilibrium service prices are* $p_l^{sl} = p_h^{sl} = (a - b\delta_l) > 0$, *and their equilibrium profits are* $\pi_i^{sl} = \delta_l\left[a - b\delta_l - c_i - r_l\right] - f_l$ *for* $i = l, h$.

<u>**Proof of Lemma 4:**</u> Denote L_1 and L_2 the respective Lagrange functions for the *l*-type and the *h*-type terminal operators in problem (10A),

$$L_1 = (a - bq_l - c_l - r_l) q_l - f_l + \lambda_1 (q_l - \delta_l) \text{ and } L_2 = (a - bq_h - c_h - r_l) q_h - f_l + \lambda_2 (q_h - \delta_l),$$

where λ_1 and λ_2 are their associated Lagrange multipliers. Then, the corresponding Kuhn-Tucker conditions for the *l*-type operator are

$$\frac{\partial L_1}{\partial q_l} = a - 2bq_l - c_l - r_l + \lambda_1 \leq 0,\ q_l \cdot \frac{\partial L_1}{\partial q_l} = 0 \text{ and} \tag{A50}$$

$$\frac{\partial L_1}{\partial \lambda_1} = q_l - \delta_l \geq 0,\ \lambda_1 \cdot \frac{\partial L_1}{\partial \lambda_1} = 0, \tag{A51}$$

and for the *h*-type operator are

$$\frac{\partial L_2}{\partial q_h} = a - 2bq_h - c_h - r_l + \lambda_2 \leq 0,\ q_h \cdot \frac{\partial L_2}{\partial q_h} = 0 \text{ and} \tag{A52}$$

$$\frac{\partial L_2}{\partial \lambda_2} = q_h - \delta_l \geq 0,\ \lambda_2 \cdot \frac{\partial L_2}{\partial \lambda_2} = 0. \tag{A53}$$

Based on the values of λ_1 and λ_2, there are four cases below.

<u>Case 1</u>: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then, (A50) and (A52) suggest

$q_l^{sl} = \frac{a - c_l - r_l}{2b}$ and $q_h^{sl} = \frac{a - c_h - r_l}{2b}$. To have $q_l^{sl} \geq \delta_l$ and $q_h^{sl} \geq \delta_l$, we need to impose condition $0 \leq \delta_l \leq \delta_l' \equiv \frac{a - c_h - r_l}{2b} = q_h^{sl}$. That is because $c_l < c_h$ implies $q_l^{sl} > q_h^{sl}$, and hence $q_h^{sl} \geq \delta_l$ implies $q_l^{sl} \geq \delta_l$. Substituting q_l^{sl} and q_h^{sl} into (13) yields $p_h^{sl} = \frac{a + c_h + r_l}{2} > p_l^{sl} = \frac{a + c_l + r_l}{2} > 0$, and into (14) yields $\pi_i^{sl} = b\left(q_i^{sl}\right)^2 - f_l$ for $i = l, h$. These prove Lemma 4(i).

<u>Case 2</u>: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then, (A50), (A52) and (A53) suggest $(a - 2bq_l - c_l - r_l) = 0$, $(a - 2bq_h - c_h - r_l + \lambda_2) = 0$ and $(q_h - \delta_l) = 0$. Solving these equations yields $q_l^{sl} = \frac{a - c_l - r_l}{2b}$, $q_h^{sl} = \delta_l$ and $\lambda_2^* = 2b\delta_l - \left(a - c_h - r_l\right)$. To guarantee $\lambda_2^* > 0$, condition $\delta_l > \delta_l'$ is needed; and to have $q_l^{sl} \geq \delta_l$, condition $\delta_l \leq \delta_l'' \equiv \frac{a - c_l - r_l}{2b}$ is needed. Thus, the plausible range for δ_l is $\delta_l \in (\delta_l', \delta_l'']$. Substituting q_l^{sl} and q_h^{sl} into (13) produces $p_h^{sl} = (a - b\delta_l) > p_l^{sl} = \frac{a + c_l + r_l}{2} > 0$ if $\delta_l \leq \delta_l''$, and into (14) gives $\pi_l^{sl} = b\left(q_l^{sl}\right)^2 - f_l$ and $\pi_h^{sl} = \delta_l\left[a - b\delta_l - c_h - r_l\right] - f_l$. These prove Lemma 4(ii).

<u>Case 3</u>: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then, (A50)-(A52) suggest $(q_l - \delta_l) = 0$, $(a - 2bq_l - c_l - r_l + \lambda_1) = 0$ and $(a - 2bq_h - c_h - r_l) = 0$. Solving these equations yields $q_l^{sl} = \delta_l$, $q_h^{sl} = \frac{a - r_l - c_h}{2b}$ and $\lambda_1^* = 2b\delta_l - \left(a - c_l - r_l\right)$. To guarantee $\lambda_1^* > 0$, condition $\delta_l > \frac{a - c_l - r_l}{2b}$ is needed. On the other hand, $q_h^{sl} \geq \delta_l$ holds if $\delta_l \leq \frac{a - r_l - c_h}{2b}$. However, these two cannot hold simultaneously because $\frac{a - r_l - c_h}{2b} - \frac{a - c_l - r_l}{2b} = \frac{-\left(c_h - c_l\right)}{2b} < 0$. Thus, no solution exists in this case.

<u>Case 4</u>: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. Thus, (A50)-(A53) suggest $q_l^{sl} = q_h^{sl} = \delta_l$, $\lambda_1^* = 2b\delta_l - \left(a - c_l - r_l\right)$ and $\lambda_2^* = 2b\delta_l - \left(a - c_h - r_l\right)$. To have $\lambda_1^* > 0$ and $\lambda_2^* > 0$, conditions $\delta_l > \delta_l''$

and $r<(a-c_l)$ are needed. Substituting $q_l^{sl}=q_h^{sl}=\delta_l$ into (13) produces $p_l^{sl}=p_h^{sl}=(a-b\delta_l)>0$ if $\delta_l<\dfrac{a}{b}$, and into (14) gives $\pi_i^{sl}=\delta_l\left[a-b\delta_l-c_i-r_l\right]-f_l$ for $i=l,h$. These prove Lemma 4(iii).

Under the new set-up, problem (11) becomes

$$\max_{q_i\geq 0}\quad \pi_i=\left(a-bq_i-c_i-r_h\right)q_i-f_h \tag{11A}$$

$$\text{s.t. } q_i\geq\delta_h$$

for $i=l,h$. Its solutions are presented below.

Lemma 5. *Suppose the conditions in (6A) hold. Given contract* $\left(r_h,f_h,\delta_h\right)$, *operator i's optimal behaviors are as follows.*

(i) *For* $\delta_h\in[0,\delta_h']$ *with* $\delta_h'=\frac{(a-c_h-r_h)}{2b}$, *both-type operators' equilibrium cargo-handling amounts are* $q_l^{sh}=\dfrac{\left(a-c_l-r_h\right)}{2b}>\delta_h'$ *and* $q_h^{sh}=\dfrac{\left(a-c_h-r_h\right)}{2b}=\delta_h'$, *their equilibrium service prices are* $p_i^{sh}=\frac{(a+c_i+r_h)}{2}>0$, *and their equilibrium profits are* $\pi_i^{sh}=b(q_i^{sh})^2-f_h$ *for* $i=l,h$.

(ii) *For* $\delta_h\in\left(\delta_h',\delta_h''\right]$ *with* $\delta_h''=\frac{(a-c_l-r_h)}{2b}>\delta_h'$, *both-type operators' equilibrium cargo-handling amounts are* $q_l^{sh}=\dfrac{\left(a-c_l-r_h\right)}{2b}$ *and* $q_h^{sh}=\delta_h$, *their equilibrium service prices are* $p_l^{sh}=\frac{(a+c_l+r_h)}{2}>0$ *and* $p_h^{sh}=a-b\delta_h>0$, *and their equilibrium profits are* $\pi_l^{sh}=b\left(q_l^{sh}\right)^2-f_h$ *and* $\pi_h^{sh}=\delta_h\left[a-b\delta_h-c_h-r_h\right]-f_h$.

(iii) *For* $\delta_h\in\left(\delta_h'',\dfrac{a}{b}\right)$, *both-type operators' equilibrium cargo-handling amounts are* $q_l^{sh}=\delta_h$ *and* $q_h^{sh}=\delta_h$, *their equilibrium service prices are* $p_l^{sh}=p_h^{sh}=(a-b\delta_h)>0$, *and their equilibrium profits are* $\pi_i^{sh}=\delta_h\left[a-b\delta_h-c_i-r_h\right]-f_h$ *for* $i=l,h$.

<u>**Proof of Lemma 5**</u>**:** Since the proofs for Lemma 5 and Lemma 4 are similar, it is omitted.

Under the new set-up, problem (12) becomes

$$\max_{(r_l,f_l,\delta_l),(r_h,f_h,\delta_h)} R=\theta\left(r_lq_l^{sl}+f_l\right)+\left(1-\theta\right)\left(r_hq_h^{sh}+f_h\right) \tag{12A}$$

s.t. $0\leq\delta_l<\dfrac{a}{b},0\leq\delta_h<\dfrac{a}{b}$, $f_l\geq 0$, $f_h\geq 0$, $\pi_l^{sl}\geq 0$, $\pi_h^{sh}\geq 0$, $\pi_l^{sl}\geq\pi_l^{sh}$ and $\pi_h^{sh}\geq\pi_h^{sl}$.

It solutions are listed below.

Proposition 2. *Suppose the conditions in (6) hold. Then, we have the following.*

(i) *If* $c_h \in \left(c_l, c'_h\right]$ *with* $c'_h = \frac{(1-\theta)a + (2-\theta)c_l}{(3-2\theta)}$*, the port authority will offer the two-part tariff scheme and minimum throughput requirement* $\left(r_h^s, f_h^s, \delta_h^s\right)^T$*, or the unit-fee scheme and minimum throughput requirement* $\left(r_h^s, \delta_h^s\right)^U$ *to the h-type operator; and offer the two-part tariff scheme and minimum throughput requirement* $\left(r_l^s, f_l^s, \delta_l^s\right)^T$*, the unit-fee scheme and minimum throughput requirement* $\left(r_l^s, \delta_l^s\right)^U$*, or the fixed-fee scheme and minimum throughput requirement* $\left(f_l^s, \delta_l^s\right)^F$ *to the l-type operator. Here*

$$\left(r_h^s, f_h^s, \delta_h^s\right)^T = \left\{ \begin{array}{l} r_h^s \in \left[\frac{c_h - c_l}{(1-\theta)}, \frac{(1-\theta)a - \theta c_l - (1-2\theta)c_h}{2(1-\theta)} \right), \\ f_h^s = \frac{\left[(1-\theta)a + \theta c_l - c_h\right] \cdot \left[(1-\theta)a - \theta c_l - (1-2\theta)c_h - 2(1-\theta)r_h^s\right]}{4b(1-\theta)^2}, \\ \delta_h^s = \frac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)} \end{array} \right\},$$

$$\left(r_h^s, \delta_h^s\right)^U = \left\{ r_h^s = \frac{(1-\theta)a - \theta c_l - (1-2\theta)c_h}{2(1-\theta)},\ \delta_h^s = \frac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)} \right\},$$

$$\left(r_l^s, f_l^s, \delta_l^s\right)^T = \left\{ \begin{array}{l} r_l^s \in \left(0, \frac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2(1+\theta)c_l c_h + (1+\theta)c_l^2 + 2c_h^2}{2(1-\theta)(a-c_l)} \right), \\ f_l^s = \frac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2(1+\theta)c_l c_h + (1+\theta)c_l^2 + 2c_h^2 - 2(1-\theta)(a-c_l)r_l^s}{4b(1-\theta)}, \\ \delta_l^s = \frac{a-c_l}{2b} \end{array} \right\},$$

$$\left(r_l^s, \delta_l^s\right)^U = \left\{ r_l^s = \frac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2(1+\theta)c_l c_h + (1+\theta)c_l^2 + 2c_h^2}{2(1-\theta)(a-c_l)}, \delta_l^s = \frac{a-c_l}{2b} \right\},$$ *and*

$$\left(f_l^s, \delta_l^s\right)^F = \left\{ f_l^s = \frac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2(1+\theta)c_l c_h + (1+\theta)c_l^2 + 2c_h^2}{4b(1-\theta)}, \delta_l^s \in \left[0, \frac{a-c_l}{2b}\right] \right\}.$$

At equilibrium, the port authority's fee revenue always equals

$$R^s = \frac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2\theta c_l c_h + \theta c_l^2 + c_h^2}{4b(1-\theta)}.$$

(ii) *If* $c_h \in \left(c'_h, c''_h\right)$ *with* $c''_h \equiv \frac{2(1-\theta)a + \theta c_l}{2-\theta}$*, then the optimal contract for the h-type operator is the unit-fee scheme* $r_h^s = \frac{(2-\theta)a - \theta c_l - 2(1-\theta)c_h}{(4-3\theta)}$ *with minimum throughput guarantee* $\delta_h^s = \frac{2(1-\theta)a + \theta c_l - (2-\theta)c_h}{b(4-3\theta)}$*. By contrast, the optimal contract for the l-type operator can be the* *two-part tariff scheme and minimum throughput requirement* $\left(r_l^s,\, f_l^s, \delta_l^s\right)^T$ *, the unit-fee scheme and minimum throughput requirement* $\left(r_l^s, \delta_l^s\right)^U$ *, or the fixed-fee scheme and minimum throughput requirement* $\left(f_l^s, \delta_l^s\right)^F$ *. Here* $\left(r_l^s,\, f_l^s, \delta_l^s\right)^T = \left\{ r_l^s \in \left(0, \frac{2bB}{(a-c_l)}\right),\, f_l^s = B - \frac{1}{2b}(a-c_l) r_l^s,\, \delta_l^s = \frac{a-c_l}{2b} \right\}$,

$\left(r_l^s, \delta_l^s\right)^U = \left\{ r_l^s = \frac{2bB}{(a-c_l)},\, \delta_l^s = \frac{a-c_l}{2b} \right\}$, $\left(f_l^s, \delta_l^s\right)^F = \left\{ f_l^s = B,\, \delta_l^s \in \left[0, \frac{a-c_l}{2b}\right]\right\}$*, and*

$B = \frac{\left[(6-5\theta)a - (8-7\theta)c_l + 2(1-\theta)c_h\right]\left[(2-\theta)a - \theta c_l - 2(1-\theta)c_h\right]}{4b(4-3\theta)^2}$*. At equilibrium, the port authority's fee revenue always equals* $R^s = \frac{\left[(2-\theta)a - \theta c_l - 2(1-\theta)c_h\right]^2}{4b(4-3\theta)}$.

(iii) *If* $c_h \in \left[c_h'', a\right)$*, then the optimal contract for the h-type operator is the unit-fee scheme* $r_h^s = \overline{r}_h = (a-c_h)$ *with minimum throughput guarantee* $\delta_h^s = 0$*. By contrast, the optimal contract for the l-type operator can be the two-part tariff scheme and minimum throughput requirement* $\left(r_l^s,\, f_l^s, \delta_l^s\right)^T$*, the unit-fee scheme and minimum throughput requirement* $\left(r_l^s, \delta_l^s\right)^U$ *, or the fixed-fee scheme and minimum throughput requirement* $\left(f_l^s, \delta_l^s\right)^F$ *. Here*

$\left(r_l^s,\, f_l^s, \delta_l^s\right)^T = \left\{ r_l^s \in \left(0, \frac{(a-c_h)(a-2c_l+c_h)}{2(a-c_l)}\right),\, f_l^s = \frac{(a-c_h)(a-2c_l+c_h) - 2(a-c_l)r_l^s}{4b},\, \delta_l^s = \frac{a-c_l}{2b} \right\}$,

$\left(r_l^s, \delta_l^s\right)^U = \left\{ r_l^s = \frac{(a-c_h)(a-2c_l+c_h)}{2(a-c_l)},\, \delta_l^s = \frac{a-c_l}{2b} \right\}$*, and*

$\left(f_l^s, \delta_l^s\right)^F = \left\{ f_l^s = \frac{(a-c_h)(a-2c_l+c_h)}{4b}, \delta_l^s \in \left[0, \frac{a-c_l}{2b}\right]\right\}$*. At equilibrium, the port authority's fee revenue always equals* $R^s = \frac{\theta(a-c_h)(a-2c_l+c_h)}{4b}$.

<u>Proof of Proposition 2</u>: We first explore whether the constraints of individual rationality and incentive compatibility bind at equilibrium. To simplify the analyses, we define $\pi_l^{sl} = \Pi_l^{sl} - f_l$, $\pi_l^{sh} = \Pi_l^{sh} - f_h$, $\pi_h^{sh} = \Pi_h^{sh} - f_h$, and $\pi_h^{sl} = \Pi_h^{sl} - f_l$. The Π_l^{sl} is $b(q_l^{sl})^2$ in Lemma 4(i)-(ii), or $\delta_l[a - b\delta_l - c_l - r_l]$ in Lemma 4(iii); the Π_l^{sh} is $b(q_l^{sh})^2$ in Lemma 5(i)-(ii), or $\delta_h[a - b\delta_h - c_l - r_h]$ in Lemma 5(iii); the Π_h^{sl} is $b(q_h^{sl})^2$ in Lemma 4(i), or $\delta_l[a - b\delta_l - c_h - r_l]$ in Lemma 4(ii)-(iii); and the Π_h^{sh} is $b(q_h^{sh})^2$ in Lemma 5(i), or $\delta_h[a - b\delta_h - c_h - r_h]$ in Lemma 5(ii)-(iii). Then, we can rewrite the four constraints in problem (12A) as follows.

$$\pi_l^{sl} = \Pi_l^{sl} - f_l \geq 0, \qquad (IR_L)$$

$$\pi_h^{sh} = \Pi_h^{sh} - f_h \geq 0, \qquad (IR_H)$$

$$\pi_l^{sl} = \Pi_l^{sl} - f_l \geq \pi_l^{sh} = \Pi_l^{sh} - f_h, \quad (IC_L)$$

$$\pi_h^{sh} = \Pi_h^{sh} - f_h \geq \pi_h^{sl} = \Pi_h^{sl} - f_l. \quad (IC_H)$$

Based on these, we acquire the following lemmas.

Lemma A. *If* (IC_L) *and* (IR_H) *hold, then* (IR_L) *holds as well.*

Proof. Since $c_l < c_h$, we have $\pi_l^{sl} > \pi_h^{sl}$ and $\pi_l^{sh} > \pi_h^{sh}$. Accordingly, if (IC_L) and (IR_H) hold and $\pi_l^{sh} > \pi_h^{sh}$, then we can get $\pi_l^{sl} \geq \pi_l^{sh} > \pi_h^{sh} \geq 0$. This implies (IR_L).

Lemma B. *Constraint* (IR_L) *will not bind at equilibrium.*

Proof. If (IR_L) binds at equilibrium, then $\pi_l^{sl} = 0$. We thus have $\pi_l^{sh} \leq \pi_l^{sl} = 0$ by (IC_L) and $\pi_h^{sh} < \pi_l^{sh} \leq \pi_l^{sl} = 0$ by $\pi_l^{sh} > \pi_h^{sh}$. These contradict (IR_H). Therefore, constraint (IR_L) does not bind at equilibrium.

Lemma C. *Constraint* (IC_L) *will bind at equilibrium.*

Proof. Since $\frac{\partial R}{\partial f_l} > 0$ and $f_l \leq \Pi_l^{sl} - \Pi_l^{sh} + f_h$ by (IC_L), the optimal fixed-fee will be $f_l = \Pi_l^{sl} - \Pi_l^{sh} + f_h$. This implies (IC_L).

Lemma D. *Constraint* (IR_H) *will bind at equilibrium.*

Proof. Since $\frac{\partial R}{\partial f_h} > 0$ and $f_h \leq \Pi_h^{sh}$ by (IR_H), the optimal fixed-fee will be $f_h = \Pi_h^{sh}$. This implies (IR_H).

Lemmas A-D suggest that in deriving the ensuing separating equilibria, we can ignore (IR_L) and substitute $\pi_l^{sl} = \pi_l^{sh}$ and $\pi_h^{sh} = 0$ into the port authority's fee revenue function, which then becomes $R = \theta[r_l q_l^{sl} + \Pi_l^{sl} - \Pi_l^{sh} + \Pi_h^{sh}] + (1-\theta)[r_h q_h^{sh} + \Pi_h^{sh}]$ with Π_l^{sl} a function of q_l^{sl}, Π_l^{sh} a function of q_l^{sh}, and Π_h^{sh} a function of q_h^{sh}. After obtaining the optimal concession contracts and their minimum throughput requirements, we can verify that (IC_H) holds at equilibrium. On the other hand, according to the ranges of δ_l in Lemma 4 and the ranges of δ_h in Lemma 5, we have nine cases. Note that q_l^{sl} in Lemmas 4(i) and 4(ii) are the same, but q_h^{sl} in Lemmas 4(i) and 4(ii) differ. Moreover, since q_h^{sl} will not appear in the port authority's objective function in (A54), the optimal contracts derived under constraint $\delta_l \in [0, \delta_l']$ will always be the same as those derived under constraint $\delta_l \in [\delta_l', \delta_l'']$. Thus, we only need to consider the six cases below.

Case 1: Suppose $\delta_l \in [0, \delta_l'']$ with $\delta_l'' = \dfrac{a - c_l - r_l}{2b}$, and $\delta_h \in [0, \delta_h']$ with $\delta_h' = \dfrac{a - c_h - r_h}{2b}$. Lemmas 4(i)-(ii) and Lemma 5(i) imply that $\pi_l^{sl} = b\left(q_l^{sl}\right)^2 - f_l$ and $\pi_h^{sh} = b\left(q_h^{sh}\right)^2 - f_h$. This in turn suggests $f_h = b\left(q_h^{sh}\right)^2 > 0$ and $f_l = b\left[\left(q_l^{sl}\right)^2 - \left(q_l^{sh}\right)^2 + \left(q_h^{sh}\right)^2\right]$ by (IC_L) and (IR_H). Thus, problem (12A) becomes

$$\max_{\substack{(r_l, f_l, \delta_l),\\ (r_h, f_h, \delta_h)}} \; R = \theta\left[r_l q_l^{sl} + b\left(q_l^{sl}\right)^2 - b\left(q_l^{sh}\right)^2 + b\left(q_h^{sh}\right)^2\right] + (1-\theta)\left[r_h q_h^{sh} + b\left(q_h^{sh}\right)^2\right]$$

$$\text{s.t.} \quad 0 \le \delta_l \le \delta_l'', \; 0 \le \delta_h \le \delta_h', \; 0 \le r_l \le (a - c_l) \text{ and } 0 \le r_h \le \overline{r}_h. \qquad \text{(A54)}$$

Its Lagrange function is

$$L = \theta\left[r_l q_l^{sl} + b\left(q_l^{sl}\right)^2 - b\left(q_l^{sh}\right)^2 + b\left(q_h^{sh}\right)^2\right] + (1-\theta)\left[r_h q_h^{sh} + b\left(q_h^{sh}\right)^2\right]$$
$$+ \lambda_1\left(\delta_l'' - \delta_l\right) + \lambda_2\left(\delta_h' - \delta_h\right) + \lambda_3\left(a - c_l - r_l\right) + \lambda_4\left(\overline{r}_h - r_h\right),$$

where λ_i, $i = 1, 2, 3, 4$, are the respective Lagrange multipliers associated with the four inequality constraints in (A54). The corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r_l} = -\frac{1}{2b}\theta r_l - \frac{1}{2b}\lambda_1 - \lambda_3 \le 0, \; r_l \cdot \frac{\partial L}{\partial r_l} = 0, \qquad \text{(A55)}$$

$$\frac{\partial L}{\partial \delta_l} = -\lambda_1 \le 0, \; \delta_l \cdot \frac{\partial L}{\partial \delta_l} = 0, \qquad \text{(A56)}$$

$$\frac{\partial L}{\partial r_h} = \frac{1}{2b}\left[\theta\left(c_h - c_l\right) - (1-\theta) r_h\right] - \frac{1}{2b}\lambda_2 - \lambda_4 \le 0, \; r_h \cdot \frac{\partial L}{\partial r_h} = 0, \qquad \text{(A57)}$$

$$\frac{\partial L}{\partial \delta_h} = -\lambda_2 \le 0, \; \delta_h \cdot \frac{\partial L}{\partial \delta_h} = 0, \qquad \text{(A58)}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta_l'' - \delta_l \ge 0, \; \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \qquad \text{(A59)}$$

$$\frac{\partial L}{\partial \lambda_2} = \delta_h' - \delta_h \ge 0, \; \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \qquad \text{(A60)}$$

$$\frac{\partial L}{\partial \lambda_3} = a - c_l - r_l \ge 0, \; \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \text{ and} \qquad \text{(A61)}$$

$$\frac{\partial L}{\partial \lambda_4} = \overline{r}_h - r_h \ge 0, \; \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0. \qquad \text{(A62)}$$

Since $\frac{\partial L}{\partial r_l}=-\frac{1}{2b}\theta r_l-\frac{1}{2b}\lambda_1-\lambda_3<0$ by (A55), we have $r_l^s=0$ and $\lambda_3^*=0$ by (A61). If $\lambda_1^*>0$, then $\delta_l^s=\delta_l''>0$, which implies $\lambda_1^*=0$ by (A56). This is a contradiction. Thus, we must have $\lambda_1^*=0$. Based on the values of λ_2 and λ_4, there are four sub-cases.

<u>Case 1a</u>: Suppose $\lambda_2^*=0$ and $\lambda_4^*=0$. Then $r_h^s=\frac{\theta(c_h-c_l)}{(1-\theta)}>0$ by (A57). It remains to check whether $r_h^s\le\overline{r}_h$ holds. By some calculations, we have $r_h^s\le\overline{r}_h$ iff $c_h\le c_{hs1}\equiv(1-\theta)a+\theta c_l$. Thus, we can obtain $\delta_h^s\in[0,\delta_h']$ with $\delta_h'=\frac{a-c_h-r_h^s}{2b}=\frac{(1-\theta)a+\theta c_l-c_h}{2b(1-\theta)}$, $\delta_l^s\in[0,\delta_l'']$ with $\delta_l''=\frac{a-c_l-r_l^s}{2b}=\frac{a-c_l}{2b}$, $f_h^s=b\left[\frac{(1-\theta)a+\theta c_l-c_h}{2b(1-\theta)}\right]^2>0$, and $f_l^s=\frac{(a-c_l)^2}{4b}-b\left[\frac{(1-\theta)a-(1-2\theta)c_l-\theta c_h}{2b(1-\theta)}\right]^2+b\left[\frac{(1-\theta)a+\theta c_l-c_h}{2b(1-\theta)}\right]^2>0$ by (A58), (A59) and (A60). It remains to show $\pi_h^{sh}\ge\pi_h^{sl}$ at equilibrium. Note that $f_h^s=b\left(\frac{a-c_h-r_h^s}{2b}\right)^2$, $f_l^s=b\left(\frac{a-c_l-r_l^s}{2b}\right)^2-b\left(\frac{a-c_l-r_h^s}{2b}\right)^2+f_h^s$, $\pi_h^{sh}=b\left(\frac{a-c_h-r_h^s}{2b}\right)^2-f_h^s$, $\pi_h^{sl}=b\left(\frac{a-c_h-r_l^s}{2b}\right)^2-f_l^s$ if $\delta_l^s\in[0,\delta_l']$, $\pi_h^{sl}=\delta_l^s\left[a-b\delta_l^s-c_h-r_l^s\right]-f_l^s$ if $\delta_l^s\in(\delta_l',\delta_l'']$, and $\delta_l^s\left[a-b\delta_l^s-c_h-r_l^s\right]<b\left(\frac{a-c_h-r_l^s}{2b}\right)^2$ if $\delta_l^s>\delta_l'$. Thus, for $\delta_l^s\in[0,\delta_l'']$, we have

$$\pi_h^{sh}-\pi_h^{sl}\ge b\left[\left(\frac{a-c_l-r_l^s}{2b}\right)^2-\left(\frac{a-c_l-r_h^s}{2b}\right)^2\right]-\left[\left(\frac{a-c_h-r_l^s}{2b}\right)^2-\left(\frac{a-c_h-r_h^s}{2b}\right)^2\right]$$

$=b\left[\int_{r_h^s}^{r_l^s}-\frac{a-r-c_l}{2b}dr-\int_{r_h^s}^{r_l^s}-\frac{a-r-c_h}{2b}dr\right]=b\int_{r_h^s}^{r_l^s}-\frac{(c_h-c_l)}{2b}dr>0$ due to $r_h^s>r_l^s$. In summary, for $c_h\le c_{hs1}$, the port authority's equilibrium fee revenue equals

$$R_1^s\equiv\frac{(1-\theta)a^2-2(1-\theta)ac_h-2\theta^2c_lc_h+\theta^2c_l^2+(1-\theta+\theta^2)c_h^2}{4b(1-\theta)}. \qquad \text{(A63)}$$

<u>Case 1b</u>: Suppose $\lambda_2^*=0$ and $\lambda_4^*>0$. Then, (A57) and (A62) suggest $r_h^s=\overline{r}_h=(a-c_h)$, and $\lambda_4^*=\frac{-(1-\theta)a-\theta c_l+c_h}{2b}>0$ iff $c_h>c_{hs1}$. Moreover, (A59) and (A60) imply $\delta_h^s=0$ by $\delta_h'=0$, $\delta_l^s\in[0,\delta_l'']$ with $\delta_l''=\frac{a-r_l^s-c_l}{2b}=\frac{a-c_l}{2b}$, $f_h^s=0$ and $f_l^s=\frac{(a-c_h)(a-2c_l+c_h)}{4b}$. As in Case 1a, we can prove that $\pi_h^{sh}\ge\pi_h^{sl}$ holds at equilibrium. Thus, for $c_h>c_{hs1}$, the port authority's equilibrium fee revenue equals

$$R_{12}^{s} \equiv \frac{\theta\left(a-c_h\right)\left(a-2c_l+c_h\right)}{4b}. \tag{A64}$$

Case 1c: Suppose $\lambda_2^* > 0$ and $\lambda_4^* = 0$. Then (A60) suggests $\delta_h^s = \delta_h' \geq 0$. If $\delta_h' > 0$, then $\lambda_2^* = 0$ by (A59). This is a contradiction. Thus, no solution exists in this case. If $\delta_h' = 0$, then $r_h^s = \overline{r}_h$ and the solution is same as that in Case 1b.

Case 1d: Suppose $\lambda_2^* > 0$ and $\lambda_4^* > 0$. The solution is the same as that in Case 1b for $c_h > c_{hs1}$.

Case 2: Suppose $\delta_l \in [0, \delta_l'']$ with $\delta_l'' = \frac{a-c_l-r_l}{2b}$, and $\delta_h \in (\delta_h', \delta_h'']$ with $\delta_h'' = \frac{a-c_l-r_h}{2b}$. Then, Lemmas 4(i)-(ii) and Lemma 5(ii) imply that the *l*-type operator's equilibrium profit is $\pi_l^{sl} = b\left(\frac{a-c_l-r_l}{2b}\right)^2 - f_l$, and the *h*-type operator's equilibrium profit is $\pi_h^{sh} = \delta_h\left[a-b\delta_h-r_h-c_h\right]-f_h$. The binding ($IC_L$) and ($IR_H$) suggest $f_l^s = b\left(\frac{a-c_l-r_l}{2b}\right)^2 - \left(\frac{a-c_l-r_h}{2b}\right)^2 + \delta_h\left(a-b\delta_h-r_h-c_h\right) > 0$ and $f_h^s = \delta_h\left(a-b\delta_h-r_h-c_h\right)$. We have $f_h^s \geq 0$ iff $\delta_h \leq \tilde{\delta}_h \equiv \frac{(a-r_h-c_h)}{b}$ and $r_h \leq \overline{r}_h$. Moreover, $(\tilde{\delta}_h - \delta_h'') = \frac{a+c_l-2c_h-r_h}{2b} \geq (<) 0$ iff $r_h \leq (>) \tilde{r}_h \equiv (a+c_l-2c_h)$. Thus, there are several sub-cases.

Case 2a: Suppose $r_h \leq \tilde{r}_h \equiv a+c_l-2c_h$. Then, problem (12A) becomes

$$\max_{(r_l, f_l, \delta_l), (r_h, f_h, \delta_h)} R = \theta\left(r_l q_l^{sl} + f_l\right) + (1-\theta)\left(r_h q_h^{sh} + f_h\right)$$

$$\text{s.t. } 0 \leq \delta_l \leq \delta_l'',\ \delta_h' < \delta_h \leq \delta_h'', 0 \leq r_l \leq (a-c_l) \text{ and } 0 \leq r_h \leq \tilde{r}_h. \tag{A65}$$

Its Lagrange function is

$$L = \theta\left[\frac{r_l\cdot\left(a-c_l-r_l\right)}{2b} + \frac{\left(a-c_l-r_l\right)^2}{4b} - \frac{\left(a-c_l-r_h\right)^2}{4b} + \delta_h\left(a-b\delta_h-r_h-c_h\right)\right]$$

$$+(1-\theta)\left[r_h\cdot\delta_h + \delta_h\left(a-b\delta_h-r_h-c_h\right)\right] + \lambda_1\left(\delta_l''-\delta_l\right) + \lambda_2\left(\delta_h-\delta_h'\right)$$

$$+\lambda_3\left(\delta_h''-\delta_h\right) + \lambda_4\left(a-c_l-r_l\right) + \lambda_5\left(\tilde{r}_h-r_h\right),$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 are the respective Lagrange multipliers associated the inequalities in (A65). The corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r_l} = -\frac{\theta}{2b}r_l - \frac{1}{2b}\lambda_1 - \lambda_4 \leq 0,\ r_l\cdot\frac{\partial L}{\partial r_l} = 0, \tag{A66}$$

$$\frac{\partial L}{\partial r_h} = \frac{\theta}{2b}\left(a - c_l - r_h - 2b\delta_h\right) + \frac{1}{2b}\lambda_2 - \frac{1}{2b}\lambda_3 - \lambda_5 \le 0,\ r_h \cdot \frac{\partial L}{\partial r_h} = 0, \tag{A67}$$

$$\frac{\partial L}{\partial \delta_l} = -\lambda_1 \le 0,\ \delta_l \cdot \frac{\partial L}{\partial \delta_l} = 0, \tag{A68}$$

$$\frac{\partial L}{\partial \delta_h} = a - c_h - \theta r_h - 2b\delta_h + \lambda_2 - \lambda_3 \le 0,\ \delta_h \frac{\partial L}{\partial \delta_h} = 0, \tag{A69}$$

$$\frac{\partial L}{\partial \lambda_1} = \delta_l'' - \delta_l \ge 0,\ \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \tag{A70}$$

$$\frac{\partial L}{\partial \lambda_2} = \delta_h - \delta_h' \ge 0,\ \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0, \tag{A71}$$

$$\frac{\partial L}{\partial \lambda_3} = \delta_h'' - \delta_h \ge 0,\ \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \tag{A72}$$

$$\frac{\partial L}{\partial \lambda_4} = a - c_l - r_l \ge 0,\ \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0, \text{and} \tag{A73}$$

$$\frac{\partial L}{\partial \lambda_5} = \tilde{r}_h - r_h \ge 0,\ \lambda_5 \cdot \frac{\partial L}{\partial \lambda_5} = 0. \tag{A74}$$

Since $\frac{\partial L}{\partial r_l} = -\frac{1}{2b}\theta r_l - \frac{1}{2b}\lambda_1 - \lambda_4 < 0$ by (A66), we have $r_l^s = 0$ and $\lambda_4^* = 0$ by (A73). If $\lambda_1^* > 0$, then $\delta_l^s = \delta_l'' > 0$ by (A68), which implies $\lambda_1^* = 0$. This is a contradiction. Thus, we must have $\lambda_1^* = 0$. Moreover, constraint $\delta_h > \delta_h'$ suggests $\lambda_2^* = 0$ by (A71).

Based on the values of λ_3 and λ_5, there are four sub-cases.

<u>Case 2a-1</u>: Suppose $\lambda_3^* = 0$ and $\lambda_5^* = 0$. Then, (A67) and (A69) become $\frac{1}{2b}\theta\left(a - c_l - r_h - 2b\delta_h\right) = 0$ and $(a - c_h - \theta r_h - 2b\delta_h) = 0$. Solving the two equations yields $r_l^s = 0$, $r_h^s = \frac{c_h - c_l}{(1-\theta)}$ and $\delta_h^s = \frac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)}$. We have $(\tilde{r}_h - r_h^s) = \frac{(1-\theta)a + (2-\theta)c_l - (3-2\theta)c_h}{(1-\theta)} \ge 0$ iff $c_h \le c_h' \equiv \frac{(1-\theta)a + (2-\theta)c_l}{(3-2\theta)}$, $(\delta_h^s - \delta_h') = \frac{c_h - c_l}{2b} > 0$, and $(\delta_h'' - \delta_h^s) = 0$ through calculations.

Moreover, we have $\delta_l^s \in [0, \delta_l'']$ with $\delta_l'' = \frac{a - r_l^s - c_l}{2b} = \frac{a - c_l}{2b}$ by (A70),

$$f_h^s = \frac{\left[(1-\theta)a + \theta c_l - c_h\right] \times \left[(1-\theta)a + (2-\theta)c_l - (3-2\theta)c_h\right]}{4b(1-\theta)^2}, \text{ and}$$

$f_l^s = \frac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2(1+\theta)c_l c_h + (1+\theta)c_l^2 + 2c_h^2}{4b(1-\theta)}$. It remains to show $\pi_h^{sh} \geq \pi_h^{sl}$ at equilibrium. Note that $f_h^s = \delta_h^s\left(a - b\delta_h^s - r_h^s - c_h\right)$, $f_l^s = b\left[\left(\frac{a-c_l-r_l^s}{2b}\right)^2 - \left(\frac{a-c_l-r_h^s}{2b}\right)^2\right] + f_h^s$, $\delta_h^s = \delta_h''$, $\pi_h^{sl} = b\left(\frac{a-c_h-r_l^s}{2b}\right)^2 - f_l^s$ if $\delta_l^s \in [0, \delta_l']$, $\pi_h^{sl} = \delta_l^s\left[a - b\delta_l^s - c_h - r_l^s\right] - f_l^s$ if $\delta_l^s \in (\delta_l', \delta_l'']$, and $\delta_l^s\left[a - b\delta_l^s - c_h - r_l^s\right] < b\left(\frac{a-c_h-r_l^s}{2b}\right)^2$ if $\delta_l^s > \delta_l'$. Thus, for $\delta_l^s \in [0, \delta_l'']$, we have

$$(\pi_h^{sh} - \pi_h^{sl}) \geq b\left[\left(\frac{a-c_l-r_l^s}{2b}\right)^2 - \left(\frac{a-c_l-r_h^s}{2b}\right)^2\right] - \left[b\left(\frac{a-c_h-r_l^s}{2b}\right)^2 - \delta_h''\left(a - b\delta_h'' - c_h - r_h^s\right)\right]$$

$= \frac{(c_h - c_l)\left[2\left(r_h^s - r_l^s\right) - (c_h - c_l)\right]}{4b} > 0$ by $(r_h^s - r_l^s) = \frac{c_h - c_l}{(1-\theta)} > \frac{c_h - c_l}{2}$. In summary, a solution exists under $c_h \leq \frac{(1-\theta)a + (2-\theta)c_l}{(3-2\theta)}$ with the port authority's equilibrium fee revenue

$$R_{21}^s \equiv \frac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2\theta c_l c_h + \theta c_l^2 + c_h^2}{4b(1-\theta)}. \tag{A75}$$

<u>Case 2a-2</u>: Suppose $\lambda_3^* = 0$ and $\lambda_5^* > 0$. Then, (A67), (A69) and (A74) become $\frac{\theta}{2b}\left(a - c_l - r_h - 2b\delta_h\right) - \lambda_5 = 0$, $(a - c_h - \theta r_h - 2b\delta_h) = 0$ and $(\tilde{r}_h - r_h) = 0$. Solving these equations yields $r_l^s = 0$, $r_h^s = \tilde{r}_h = (a + c_l - 2c_h)$, $\delta_h^s = \frac{1}{2b}\left[(1-\theta)a - \theta c_l - (1-2\theta)c_h\right]$ and $\lambda_5^* = \frac{\theta}{2b}\left[-(1-\theta)a - (2-\theta)c_l + (3-2\theta)c_h\right]$. We obtain $r_h^s = \tilde{r}_h = (a + c_l - 2c_h) \geq 0$ iff $c_h \leq c_{hs2} \equiv \frac{a+c_l}{2}$, $(\delta_h^s - \delta_h') = \frac{1}{2b}(1-\theta)\left(a + c_l - 2c_h\right) > 0$ iff $c_h < c_{hs2} \equiv \frac{a+c_l}{2}$, $(\delta_h'' - \delta_h^s) = \frac{1}{2b}\left[-(1-\theta)a - (2-\theta)c_l + (3-2\theta)c_h\right] \geq 0$ iff $c_h \geq c_h' \equiv \frac{(1-\theta)a + (2-\theta)c_l}{(3-2\theta)}$, and $\lambda_5^* > 0$ iff $c_h > c_h'$ with $c_h' < c_{hs2}$ through calculations. Hence

$f_h^s = \frac{\left[(1-\theta)a - \theta c_l + (2\theta - 1)c_h\right] \times \left[-(1-\theta)a - (2-\theta)c_l + (3-2\theta)c_h\right]}{4b}$,

$f_l^s = \frac{(a-c_l)^2 - 4(c_h - c_l)^2}{4b} + f_h^s$ and $\delta_l^s \in [0, \delta_l'']$ with $\delta_l'' = \frac{a - r_l^s - c_l}{2b} = \frac{a - c_l}{2b}$ by (A70). It remains to check whether $\pi_h^{sh} \geq \pi_h^{sl}$ holds at equilibrium. Because $f_h^s = \delta_h^s\left(a - b\delta_h^s - r_h^s - c_h\right)$, $f_l^s = b\left[\left(\frac{a-c_l-r_l^s}{2b}\right)^2 - \left(\frac{a-c_l-r_h^s}{2b}\right)^2\right] + f_h^s$, $\pi_h^{sl} = b\left(\frac{a-c_h-r_l^s}{2b}\right)^2 - f_l^s$ if $\delta_l^s \in [0, \delta_l']$,

$\pi_h^{sl} = \delta_l^s\left[a - b\delta_l^s - c_h - r_l^s\right] - f_l^s$ if $\delta_l^s \in \left(\delta_l', \delta_l''\right]$, and $\delta_l^s\left[a - b\delta_l^s - c_h - r_l^s\right] < b\left(\dfrac{a - c_h - r_l^s}{2b}\right)^2$ if $\delta_l^s > \delta_l'$;

for $\delta_l^s \in [0, \delta_l'']$, we have

$$(\pi_h^{sh} - \pi_h^{sl}) \ge b\left[\left(\frac{a - c_l - r_l^s}{2b}\right)^2 - \left(\frac{a - c_l - r_h^s}{2b}\right)^2\right] - \left[b\left(\frac{a - c_h - r_l^s}{2b}\right)^2 - \delta_h^s\left(a - b\delta_h^s - r_h^s - c_h\right)\right]$$

$$= \frac{\left(a + c_l - 2c_h\right)\left[-(1-\theta)^2 a - \left(3 - 2\theta + \theta^2\right)c_l + \left(4 - 4\theta + 2\theta^2\right)c_h\right]}{4b} > 0 \text{ iff}$$

$c_h > \dfrac{(1-\theta)^2 a + \left(3 - 2\theta + \theta^2\right)c_l}{\left(4 - 4\theta + 2\theta^2\right)}$ with $\dfrac{(1-\theta)^2 a + \left(3 - 2\theta + \theta^2\right)c_l}{\left(4 - 4\theta + 2\theta^2\right)} < \dfrac{(1-\theta)a + (2-\theta)c_l}{(3 - 2\theta)} = c_h'$. This suggests that $\pi_h^{sh} > \pi_h^{sl}$ when $c_h > c_h'$. In summary, a solution exists under $c_h' < c_h < c_{hs2}$ with the port authority's equilibrium fee revenue

$$R_{22}^s \equiv \frac{1}{4b}\begin{bmatrix}\left(1 - \theta + \theta^2\right)a^2 - 2\theta(2-\theta)ac_l - 2\left(1 - 3\theta + 2\theta^2\right)ac_h \\ +2\theta(5 - 2\theta)c_l c_h - \theta(3-\theta)c_l^2 + \left(1 - 8\theta + 4\theta^2\right)c_h^2\end{bmatrix}. \tag{A76}$$

Case 2a-3: Suppose $\lambda_3^* > 0$ and $\lambda_5^* = 0$. Then, (A67), (A69) and (A72) become $\dfrac{\theta}{2b}\left(a - c_l - r_h - 2b\delta_h\right) - \dfrac{1}{2b}\lambda_3 = 0$, $(a - c_h - \theta r_h - 2b\delta_h - \lambda_3) = 0$ and $(\delta_h'' - \delta_h) = 0$. Solving these equations yields $\lambda_3^* = 0$, which contradicts $\lambda_3^* > 0$. Thus, no solution exists in this case.

Case 2a-4: Suppose $\lambda_3^* > 0$ and $\lambda_5^* > 0$. Then, (A67), (A69), (A72) and (A74) become $\dfrac{\theta}{2b}\left(a - c_l - r_h - 2b\delta_h\right) - \dfrac{1}{2b}\lambda_3 - \lambda_5 = 0$, $(a - c_h - \theta r_h - 2b\delta_h - \lambda_3) = 0$, $(\delta_h'' - \delta_h) = 0$ and $(\tilde{r}_h - r_h) = 0$. Solving these equations yields $\lambda_3^* = 0$. This is a contradiction. Thus, no solution exists in this case.

Case 2b: Suppose $r_h > \tilde{r}_h \equiv a + c_l - 2c_h$. Since $f_h^s = \delta_h\left(a - b\delta_h - r_h - c_h\right) \ge 0$ iff $\delta_h \le \tilde{\delta}_h \equiv \dfrac{(a - c_h - r_h)}{b}$. Under condition $r_h > \tilde{r}_h$, we have $\tilde{\delta}_h < \delta_h''$. Thus, conditions $\delta_h' < \delta_h \le \delta_h''$ and $\tilde{\delta}_h < \delta_h''$ can be combined into $\delta_h' < \delta_h \le \tilde{\delta}_h$. Thus, problem (12A) becomes

$$\max_{(r_l, f_l, \delta_l), (r_h, f_h, \delta_h)} R = \theta\left(r_l q_l^{sl} + f_l\right) + (1-\theta)\left(r_h q_h^{sh} + f_h\right)$$

$$\text{s.t. } 0 \le \delta_l \le \delta_l'',\ \delta_h' < \delta_h \le \tilde{\delta}_h, 0 \le r_l \le a - c_l \text{ and } \tilde{r}_h < r_h \le \overline{r}_h. \tag{A77}$$

Its Lagrange function is

$$L = \theta\left[\frac{r_l \cdot (a - c_l - r_l)}{2b} + \frac{(a - c_l - r_l)^2}{4b} - \frac{(a - c_l - r_h)^2}{4b} + \delta_h\left(a - b\delta_h - r_h - c_h\right)\right]$$

$$+(1-\theta)\left[r_h\cdot\delta_h+\delta_h\left(a-b\delta_h-r_h-c_h\right)\right]+\lambda_1\left(\delta_l''-\delta_l\right)+\lambda_2\left(\delta_h-\delta_h'\right)$$

$$+\lambda_3\left(\tilde{\delta}_h-\delta_h\right)+\lambda_4\left(a-c_l-r_l\right)+\lambda_5\left(r_h-\tilde{r}_h\right)+\lambda_6\left(\overline{r}_h-r_h\right),$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ and λ_6 are the respective Lagrange multipliers associated with the inequalities in (A77). The corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r_l}=-\frac{\theta}{2b}r_l-\frac{1}{2b}\lambda_1-\lambda_4\le 0,\ r_l\cdot\frac{\partial L}{\partial r_l}=0, \tag{A78}$$

$$\frac{\partial L}{\partial r_h}=\frac{\theta}{2b}\left(a-c_l-r_h-2b\delta_h\right)+\frac{1}{2b}\lambda_2-\frac{1}{b}\lambda_3+\lambda_5-\lambda_6\le 0,\ r_h\cdot\frac{\partial L}{\partial r_h}=0, \tag{A79}$$

$$\frac{\partial L}{\partial \delta_l}=-\lambda_1\le 0,\ \delta_l\cdot\frac{\partial L}{\partial \delta_l}=0, \tag{A80}$$

$$\frac{\partial L}{\partial \delta_h}=a-c_h-\theta r_h-2b\delta_h+\lambda_2-\lambda_3\le 0,\ \delta_h\frac{\partial L}{\partial \delta_h}=0, \tag{A81}$$

$$\frac{\partial L}{\partial \lambda_1}=\delta_l''-\delta_l\ge 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1}=0, \tag{A82}$$

$$\frac{\partial L}{\partial \lambda_2}=\delta_h-\delta_h'\ge 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2}=0, \tag{A83}$$

$$\frac{\partial L}{\partial \lambda_3}=\tilde{\delta}_h-\delta_h\ge 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3}=0, \tag{A84}$$

$$\frac{\partial L}{\partial \lambda_4}=a-c_l-r_l\ge 0,\ \lambda_4\cdot\frac{\partial L}{\partial \lambda_4}=0, \tag{A85}$$

$$\frac{\partial L}{\partial \lambda_5}=r_h-\tilde{r}_h\ge 0,\ \lambda_5\cdot\frac{\partial L}{\partial \lambda_5}=0,\ \text{and} \tag{A86}$$

$$\frac{\partial L}{\partial \lambda_6}=\overline{r}_h-r_h\ge 0,\ \lambda_6\cdot\frac{\partial L}{\partial \lambda_6}=0. \tag{A87}$$

Since $\frac{\partial L}{\partial r_l}=-\frac{1}{2b}\theta r_l-\frac{1}{2b}\lambda_1-\lambda_4<0$ by (A78), we have $r_l^s=0$. Hence $\lambda_4^*=0$ by (A85). If $\lambda_1^*>0$, then $\delta_l^s=\delta_l''>0$ by (A82), which implies $\lambda_1^*=0$ by (A80). This is a contradiction. Thus, we must have $\lambda_1^*=0$. If $\lambda_6^*>0$, then $r_h=\overline{r}_h$ by (A87). This suggests $\delta_h'=\frac{(a-c_h-r_h)}{2b}=\frac{(a-c_h-\overline{r}_h)}{2b}=0=\tilde{\delta}_h$ at $r_h=\overline{r}_h=(a-c_h)$, which contradicts $\delta_h'<\delta_h\le\tilde{\delta}_h$. Thus, we must have $\lambda_6^*=0$. Moreover, constraints $\delta_h>\delta_h'$ and $\tilde{r}_h<r_h$ suggest $\lambda_2^*=\lambda_5^*=0$ by (A83) and (A86).

Based on the values of λ_3, there are two sub-cases.

<u>Case 2b-1</u>: Suppose $\lambda_3^* = 0$. Then, (A79) and (A81) become $\frac{\theta}{2b}\left(a - c_l - r_h - 2b\delta_h\right) = 0$ and $(a - c_h - \theta r_h - 2b\delta_h) = 0$. Solving the two equations yields $r_l^s = 0$, $r_h^s = \frac{c_h - c_l}{(1-\theta)}$ and $\delta_h^s = \frac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)}$. Since $(r_h^s - \tilde{r}_h) = \frac{-(1-\theta)a - (2-\theta)c_l + (3-2\theta)c_h}{(1-\theta)} > 0$ iff $c_h > c_h' \equiv \frac{(1-\theta)a + (2-\theta)c_l}{(3-2\theta)}$, $(\tilde{\delta}_h - \delta_h^s) = \frac{(1-\theta)a + (2-\theta)c_l - (3-2\theta)c_h}{2b(1-\theta)} \geq 0$ iff $c_h \leq c_h'$, we cannot have $r_h^s > \tilde{r}_h$ and $\tilde{\delta}_h \geq \delta_h^s$ at the same time. Thus, no solution exists in this case.

<u>Case 2b-2</u>: Suppose $\lambda_3^* > 0$. Then, (A79), (A81) and (A84) become $\frac{\theta}{2b}\left(a - c_l - r_h - 2b\delta_h\right) - \frac{1}{b}\lambda_3 = 0$, $(a - c_h - \theta r_h - 2b\delta_h - \lambda_3) = 0$ and $(\tilde{\delta}_h - \delta_h) = 0$. Solving these equations yields $r_l^s = 0$, $r_h^s = \frac{(2-\theta)a - \theta c_l - 2(1-\theta)c_h}{4-3\theta}$,

$\delta_h^s = \tilde{\delta}_h = \frac{(a - c_h - r_h^s)}{b} = \frac{2(1-\theta)a + \theta c_l - (2-\theta)c_h}{b(4-3\theta)}$ and $\lambda_3^* = \frac{\theta\left[-(1-\theta)a - (2-\theta)c_l + (3-2\theta)c_h\right]}{(4-3\theta)}$.

Note that $(r_h^s - \tilde{r}_h) = \frac{-2\left[(1-\theta)a + (2-\theta)c_l - (3-2\theta)c_h\right]}{4-3\theta} > 0$ iff $c_h > c_h'$,

$(\delta_h^s - \delta_h') = \frac{2(1-\theta)a + \theta c_l - (2-\theta)c_h}{2b(4-3\theta)} > 0$ iff $c_h < c_h'' \equiv \frac{2(1-\theta)a + \theta c_l}{2-\theta}$, and

$(\overline{r}_h - r_h^s) = \frac{2(1-\theta)a + \theta c_l - (2-\theta)c_h}{(4-3\theta)} \geq 0$ iff $c_h \leq c_h''$. We have $(c_h'' - c_h') = \frac{(1-\theta)(4-3\theta)(a - c_l)}{(2-\theta)(3-2\theta)} > 0$ through calculations. It remains to show $\pi_h^{sh} \geq \pi_h^{sl}$ at equilibrium. Because $f_h^s = 0$,

$f_l^s = b\left[\left(\frac{a - c_l - r_l^s}{2b}\right)^2 - \left(\frac{a - c_l - r_h^s}{2b}\right)^2\right] + f_h^s$, $\pi_h^{sl} = b\left(\frac{a - c_h - r_l^s}{2b}\right)^2 - f_l^s$ if $\delta_l^s \in [0, \delta_l']$,

$\pi_h^{sl} = \delta_l^s\left[a - b\delta_l^s - c_h - r_l^s\right] - f_l^s$ if $\delta_l^s \in (\delta_l', \delta_l'']$, and $\delta_l^s\left[a - b\delta_l^s - c_h - r_l^s\right] < b\left(\frac{a - c_h - r_l^s}{2b}\right)^2$ if $\delta_l^s > \delta_l'$; for $\delta_l^s \in [0, \delta_l'']$, we have $(\pi_h^{sh} - \pi_h^{sl}) \geq b\left[\left(\frac{a - c_l - r_l^s}{2b}\right)^2 - \left(\frac{a - c_l - r_h^s}{2b}\right)^2 - \left(\frac{a - c_h - r_l^s}{2b}\right)^2\right]$

$= H \equiv \frac{1}{4b(4-3\theta)^2}\begin{bmatrix} -4(1-\theta)^2 a^2 - 2(8 - 8\theta + \theta^2)ac_l + 2(12 - 16\theta + 5\theta^2)ac_h \\ +16(1-\theta)^2 c_l c_h + \theta(8-7\theta)c_l^2 - (20 - 32\theta + 13\theta^2)c_h^2 \end{bmatrix}$. On the other hand,

$\frac{\partial^2 H}{\partial c_h{}^2} = \frac{-(20 - 32\theta + 13\theta^2)}{2b(4-3\theta)^2} < 0$, $H = \frac{(1-\theta^2)(a - c_l)^2}{4b(3-2\theta)^2} > 0$ at $c_h = c_h'$, and $H = \frac{\theta(1-\theta)(a - c_l)^2}{b(2-\theta)^2} > 0$ at $c_h = c_h''$. Thus, we get $(\pi_h^{sh} - \pi_h^{sl}) \geq H > 0$ for $c_h' < c_h < c_h''$, and a solution exists under $c_h' < c_h < c_h''$.

The optimal fee contract and minimum throughput are $r_l^s = 0$,

$f_l^s = \frac{\left[(6-5\theta)a-(8-7\theta)c_l+2(1-\theta)c_h\right]\left[(2-\theta)a-\theta c_l-2(1-\theta)c_h\right]}{4b(4-3\theta)^2}$, $\delta_l^s \in [0, \delta_l'']$ with

$\delta_l'' = \frac{a-r_l^s-c_l}{2b} = \frac{a-c_l}{2b}$, $r_h^s = \frac{(2-\theta)a-\theta c_l-2(1-\theta)c_h}{4-3\theta}$, $f_h^s = 0$ and

$\delta_h^s = \tilde{\delta}_h = \frac{2(1-\theta)a+\theta c_l-(2-\theta)c_h}{(4-3\theta)b}$, and the port authority's equilibrium fee revenue equals

$$R_{23}^s = \frac{\left[(2-\theta)a-\theta c_l-2(1-\theta)c_h\right]^2}{4b(4-3\theta)}. \tag{A88}$$

<u>Case 3</u>: Suppose $\delta_l \in [0, \delta_l'']$ with $\delta_l'' = \frac{a-r_l-c_l}{2b}$, and $\delta_h \in \left(\delta_h'', \frac{a}{b}\right)$ with $\delta_h'' = \frac{a-r_h-c_l}{2b}$. Lemmas 4(i)-(ii) and Lemma 5(iii) imply that the *l*-type operator's equilibrium profit is $\pi_l^{sl} = b\left(\frac{a-c_l-r_l}{2b}\right)^2 - f_l$ and the *h*-type operator's profit is $\pi_h^{sh} = \delta_h\left[a-b\delta_h-r_h-c_h\right]-f_h$. Again, the binding ($IC_L$) and ($IR_H$) suggests

$f_l^s = b\left(\frac{a-c_l-r_l}{2b}\right)^2 - \delta_h\cdot(a-b\delta_h-r_h-c_l)+\delta_h(a-b\delta_h-r_h-c_h)>0$ and $f_h^s = \delta_h(a-b\delta_h-r_h-c_h)$.

We have $f_h^s \geq 0$ iff $\delta_h \leq \tilde{\delta}_h \equiv \frac{(a-r_h-c_h)}{b}$ and $r_h \leq \overline{r}_h$. Moreover, $(\tilde{\delta}_h - \delta_h'') = \frac{a+c_l-2c_h-r_h}{2b} > 0$ iff $r_h < \tilde{r}_h \equiv a+c_l-2c_h$. Thus, problem (12A) becomes

$$\max_{(r_l, f_l, \delta_l), (r_h, f_h, \delta_h)} R = \theta\left(r_l q_l^{sl} + f_l\right)+(1-\theta)\left(r_h q_h^{sh} + f_h\right)$$

$$\text{s.t. } 0 \leq \delta_l \leq \delta_l'',\ \delta_h'' < \delta_h \leq \tilde{\delta}_h, 0 \leq r_l \leq a-c_l \text{ and } 0 \leq r_h < \tilde{r}_h. \tag{A89}$$

We rewrite the port authority's objective function as

$$R = \theta\left[\frac{r_l\cdot(a-c_l-r_l)}{2b}+\frac{(a-c_l-r_l)^2}{4b}-\delta_h\cdot(a-b\delta_h-r_h-c_l)+\delta_h(a-b\delta_h-r_h-c_h)\right]$$

$$+(1-\theta)\left[r_h\cdot\delta_h+\delta_h(a-b\delta_h-r_h-c_h)\right]$$

$$=\theta\left[\frac{r_l\cdot(a-c_l-r_l)}{2b}+\frac{(a-c_l-r_l)^2}{4b}-\delta_h\cdot(c_h-c_l)\right]+(1-\theta)\delta_h(a-b\delta_h-c_h);$$

and its Lagrange function is

$$L=\theta\left[\frac{r_l\cdot(a-c_l-r_l)}{2b}+\frac{(a-c_l-r_l)^2}{4b}-\delta_h\cdot(c_h-c_l)\right]+(1-\theta)\delta_h(a-b\delta_h-c_h)+\lambda_1\left(\delta_l''-\delta_l\right)$$
$$+\lambda_2\left(\delta_h-\delta_h''\right)+\lambda_3\left(\tilde{\delta}_h-\delta_h\right)+\lambda_4\left(a-c_l-r_l\right)+\lambda_5\left(\tilde{r}_h-r_h\right),$$

where $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ and λ_5 are the respective Lagrange multipliers associated with the inequalities in (A89). The corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial r_l}=-\frac{\theta}{2b}r_l-\frac{1}{2b}\lambda_1-\lambda_4\le 0,\ r_l\cdot\frac{\partial L}{\partial r_l}=0, \quad \text{(A90)}$$

$$\frac{\partial L}{\partial r_h}=\frac{1}{2b}\lambda_2-\frac{1}{b}\lambda_3-\lambda_5\le 0,\ r_h\cdot\frac{\partial L}{\partial r_h}=0, \quad \text{(A91)}$$

$$\frac{\partial L}{\partial \delta_l}=-\lambda_1\le 0,\ \delta_l\cdot\frac{\partial L}{\partial \delta_l}=0, \quad \text{(A92)}$$

$$\frac{\partial L}{\partial \delta_h}=(1-\theta)a+\theta c_l-c_h-2b(1-\theta)\delta_h+\lambda_2-\lambda_3\le 0,\ \delta_h\frac{\partial L}{\partial \delta_h}=0, \quad \text{(A93)}$$

$$\frac{\partial L}{\partial \lambda_1}=\delta_l''-\delta_l\ge 0,\ \lambda_1\cdot\frac{\partial L}{\partial \lambda_1}=0, \quad \text{(A94)}$$

$$\frac{\partial L}{\partial \lambda_2}=\delta_h-\delta_h''\ge 0,\ \lambda_2\cdot\frac{\partial L}{\partial \lambda_2}=0, \quad \text{(A95)}$$

$$\frac{\partial L}{\partial \lambda_3}=\tilde{\delta}_h-\delta_h\ge 0,\ \lambda_3\cdot\frac{\partial L}{\partial \lambda_3}=0, \quad \text{(A96)}$$

$$\frac{\partial L}{\partial \lambda_4}=a-c_l-r_l\ge 0,\ \lambda_4\cdot\frac{\partial L}{\partial \lambda_4}=0,\text{and} \quad \text{(A97)}$$

$$\frac{\partial L}{\partial \lambda_5}=\tilde{r}_h-r_h\ge 0,\ \lambda_5\cdot\frac{\partial L}{\partial \lambda_5}=0. \quad \text{(A98)}$$

Since $\frac{\partial L}{\partial r_l}=-\frac{1}{2b}\theta r_l-\frac{1}{2b}\lambda_1-\lambda_4<0$ by (A90), we have $r_l^s=0$ and $\lambda_4^*=0$ by (A97). If $\lambda_1^*>0$, then $\delta_l^s=\delta_l''>0$ by (A94), which implies $\lambda_1^*=0$ by (A92). This is a contradiction. Thus, we must have $\lambda_1^*=0$. Moreover, constraints $\delta_h''<\delta_h$ and $r_h<\tilde{r}_h$ suggest $\lambda_2^*=\lambda_5^*=0$ by (A95) and (A98). If $\lambda_3^*>0$, then $r_h^s=0$ by (A91) and $\delta_h^s=\tilde{\delta}_h=\frac{(a-r_h^s-c_h)}{b}=\frac{(a-c_h)}{b}$ by (A96). Hence (A93) suggests $\lambda_3^*=-(1-\theta)a+\theta c_l+(1-2\theta)c_h<-(1-\theta)a+\theta c_h+(1-2\theta)c_h=-(1-\theta)(a-c_h)<0$. This is a contradiction. Thus, we must have $\lambda_3^*=0$. Therefore, $\delta_h^s=\frac{(1-\theta)a+\theta c_l-c_h}{2b(1-\theta)}$ by (A93). Note that

$(\delta_h^s - \delta_h'') = \dfrac{(1-\theta) r_h - (c_h - c_l)}{2b(1-\theta)} > 0$ if $r_h > \dfrac{c_h - c_l}{(1-\theta)}$,

$(\tilde{\delta}_h - \delta_h^s) = \dfrac{(1-\theta) a - \theta c_l - (1-2\theta) c_h - 2(1-\theta) r_h}{2b(1-\theta)} \geq 0$ if $r_h \leq \dfrac{(1-\theta) a - \theta c_l - (1-2\theta) c_h}{2(1-\theta)}$, and

$r_h^s < (a + c_l - 2c_h)$. By some calculations, we have

$\dfrac{(1-\theta) a - \theta c_l - (1-2\theta) c_h}{2(1-\theta)} - \dfrac{c_h - c_l}{(1-\theta)} = \dfrac{(1-\theta) a + (2-\theta) c_l - (3-2\theta) c_h}{2(1-\theta)} > 0$ iff

$c_h < \dfrac{(1-\theta) a + (2-\theta) c_l}{(3-2\theta)}$, and $(a + c_l - 2c_h) > \dfrac{(1-\theta) a - \theta c_l - (1-2\theta) c_h}{2(1-\theta)}$ if $c_h < \dfrac{(1-\theta) a + (2-\theta) c_l}{(3-2\theta)}$.

It remains to check whether $\pi_h^{sh} \geq \pi_h^{sl}$ at equilibrium. Because $f_h^s = \delta_h^s \left(a - b\delta_h^s - r_h^s - c_h\right)$,

$f_l^s = b\left(\dfrac{a - c_l - r_l^s}{2b}\right)^2 - \delta_h^s \left(a - b\delta_h^s - r_h^s - c_l\right) + f_h^s$, $\pi_h^{sl} = b\left(\dfrac{a - c_h - r_l^s}{2b}\right)^2 - f_l^s$ if $\delta_l^s \in [0, \delta_l']$,

$\pi_h^{sl} = \delta_l^s \left[a - b\delta_l^s - c_h - r_l^s\right] - f_l^s$ if $\delta_l^s \in (\delta_l', \delta_l'']$, and $\delta_l^s \left[a - b\delta_l^s - c_h - r_l^s\right] < b\left(\dfrac{a - c_h - r_l^s}{2b}\right)^2$ if $\delta_l^s > \delta_l'$;

for $\delta_l^s \in [0, \delta_l'']$, we have

$$\pi_h^{sh} - \pi_h^{sl} \geq b\left[\left(\frac{a - c_l - r_l^s}{2b}\right)^2 - \left(\frac{a - c_h - r_l^s}{2b}\right)^2\right] - \left[\delta_h^s \left(a - b\delta_h^s - r_h^s - c_l\right) - \delta_h^s \left(a - b\delta_h^s - r_h^s - c_h\right)\right]$$

$= \dfrac{(c_h - c_l)\left[2a - c_l - c_h - 2r_l^s - 4\delta_h^s\right]}{4b} = \dfrac{(1+\theta)(c_h - c_l)^2}{4b(1-\theta)} > 0$. The optimal fee contract and minimum

throughout exist for $c_h \leq \dfrac{(1-\theta) a + (2-\theta) c_l}{(3-2\theta)}$ with $r_h^s \in \left(\dfrac{c_h - c_l}{(1-\theta)}, \dfrac{(1-\theta) a - \theta c_l - (1-2\theta) c_h}{2(1-\theta)}\right]$,

$\delta_l^s \in [0, \delta_l'']$ with $\delta_l'' = \dfrac{a - r_l^s - c_l}{2b} = \dfrac{a - c_l}{2b}$,

$$f_h^s = \frac{\left[(1-\theta) a + \theta c_l - c_h\right] \times \left[(1-\theta) a - \theta c_l - (1-2\theta) c_h - 2(1-\theta) r_h^s\right]}{4b(1-\theta)^2},$$

$f_l^s = \dfrac{(1-\theta) a^2 - 2(1-\theta) a c_h - 2(1+\theta) c_l c_h + (1+\theta) c_l^2 + 2c_h^2}{4b(1-\theta)}$, and the port authority's equilibrium fee

revenue equals

$$R_{21}^s = \frac{(1-\theta) a^2 - 2(1-\theta) a c_h - 2\theta c_l c_h + \theta c_l^2 + c_h^2}{4b(1-\theta)}. \qquad \text{(A99)}$$

Case 4: Suppose $\delta_l \in \left(\delta_l'', \dfrac{a}{b}\right)$ with $\delta_l'' = \dfrac{a - r_l - c_l}{2b}$, and $\delta_h \in [0, \delta_h']$ with $\delta_h' = \dfrac{a - r_h - c_h}{2b}$. Lemma 4(iii) and Lemma 5(i) then imply that $\pi_l^{sl} = \delta_l \left[a - b\delta_l - r_l - c_l\right] - f_l$, $\pi_h^{sh} = b\left(q_h^{sh}\right)^2 - f_h$, $f_h = b\left(q_h^{sh}\right)^2$, and $f_l = \delta_l \left[a - b\delta_l - r_l - c_l \right] - b\left(q_l^{sh}\right)^2 + f_h$ by the binding (IC_L) and (IR_H). Problem (12A) thus becomes

$$\max_{\substack{(r_l, f_l, \delta_l),\\(r_h, f_h, \delta_h)}} R = \theta\left[r_l\delta_l + \delta_l\left(a - b\delta_l - r_l - c_l\right) - b\left(q_l^{sh}\right)^2 + b\left(q_h^{sh}\right)^2 \right] + (1-\theta)\left[r_h q_h^{sh} + b\left(q_h^{sh}\right)^2 \right]$$

$$\text{s.t. } \delta_l'' < \delta_l < \frac{a}{b}, 0 \le \delta_h \le \delta_h', \; 0 \le r_l \le a - c_l \text{ and } 0 \le r_h \le \overline{r}_h. \tag{A100}$$

Its Lagrange function is

$$L = \theta\left[\delta_l\left(a - b\delta_l - c_l\right) - b\left(q_l^{sh}\right)^2 + b\left(q_h^{sh}\right)^2 \right] + (1-\theta)\left[r_h q_h^{sh} + b\left(q_h^{sh}\right)^2 \right]$$
$$+\lambda_1\left(\delta_l - \delta_l''\right) + \lambda_2\left(\delta_h' - \delta_h\right) + \lambda_3\left(\frac{a}{b} - \delta_l\right) + \lambda_4\left(\overline{r}_h - r_h\right),$$

where $\lambda_1, \lambda_2, \lambda_3$ and λ_4 are the respective Lagrange multipliers associated with the inequalities in (A100). The corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial \delta_l} = \theta \cdot \left(a - 2b\delta_l - c_l\right) + \lambda_1 - \lambda_3 \le 0, \; \delta_l \cdot \frac{\partial L}{\partial \delta_l} = 0, \tag{A101}$$

$$\frac{\partial L}{\partial r_h} = \frac{1}{2b}\left[\theta\left(c_h - c_l\right) - (1-\theta) r_h\right] - \frac{1}{2b}\lambda_2 - \lambda_4 \le 0, \; r_h \cdot \frac{\partial L}{\partial r_h} = 0,$$
$$\frac{\partial L}{\partial \delta_h} = -\lambda_2 \le 0, \; \delta_h \cdot \frac{\partial L}{\partial \delta_h} = 0,$$
$$\frac{\partial L}{\partial \lambda_1} = \delta_l - \delta_l'' \ge 0, \; \lambda_1 \cdot \frac{\partial L}{\partial \lambda_1} = 0, \tag{A102}$$

$$\frac{\partial L}{\partial \lambda_2} = \delta_h' - \delta_h \ge 0, \; \lambda_2 \cdot \frac{\partial L}{\partial \lambda_2} = 0,$$
$$\frac{\partial L}{\partial \lambda_3} = \frac{a}{b} - \delta_l \ge 0, \; \lambda_3 \cdot \frac{\partial L}{\partial \lambda_3} = 0, \text{ and} \tag{A103}$$

$$\frac{\partial L}{\partial \lambda_4} = \overline{r}_h - r_h \ge 0, \; \lambda_4 \cdot \frac{\partial L}{\partial \lambda_4} = 0.$$

Constraint $\delta_l'' < \delta_l < a$ suggests $\lambda_1^* = \lambda_3^* = 0$ by (A102) and (A103), and hence $\delta_l^s = \frac{a - c_l}{2b}$ by (A101). Problem (12A) then becomes

$$\max_{(r_h, f_h, \delta_h)} R = \theta\left[\frac{\left(a - c_l\right)^2}{4b} - b\left(q_l^{sh}\right)^2 + b\left(q_h^{sh}\right)^2 \right] + (1-\theta)\left[r_h q_h^{sh} + b\left(q_h^{sh}\right)^2 \right],$$

in which the port authority's objective function is not affected by r_l. Note that the port authority's maximization problem in (A54) given $r_l = 0$ is

$$\max_{(r_h, f_h, \delta_h)} R = \theta\left[\frac{(a-c_l)^2}{4b} - b\left(q_l^{sh}\right)^2 + b\left(q_h^{sh}\right)^2\right] + (1-\theta)\left[r_h q_h^{sh} + b\left(q_h^{sh}\right)^2\right],$$

because $q_l^{sl} = \dfrac{(a-c_l)}{2b}$ at $r_l = 0$ by Lemmas 4(i)-(ii). This implies that the optimal concession contract $\left(r_h^s, f_h^s, \delta_h^s\right)$ here is the same as that in Case 1. By contrast, the optimal contract $\left(r_l^s, f_l^s, \delta_l^s\right)$ for the *l*-type operator in Case 1 is $r_l^s = 0$, and f_l^s and δ_l^s satisfy the constraints of Case 1, while the optimal contract $\left(r_l^s, f_l^s, \delta_l^s\right)$ for the *l*-type operator here is $\delta_l^s = \dfrac{a-c_l}{2b}$, and r_l^s and f_l^s satisfy the constraints of this case. Nevertheless, these differences do not affect the port authority's equilibrium profits. They are the same in Case 1 and here. The optimal contract $\left(r_h^s, f_h^s, \delta_h^s\right)$ for the *h*-type operator here is the same as that in Case 1, but the optimal contract $\left(r_l^s, f_l^s, \delta_l^s\right)$ for the *l*-type operator is different. Accordingly, we also have two sub-cases.

Case 4a: As in Case 1a, under $c_h \le c_{hs1}$ with $c_{hs1} = (1-\theta)a + \theta c_l$, the optimal concession contract $\left(r_h^s, f_h^s, \delta_h^s\right)$ is $r_h^s = \dfrac{\theta(c_h - c_l)}{(1-\theta)} > 0$, $\delta_h^s \in [0, \delta_h']$ with $\delta_h' = \dfrac{a - r_h^s - c_h}{2b} = \dfrac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)}$ and $f_h^s = b\left[\dfrac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)}\right]^2 > 0$. The optimal concession contract $\left(r_l^s, f_l^s, \delta_l^s\right)$ is $\delta_l^s = \dfrac{a-c_l}{2b}$, $r_l^s \in \left(0, \dfrac{2bA_1}{(a-c_l)}\right]$ with $A_1 = \dfrac{(a-c_l)^2}{4b} - b\left[\dfrac{(1-\theta)a - (1-2\theta)c_l - \theta c_h}{2b(1-\theta)}\right]^2 + b\left[\dfrac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)}\right]^2$, and $f_l^s = \dfrac{(a-c_l)^2}{4b} - b\left[\dfrac{(1-\theta)a - (1-2\theta)c_l - \theta c_h}{2b(1-\theta)}\right]^2 + b\left[\dfrac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)}\right]^2 - \dfrac{(a-c_l)\cdot r_l^s}{2b}$. It remains to show $\pi_h^{sh} \ge \pi_h^{sl}$ at equilibrium. Since $f_h^s = b\left(\dfrac{a - c_h - r_h^s}{2b}\right)^2$,

$f_l^s = \delta_l^s \cdot \left(a - b\delta_l^s - r_l^s - c_l\right) - b\left(\dfrac{a - c_l - r_h^s}{2b}\right)^2 + f_h^s$, $\pi_h^{sh} = b\left(\dfrac{a - c_h - r_h^s}{2b}\right)^2 - f_h^s$, and $\pi_h^{sl} = \delta_l^s\left[a - b\delta_l^s - c_h - r_l^s\right] - f_l^s$, we have

$$(\pi_h^{sh} - \pi_h^{sl}) = \left[\delta_l^s \cdot \left(a - b\delta_l^s - r_l^s - c_l\right) - b\left(\frac{a - c_l - r_h^s}{2b}\right)^2\right] - \left[\delta_l^s\left(a - b\delta_l^s - c_h - r_l^s\right) - b\left(\frac{a - c_h - r_h^s}{2b}\right)^2\right]$$

$= \dfrac{(c_h - c_l)\left[2r_h^s + (c_h - c_l)\right]}{4b} > 0$. Thus, the port authority's equilibrium fee revenue is R_1^s as in (A63).

Case 4b: As in Case 1b, under $c_h > c_{hs1}$ with $c_{hs1} = (1-\theta)a + \theta c_l$, the optimal concession contract $(r_h^s, f_h^s, \delta_h^s)$ is $r_h^s = \overline{r}_h = (a - c_h)$, $\delta_h^s = 0$ and $f_h^s = 0$. The optimal concession contract $(r_l^s, f_l^s, \delta_l^s)$ is $\delta_l^s = \frac{a-c_l}{2b}$, $r_l^s \in \left(0, \frac{(a-c_h)[a-2c_l+c_h]}{2(a-c_l)}\right]$, and

$f_l^s = \frac{(a-c_h)[a-2c_l+c_h] - 2(a-c_l)r_l^s}{4b}$. In addition, $\pi_h^{sh} \geq \pi_h^{sl}$ holds at equilibrium as in Case 4a. Thus, the port authority's equilibrium fee revenue equals R_{12}^s as in (A64).

Case 5: Suppose $\delta_l \in \left(\delta_l'', \frac{a}{b}\right)$ with $\delta_l'' = \frac{a - r_l - c_l}{2b}$, and $\delta_h \in (\delta_h', \delta_h'']$ with $\delta_h'' = \frac{a - r_h - c_l}{2b}$. Lemma 4(iii) and Lemma 5(ii) then imply $\pi_l^{sl} = \delta_l[a - b\delta_l - r_l - c_l] - f_l$ and $\pi_h^{sh} = \delta_h[a - b\delta_h - r_h - c_h] - f_h$ with $f_l^s = \delta_l(a - b\delta_l - r_l - c_l) - b\left(\frac{a - c_l - r_h}{2b}\right)^2 + \delta_h(a - b\delta_h - r_h - c_h)$ and $f_h^s = \delta_h(a - b\delta_h - r_h - c_h)$. Problem (12A) thus becomes

$$\max_{\substack{(r_l, f_l, \delta_l), \\ (r_h, f_h, \delta_h)}} R = \theta\left[r_l\delta_l + \delta_l(a - b\delta_l - r_l - c_l) - \frac{(a - c_l - r_h)^2}{4b} + \delta_h(a - b\delta_h - r_h - c_h)\right]$$

$$+(1-\theta)\left[r_h \cdot \delta_h + \delta_h(a - b\delta_h - r_h - c_h)\right]$$

$$= \theta\left[\delta_l(a - b\delta_l - c_l) - \frac{(a - c_l - r_h)^2}{4b} + \delta_h(a - b\delta_h - r_h - c_h)\right] + (1-\theta)\delta_h(a - b\delta_h - c_h)$$

$$\text{s.t. } \delta_l'' < \delta_l < \frac{a}{b}, \delta_h' < \delta_h \leq \delta_h'', \ 0 \leq r_l \leq a - c_l \text{ and } 0 \leq r_h \leq \overline{r}_h. \qquad \text{(A104)}$$

Similar to Case 4, we discover that $\delta_l^s = \frac{a-c_l}{2b}$, and r_l^s and f_l^s meet the constraints in (A104). Thus, the optimal concession contract $(r_h^s, f_h^s, \delta_h^s)$ in Case 5 is same as that in Case 2, and there are three similar sub-cases.

Case 5a: As in Case 2a-1, when $c_h \leq \frac{(1-\theta)a + (2-\theta)c_l}{(3-2\theta)}$, the optimal contract for the *h*-type operator is $(r_h^s, f_h^s, \delta_h^s)$ with $r_h^s = \frac{c_h - c_l}{(1-\theta)}$, $\delta_h^s = \frac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)}$ and

$f_h^s = \frac{[(1-\theta)a + \theta c_l - c_h] \times [(1-\theta)a + (2-\theta)c_l - (3-2\theta)c_h]}{4b(1-\theta)^2}$. The optimal contract for the *l*-type operator is $(r_l^s, f_l^s, \delta_l^s)$ with $r_l^s \in \left(0, \frac{2bA_2}{(a-c_l)}\right]$,

$$A_2 = \frac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2(1+\theta)c_l c_h + (1+\theta)c_l^2 + 2c_h^2}{4b(1-\theta)},$$

$f_l^s = \frac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2(1+\theta)c_l c_h + (1+\theta)c_l^2 + 2c_h^2}{4b(1-\theta)} - \frac{(a-c_l)\cdot r_l^s}{2b}$ and $\delta_l^s = \frac{a-c_l}{2b}$. It remains to show $\pi_h^{sh} \ge \pi_h^{sl}$ at equilibrium. Since $f_h^s = \delta_h^s \cdot (a - b\delta_h^s - r_h^s - c_h)$,

$f_l^s = \delta_l^s \cdot (a - b\delta_l^s - r_l^s - c_l) - b\left(\frac{a-c_l-r_h^s}{2b}\right)^2 + f_h^s$, $\pi_h^{sh} = \delta_h^s \cdot (a - b\delta_h^s - r_h^s - c_h) - f_h^s$ and

$\pi_h^{sl} = \delta_l^s \left[a - b\delta_l^s - c_h - r_l^s\right] - f_l^s$, we have

$$(\pi_h^{sh} - \pi_h^{sl}) = \left[\delta_l^s \cdot (a - b\delta_l^s - r_l^s - c_l) - b\left(\frac{a-c_l-r_h^s}{2b}\right)^2\right] - \left[\delta_l^s (a - b\delta_l^s - c_h - r_l^s) - \delta_h^s \cdot (a - b\delta_h^s - r_h^s - c_h)\right]$$

$= \frac{(c_h - c_l)^2}{2b(1-\theta)} > 0$. Thus, the port authority's equilibrium fee revenue equals R_{21}^s as in (A75).

Case 5b: As in Case 2a-2, when $c_h' < c_h < c_{hs2}$ with $c_{hs2} = \frac{a+c_l}{2}$ and $c_h' = \frac{(1-\theta)a + (2-\theta)c_l}{(3-2\theta)}$, the optimal contract for the *h*-type operator is $(r_h^s, f_h^s, \delta_h^s)$ with $r_h^s = \tilde{r}_h = (a + c_l - 2c_h)$,

$f_h^s = \frac{\left[(1-\theta)a - \theta c_l + (2\theta-1)c_h\right] \times \left[-(1-\theta)a - (2-\theta)c_l + (3-2\theta)c_h\right]}{4b}$ and

$\delta_h^s = \frac{1}{2b}\left[(1-\theta)a - \theta c_l - (1-2\theta)c_h\right]$. The optimal contract for the *l*-type operator is $(r_l^s, f_l^s, \delta_l^s)$ with

$r_l^s \in \left(0, \frac{2bA_3}{(a-c_l)}\right]$,

$$A_3 = \frac{(a-c_l)^2 - 4(c_h - c_l)^2}{4b} + \frac{\left[(1-\theta)a - \theta c_l + (2\theta-1)c_h\right] \times \left[-(1-\theta)a - (2-\theta)c_l + (3-2\theta)c_h\right]}{4b},$$

$f_l^s = A_3 - \frac{(a-c_l)\cdot r_l^s}{2b}$ and $\delta_l^s = \frac{a-c_l}{2b}$. It remains to check whether $\pi_h^{sh} \ge \pi_h^{sl}$ at equilibrium. Since

$f_h^s = \delta_h^s \cdot (a - b\delta_h^s - r_h^s - c_h)$, $f_l^s = \delta_l^s \cdot (a - b\delta_l^s - r_l^s - c_l) - b\left(\frac{a-c_l-r_h^s}{2b}\right)^2 + f_h^s$,

$\pi_h^{sh} = \delta_h^s \cdot (a - b\delta_h^s - r_h^s - c_h) - f_h^s$ and $\pi_h^{sl} = \delta_l^s \left[a - b\delta_l^s - c_h - r_l^s\right] - f_l^s$, we have

$$(\pi_h^{sh} - \pi_h^{sl}) = \left[\delta_l^s \cdot (a - b\delta_l^s - r_l^s - c_l) - b\left(\frac{a-c_l-r_h^s}{2b}\right)^2\right] - \left[\delta_l^s (a - b\delta_l^s - c_h - r_l^s) - \delta_h^s \cdot (a - b\delta_h^s - r_h^s - c_h)\right]$$

$$= \frac{(a-c_l)(c_h-c_l)}{2b} - b\left(\frac{a-c_l-r_h^s}{2b}\right)^2 + \delta_h^s \cdot (a - b\delta_h^s - r_h^s - c_h)$$

$$> b\left[\left(\frac{a-c_l}{2b}\right)^2 - \left(\frac{a-c_l-r_h^s}{2b}\right)^2\right] - \left[b\left(\frac{a-c_h}{2b}\right)^2 - \delta_h^s (a - b\delta_h^s - r_h^s - c_h)\right]$$

$$=\frac{\left(a+c_l-2c_h\right)\left[-\left(1-\theta\right)^2 a-\left(3-2\theta+\theta^2\right)c_l+\left(4-4\theta+2\theta^2\right)c_h\right]}{4b}>0$$ if $c_h>c_h'$. Thus, the port authority's equilibrium fee revenue equals R_{22}^s as in (A76).

Case 5c: As in Case 2b-2, when $c_h'<c_h<c_h''$ with $c_h'=\frac{(1-\theta)a+(2-\theta)c_l}{(3-2\theta)}$ and $c_h''=\frac{2(1-\theta)a+\theta c_l}{2-\theta}$, the optimal contract for the *h*-type operator is $\left(r_h^s, f_h^s, \delta_h^s\right)$ with $r_h^s=\frac{(2-\theta)a-\theta c_l-2(1-\theta)c_h}{4-3\theta}$, $f_h^s=0$ and $\delta_h^s=\tilde{\delta}_h=\frac{2(1-\theta)a+\theta c_l-(2-\theta)c_h}{(4-3\theta)b}$. The optimal contract for the *l*-type operator is $\left(r_l^s, f_l^s, \delta_l^s\right)$ with

$$r_l^s\in\left(0,\frac{\left[(6-5\theta)a-(8-7\theta)c_l+2(1-\theta)c_h\right]\left[(2-\theta)a-\theta c_l-2(1-\theta)c_h\right]}{2(4-3\theta)^2(a-c_l)}\right],$$

$$f_l^s=\frac{\left[(6-5\theta)a-(8-7\theta)c_l+2(1-\theta)c_h\right]\left[(2-\theta)a-\theta c_l-2(1-\theta)c_h\right]}{4b(4-3\theta)^2}-\frac{(a-c_l)\cdot r_l^s}{2b}$$ and $\delta_l^s=\frac{a-c_l}{2b}$.

It remains to check whether $\pi_h^{sh}\geq\pi_h^{sl}$. Since $f_h^s=\delta_h^s\cdot\left(a-b\delta_h^s-r_h^s-c_h\right)=0$,

$f_l^s=\delta_l^s\cdot\left(a-b\delta_l^s-r_l^s-c_l\right)-b\left(\frac{a-c_l-r_h^s}{2b}\right)^2$, $\pi_h^{sh}=\delta_h^s\cdot\left(a-b\delta_h^s-r_h^s-c_h\right)-f_h^s$ and $\pi_h^{sl}=\delta_l^s\left[a-b\delta_l^s-c_h-r_l^s\right]-f_l^s$, we have

$$(\pi_h^{sh}-\pi_h^{sl})=\delta_l^s\cdot\left(a-b\delta_l^s-r_l^s-c_l\right)-b\left(\frac{a-c_l-r_h^s}{2b}\right)^2-\delta_l^s\left(a-b\delta_l^s-c_h-r_l^s\right)$$

$$=\frac{1}{b}\left[\frac{(a-c_l)(c_h-c_l)}{2}-\left(\frac{a-c_l-r_h^s}{2}\right)^2\right]>\frac{1}{b}\left[\left(\frac{a-c_l}{2}\right)^2-\left(\frac{a-c_h}{2}\right)^2-\left(\frac{a-c_l-r_h^s}{2}\right)^2\right]$$

$$=\frac{1}{4b(4-3\theta)^2}\begin{bmatrix}-4(1-\theta)^2a^2-2\left(8-8\theta+\theta^2\right)ac_l+2\left(12-16\theta+5\theta^2\right)ac_h\\+16(1-\theta)^2c_lc_h+\theta(8-7\theta)c_l^2-\left(20-32\theta+13\theta^2\right)c_h^2\end{bmatrix}>0$$ for $c_h'<c_h<c_h''$. Thus, the port authority's equilibrium fee revenue equals R_{23}^s as in (A88).

Case 6: Suppose $\delta_l\in\left(\delta_l'',\frac{a}{b}\right)$ with $\delta_l''=\frac{a-r_l-c_l}{2b}$, and $\delta_h\in\left(\delta_h'',\frac{a}{b}\right)$ with $\delta_h''=\frac{a-r_h-c_l}{2b}$. Lemma 4(iii) and Lemma 5(iii) then imply $\pi_l^{sl}=\delta_l\left[a-b\delta_l-r_l-c_l\right]-f_l$ and $\pi_h^{sh}=\delta_h\left[a-b\delta_h-r_h-c_h\right]-f_h$ with

$f_l^s=\delta_l\left(a-b\delta_l-r_l-c_l\right)-\delta_h\cdot\left(a-b\delta_h-r_h-c_l\right)+\delta_h\left(a-b\delta_h-r_h-c_h\right)>0$ and

$f_h^s=\delta_h\left(a-b\delta_h-r_h-c_h\right)$. Constraint $\delta_h\leq\tilde{\delta}_h<\frac{a}{b}$ will guarantee $f_h^s\geq 0$. Thus, constraints $\delta_h''<\delta_h<\frac{a}{b}$ and $\delta_h\leq\tilde{\delta}_h$ can be combined into $\delta_h''<\delta_h\leq\tilde{\delta}_h$. Problem (12A) thus becomes

$$\max_{\substack{(r_l, f_l, \delta_l), \\ (r_h, f_h, \delta_h)}} R = \theta\left[r_l \delta_l + \delta_l\left(a - b\delta_l - r_l - c_l\right) - \delta_h \cdot \left(a - b\delta_h - r_h - c_l\right) + \delta_h\left(a - b\delta_h - r_h - c_h\right)\right]$$

$$+ (1-\theta)\left[r_h \cdot \delta_h + \delta_h\left(a - b\delta_h - r_h - c_h\right)\right]$$

$$= \theta\left[\delta_l\left(a - b\delta_l - c_l\right) - \delta_h \cdot \left(c_h - c_l\right)\right] + (1-\theta)\delta_h\left(a - b\delta_h - c_h\right)$$

$$\text{s.t. } \delta_l'' < \delta_l < \frac{a}{b},\ \delta_h'' < \delta_h \le \tilde{\delta}_h, 0 \le r_l \le a - c_l \text{ and } 0 \le r_h < \tilde{r}_h. \qquad \text{(A105)}$$

Similar to Case 3, we discover $\delta_l^s = \dfrac{a - c_l}{2b}$, and r_l^s and f_l^s satisfy the constraints in (A105). The optimal contract $\left(r_h^s, f_h^s, \delta_h^s\right)$ is the same as that in Case 3. It remains to show $\pi_h^{sh} \ge \pi_h^{sl}$ at equilibrium. Since $f_h^s = \delta_h^s \cdot \left(a - b\delta_h^s - r_h^s - c_h\right)$, $\pi_h^{sh} = \delta_h^s \cdot \left(a - b\delta_h^s - r_h^s - c_h\right) - f_h^s$,
$f_l^s = \delta_l^s \cdot \left(a - b\delta_l^s - r_l^s - c_l\right) - \delta_h^s \cdot \left(a - b\delta_h^s - r_h^s - c_l\right) + \delta_h^s \cdot \left(a - b\delta_h^s - r_h^s - c_h\right)$ and
$\pi_h^{sl} = \delta_l^s\left[a - b\delta_l^s - c_h - r_l^s\right] - f_l^s$, we have
$(\pi_h^{sh} - \pi_h^{sl}) = \delta_l^s \cdot \left(a - b\delta_l^s - r_l^s - c_l\right) - \delta_h^s \cdot \left(a - b\delta_h^s - r_h^s - c_l\right) + \delta_h^s \cdot \left(a - b\delta_h^s - r_h^s - c_h\right) - \delta_l^s\left(a - b\delta_l^s - c_h - r_l^s\right)$
$= \left(c_h - c_l\right)\left(\delta_l^s - \delta_h^s\right) = \dfrac{\left(c_h - c_l\right)^2}{2(1-\theta)} > 0$. Thus, the port authority's equilibrium fee revenue equals R_{21}^s as in (A99).

Therefore, there exist five solutions. By comparing them, we can find the port authority's optimal contracts. Since $(c_h'' - c_{hs2}) = \dfrac{(2-3\theta)(a - c_l)}{2(2-\theta)} > (<) 0$ iff $\theta < (>) \dfrac{2}{3}$,
$(c_h'' - c_{hs1}) = \dfrac{\theta(1-\theta)(a - c_l)}{(2-\theta)} > 0$, $(c_{hs2} - c_{hs1}) = \dfrac{(2\theta - 1)(a - c_l)}{2} > (<) 0$ iff $\theta > (<) \dfrac{1}{2}$,
$(c_{hs1} - c_h') = \dfrac{2(1-\theta)^2(a - c_l)}{(3-2\theta)} > 0$ and $(c_{hs2} - c_h') = \dfrac{(a - c_l)}{2(3-2\theta)} > 0$, we have three sub-cases according to the values of θ as follows.

Case A: Suppose $\theta \le \dfrac{1}{2}$. If $\theta < \dfrac{1}{2}$, then $c_h' < c_{hs2} < c_{hs1} < c_h''$. However, if $\theta = \dfrac{1}{2}$, then $c_h' < c_{hs2} = c_{hs1} < c_h''$. In either case, we have four sub-cases.

Case A-1: For $c_h \le c_h'$, there are two solutions:
$R_{11}^s \equiv \dfrac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2\theta^2 c_l c_h + \theta^2 c_l^2 + \left(1 - \theta + \theta^2\right)c_h^2}{4b(1-\theta)}$ as $c_h \le c_{hs1}$, and
$R_{21}^s \equiv \dfrac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2\theta c_l c_h + \theta c_l^2 + c_h^2}{4b(1-\theta)}$ as $c_h \le c_h'$. Since $(R_{21}^s - R_{11}^s) = \dfrac{\theta\left(c_h - c_l\right)^2}{4} > 0$, revenue R_{21}^s is optimal.

Case A-2: For $c_h' < c_h \le c_{hs1}$, there are three solutions:

$R_{11}^s \equiv \dfrac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2\theta^2 c_l c_h + \theta^2 c_l^2 + (1-\theta+\theta^2)c_h^2}{4b(1-\theta)}$ as $c_h \le c_{hs1}$,

$R_{22}^s \equiv \dfrac{1}{4b}\left[\begin{array}{l}(1-\theta+\theta^2)a^2 - 2\theta(2-\theta)ac_l - 2(1-3\theta+2\theta^2)ac_h \\ +2\theta(5-2\theta)c_l c_h - \theta(3-\theta)c_l^2 + (1-8\theta+4\theta^2)c_h^2\end{array}\right]$ as $c_h' < c_h < c_{hs2}$, and

$R_{23}^s = \dfrac{\left[(2-\theta)a - \theta c_l - 2(1-\theta)c_h\right]^2}{4b(4-3\theta)}$ as $c_h' < c_h < c_h''$. Since

$\dfrac{\partial(R_{23}^s - R_{11}^s)}{\partial c_h} = \dfrac{\theta\left[(3-5\theta+2\theta^2)a + (2-\theta^2)c_l - (5-5\theta+\theta^2)c_h\right]}{2b(1-\theta)(4-3\theta)}$,

$\dfrac{\partial^2(R_{23}^s - R_{11}^s)}{\partial {c_h}^2} = \dfrac{-\theta(5-5\theta+\theta^2)}{2b(1-\theta)(4-3\theta)} < 0$, $\dfrac{\partial(R_{23}^s - R_{11}^s)}{\partial c_h} = \dfrac{\theta(1-\theta)(a-c_l)}{2b(3-2\theta)} > 0$,

$(R_{23}^s - R_{11}^s) = \dfrac{\theta(1-\theta)^2(a-c_l)^2}{4b(3-2\theta)^2} > 0$ at $c_h = c_h'$, $\dfrac{\partial(R_{23}^s - R_{11}^s)}{\partial c_h} = \dfrac{-\theta(1-\theta)(2-\theta)(a-c_l)}{2b(4-3\theta)} < 0$ and

$(R_{23}^s - R_{11}^s) = \dfrac{\theta^2(1-\theta)^2(a-c_l)^2}{4b(4-3\theta)} > 0$ at $c_h = c_{hs1}$, we have $R_{23}^s > R_{11}^s$ for $c_h' < c_h < c_{hs1}$. Moreover, by

$(R_{23}^s - R_{22}^s) = \dfrac{3\theta\left[(1-\theta)a + (2-\theta)c_l - (3-2\theta)c_h\right]^2}{4b(4-3\theta)} > 0$, we obtain $R_{23}^s > R_{11}^s$ and $R_{23}^s > R_{22}^s$ for $c_h' < c_h \le c_{hs1}$.

Case A-3: For $c_{hs1} < c_h < c_h''$, there are two solutions: $R_{12}^s = \dfrac{\theta(a-c_h)(a-2c_l+c_h)}{4b}$ as $c_h > c_{hs1}$, and $R_{23}^s = \dfrac{\left[(2-\theta)a - \theta c_l - 2(1-\theta)c_h\right]^2}{4b(4-3\theta)}$ as $c_h' < c_h < c_h''$. Since

$(R_{23}^s - R_{12}^s) = \dfrac{\left[2(1-\theta)a + \theta c_l - (2-\theta)c_h\right]^2}{4b(4-3\theta)} > 0$, we have $R_{23}^s > R_{12}^s$ for $c_{hs1} < c_h < c_h''$.

Case A-4: For $c_h \ge c_h''$, there is a unique solution: $R_{12}^s = \dfrac{\theta(a-c_h)(a-2c_l+c_h)}{4b}$ as $c_h > c_{hs1}$. Thus, revenue R_{12}^s is optimal for $c_h \ge c_h''$.

Case A-1 to Case A-4 imply that R_{21}^s is the largest when $c_h \le c_h'$, that R_{23}^s is the largest when $c_h' < c_h < c_h''$, and that R_{12}^s is the largest when $c_h \ge c_h''$ for $\theta < \dfrac{1}{2}$.

Case B: Suppose $\dfrac{1}{2} < \theta \le \dfrac{2}{3}$. If $\dfrac{1}{2} < \theta < \dfrac{2}{3}$, then $c_h' < c_{hs1} < c_{hs2} < c_h''$. However, if $\theta = \dfrac{2}{3}$, then $c_h' < c_{hs1} < c_{hs2} = c_h''$. In either case, we have three sub-cases.

Case B-1: For $c_h \le c_h'$, there are two solutions: R_{11}^s and R_{21}^s. As in Case A-1, revenue R_{21}^s is larger.

Case B-2: For $c_h' < c_h < c_h''$, there are four solutions: R_{11}^s for $c_h \le c_{hs1}$, R_{12}^s for $c_h > c_{hs1}$, R_{22}^s for $c_h' < c_h < c_{hs2}$, and R_{23}^s for $c_h' < c_h < c_h''$. As in Case A-2, we have $R_{23}^s > R_{11}^s$ for $c_h' < c_h \le c_{hs1}$, and $R_{23}^s > R_{22}^s$ for $c_h' < c_h < c_{hs2}$. As in Case A-3, we have $R_{23}^s > R_{12}^s$ for $c_{hs1} < c_h < c_h''$. Thus, revenue R_{23}^s is optimal for $c_h' < c_h < c_h''$.

Case B-3: For $c_h \ge c_h''$, there is a unique solution: R_{12}^s for $c_h > c_{hs1}$. Thus, revenue R_{12}^s is optimal for $c_h \ge c_h''$.

Case C: Suppose $\frac{2}{3} < \theta < 1$. Then $c_h' < c_{hs1} < c_h'' < c_{hs2}$. There are four sub-cases as follow.

Case C-1: For $c_h \le c_h'$, there are two solutions: R_{11}^s and R_{21}^s. As in Case A-1, revenue R_{21}^s is larger.

Case C-2: For $c_h' < c_h < c_h''$, there are four solutions: R_{11}^s for $c_h \le c_{hs1}$, R_{12}^s for $c_h > c_{hs1}$, R_{22}^s for $c_h' < c_h < c_{hs2}$, and R_{23}^s for $c_h' < c_h < c_h''$. As in Case A-2, we have $R_{23}^s > R_{11}^s$ when $c_h' < c_h \le c_{hs1}$, and $R_{23}^s > R_{22}^s$ when $c_h' < c_h < c_h''$. As in Case A-3, we have $R_{23}^s > R_{12}^s$ when $c_{hs1} < c_h < c_h''$. Thus, revenue R_{23}^s is optimal when $c_h' < c_h < c_h''$.

Case C-3: For $c_h'' \le c_h < c_{hs2}$, there are two solutions: R_{12}^s and R_{22}^s. Since

$$\frac{\partial\left(R_{12}^s - R_{22}^s\right)}{\partial c_h} = \frac{1}{2b}\left[\left(1-3\theta+2\theta^2\right)a - 2\theta\left(2-\theta\right)c_l + \left(-1+7\theta-4\theta^2\right)c_h\right]$$

$$> \frac{1}{2b}\left[\left(1-3\theta+2\theta^2\right)a - 2\theta\left(2-\theta\right)c_l + \left(-1+7\theta-4\theta^2\right)c_h''\right] = \frac{3\theta\left(1-\theta\right)\left(3-2\theta\right)\left(a-c_l\right)}{2b\left(2-\theta\right)} > 0$$ due to

$\left(-1+7\theta-4\theta^2\right) > 0$ when $\frac{2}{3} < \theta < 1$, we have $\frac{\partial\left(R_{12}^s - R_{22}^s\right)}{\partial c_h} > 0$ for $c_h'' \le c_h < c_{hs2}$ and $\frac{2}{3} < \theta < 1$.

Moreover, we have $(R_{12}^s - R_{22}^s) = \frac{3\theta\left(4-3\theta\right)\left(1-\theta\right)^2\left(a-c_l\right)^2}{4b\left(2-\theta\right)^2} > 0$ at $c_h = c_h''$, and hence $R_{12}^s > R_{22}^s$ when $c_h'' \le c_h < c_{hs2}$ and $\frac{2}{3} < \theta < 1$.

Case C-4: For $c_h \ge c_{hs2}$, the unique solution is R_{12}^s.

Based on Case C-1 to Case C-4, we find that R_{21}^s is the largest when $c_h \le c_h'$, that R_{23}^s is the largest when $c_h' < c_h < c_h''$, and that R_{12}^s is the largest when $c_h \ge c_h''$ and $\frac{2}{3} < \theta < 1$.

In summary, the results of Cases A-C suggest that R_{21}^s is the largest when $c_h \le c'_h$, that R_{23}^s is the largest when $c'_h < c_h < c''_h$, and that R_{12}^s is the largest when $c_h \ge c''_h$ for all values of θ. Note that R_{21}^s occurs in Case 2a-1, Case 3, Case 5a, and Case 6; that R_{23}^s appears in Case 2b-2 and Case 5c; and that R_{12}^s exists in Case 1b and Case 4b. Thus, for $c_h \in (c_l, c'_h]$ with $c'_h = \dfrac{(1-\theta)a + (2-\theta)c_l}{(3-2\theta)}$, the port authority's best choice $(r_l^s, f_l^s, \delta_l^s)$ for the *l*-type operator is the fixed-fee scheme with $f_l^s = \dfrac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2(1+\theta)c_l c_h + (1+\theta)c_l^2 + 2c_h^2}{4b(1-\theta)}$ and $\delta_l^s \in \left[0, \dfrac{a-c_l}{2b}\right]$ as in Case 2a-1 and Case 3; or the two-part tariff scheme with

$r_l^s \in \left(0, \dfrac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2(1+\theta)c_l c_h + (1+\theta)c_l^2 + 2c_h^2}{2(1-\theta)(a-c_l)}\right]$,

$f_l^s = \dfrac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2(1+\theta)c_l c_h + (1+\theta)c_l^2 + 2c_h^2 - 2(1-\theta)(a-c_l)r_l^s}{4b(1-\theta)}$ and $\delta_l^s = \dfrac{a-c_l}{2b}$ as in Case 5a and Case 6. In particular, at $r_l^s = \dfrac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2(1+\theta)c_l c_h + (1+\theta)c_l^2 + 2c_h^2}{2(1-\theta)(a-c_l)}$, we will have $f_l^s = 0$ and $\delta_l^s = \dfrac{a-c_l}{2b}$, which is an optimal unit-fee contract. On the other hand, the optimal two-part tariff contract and minimum throughput guarantee $(r_h^s, f_h^s, \delta_h^s)$ for the *h*-type operator are $r_h^s = \dfrac{c_h - c_l}{(1-\theta)}$, $f_h^s = \dfrac{\left[(1-\theta)a + \theta c_l - c_h\right] \times \left[(1-\theta)a + (2-\theta)c_l - (3-2\theta)c_h\right]}{4b(1-\theta)^2}$ and $\delta_h^s = \dfrac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)}$ as in Case 2a-1 and Case 5a; or $r_h^s \in \left(\dfrac{c_h - c_l}{(1-\theta)}, \dfrac{(1-\theta)a - \theta c_l - (1-2\theta)c_h}{2(1-\theta)}\right]$, $f_h^s = \dfrac{\left[(1-\theta)a + \theta c_l - c_h\right] \times \left[(1-\theta)a - \theta c_l - (1-2\theta)c_h - 2(1-\theta)r_h^s\right]}{4b(1-\theta)^2}$ and $\delta_h^s = \dfrac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)}$ as in Case 3 and Case 6. In particular, at $r_h^s = \dfrac{(1-\theta)a - \theta c_l - (1-2\theta)c_h}{2(1-\theta)}$, we will have $f_h^s = 0$ and $\delta_h^s = \dfrac{(1-\theta)a + \theta c_l - c_h}{2b(1-\theta)}$, which is an optimal unit-fee contract. No matter which contracts are offered to the operators, the port authority's equilibrium fee revenue is always $R^s = \dfrac{(1-\theta)a^2 - 2(1-\theta)ac_h - 2\theta c_l c_h + \theta c_l^2 + c_h^2}{4b(1-\theta)}$. This is what Proposition 2(i) shows.

When $c_h \in (c'_h, c''_h)$ with $c''_h \equiv \dfrac{2(1-\theta)a + \theta c_l}{2-\theta}$, the port authority's optimal contract is the fixed-fee scheme for the *l*-type operator with $f_l^s = \dfrac{\left[(6-5\theta)a - (8-7\theta)c_l + 2(1-\theta)c_h\right]\left[(2-\theta)a - \theta c_l - 2(1-\theta)c_h\right]}{4b(4-3\theta)^2}$ and $\delta_l^s \in \left[0, \dfrac{a-c_l}{2b}\right]$ as in

Case 2b-2; or the two-part tariff scheme with

$$r_l^s \in \left(0, \frac{\left[(6-5\theta)a-(8-7\theta)c_l+2(1-\theta)c_h\right]\left[(2-\theta)a-\theta c_l-2(1-\theta)c_h\right]}{2(4-3\theta)^2(a-c_l)}\right],$$

$$f_l^s = \frac{\left[(6-5\theta)a-(8-7\theta)c_l+2(1-\theta)c_h\right]\left[(2-\theta)a-\theta c_l-2(1-\theta)c_h\right]-2(4-3\theta)^2(a-c_l)r_l^s}{4b(4-3\theta)^2}$$ and

$\delta_l^s = \frac{a-c_l}{2b}$ as in Case 5c. In particular, at

$r_l^s = \frac{\left[(6-5\theta)a-(8-7\theta)c_l+2(1-\theta)c_h\right]\left[(2-\theta)a-\theta c_l-2(1-\theta)c_h\right]}{2(4-3\theta)^2(a-c_l)}$, we will have $f_l^s = 0$ and

$\delta_l^s = \frac{a-c_l}{2b}$, which is a unit-fee scheme. On the other hand, the optimal unit-fee contract and minimum throughput requirement for the *h*-type operator are $r_h^s = \frac{(2-\theta)a-\theta c_l-2(1-\theta)c_h}{(4-3\theta)}$ and

$\delta_h^s = \frac{2(1-\theta)a+\theta c_l-(2-\theta)c_h}{b(4-3\theta)}$ as in Case 2b-2 and Case 5c. No matter which contracts are offered to the operators, the port authority's equilibrium fee revenue is always

$R^s = \frac{\left[(2-\theta)a-\theta c_l-2(1-\theta)c_h\right]^2}{4b(4-3\theta)}$. This is the content of Proposition 2(ii).

Finally, when $c_h \in \left[c_h'', a\right)$, the best offer to the *l*-type operator is the fixed-fee contract with $f_l^s = \frac{(a-c_h)\left[a-2c_l+c_h\right]}{4b}$ and $\delta_l^s \in \left[0, \frac{a-c_l}{2b}\right]$ as in Case 1b, or the two-part tariff contract with $r_l^s \in \left(0, \frac{(a-c_h)\left[a-2c_l+c_h\right]}{2(a-c_l)}\right]$, $f_l^s = \frac{(a-c_h)\left[a-2c_l+c_h\right]-2(a-c_l)r_l^s}{4b}$ and $\delta_l^s = \frac{a-c_l}{2b}$ as in Case 4b. In particular, at $r_l^s = \frac{(a-c_h)\left[a-2c_l+c_h\right]}{2(a-c_l)}$, we will have $f_l^s = 0$ and $\delta_l^s = \frac{a-c_l}{2b}$, which is an optimal unit-fee contract. On the other hand, the optimal unit-fee contract and minimum throughput requirement for the *h*-type operator are $r_h^s = \overline{r}_h = (a-c_h)$ and $\delta_h^s = 0$ as in Case 1b and Case 4b. No matter which contracts are offered to the operators, the port authority's equilibrium fee revenue always equals $R^s = \frac{\theta(a-c_h)\left[a-2c_l+c_h\right]}{4b}$. This is what Proposition 2(iii) states. □

To sum up, we discover that the change from equation (1) to equation (13) only alter some constant terms of the optimal concession contracts derived in Section 4. This implies that the port authority's optimal concession contracts stay the same qualitatively when the demand function becomes more general as in (13). Thus, it is obvious that the results derived in Section 5 remain true qualitatively as well under the general demand function (13). This proves Proposition 7.