Proof of Proposition 8: Under (3), (4), (15) and (16), we re-derive all the Lemmas and Propositions in Section 4. First, the changed problems are stated. Then, the associated Lemmas and Propositions are provided.

Under the new set-up, problem (5) becomes

$$\max_{p_i \geq 0} \; \pi_i = \left(p_i - c_i - r\right)\left(1 - p_i\right) - f \tag{5B}$$

$$\text{s.t.} \quad q_i \geq \delta$$

for $i = l, h$. Its solutions are listed below.

Lemma 1. *Given concession contract* $\left(r, f, \delta\right)$, *operator i's optimal behaviors are as follows.*

(i) *If* $\delta \in [0, \overline{\delta}_i]$ *with* $\overline{\delta}_i = \frac{(1-c_i-r)}{2}$, *then we have* $p_i^c = \frac{(1+c_i+r)}{2} > 0$ *with equilibrium cargo-handling amounts* $q_i^c = \frac{(1-c_i-r)}{2} = \overline{\delta}_i$ *and equilibrium profits* $\pi_i^c = (q_i^c)^2 - f$ *for* $i = l, h$.

(ii) *If* $\delta \in (\overline{\delta}_i, 1)$, *then we have* $p_i^c = (1-\delta) > 0$ *with equilibrium cargo-handling amounts* $q_i^c = \delta$ *and equilibrium profits* $\pi_i^c = (1-\delta-c_i-r)\delta - f$ *for* $i = l, h$.

Proof of Lemma 1: Denote L the terminal operator's Lagrange function in problem (5B),

$$L = \left(p_i - c_i - r\right)\left(1 - p_i\right) - f + \lambda\left(1 - p_i - \delta\right),$$

where λ is the Lagrange multiplier associated with the constraint in problem (5B). Then, the corresponding Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial p_i} = 1 - 2p_i + c_i + r - \lambda \leq 0, \; p_i \cdot \frac{\partial L}{\partial p_i} = 0, \text{ and} \tag{A1}$$

$$\frac{\partial L}{\partial \lambda} = 1 - p_i - \delta \geq 0, \; \lambda \cdot \frac{\partial L}{\partial \lambda} = 0. \tag{A2}$$

Based on the values of λ, there are two cases below.

Case 1: Suppose $\lambda^* = 0$. We have $p_i^c = \dfrac{\left(1 + c_i + r\right)}{2} > 0$. Substituting p_i^c into (15) yields $q_i^c = \dfrac{\left(1 - c_i - r\right)}{2}$. To guarantee $q_i^c \geq \delta$, condition $0 \leq \delta \leq \overline{\delta}_i \equiv \dfrac{\left(1 - c_i - r\right)}{2} = q_i^c$ should be met. Substituting p_i^c into (16) yields $\pi_i^c = \left(q_i^c\right)^2 - f$ for $i = l, h$. These prove Lemma 1(i).

Case 2: Suppose $\lambda^* > 0$ .We have $p_i^c = (1-\delta) > 0$ and $\lambda^* = 2\left[\delta - \dfrac{\left(1 - c_i - r\right)}{2}\right]$ by (A1) and (A2).

To guarantee $\lambda^* > 0$, conditions $\delta > \dfrac{\left(1 - c_i - r\right)}{2} = \overline{\delta}_i$ and $r_i \leq \overline{r}_i = 1 - c_i$ are needed. Substituting

p_i^c into (15) yields $q_i^c = \delta$ if $\overline{\delta}_i < 1$, and into (16) gives $\pi_i^c = \delta(1-\delta-c_i-r)-f$ for $i = l, h$. These prove Lemma 1(ii). □

Under the new set-up, problem (7) becomes

$$\max_{r_i, f_i, \delta_i} r_i q_i^c + f_i \tag{7B}$$

$$\text{s.t. } 0 \le \delta_i < 1, \; 0 \le r_i \le \overline{r}_i, \; f_i \ge 0 \text{ and } \pi_i^c \ge 0$$

for $i = l, h$. Its solutions are as follows.

Lemma 2. *Suppose the conditions in (6) hold. The optimal concession contract* $(r_i^c, f_i^c, \delta_i^c)$ *offered to the operator with marginal service cost* c_i, $i = l, h$, *can be the fixed-fee contract* $f_i^c = \frac{(1-c_i)^2}{4}$ *with minimum throughput requirement* $\delta_i^c \in \left[0, \frac{1-c_i}{2}\right]$, *the unit-fee contract* $r_i^c = \frac{1-c_i}{2}$ *with minimum throughput requirement* $\delta_i^c = \frac{1-c_i}{2}$, *or the two-part tariff contract with* $r_i^c \in \left(0, \frac{1-c_i}{2}\right)$, $f_i^c = \frac{(1-c_i)(1-c_i-2r_i^c)}{4}$, *and minimum throughput requirement* $\delta_i^c = \frac{1-c_i}{2}$. *However, the port authority's equilibrium fee revenue always equals* $R_i^c = \frac{(1-c_i)^2}{4}$, $i = l, h$.

<u>Proof of Lemma 2</u>: As shown in (7B), the port authority's maximization problem is

$$\max_{r_i, f_i, \delta_i} r_i q_i^c + f_i$$

$$\text{s.t. } 0 \le \delta_i < 1, \; 0 \le r_i \le \overline{r}_i, \; f_i \ge 0 \text{ and } \pi_i^c \ge 0.$$

Obviously, the operator's optimal cargo-handling amounts and equilibrium profits (q_i^c, π_i^c) used in the above objective function and constraints are equivalent to those in the quantity competition. This implies that the optimal concession contracts under price competition are the same as those in the quantity competition. □

Under the new set-up, problem (8) becomes

$$\max_{p_i \ge 0} \pi_i = (p_i - c_i - r)(1-p_i) - f \tag{8B}$$

$$\text{s.t. } q_i \ge \delta, i = l, h.$$

Its solutions are listed below.

Lemma 3. *Given contract* (r, f, δ), *operator i's optimal behaviors are as follows for* $i = l, h$.

(i) *For* $\delta \in [0, \delta_1^p]$ *with* $\delta_1^p = \frac{(1-c_h-r)}{2} > 0$, *both-type operators' equilibrium service prices are* $p_l^p = \frac{(1+c_l+r)}{2} > 0$ *and* $p_h^p = \frac{(1+c_h+r)}{2} > 0$, *their equilibrium cargo-handling amounts are* $q_l^p = \frac{(1-c_l-r)}{2} > \delta_1^p$ *and* $q_h^p = \frac{(1-c_h-r)}{2} = \delta_1^p$, *and their equilibrium profits are* $\pi_i^p = (q_i^p)^2 - f$ *for* $i = l, h$.

(ii) *For* $\delta \in (\delta_1^p, \delta_2^p]$ *with* $\delta_2^p = \frac{(1-c_l-r)}{2} > \delta_1^p$, *both-type operators' equilibrium service prices are* $p_l^p = \frac{(1+c_l+r)}{2}$ *and* $p_h^p = 1-\delta$, *their equilibrium cargo-handling amounts are* $q_l^p = \frac{(1-c_l-r)}{2}$ *and* $q_h^p = \delta$, *and their equilibrium profits are* $\pi_l^p = (q_l^p)^2 - f$ *and* $\pi_h^p = \delta[1-\delta-c_h-r] - f$.

(iii) *For* $\delta \in (\delta_2^p, 1)$, *both-type operators' equilibrium service prices are* $p_l^p = p_h^p = 1-\delta > 0$, *their equilibrium cargo-handling amounts are* $q_l^p = \delta$ *and* $q_h^p = \delta$, *and their equilibrium profits are* $\pi_i^p = \delta[1-\delta-c_i-r] - f$ *for* $i = l, h$.

Proof of Lemma 3: Denote L_1 and L_2 the respective Lagrange functions for the *l*-type and the *h*-type terminal operators in problem (8B),

$$L_1 = (p_l - c_l - r)(1-p_l) - f + \lambda_1(1-p_l-\delta) \text{ and}$$
$$L_2 = (p_h - c_h - r)(1-p_h) - f + \lambda_2(1-p_h-\delta),$$

where λ_1 and λ_2 are Lagrange multipliers for the *l*-type and the *h*-type operators, respectively. Then, the Kuhn-Tucker conditions for the *l*-type operator are

$$\frac{\partial L_1}{\partial p_l} = 1 - 2p_l + c_l + r - \lambda_1 \le 0,\ p_l \cdot \frac{\partial L_1}{\partial p_l} = 0 \text{ and} \tag{A3}$$

$$\frac{\partial L_1}{\partial \lambda_1} = 1 - p_l - \delta \ge 0,\ \lambda_1 \cdot \frac{\partial L_1}{\partial \lambda_1} = 0, \tag{A4}$$

and for the *h*-type operator are

$$\frac{\partial L_2}{\partial p_h} = 1 - 2p_h + c_h + r - \lambda_2 \le 0,\ p_h \cdot \frac{\partial L_2}{\partial p_h} = 0 \text{ and} \tag{A5}$$

$$\frac{\partial L_2}{\partial \lambda_2} = 1 - p_h - \delta \ge 0,\ \lambda_2 \cdot \frac{\partial L_2}{\partial \lambda_2} = 0. \tag{A6}$$

Based on the values of λ_1 and λ_2, there are four cases below.

Case 1: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$.Then (A3) and (A5) become $(1-2p_l + c_l + r) = 0$ and $(1-2p_h + c_h + r) = 0$, respectively. Solving the two equations yields $p_l^p = \frac{(1+c_l+r)}{2} > 0$ and $p_h^p = \frac{(1+c_h+r)}{2} > 0$. Substituting p_l^p and p_h^p into (15) yields $q_l^p = \frac{(1-c_l-r)}{2}$ and $q_h^p = \frac{(1-c_h-r)}{2}$. To guarantee $q_l^p \geq \delta$ and $q_h^p \geq \delta$, condition $0 \leq \delta \leq \delta_1^p \equiv \frac{(1-c_h-r)}{2} = q_h^p$ should be imposed, because $c_l < c_h$ suggests $q_l^p > q_h^p$ and $q_h^p \geq \delta$ suggests $q_l^p \geq \delta$. Substituting p_l^p and p_h^p into (16) yields $\pi_i^p = \left(q_i^p\right)^2 - f$ for $i = l,h$. These prove Lemma 3(i).

Case 2: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then (A3), (A5) and (A6) imply $(1-2p_l + c_l + r) = 0$, $(1-2p_h + c_h + r - \lambda_2) = 0$ and $(1 - p_h - \delta) = 0$. Solving these equations yields $p_l^p = \frac{(1+c_l+r)}{2}$, $p_h^p = 1-\delta$ and $\lambda_2^* = 2\left[\delta - \frac{(1-c_h-r)}{2}\right]$. Substituting p_l^p and p_h^p into (15) produces $q_l^p = \frac{(1-c_l-r)}{2}$ and $q_h^p = \delta$. To guarantee $\lambda_2^* > 0$, conditions $\delta > \frac{(1-c_h-r)}{2} = \delta_1^p$ and $r \leq \overline{r}_h$ are needed. On the other hand, to have $q_l^p \geq \delta$, condition $\delta \leq \delta_2^p \equiv \frac{1-c_l-r}{2}$ should be imposed. Accordingly, the plausible range for δ is $\left(\delta_1^p, \delta_2^p\right]$, and $p_h^p = 1-\delta \geq p_l^p = \frac{(1+c_l+r)}{2} > 0$ if $\delta \leq \delta_2^p$. Substituting p_l^p and p_h^p into (16) gives $\pi_l^p = \left(q_l^p\right)^2 - f$ and $\pi_h^p = \delta\left[1-\delta-c_h-r\right] - f$. These prove Lemma 3(ii).

Case 3: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then (A3)-(A5) suggest $(1-p_l-\delta) = 0$, $(1-2p_l + c_l + r - \lambda_1) = 0$ and $(1-2p_h + c_h + r) = 0$. Solving these equations yields $p_l^p = 1-\delta$, $p_h^p = \frac{(1+c_h+r)}{2}$ and $\lambda_1^* = 2\left[\delta - \frac{(1-c_l-r)}{2}\right]$. Substituting p_l^p and p_h^p into (15) produces $q_l^p = \delta$ and $q_h^p = \frac{(1-c_h-r)}{2}$. To guarantee $\lambda_1^* > 0$, condition $\delta > \frac{(1-c_l-r)}{2}$ is needed; and $q_h^p \geq \delta$ is guaranteed if $\delta \leq \frac{(1-c_h-r)}{2}$. However, the two conditions are incompatible with each other because $\frac{(1-c_h-r)}{2} - \frac{(1-c_l-r)}{2} = \frac{-(c_h-c_l)}{2} < 0$. Thus, no solution exists in this case.

Case 4: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. Then (A3)-(A6) suggest $p_l^p = p_h^p = 1-\delta$, $\lambda_1^* = 2\delta - \left(1-c_l-r\right)$ and $\lambda_2^* = 2\delta - \left(1-c_h-r\right)$. To guarantee $p_l^p = p_h^p > 0$, condition $\delta < 1$ should be met. To have $\lambda_1^* > 0$ and $\lambda_2^* > 0$, conditions $\delta > \delta_2^p \equiv \frac{1-c_l-r}{2}$ and $r < (1-c_l)$ are needed.

Note that we have $r < (1 - c_l)$ because of $r \le \overline{r}_h \equiv 1 - c_h$ and $c_l < c_h$. Substituting p_l^p and p_h^p into (15) produces $q_l^p = q_h^p = \delta$, and into (16) gives $\pi_i^p = \delta[1 - \delta - c_i - r] - f$ for $i = l, h$. These prove Lemma 3(iii). □

Under the new set-up, problem (9) becomes

$$\max_{r,f,\delta} \; R = \theta\left(rq_l^p + f\right) + (1-\theta)\left(rq_h^p + f\right) \tag{9B}$$

$$\text{s.t.} \;\; 0 \le \delta < 1, \;\; r \ge 0, \;\; f \ge 0, \; \pi_l^p \ge 0 \;\; \text{and} \; \pi_h^p \ge 0.$$

Its solutions are given below.

Proposition 1. *Suppose the conditions in (6) hold. Then we have the following.*

(i) *If* $c_h \in \left(c_l, \dot{c}_h\right]$ *with* $\dot{c}_h = \dfrac{1+c_l}{2}$, *then the two-part tariff contract is the port authority's best choice. The optimal scheme and minimum throughput guarantee are* $r^p = \dfrac{\left(c_h - c_l\right)}{2-\theta}$, $f^p = \dfrac{\left(1 + c_l - 2c_h\right)\left[(2-\theta) + \theta c_l - 2c_h\right]}{4(2-\theta)}$, *and* $\delta^p = \dfrac{(2-\theta) + \theta c_l - 2c_h}{2(2-\theta)}$. *At the equilibrium, the port authority's fee revenue equals* $R^p = \dfrac{(2-\theta) - 2(2-\theta)c_h - 2\theta c_l c_h + \theta c_l^2 + 2c_h^2}{4(2-\theta)}$.

(ii) *If* $c_h \in \left(\dot{c}_h, \hat{c}_h\right)$ *with* $\hat{c}_h \equiv \dfrac{(2-\theta) + \theta c_l}{2}$, *then the unit-fee scheme is the port authority's best choice. The optimal scheme and minimum throughput guarantee are* $r^p = \dfrac{(2-\theta) - \theta c_l - 2(1-\theta)c_h}{2(2-\theta)}$ *and* $\delta^p = \dfrac{(2-\theta) + \theta c_l - 2c_h}{2(2-\theta)}$. *At the equilibrium, the port authority's fee revenue equals* $R^p = \dfrac{\left[(2-\theta) - \theta c_l - 2(1-\theta)c_h\right]^2}{8(2-\theta)}$.

(iii) *If* $c_h \in \left[\hat{c}_h, 1\right)$, *then the unit-fee scheme is the port authority's best choice. The optimal scheme and minimum throughput guarantee are* $r^p = \overline{r}_h \equiv (1 - c_h)$ *and* $\delta^p = 0$. *At the equilibrium, the port authority's fee revenue equals* $R^p = \dfrac{\theta\left(1-c_h\right)\left(c_h - c_l\right)}{2}$.

<u>Proof of Proposition 1</u>: As shown in (9B), the port authority's maximization problem is

$$\max_{r,f,\delta} \; R = \theta\left(rq_l^p + f\right) + (1-\theta)\left(rq_h^p + f\right)$$

$$\text{s.t.} \;\; 0 \le \delta < 1, \;\; r \ge 0, \;\; f \ge 0, \; \pi_l^p \ge 0 \;\; \text{and} \; \pi_h^p \ge 0.$$

Obviously, the operator's optimal cargo-handling amounts and equilibrium profits $(q_l^p, q_h^p, \pi_l^p, \pi_h^p)$ used in the above objective function and constraints are equivalent to those in the quantity competition. This implies that the optimal concession contracts under price competition are the same as those under quantity competition. □

Under the new set-up, problem (10) becomes

$$\max_{p_i \geq 0} \quad \pi_i = \left(p_i - c_i - r_l\right)\left(1 - p_i\right) - f_l \tag{10B}$$

$$\text{s.t.} \quad q_i \geq \delta_l$$

for $i = l, h$. Its solutions are exhibited below.

Lemma 4. *Given contract* $\left(r_l, f_l, \delta_l\right)$, *operator i's optimal behaviors are as follows.*

(i) *For* $\delta_l \in [0, \delta_l']$ *with* $\delta_l' = \frac{(1-c_h-r_l)}{2}$, *both-type operators' equilibrium service prices are* $p_l^{sl} = \frac{\left(1+c_l+r_l\right)}{2} > 0$ *and* $p_h^{sl} = \frac{\left(1+c_h+r_l\right)}{2} > 0$, *their equilibrium cargo-handling amounts are* $q_l^{sl} = \frac{\left(1-c_l-r_l\right)}{2}$ *and* $q_h^{sl} = \frac{\left(1-c_h-r_l\right)}{2} = \delta_l'$, *and their equilibrium profits are* $\pi_i^{sl} = (q_i^{sl})^2 - f_l$ *for* $i = l, h$.

(ii) *For* $\delta_l \in \left(\delta_l', \delta_l''\right]$ *with* $\delta_l'' = \frac{(1-c_l-r_l)}{2} > \delta_l'$, *both-type operators' equilibrium service prices are* $p_l^{sl} = \frac{1+c_l+r_l}{2} > 0$ *and* $p_h^{sl} = (1-\delta_l) > 0$, *their equilibrium cargo-handling amounts are* $q_l^{sl} = \frac{1-c_l-r_l}{2} > \delta_l$ *and* $q_h^{sl} = \delta_l$, *and their equilibrium profits are* $\pi_l^{sl} = \left(q_l^{sl}\right)^2 - f_l$ *and* $\pi_h^{sl} = \delta_l\left[1-\delta_l-c_h-r_l\right] - f_l$.

(iii) *For* $\delta_l \in \left(\delta_l'', 1\right)$, *both-type operators' equilibrium service prices are* $p_l^{sl} = p_h^{sl} = (1-\delta_l) > 0$, *their equilibrium cargo-handling amounts are* $q_l^{sl} = \delta_l$ *and* $q_h^{sl} = \delta_l$, *and their equilibrium profits are* $\pi_i^{sl} = \delta_l\left[1-\delta_l-c_i-r_l\right] - f_l$ *for* $i = l, h$.

Proof of Lemma 4: Denote L_1 and L_2 the respective Lagrange functions for the *l*-type and the *h*-type terminal operators in problem (10B),

$$L_1 = \left(p_l - c_l - r_l\right)\left(1 - p_l\right) - f_l + \lambda_1\left(1 - p_l - \delta_l\right) \quad \text{and} \quad L_2 = \left(p_h - c_h - r_l\right)\left(1 - p_h\right) - f_l + \lambda_2\left(1 - p_h - \delta_l\right),$$

where λ_1 and λ_2 are their associated Lagrange multipliers. Then, the corresponding Kuhn-Tucker conditions for the *l*-type operator are

$$\frac{\partial L_1}{\partial p_l} = 1 - 2p_l + c_l + r_l - \lambda_1 \leq 0, \; p_l \cdot \frac{\partial L_1}{\partial p_l} = 0 \quad \text{and} \tag{A7}$$

$$\frac{\partial L_1}{\partial \lambda_1} = 1 - p_l - \delta_l \geq 0,\ \lambda_1 \cdot \frac{\partial L_1}{\partial \lambda_1} = 0\,, \qquad \text{(A8)}$$

and for the *h*-type operator are

$$\frac{\partial L_2}{\partial p_h} = 1 - 2p_h + c_h + r_l - \lambda_2 \leq 0,\ p_h \cdot \frac{\partial L_2}{\partial p_h} = 0 \ \text{ and} \qquad \text{(A9)}$$

$$\frac{\partial L_2}{\partial \lambda_2} = 1 - p_h - \delta_l \geq 0,\ \lambda_2 \cdot \frac{\partial L_2}{\partial \lambda_2} = 0. \qquad \text{(A10)}$$

Based on the values of λ_1 and λ_2, there are four cases below.

<u>Case 1</u>: Suppose $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Then (A7) and (A9) become $(1 - 2p_l + c_l + r_l) = 0$ and $(1 - 2p_h + c_h + r_l) = 0$, respectively. Solving the two equations yields $p_l^{sl} = \frac{(1 + c_l + r_l)}{2} > 0$ and $p_h^{sl} = \frac{(1 + c_h + r_l)}{2} > 0$. Substituting p_l^{sl} and p_h^{sl} into (15) yields $q_l^{sl} = \frac{(1 - c_l - r_l)}{2}$ and $q_h^{sl} = \frac{(1 - c_h - r_l)}{2}$. To guarantee $q_l^{sl} \geq \delta$ and $q_h^{sl} \geq \delta$, condition $0 \leq \delta_l \leq \delta_l' \equiv \frac{1 - c_h - r_l}{2} = q_h^{sl}$ should be imposed, because $c_l < c_h$ suggests $q_l^{sl} > q_h^{sl}$ and $q_h^{sl} \geq \delta_l$ suggests $q_l^{sl} \geq \delta_l$. Substituting p_l^{sl} and p_h^{sl} into (16) yields $\pi_i^{sl} = \left(q_i^{sl}\right)^2 - f_l$ for $i = l, h$. These prove Lemma 4(i).

<u>Case 2</u>: Suppose $\lambda_1^* = 0$ and $\lambda_2^* > 0$. Then (A7), (A9) and (A10) imply $(1 - 2p_l + c_l + r_l) = 0$, $(1 - 2p_h + c_h + r_l - \lambda_2) = 0$ and $(1 - p_h - \delta_l) = 0$. Solving these equations yields $p_l^{sl} = \frac{1 + c_l + r_l}{2}$, $p_h^{sl} = (1 - \delta_l)$ and $\lambda_2^* = 2\left[\delta_l - \frac{(1 - c_h - r_l)}{2}\right]$. Substituting p_l^{sl} and p_h^{sl} into (15) produces $q_l^{sl} = \frac{1 - c_l - r_l}{2}$ and $q_h^{sl} = \delta_l$. To guarantee $\lambda_2^* > 0$, condition $\delta > \frac{(1 - c_h - r)}{2} = \delta_l'$ is needed. On the other hand, to have $q_l^{sl} \geq \delta_l$, condition $\delta_l \leq \delta_l'' \equiv \frac{1 - c_l - r_l}{2}$ should be imposed. Accordingly, the plausible range for δ_l is $\delta_l \in (\delta_l', \delta_l'']$, $p_h^{sl} = (1 - \delta_l) \geq p_l^{sl} = \frac{1 + c_l + r_l}{2} > 0$ if $\delta_l \leq \delta_l''$. Substituting p_l^{sl} and p_h^{sl} into (16) gives $\pi_l^{sl} = \left(q_l^{sl}\right)^2 - f_l$ and $\pi_h^{sl} = \delta_l\left[1 - \delta_l - c_h - r_l\right] - f_l$. These prove Lemma 4(ii).

<u>Case 3</u>: Suppose $\lambda_1^* > 0$ and $\lambda_2^* = 0$. Then (A7)-(A9) suggest $(1 - p_l - \delta_l) = 0$, $(1 - 2p_l + c_l + r_l - \lambda_1) = 0$ and $(1 - 2p_h + c_h + r_l) = 0$. Solving these equations yields $p_l^{sl} = 1 - \delta_l$, $p_h^{sl} = \frac{(1 + c_h + r_l)}{2}$ and $\lambda_1^* = 2\left[\delta_l - \frac{(1 - c_l - r_l)}{2}\right]$. Substituting p_l^{sl} and p_h^{sl} into (15) produces

$q_l^{sl} = \delta_l$ and $q_h^{sl} = \frac{1-r_l-c_h}{2}$. To guarantee $\lambda_1^* > 0$, condition $\delta_l > \frac{1-c_l-r_l}{2}$ is needed. On the other hand, $q_h^{sl} \geq \delta_l$ holds if $\delta_l \leq \frac{1-r_l-c_h}{2}$. However, these two cannot hold simultaneously because $\frac{1-r_l-c_h}{2} - \frac{1-c_l-r_l}{2} = \frac{-(c_h-c_l)}{2} < 0$. Thus, no solution exists in this case.

<u>Case 4</u>: Suppose $\lambda_1^* > 0$ and $\lambda_2^* > 0$. Then (A7)-(A10) suggest $p_l^{sl} = p_h^{sl} = 1-\delta_l$, $\lambda_1^* = 2\delta_l - (1-c_l-r_l)$ and $\lambda_2^* = 2\delta_l - (1-c_h-r_l)$. To guarantee $p_l^{sl} = p_h^{sl} > 0$, condition $\delta_l < 1$ should be met. To have $\lambda_1^* > 0$ and $\lambda_2^* > 0$, conditions $\delta_l > \delta_l''$ and $r < (1-c_l)$ are needed. Substituting p_l^{sl} and p_h^{sl} into (15) produces $q_l^{sl} = q_h^{sl} = \delta_l$, and into (16) gives $\pi_i^{sl} = \delta_l[1-\delta_l-c_i-r_l] - f_l$ for $i = l, h$. These prove Lemma 4(iii). □

Under the new set-up, problem (11) becomes

$$\max_{p_i \geq 0} \quad \pi_i = (p_i - c_i - r_h)(1-p_i) - f_h \tag{11B}$$

$$\text{s.t.} \quad q_i \geq \delta_h$$

for $i = l, h$. Its solutions are presented below.

Lemma 5. *Given contract* (r_h, f_h, δ_h), *operator i's optimal behaviors are as follows.*

(i) *For* $\delta_h \in [0, \delta_h']$ *with* $\delta_h' = \frac{(1-c_h-r_h)}{2}$, *both-type operators' equilibrium service prices are* $p_l^{sh} = \frac{(1+c_l+r_h)}{2} > 0$ *and* $p_h^{sh} = \frac{(1+c_h+r_h)}{2} > 0$, *their equilibrium cargo-handling amounts are* $q_l^{sh} = \frac{(1-c_l-r_h)}{2} > \delta_h'$ *and* $q_h^{sh} = \frac{(1-c_h-r_h)}{2} = \delta_h'$, *and their equilibrium profits are* $\pi_i^{sh} = (q_i^{sh})^2 - f_h$ *for* $i = l, h$.

(ii) *For* $\delta_h \in (\delta_h', \delta_h'']$ *with* $\delta_h'' = \frac{(1-c_l-r_h)}{2} > \delta_h'$, *both-type operators' equilibrium service prices are* $p_l^{sh} = \frac{1+c_l+r_h}{2} > 0$ *and* $p_h^{sh} = 1-\delta_h > 0$, *their equilibrium cargo-handling amounts are* $q_l^{sh} = \frac{(1-c_l-r_h)}{2} > \delta_h$ *and* $q_h^{sh} = \delta_h$, *and their equilibrium profits are* $\pi_l^{sh} = (q_l^{sh})^2 - f_h$ *and* $\pi_h^{sh} = \delta_h[1-\delta_h-c_h-r_h] - f_h$.

(iii) *For* $\delta_h \in (\delta_h'', 1)$, *both-type operators' equilibrium service prices are* $p_l^{sh} = p_h^{sh} = (1-\delta_h) > 0$, *their equilibrium cargo-handling amounts are* $q_l^{sh} = \delta_h$ *and* $q_h^{sh} = \delta_h$, *and their equilibrium profits are* $\pi_i^{sh} = \delta_h[1-\delta_h-c_i-r_h] - f_h$ *for* $i = l, h$.

<u>**Proof of Lemma 5:**</u> Since the proofs for Lemma 5 and Lemma 4 are similar, it is omitted. □

Finally, under the new set-up, problem (12) becomes

$$\max_{(r_l, f_l, \delta_l), (r_h, f_h, \delta_h)} R = \theta\left(r_l q_l^{sl} + f_l\right) + (1-\theta)\left(r_h q_h^{sh} + f_h\right) \tag{12B}$$

s.t. $0 \le \delta_l < 1, 0 \le \delta_h < 1$, $f_l \ge 0$, $f_h \ge 0$, $\pi_l^{sl} \ge 0$, $\pi_h^{sh} \ge 0$, $\pi_l^{sl} \ge \pi_l^{sh}$ and $\pi_h^{sh} \ge \pi_h^{sl}$.

Its solutions are listed below.

Proposition 2. *Suppose the conditions in (6) hold. Then we have the following.*

(i) *If* $c_h \in \left(c_l, c_h'\right]$ *with* $c_h' = \dfrac{(1-\theta)+(2-\theta)c_l}{(3-2\theta)}$*, the port authority will offer the two-part tariff scheme and minimum throughput requirement* $\left(r_h^s, f_h^s, \delta_h^s\right)^T$*, or the unit-fee scheme and minimum throughput requirement* $\left(r_h^s, \delta_h^s\right)^U$ *to the h-type operator; and offer the two-part tariff scheme and minimum throughput requirement* $\left(r_l^s, f_l^s, \delta_l^s\right)^T$*,the unit-fee scheme and minimum throughput requirement* $\left(r_l^s, \delta_l^s\right)^U$*, or the fixed-fee scheme and minimum throughput requirement* $\left(f_l^s, \delta_l^s\right)^F$ *to the l-type operator. Here*

$$\left(r_h^s, f_h^s, \delta_h^s\right)^T = \left\{\begin{array}{l} r_h^s \in \left[\dfrac{c_h - c_l}{(1-\theta)}, \dfrac{(1-\theta)-\theta c_l-(1-2\theta)c_h}{2(1-\theta)}\right), \\ f_h^s = \dfrac{\left[(1-\theta)+\theta c_l - c_h\right]\cdot\left[(1-\theta)-\theta c_l-(1-2\theta)c_h - 2(1-\theta)r_h^s\right]}{4(1-\theta)^2}, \\ \delta_h^s = \dfrac{(1-\theta)+\theta c_l - c_h}{2(1-\theta)} \end{array}\right\},$$

$$\left(r_h^s, \delta_h^s\right)^U = \left\{r_h^s = \frac{(1-\theta)-\theta c_l-(1-2\theta)c_h}{2(1-\theta)},\ \delta_h^s = \frac{(1-\theta)+\theta c_l - c_h}{2(1-\theta)}\right\},$$

$$\left(r_l^s, f_l^s, \delta_l^s\right)^T = \left\{\begin{array}{l} r_l^s \in \left(0, \dfrac{(1-\theta)-2(1-\theta)c_h-2(1+\theta)c_l c_h+(1+\theta)c_l^2+2c_h^2}{2(1-\theta)(1-c_l)}\right), \\ f_l^s = \dfrac{(1-\theta)-2(1-\theta)c_h-2(1+\theta)c_l c_h+(1+\theta)c_l^2+2c_h^2-2(1-\theta)(1-c_l)r_l^s}{4(1-\theta)}, \\ \delta_l^s = \dfrac{1-c_l}{2} \end{array}\right\},$$

$$\left(r_l^s, \delta_l^s\right)^U = \left\{r_l^s = \frac{(1-\theta)-2(1-\theta)c_h-2(1+\theta)c_l c_h+(1+\theta)c_l^2+2c_h^2}{2(1-\theta)(1-c_l)}, \delta_l^s = \frac{1-c_l}{2}\right\}, \text{ and}$$

$$\left(f_l^s, \delta_l^s\right)^F = \left\{f_l^s = \frac{(1-\theta)-2(1-\theta)c_h-2(1+\theta)c_l c_h+(1+\theta)c_l^2+2c_h^2}{4(1-\theta)}, \delta_l^s \in \left[0, \frac{1-c_l}{2}\right]\right\}.$$ *At the*

equilibrium, the port authority's fee revenue always equals $R^s = \dfrac{(1-\theta)-2(1-\theta)c_h - 2\theta c_l c_h + \theta c_l^2 + c_h^2}{4(1-\theta)}$.

(ii) *If* $c_h \in (c_h', c_h'')$ *with* $c_h'' \equiv \dfrac{2(1-\theta)+\theta c_l}{2-\theta}$*, then the optimal contract for the h-type operator is the unit-fee scheme* $r_h^s = \dfrac{(2-\theta)-\theta c_l - 2(1-\theta)c_h}{(4-3\theta)}$ *with minimum throughput guarantee* $\delta_h^s = \dfrac{2(1-\theta)+\theta c_l-(2-\theta)c_h}{(4-3\theta)}$*. By contrast, the optimal contract for the l-type operator can be the two-part tariff scheme and minimum throughput requirement* $\left(r_l^s, f_l^s, \delta_l^s\right)^T$*, the unit-fee scheme and minimum throughput requirement* $\left(r_l^s, \delta_l^s\right)^U$*, or the fixed-fee scheme and minimum throughput requirement* $\left(f_l^s, \delta_l^s\right)^F$*. Here* $\left(r_l^s, f_l^s, \delta_l^s\right)^T = \left\{ r_l^s \in \left(0, \dfrac{2B}{(1-c_l)}\right), f_l^s = B - \dfrac{1}{2}(1-c_l) r_l^s,\ \delta_l^s = \dfrac{1-c_l}{2} \right\}$,
$\left(r_l^s, \delta_l^s\right)^U = \left\{ r_l^s = \dfrac{2B}{(1-c_l)},\ \delta_l^s = \dfrac{1-c_l}{2} \right\}$, $\left(f_l^s, \delta_l^s\right)^F = \left\{ f_l^s = B,\ \delta_l^s \in \left[0, \dfrac{1-c_l}{2}\right] \right\}$*, and*
$B = \dfrac{\left[(6-5\theta)-(8-7\theta)c_l + 2(1-\theta)c_h\right]\left[(2-\theta)-\theta c_l - 2(1-\theta)c_h\right]}{4(4-3\theta)^2}$*. At the equilibrium, the port authority's fee revenue always equals* $R^s = \dfrac{\left[(2-\theta)-\theta c_l - 2(1-\theta)c_h\right]^2}{4(4-3\theta)}$.

(iii) *If* $c_h \in [c_h'', 1)$*, then the optimal contract for the h-type operator is the unit-fee scheme* $r_h^s = \overline{r}_h = (1-c_h)$ *with minimum throughput guarantee* $\delta_h^s = 0$*. By contrast, the optimal contract for the l-type operator can be the two-part tariff scheme and minimum throughput requirement* $\left(r_l^s, f_l^s, \delta_l^s\right)^T$*, the unit-fee scheme and minimum throughput requirement* $\left(r_l^s, \delta_l^s\right)^U$*, or the fixed-fee scheme and minimum throughput requirement* $\left(f_l^s, \delta_l^s\right)^F$*. Here*
$\left(r_l^s, f_l^s, \delta_l^s\right)^T = \left\{ r_l^s \in \left(0, \dfrac{(1-c_h)(1-2c_l+c_h)}{2(1-c_l)}\right), f_l^s = \dfrac{(1-c_h)(1-2c_l+c_h)-2(1-c_l)r_l^s}{4},\ \delta_l^s = \dfrac{1-c_l}{2} \right\}$,
$\left(r_l^s, \delta_l^s\right)^U = \left\{ r_l^s = \dfrac{(1-c_h)(1-2c_l+c_h)}{2(1-c_l)},\ \delta_l^s = \dfrac{1-c_l}{2} \right\}$*, and*
$\left(f_l^s, \delta_l^s\right)^F = \left\{ f_l^s = \dfrac{(1-c_h)(1-2c_l+c_h)}{4}, \delta_l^s \in \left[0, \dfrac{1-c_l}{2}\right] \right\}$*. At the equilibrium, the port authority's fee revenue always equals* $R^s = \dfrac{\theta(1-c_h)(1-2c_l+c_h)}{4}$.

<u>Proof of Proposition 2</u>: As shown in (12B), the port authority's maximization problem is

$$\max_{(r_l, f_l, \delta_l), (r_h, f_h, \delta_h)} R = \theta\left(r_l q_l^{sl} + f_l\right) + \left(1-\theta\right)\left(r_h q_h^{sh} + f_h\right)$$

s.t. $0 \le \delta_l < 1, 0 \le \delta_h < 1$, $f_l \ge 0$, $f_h \ge 0, \pi_l^{sl} \ge 0$, $\pi_h^{sh} \ge 0$, $\pi_l^{sl} \ge \pi_l^{sh}$ and $\pi_h^{sh} \ge \pi_h^{sl}$.

Obviously, the operator's optimal cargo-handling amounts and equilibrium profits $\{(q_i^{sl}, \pi_i^{sl}), (q_i^{sh}, \pi_i^{sh})\}_{i=l,h}$ used in the above objective function and constraints are equivalent to those in the quantity competition. This implies that the optimal concession contracts under price competition are the same as those obtained in Proposition 2 for quantity competition. □

In summary, all the Lemmas and Propositions show that the port authority's optimal concession contracts are exactly the same whether terminal operators compete in service prices or in cargo-handling amounts. Accordingly, the outcomes derived in Section 5 still hold even when the operator chooses service price to maximize its profit. This proves Proposition 8.