Finance Theory

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MBA, IM, NTPU (M6132) (Fall 2022)
Tue 2, 3, 4 (9:10-12:00) (B8F40)

https://meet.google.com/paj-zhhj-myap
## Syllabus

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# Syllabus

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Financial Theories
Financial Theories

• Uncertainty and Risk
• Expected Utility Theory (EUT)
• Mean-Variance Portfolio Theory (MVPT)
• Capital Asset Pricing Model (CAPM)
• Arbitrage Pricing Theory (APT)

Major Normative Financial Theories and Models

• Normative Theory
  • Based on assumptions (mathematically, axioms) and derives insights, results, and more from the set of relevant assumptions.

• Positive theory
  • Based on observation, experiments, data, relationships, and the like and describes phenomena given the insights gained from the available information and the derived results.

Normative Finance

• The **CAPM** is based on many unrealistic assumptions.
  • The assumption that investors care only about the mean and variance of one-period portfolio returns is extreme.
  (Eugene Fama and Kenneth French, 2004)
Uncertainty and Risk

• Financial theory deals with investment, trading, and valuation in the presence of uncertainty and risk.

• The focus is on fundamental concepts from probability theory that build the backbone of quantitative finance.

Risk and Return

\[
\text{Sharpe Ratio} = \frac{\text{Portfolio Return} - \text{Risk Free Return}}{\text{Portfolio Risk}}
\]

Sharpe Ratio

\[ SR = \frac{r_P - r_F}{\sigma_P} \]

Where

- \( r_P \) = portfolio return
- \( r_F \) = risk free rate
- \( \sigma_P \) = portfolio risk
  
  (variability, standard deviation of return)

**Sortino Ratio**

\[
\text{Sortino Ratio} = \frac{r_P - r_T}{\sigma_D}
\]

Where

- \(r_P\) = portfolio return
- \(r_T\) = Minimum Target Return
- \(\sigma_D\) = Downside Risk

**Downside Risk \(\sigma_D\)**

\[
\sigma_D = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \min[(r_i - r_T), 0]^2}
\]

Max Drawdown

Traded Assets

• In the economy, **two assets** are traded.
  • The first is a **risky asset**, the **stock**, with a certain price today of $S_0 = 10$ and an uncertain payoff tomorrow.
  • The second is a **risk-less asset**, the **bond**, with a certain price today of $B_0 = 10$ and a certain payoff tomorrow.

Arbitrage Pricing

- Deriving the fair value of a European call option on the stock with a strike price of $K = 14.5$

- *Arbitrage pricing theory* can be considered one of the strongest financial theories with some of the most robust mathematical results, such as the *fundamental theorem of asset pricing (FTAP)*.
Expected Utility Theory (EUT)

- Expected utility theory (EUT)
  - 1940s
  - Cornerstone of financial theory
  - One of the central paradigms for modeling decision making under uncertainty

Expected Utility Theory

• Expected utility theory (EUT)
  • EUT is an axiomatic theory
    • von Neumann and Morgenstern (1944)

• Axiomatic
  • Major results of the theory can be deduced from a small number of axioms only

• Axioms and normative theory
  • An axiom is a proposition regarded as self-evidently true without proof.

Preferences of an Agent

• Assume an agent with preferences $\succeq$ is faced with the problem of investing in the two traded assets of the model economy $M^2$.

• One possible set of axioms leading to EUT
  • Completeness
  • Transitivity
  • Continuity
  • Independence
  • Dominance

Utility functions

• A utility function is a way to represent the preferences of an agent in a mathematical and numerical way in that such a function assigns a numerical value to a certain payoff.

Expected Utility Functions

• Von Neumann and Morgenstern (1944) show that if the preferences of an agent $\preceq$ satisfy the preceding five axioms, then there exists an expected utility function.
Risk aversion

• In finance, the concept of risk aversion is important.

• The most commonly used measure of risk aversion is the Arrow-Pratt measure of absolute risk aversion (ARA) (Pratt, 1964).
  • $\text{ARA}(x) > 0$, risk-averse
  • $\text{ARA}(x) = 0$, risk-neutral
  • $\text{ARA}(x) < 0$, risk-loving

Mean-Variance Portfolio Theory (MVPT)

- Mean-variance portfolio (MVP) theory
  - Markowitz (1952)
  - cornerstone in financial theory
- One of the first theories of investment under uncertainty that focused on statistical measures only for the construction of stock investment portfolios.
- MVP completely abstracts from fundamentals of a company that might drive its stock performance or assumptions about the future competitiveness of a company that might be important for the growth prospects of a company.

Mean-Variance Portfolio Theory (MVPT)

• The only input data that counts is the time series of share prices and statistics derived therefrom, such as the (historical) annualized mean return and the (historical) annualized variance of the returns.

• The central assumption of MVP, according to Markowitz (1952), is that investors only care about expected returns and the variance of these returns.

Mean-Variance Portfolio Theory (MVPT)

- Portfolio statistics
  - returns vector
  - expected return
  - vector of expected returns
  - expected return of the portfolio
  - covariance matrix
  - expected variance of the portfolio
  - expected volatility of the portfolio

Sharpe ratio

• Sharpe (1966) introduces a measure to judge the risk-adjusted performance of mutual funds and other portfolios, or even single risky assets.

• It relates the (expected, realized) return of a portfolio to its (expected, realized) volatility.

  • Sharpe ratio \[ \pi = \frac{\mu}{\sigma} \]

• If \( r \) represents the risk-less short rate, the risk premium or excess return of a portfolio \( \phi \) over a risk-free alternative is defined by \( \mu_{\phi} - r \)

  • Sharpe ratio \[ \pi = \frac{\mu - r}{\sigma} \]

Investment Opportunity Set

Simulated expected portfolio volatility and return (one risky asset)

Investment Opportunity Set
Simulated expected portfolio volatility and return (two risky assets)

Minimum volatility and maximum Sharpe ratio portfolios

• An efficient portfolio
  • has a maximum expected return (risk) given its expected risk (return)

• All those portfolios that have a lower expected return than the minimum risk portfolio are inefficient.

• Efficient frontier
  • The set of all efficient portfolios
  • Agents will only choose a portfolio that lies on the efficient frontier
Portfolio Optimization
Efficient Frontier

Efficient Frontier

Source: Tucker Balch (2012), Investment Science: Portfolio Optimization,
https://www.youtube.com/watch?v=5qbMhXXg0vl
Portfolio Optimization

Efficient Frontier

Portfolio Optimization with Individual Stocks

https://tinyurl.com/aintpupython101
Efficient Frontier

Portfolio Optimization and Algorithmic Trading

https://colab.research.google.com/drive/1FEG6DnGvufUbeo4zJ1zTunjMqf2RkCrT

https://tinyurl.com/aintpupython101
Capital Asset Pricing Model (CAPM)

- Capital Asset Pricing Model (CAPM)
  - One of the most widely documented and applied models in finance
  - It relates in linear fashion the expected return for a single stock to the expected return of the market portfolio, usually approximated by a broad stock index such as the S&P 500.
  - Sharpe (1964) and Lintner (1965)

ONE OF THE PROBLEMS which has plagued those attempting to predict the behavior of capital markets is the absence of a body of positive microeconomic theory dealing with conditions of risk. Although many useful insights can be obtained from the traditional models of investment under conditions of certainty, the pervasive influence of risk in financial transactions has forced those working in this area to adopt models of price behavior which are little more than assertions. A typical classroom explanation of the determination of ...
CAPITAL ASSET PRICES: A THEORY OF MARKET EQUILIBRIUM UNDER CONDITIONS OF RISK*

WILLIAM F. SHARPE†

I. Introduction

One of the problems which has plagued those attempting to predict the behavior of capital markets is the absence of a body of positive microeconomic theory dealing with conditions of risk. Although many useful insights can be obtained from the traditional models of investment under conditions of certainty, the pervasive influence of risk in financial transactions has forced those working in this area to adopt models of price behavior which are little more than assertions. A typical classroom explanation of the determination of capital asset prices, for example, usually begins with a careful and relatively rigorous description of the process through which individual preferences and physical relationships interact to determine an equilibrium pure interest rate. This is generally followed by the assertion that somehow a market risk-premium is also determined, with the prices of assets adjusting accordingly to account for differences in their risk.
Capital Asset Pricing Model (CAPM)

Capital Asset Pricing Model (CAPM)

*Journal of Economic Perspectives—Volume 18, Number 3—Summer 2004—Pages 25–46*

The Capital Asset Pricing Model: Theory and Evidence

Eugene F. Fama and Kenneth R. French

The capital asset pricing model (CAPM) of William Sharpe (1964) and John Lintner (1965) marks the birth of asset pricing theory (resulting in a Nobel Prize for Sharpe in 1990). Four decades later, the CAPM is still widely used in applications, such as estimating the cost of capital for firms and evaluating the performance of managed portfolios. It is the centerpiece of MBA investment courses. Indeed, it is often the only asset pricing model taught in these courses.¹

Investment Opportunities

Capital Asset Pricing Model (CAPM)

\[ E(R_i) = R_f + \beta_{iM} [E(R_M) - R_f] \]

The expected return on any asset \( i \) is the risk-free interest rate, \( R_f \), plus a risk premium, which is the asset’s market beta, \( \beta_{iM} \), times the premium per unit of beta risk, \( E(R_M) - R_f \).

Capital Asset Pricing Model (CAPM)

Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928–2003

Capital Asset Pricing Model (CAPM)

- Capital market theory is a positive theory in that it hypothesizes how investors do behave rather than how investors should behave, as in the case of modern portfolio theory (MVP).

- It is reasonable to view capital market theory as an extension of portfolio theory, but it is important to understand that MVP is not based on the validity, or lack thereof, of capital market theory.

Capital Asset Pricing Model (CAPM)

• The specific equilibrium model of interest to many investors is known as the capital asset pricing model, typically referred to as the CAPM.

• It allows us to assess the relevant risk of an individual security as well as to assess the relationship between risk and the returns expected from investing.

• The CAPM is attractive as an equilibrium model because of its simplicity and its implications.

Capital Asset Pricing Model
(CAPM)

• In the CAPM, agents are assumed to invest according to MVP, caring only about the **risk** and **return** statistics of risky assets over one period.

• In a **capital market equilibrium**, all available assets are held by all agents and the markets clear.

• **Market portfolio (set of tradable assets)** must lie on the **efficient frontier**.

• **Two fund separation theorem**
  • Every agent will hold a combination of the market portfolio and the risk-free asset in equilibrium.
  • The set of all such portfolios is called the **Capital Market Line (CML)**.

Capital Market Line (CML)

Capital Market Line with Two Risky Assets

Indifference curves in risk-return space

Optimal Portfolio on the Capital Market Line (CML)

Arbitrage Pricing Theory (APT)

• Arbitrage Pricing Theory (APT)
  • One of the major generalizations of the Capital Asset Pricing Model (CAPM)
  • Ross (1971) and Ross (1976)
  • The purpose of this paper is to examine rigorously the arbitrage model of capital asset pricing developed in Ross (1971).
    • The arbitrage model was proposed as an alternative to the mean variance capital asset pricing model, introduced by Sharpe, Lintner, and Treynor, that has become the major analytic tool for explaining phenomena observed in capital markets for risky assets.

The Arbitrage Theory of Capital Asset Pricing

STEPHEN A. ROSS

Departments of Economics and Finance, University of Pennsylvania,
The Wharton School, Philadelphia, Pennsylvania 19174

Received March 19, 1973; revised May 19, 1976

The purpose of this paper is to examine rigorously the arbitrage model of capital asset pricing developed in Ross [13, 14]. The arbitrage model was proposed as an alternative to the mean variance capital asset pricing model, introduced by Sharpe, Lintner, and Treynor, that has become the major analytic tool for explaining phenomena observed in capital markets for risky assets. The principal relation that emerges from the mean variance model holds that for any asset, \( i \), its (ex ante) expected return

\[
E_i = \rho + \lambda b_i ,
\]

(1)
Arbitrage Pricing Theory (APT)

- The APT is a generalization of the CAPM to multiple risk factors.
- APT does not assume that the market portfolio is the only relevant risk factor.
  - There are rather multiple types of risk that together are assumed to drive the performance (expected returns) of a stock.
  - Such risk factors might include size, volatility, value, and momentum.

Capital Asset Pricing Model (CAPM)
Arbitrage Pricing Theory (APT)

• Capital Asset Pricing Model (CAPM)
  • univariate ordinary least-squares (OLS) regression

• Arbitrage Pricing Theory (APT)
  • multivariate ordinary least-squares (OLS) regression

The Quant Finance PyData Stack

Quantopian

PyThalesians

Zipline

DX Analytics

PyAlgoTrade

QuantLib

SM

StatsModels

Statistics in Python

Matplotlib

Pandas

SciPy

NumPy

SymPy

IPython

Python

Jupyter

Source: http://nbviewer.jupyter.org/format/slides/github/quantopian/pyfolio/blob/master/pyfolio/examples/overview_slides.ipynb#5
Yves Hilpisch (2020),
Artificial Intelligence in Finance:
A Python-Based Guide,
O’Reilly

Source: https://www.amazon.com/Artificial-Intelligence-Finance-Python-Based-Guide/dp/1492055433
Artificial Intelligence in Finance

About this Repository

This repository provides Python code and Jupyter Notebooks accompanying the Artificial Intelligence in Finance book published by O'Reilly.

Source: https://github.com/yhilpisch/aiif
Yves Hilpisch (2020), *Artificial Intelligence in Finance: A Python-Based Guide*, O’Reilly

https://github.com/yhilpisch/aiif/tree/main/code
Python in Google Colab (Python101)

https://colab.research.google.com/drive/1FEG6DnGvfwUbeo4zJ1zTunjMqf2RkCrT

# Future Value
1. pv = 100
2. r = 0.1
3. n = 7
4. fr = pv * ((1 + (r)) ** n)
5. print(round(fr, 2))

amount = 100
interest = 0.10 # 10% - 0.01 + 0.1
years = 7
future_value = amount * ((1 + (0.01 * interest)) ** years)
print(round(future_value, 2))

# Python Function def
1. def getFV(pv, r, n):
2.     fr = pv * ((1 + (r)) ** n)
3.     return fr
4.     fr = getFV(100, 0.1, 7)
5.     print(round(fr, 2))

# Python if else
1. score = 80
2. if score >= 60:
3.     print("Pass")
4. else:
5.     print("Fail")

https://tinyurl.com/aintpupython101
Python in Google Colab (Python101)

https://colab.research.google.com/drive/1FEG6DnGvwfUbeo4zJ1zTunjMqf2RkCrT

AI in Finance

- Github: https://github.com/philipsci/allf

Normative Finance and Financial Theories

Uncertainty and Risk

Import numpy as np

# The prices of the stock and bond today.
S0 = 10
B0 = 10
print('S0', S0)
print('B0', B0)

# The uncertain payoff of the stock and bond tomorrow.
S1 = np.array([20, 5])
B1 = np.array([11, 11])
print('S1', S1)
print('B1', B1)

# The market price vector
M0 = np.array([S0, B0])

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https://colab.research.google.com/drive/1FEG6DnGvwfUbeo4zJ1zTunjMqf2RkCrT

Portfolio Optimization and Algorithmic Trading

```python
1  ! pip install pandas_datareader
2  ! import pandas as pd
3  ! import pandas_datareader.data as web
4  ! import matplotlib.pyplot as plt
5  ! import seaborn as sns
6  ! import datetime as dt
7  ! import matplotlib Inline
8  
9  #Read Stock Data from Yahoo Finance
10  end = dt.datetime.now()
11  #start = dt.datetime(2013, 1, 1)
12  start = dt.datetime(2010, 1, 1)
13  df = web.DataReader('AAPL', 'yahoo', start, end)
14  df.to_csv('AAPL.csv')
15  df = pd.read_csv('AAPL.csv')
16  print(df.head())
17  print(df.tail())
18  print(df.describe())
19  
20  df['Adj Close'].plot(legend=True, figsize=(12, 8), title='AAPL', label='Adj Close')
21  plt.figure(figsize=(12,9))
22  top = plt.subplot2grid((12,9), (0, 0), rowspan=10, colspan=9)
23  bottom = plt.subplot2grid((12,9), (10,0), rowspan=2, colspan=9)
24  top.plot(df.index, df['Adj Close'], color='blue') #df.index gives the dates
25  bottom.bar(df.index, df['Volume'])
26  
27  #set the labels
28  top.axes.set_xaxis().set_visible(False)
29  top.set_title('AAPL')
30  bottom.set_xlabel('Volume')
31  bottom.set_xlabel('Volume')
32  
33  plt.figure(figsize=(12,9))
```

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Annualised Volatility: 0.18

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<th>FB</th>
<th>GOOGL</th>
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<tr>
<td>allocation</td>
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<td>29.05</td>
<td>26.28</td>
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Minimum Volatility Portfolio Allocation

Annualised Return: 0.22
Annualised Volatility: 0.16

Calculated Portfolio Optimization based on Efficient Frontier

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Portfolio Optimization

Efficient Frontier Portfolio Optimization

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- Open Chinese Convert (OpenCC, 开放中文转换)
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- Natural Language Toolkit (NLTK)
- Stanza: A Python NLP Library for Many Human Languages

https://tinyurl.com/aintpupython101
Summary

• Uncertainty and Risk
• Expected Utility Theory (EUT)
• Mean-Variance Portfolio Theory (MVPT)
• Capital Asset Pricing Model (CAPM)
• Arbitrage Pricing Theory (APT)

• Min-Yuh Day (2022), Python 101, [https://tinyurl.com/aintpuppython101](https://tinyurl.com/aintpuppython101)