A Survey of Probability Concepts

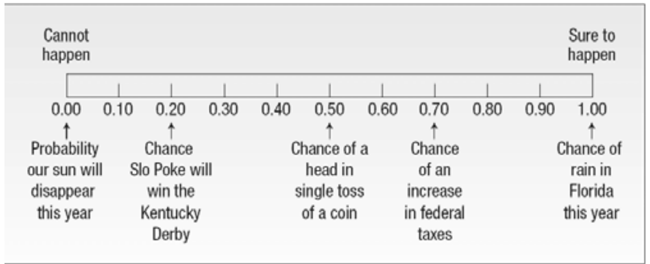
Chapter 5

Definitions

A probability is a value between 0 and 1, inclusive, describing the chance or likelihood that an event will occur.

- A value near zero means the event is not likely to happen. A value near one means it is likely.
- There are three ways of assigning probability:
  - classical
  - empirical
  - subjective

Probability Examples



Three Key Words

- There are three key words used in the study of probability:
  - **Experiment**
  - **Outcome**
  - **Event**

Definitions

- An experiment is a process that leads to the occurrence of one and only one of several possible observation.
- An outcome is the particular result of an experiment.
- An event is the collection of one or more outcomes of an experiment.

5

Experiments, Events and Outcomes

	
Experiment	Roll a die
All possible outcomes	Observe a 1 Observe a 2 Observe a 3 Observe a 4 Observe a 5 Observe a 6
Some possible events	Observe an even number Observe a number greater than 4 Observe a number 3 or less

6

集合論 (Set Theory) 簡介

- 一般常用 大寫字母表示一個集合 (Set)，例如集合 A, B,…。小寫字母表示集合裡的元素 (Element)。
  - A = {1,2,3,4}
  - B = {b| 0 < b < 1}
  - C = {統計系所有學生}
- 與隨機實驗對應：
  - 元素 (Element) 表示實驗 (Experiment) 可能的結果 (Outcome)。
  - 集合 (Set) 則代表事件 (Event)

7

集合論 (Set Theory) 簡介

- Ω : 全集合或宇集合 (Universal Set)
- $\varnothing$: 空集合（Null Set, Empty Set）
- $A^c = A' = \Omega - A$: 補集或餘集 (Complement Set)
- 交集 (Intersection of Sets) : $A \cap B$
- 聯集 (Union of Sets) : $A \cup B$

8

集合運算

假設 A, B, C, 爲三任意事件：

- $\Omega' = \varnothing, \varnothing' = \Omega, A \cup \Omega = A, A \cap \varnothing = \varnothing, A \cup A' = \Omega, A \cap A' = \varnothing$
- 交換律 (Commutative Law):
  $A \cup B = B \cup A;\ A \cap B = B \cap A$
- 結合律 (Associative Law):
  $(A \cup B) \cup C = A \cup (B \cup C);\ (A \cap B) \cap C = A \cap (B \cap C)$
- 分配律 (Distributive Law):
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C);\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- Demorgan Law :
  $(A \cup B)' = A' \cap B';\ (A \cap B)' = A' \cup B'$

9

機率測度 (Probability Measure)

- 通常以 P 表示機率測度，P(A) 表示發生事件A 的機率。$P(\Omega) = 1$, P(Null Set) = 0.
- 考慮隨機實驗：丟擲一公平骰子與一公平硬幣

  A = {骰子出現偶數點}

  B＝{骰子點數小於等於 5}

  C＝{骰子點數大於等於 3}

  P(A) = 3/6

  P(B) = 5/6

  $P(A \cap B) = 2/6$

  $P(A \cup C) = 2/6$

10

Assigning Probabilities

Three approaches to assigning probabilities

- Objective Approach
  - Classical Probability
    - The probability of each outcome is known.
  - Empirical Probability
    - Use past information to assign probabilities.
- Subjective Approach
  - Subjective Probability
    - There is no information so that the probabilities are assigned by subjected.

11

Classical Probability

- Classical probability is based on the assumption that the outcomes of an experiment are equally likely.
- The total number of outcomes is known before the experiment.
  - The die-rolling experiment
  - The coin-flipping experiment
  - Lottery
  - Poker

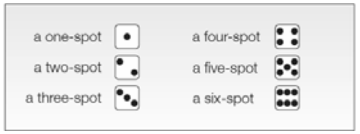
12

Classical Probability

CLASSICAL PROBABILITY $\text{Probability of an event} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$ [5-1]

Consider an experiment of rolling a six-sided die. What is the probability of the event "an even number of spots appear face up"?

The possible outcomes are:



There are three "favorable" outcomes (a two, a four, and a six) in the collection of six equally likely possible outcomes and thus the probability of getting an even number is 3/6.

13

Mutually Exclusive Events (互斥事件)

- Two events are mutually exclusive if they cannot occur at the same time.
- 實驗：丟擲一公平骰子。

  事件A：出現奇數點

  事件B：出現偶數點

  ➔ 則事件A與事件B是 互斥。

  ➔ 即 A ∩ B=Null Set 或 P(A∩B) = 0

14

Independent Events （獨立事件）

- Events are independent if the occurrence of one event does not affect the occurrence of another.
- 實驗：丟擲一公平骰子與一公平硬幣。

  事件A：骰子出現奇數點

  事件B：硬幣出現正面

  ➔ 則事件A與事件B是 獨立事件。

  ➔ P(A∩B) = P(A) P(B)

15

Collectively Exhaustive Events

- Events are collectively exhaustive if at least one of the events must occur when an experiment is conducted.
  - the events "an even number" and "an odd number" in the die-tossing experiment.

16

Empirical Probability

- The empirical approach to probability is based on the number of times an event occurs as a proportion of a known number of trials.
  - The Weather Forecasting
  - Reggie Miller of the Pacers made 250 free throws out of 268 attempts. We estimate that the likelihood of him making his next free throw is 0.933.

EMPIRICAL PROBABILITY The probability of an event happening is the fraction of the time similar events happened in the past.

17

Empirical Probability

- The empirical approach to probability is based on what is called the law of large numbers. The key to establishing probabilities empirically is that more observations will provide a more accurate estimate of the probability.

LAW OF LARGE NUMBERS Over a large number of trials the empirical probability of an event will approach its true probability.

18

Law of Large Numbers

Suppose we toss a fair coin. The result of each toss is either a head or a tail. If we toss the coin a great number of times, the probability of the outcome of heads will approach .5. The following table reports the results of an experiment of flipping a fair coin 1, 10, 50, 100, 500, 1,000 and 10,000 times and then computing the relative frequency of heads

Number of Trials	Number of Heads	Relative Frequency of Heads
1	0	.00
10	3	.30
50	26	.52
100	52	.52
500	236	.472
1,000	494	.494
10,000	5,027	.5027

19

Empirical Probability - Example

On February 1, 2003, the Space Shuttle Columbia exploded. This was the second disaster in 113 space missions for NASA. On the basis of this information, what is the probability that a future mission is successfully completed?

$$\text{Probability of a successful flight} = \frac{\text{Number of successful flights}}{\text{Total number of flights}}$$

$$= \frac{111}{113} = 0.98$$

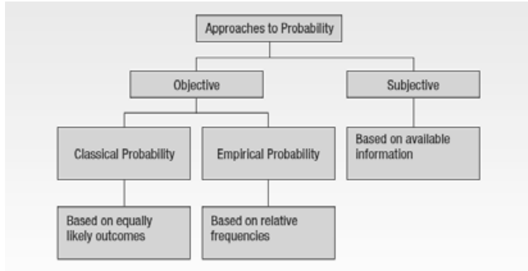
20

Subjective Probability - Example

- If there is little or no past experience or information on which to base a probability, it may be arrived at subjectively.
- Illustrations of subjective probability are:
  1. Estimating the likelihood the New England Patriots will play in the Super Bowl next year.
  2. Estimating the likelihood you will be married before the age of 30.
  3. Estimating the likelihood the U.S. budget deficit will be reduced by half in the next 10 years.

SUBJECTIVE CONCEPT OF PROBABILITY The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.

Summary of Types of Probability



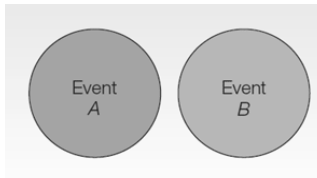
Some Rules for Computing Probabilities

- Special Rule of Addition
  - Mutually Exclusive Rule
  - Complement Rule
- General Rule of Addition

Mutually Exclusive Rule

- If two events *A* and *B* are mutually exclusive, the probability of one or the other event's occurring equals the sum of their probabilities.

P(A or B) = P(A) + P(B)



Mutually Exclusive Rule: Example

- Consider a die-tossing experiment and two events:

  A={a number 4 or larger}

  B={a number 2 or smaller}

  Then, A and B are mutually exclusive.

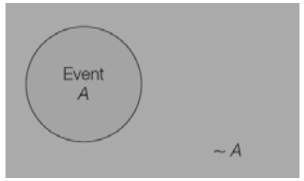
- P(A or B) = P(1,2,4,5,6)= P(A) +P(B)

  =5/6

25

Complement

The complement of a set A is the set that contains all elements not included in set A, denoted by ~A.

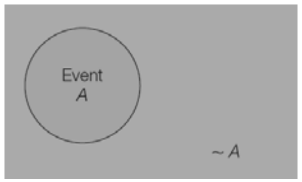


26

The Complement Rule for Computing Probabilities

The complement rule is used to determine the probability of an event occurring by subtracting the probability of the event *not* occurring from 1.

$P(A) + P(\sim A) = 1$ or $P(A) = 1 - P(\sim A)$.



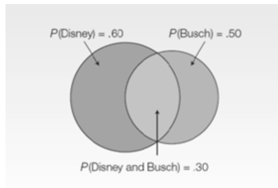
27

The General Rule for Computing Probabilities

- If A and B are two events that are not mutually exclusive, then P(A or B) is given by the following formula:

  P(A or B) = P(A) + P(B) - P(A and B)

  $\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$



28

Addition Rule - Example

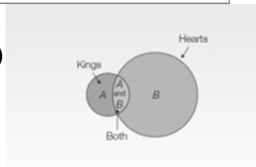
What is the probability that a card chosen at random from a standard deck of cards will be either a king or a heart?

Card	Probability		Explanation
King	$P(A)$	= 4/52	4 kings in a deck of 52 cards
Heart	$P(B)$	= 13/52	13 hearts in a deck of 52 cards
King of Hearts	$P(A \text{ and } B)$	= 1/52	1 king of hearts in a deck of 52 cards

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

= 4/52 + 13/52 - 1/52

= 16/52, or .3077



29

Special Rule of Multiplication

- The special rule of multiplication requires that two events *A* and *B* are *independent.*
- Two events *A* and *B* are independent if the occurrence of one has no effect on the probability of the occurrence of the other.

A and *B* are Independent ⟷ $P(A \cap B) = P(A)P(B)$

If A, B and C are independent, then $P(A \cap B \cap C)=P(A)P(B)P(C)$

30

Multiplication Rule-Example

A survey by the American Automobile association (AAA) revealed 60 percent of its members made airline reservations last year. Two members are selected at random. What is the probability both made airline reservations last year?

Solution:

The probability the first member made an airline reservation last year is .60, written as $P(R_1) = .60$

The probability that the second member selected made a reservation is also .60, so $P(R_2) = .60$.

Since the number of AAA members is very large, you may assume that R_1 and R_2 are independent.

$$P(R_1 \cap R_2) = P(R_1)P(R_2) = (.60)(.60) = .36$$

31

聯合機率與邊際機率 (Joint Probability and Marginal Probability)

- 聯合機率 （Joint Probability）
  ---- 兩個或兩個以上事件同發生的機率。
- 邊際機率 （Marginal Probability)
  ---- 在有兩個貨兩個以上類別的樣本空間，若僅考慮某一類別發生的機率稱之。
- 條件機率 (Conditional Probability)
  ---- 令A與B為兩事件，已知事件 B 發生的情形下，發生A的機率稱之，以 P(A|B) 表示之。

32

聯合次數分配表

A\B	B_1	B_2	⋯	B_c
A_1	$A_1 \cap B_1$	$A_1 \cap B_2$	⋯	$A_1 \cap B_c$
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
A_r	$A_r \cap B_1$	$A_r \cap B_2$	⋯	$A_r \cap B_c$

33

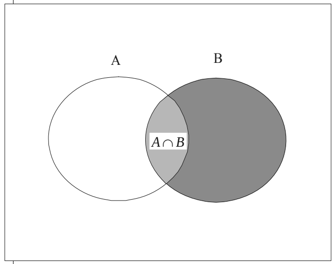
聯合機率分配表

A\B	B_1	B_2	⋯	B_c
A_1	$P(A_1 \cap B_1)$	$P(A_1 \cap B_2)$	⋯⋯	$P(A_1 \cap B_c)$
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
A_r	$P(A_r \cap B_1)$	$P(A_r \cap B_2)$	⋯	$P(A_r \cap B_c)$

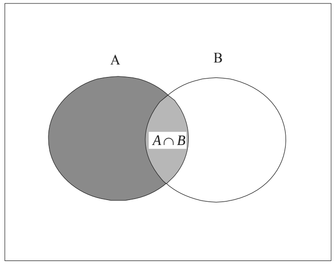
34

圖例

事件A的條件機率



事件B的條件機率



35

Conditional Probability

- A conditional probability is the probability of a particular event occurring, given that another event has occurred.
- The probability of the event *A* given that the event *B* has occurred is written $P(A|B)$.

36

General Multiplication Rule

The general rule of multiplication is used to find the joint probability that two events will occur.

Use the general rule of multiplication to find the joint probability of two events when the events are not independent.

It states that for two events, *A* and *B*, the probability that both events will happen is found by multiplying the probability that event *A* will happen by the conditional probability of event *B* occurring given that *A* has occurred.

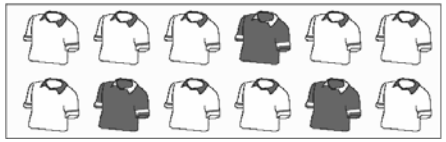
GENERAL RULE OF MULTIPLICATION $P(A \text{ and } B) = P(A)P(B|A)$ **[5-6]**

$$P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$$

General Multiplication Rule - Example

A golfer has 12 golf shirts in his closet. Suppose 9 of these shirts are white and the others blue. He gets dressed in the dark, so he just grabs a shirt and puts it on. He plays golf two days in a row and does not do laundry.

What is the likelihood both shirts selected are white?



General Multiplication Rule - Example

- The event that the first shirt selected is white is W_1. The probability is $P(W_1) = 9/12$
- The event that the second shirt selected is also white is identified as W_2. The conditional probability that the second shirt selected is white, given that the first shirt selected is also white, is $P(W_2 | W_1) = 8/11$.
- To determine the probability of 2 white shirts being selected we use formula: P(AB) = P(A) P(B|A)
- $P(W_1 \text{ and } W_2) = P(W_1)P(W_2 | W_1) = (9/12)(8/11) = 0.55$

Contingency Tables

A CONTINGENCY TABLE is a table used to classify sample observations according to two or more identifiable characteristics

E.g. A survey of 150 adults classified each as to gender and the number of movies attended last month. Each respondent is classified according to two criteria—the number of movies attended and gender.

	Gender		
Movies Attended	Men	Women	Total
0	20	40	60
1	40	30	70
2 or more	10	10	20
Total	70	80	150

Contingency Tables - Example

A sample of executives were surveyed about their loyalty to their company. One of the questions was, “If you were given an offer by another company equal to or slightly better than your present position, would you remain with the company or take the other position?” The responses of the 200 executives in the survey were cross-classified with their length of service with the company.

	Length of Service				
Loyalty	Less than 1 Year, B_1	1–5 Years, B_2	6–10 Years, B_3	More than 10 Years, B_4	Total
Would remain, A_1	10	30	5	75	120
Would not remain, A_2	25	15	10	30	80
	35	45	15	105	200

What is the probability of randomly selecting an executive who is loyal to the company (would remain) and who has more than 10 years of service?

41

Contingency Tables - Example

Event A_1 happens if a randomly selected executive will remain with the company despite an equal or slightly better offer from another company. Since there are 120 executives out of the 200 in the survey who would remain with the company

$P(A_1)$ = 120/200, or .60.

Event B_4 happens if a randomly selected executive has more than 10 years of service with the company. Thus, P(B4| A1) is the conditional probability that an executive with more than 10 years of service would remain with the company. Of the 120 executives who would remain 75 have more than 10 years of service, so P(B4| A1) = 75/120.

$$P(A_1 \text{ and } B_4) = P(A_1)P(B_4|A_1) = \left(\frac{120}{200}\right)\left(\frac{75}{120}\right) = \frac{9{,}000}{24{,}000} = .375$$

Note: A1 and B4 are not independent!!

42

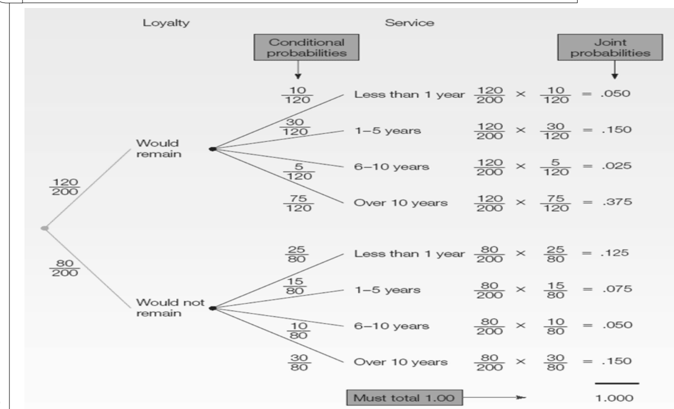
Tree Diagrams

A tree diagram is useful for portraying conditional and joint probabilities. It is particularly useful for analyzing business decisions involving several stages.

A tree diagram is a graph that is helpful in organizing calculations that involve several stages. Each segment in the tree is one stage of the problem. The branches of a tree diagram are weighted by probabilities.

43

Chart 5-2, PP. 159



44

Bayes' Theorem

- Bayes' Theorem is a method for revising a probability given additional information.
- Given A1 and A2 are mutually exclusive and collectively exhaustive. It is computed using the following formula:

BAYES' THEOREM
$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)} \quad [5\text{–}7]$$

45

Bayes' Theorem (貝氏定理)

- 條件機率是在事件 B 已知情形下，發生 A 的機率，亦即條件機率可看成：
  - 已知事前的發生事件 A 的機率，然後經由樣本的調查試驗得到新的資訊 （事件 B），再利用此新資訊將事前的機率加以修正，得到事後的機率 P(A|B)。
- 貝氏定理即是說明如何由新資訊修正事前機率的方法。

46

Bayes' Theorem (貝氏定理)

- P(A) 是在沒有任何資訊下，判斷發生A的機率，我們稱為事前機率 (Prior Probability).
- P(A|B) 是條件機率，是在已知訊息B的情形下，判斷發生A的機率，稱為事後機率 (Posterior Probability).
- 在已知的資訊越多的情形下，對機率做判斷，會更準確，因此，P(A|B) 較 P(A) 具參考性。

47

例子：貝氏定理

- 某唱片公司，依據過去發行唱片經驗，成功機率 60%，失敗 40%。
  - P(A1) = 0.6, P(A2)=0.4
    A1 = 上市成功　　A2 = 上市失敗

上市情況	機率
成功	0.6
失敗	0.4
合計	1.00

48

例子：貝氏定理

- 該公司推出唱片前，會作市場調查。調查結果以條件機率表示如下：
- B = 調查結果喜歡 ~B = 調查結果不喜歡
  - 推出成功時，調查結果喜歡90%，不喜歡10%。
    P(B|A1)=0.9, P(~B|A1)=0.1
  - 推出不成功時，調查結果喜歡30%，不喜歡70%。
    P(B|A2)=0.3, P(~B|A2)=0.7

	上市成功（A_1）	上市失敗（A_2）
客戶喜歡（B）	0.90	0.30
客戶不喜歡（$\bar{B}$）	0.10	0.70
合計	1.00	1.00

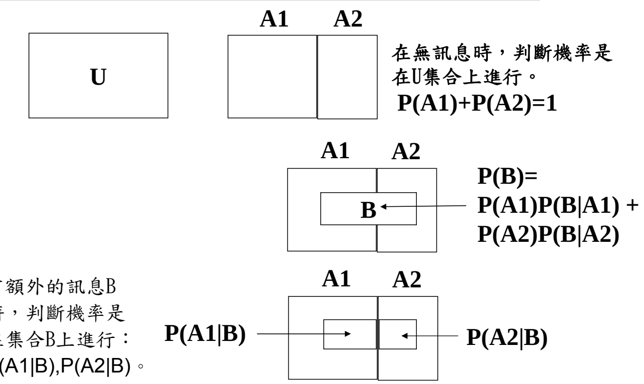
例子：貝氏定理

- 唱片行關心的是，調查報告是喜歡情形下，上市成功機率是多少，即 P(A1|B)。
- **上市成功與失敗的聯合機率分配表**

	上市成功（A_1）	上市失敗（A_2）	
客戶喜歡（B）	$P(B\cap A_1)=0.54$	$P(B\cap A_2)=0.12$	$P(B)=0.66$
客戶不喜歡（$\bar{B}$）	$P(\bar{B}\cap A_1)=0.06$	$P(\bar{B}\cap A_2)=0.28$	$P(\bar{B})=0.34$
	$P(A_1)=0.60$	$P(A_2)=0.40$	1.00

- **P(A1|B) = P(A1∩B)/P(B)=0.54/0.66 =0.82**
- **P(A2|B) = P(A2∩B)/P(B)=0.12/0.66 =0.18**

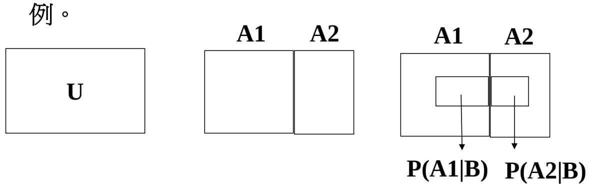
圖示：貝氏定理



有額外的訊息B時，判斷機率是在集合B上進行：P(A1|B),P(A2|B)。

計算原則：

- 原本算 **A1** 機率，是看集合 **A1** 佔集合 **U** 的比例。
- 在多了訊息 **B** 之後， 算 **A1** 機率是看集合 **A1** 佔 **B** 的比例。

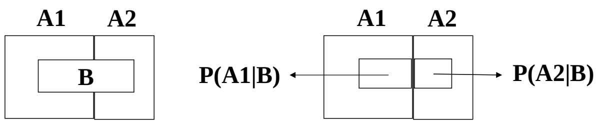


BAYES' THEOREM

$$P(A_i|B)=\frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1)+P(A_2)P(B|A_2)} \qquad [5\text{-}7]$$

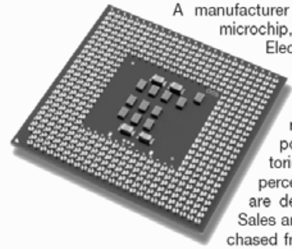
例子：貝氏定理

- P(B) = P(A1 and B) +P(A2 and B)
  = P(A1)P(B|A1) + P(A2)P(B|A2)
  = 0.6 × 0.9 + 0.4 × 0.3 = 0.66
- P(A1|B) = P(A1)P(B|A1) / P(B)
  = 0.54 / 0.66 = 0.82



53

Bayes Theorem - Example



A manufacturer of DVD players purchases a particular microchip, called the LS-24, from three suppliers: Hall Electronics, Schuller Sales, and Crawford Components. Thirty percent of the LS-24 chips are purchased from Hall Electronics, 20 percent from Schuller Sales, and the remaining 50 percent from Crawford Components. The manufacturer has extensive histories on the three suppliers and knows that 3 percent of the LS-24 chips from Hall Electronics are defective, 5 percent of chips from Schuller Sales are defective, and 4 percent of the chips purchased from Crawford Components are defective.

When the LS-24 chips arrive at the manufacturer, they are placed directly in a bin and not inspected or otherwise identified by supplier. A worker selects a chip for installation in a DVD player and finds it defective. What is the probability that it was manufactured by Schuller Sales?

54

Bayes Theorem – Example *(cont.)*

- There are three mutually exclusive and collectively exhaustive events, that is, three suppliers.

  A_1 The LS-24 was purchased from Hall Electronics.
  A_2 The LS-24 was purchased from Schuller Sales.
  A_3 The LS-24 was purchased from Crawford Components.

- The prior probabilities are:

  $P(A_1) = .30$ The probability the LS-24 was manufactured by Hall Electronics.
  $P(A_2) = .20$ The probability the LS-24 was manufactured by Schuller Sales.
  $P(A_3) = .50$ The probability the LS-24 was manufactured by Crawford Components.

- The additional information can be either:

  B_1 The LS-24 appears defective, or
  B_2 The LS-24 appears not to be defective.

55

Bayes Theorem – Example *(cont.)*

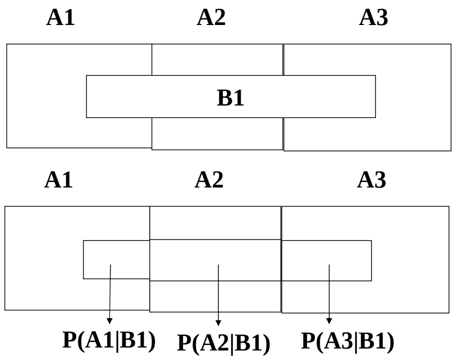
- The following conditional probabilities are given.

  $P(B_1|A_1) = .03$ The probability that an LS-24 chip produced by Hall Electronics is defective.
  $P(B_1|A_2) = .05$ The probability that an LS-24 chip produced by Schuller Sales is defective.
  $P(B_1|A_3) = .04$ The probability that an LS-24 chip produced by Crawford Components is defective.

- A chip is selected from the bin. Because the chips are not identified by supplier, we are not certain which supplier manufactured the chip. We want to determine the probability that the defective chip was purchased from Schuller Sales. The probability is written $P(A_2|B_1)$.

56

Bayes Theorem – Example *(cont.)*



57

Bayes Theorem – Example *(cont.)*

P(A2 | B1)

= P(Purchased from Schuller Sales | A defect chip)

= P (A2)P(B1|A2) / [P (A1)P(B1|A1)+ P (A2)P(B1|A2) + P (A3)P(B1|A3)]

= 0.01 / 0.039 = 0.2564

Event, A_i	Prior Probability, $P(A_i)$	Conditional Probability, $P(B_1 \mid A_i)$	Joint Probability, $P(A_i$ and $B_1)$	Posterior Probability, $P(A_i \mid B_1)$
Hall	.30	.03	.009	.009/.039 = .2308
Schuller	.20	.05	.010	.010/.039 = .2564
Crawford	.50	.04	.020	.020/.039 = .5128
			$P(B_1)$ = .039	1.0000

58

Bayes Theorem – Example *(cont.)*

The probability the defective LS-24 chip came from Schuller Sales can be formally found by using Bayes' theorem. We compute $P(A_2|B_1)$, where A_2 refers to Schuller Sales and B_1 to the fact that the selected LS-24 chip was defective.

$$P(A_2|B_1) = \frac{P(A_2)P(B_1|A_2)}{P(A_1)P(B_1|A_1) + P(A_2)P(B_1|A_2) + P(A_3)(B_1|A_3)}$$

$$= \frac{(.20)(.05)}{(.30)(.03) + (.20)(.05) + (.50)(.04)} = \frac{.010}{.039} = .2564$$

59

Bayes Theorem – Example *(cont.)*

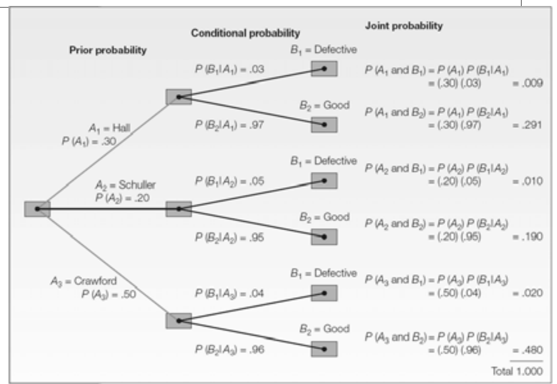


CHART 5–3 Tree Diagram of DVD Manufacturing Problem

60

Counting Rules – Multiplication

The multiplication formula indicates that if there are m ways of doing one thing and n ways of doing another thing, there are m x n ways of doing both.

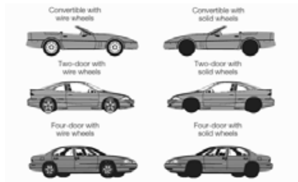
Example: Dr. Delong has 10 shirts and 8 ties. How many shirt and tie outfits does he have?

(10)(8) = 80

61

Counting Rules – Multiplication: Example

An automobile dealer wants to advertise that for $29,999 you can buy a convertible, a two-door sedan, or a four-door model with your choice of either wire wheel covers or solid wheel covers. How many different arrangements of models and wheel covers can the dealer offer?



62

Counting Rules – Multiplication: Example

MULTIPLICATION FORMULA Total number of arrangements = $(m)(n)$ [5–8]

We can employ the multiplication formula as a check (where m is the number of models and n the wheel cover type). From formula (5–8):

Total possible arrangements $= (m)(n) = (3)(2) = 6$

63

Counting Rules - Permutation

- The multiplication formula is applied to the number of possible arrangements for two or more groups.
- The permutation formula is applied to find the possible number of arrangements when there is only one group.
- A permutation is any arrangement of r objects selected from n possible objects. The order of arrangement is important in permutations.

PERMUTATION FORMULA $${}_nP_r = \frac{n!}{(n-r)!}$$ [5–9]

where:
n is the total number of objects.
r is the number of objects selected.

64

Counting - Combination

A combination is the number of ways to choose *r* objects from a group of *n* objects without regard to order.

COMBINATION FORMULA $${}_nC_r = \frac{n!}{r!(n-r)!} \quad [5\text{–}10]$$

where:
n is the total number of objects.
r is the number of objects selected.

65

Combination - Example

There are 12 players on the Carolina Forest High School basketball team. Coach Thompson must pick five players among the twelve on the team to comprise the starting lineup. How many different groups are possible?

$${}_{12}C_5 = \frac{12!}{5!(12-5)!} = 792$$

66

Permutation - Example

Suppose that in addition to selecting the group, he must also rank each of the players in that starting lineup according to their ability.

$${}_{12}P_5 = \frac{12!}{(12-5)!} = 95{,}040$$

67

Exercises

- 3,7,11,13,15,17,19,21,23,27,29,33,35,39,41, 47,49,51,53,55,57,59,61,65,67,69,71,73,75, 81,83,85

68

End of Chapter 5

69