國立臺北大學自然資源與環境管理研究所
九十九學年度第二學期
『環境系統分析』課程講義（二）

進度：線性規劃回顧與進階主題
Review of Linear Programming and Advanced Topics

● GOALS OF THE COURSE

□ Content ⇒ Systems Analysis Models and Applications Concerning the *Environment*
- ➪ Systems vs. Systems Analysis
- ➪ Systems Analysis vs. Operations Research (Operational Research)
- ➪ Systems Analysis, System Simulation, and System Dynamics
- ➪ Programming vs. Planning => Simulation and Optimization
- ➪ Conceptual models => Mathematical models => Simulation/Optimization models

□ Mathematical Models
- ➪ Classification: Prescriptive vs. Descriptive; Deterministic vs. Stochastic
- ➪ Solution Techniques: Symbolic/Graphical Interpretation; Analytical vs. Numerical
- ➪ Algorithms, Numerical Methods => Linearity, Convexity, and Complexity

● OPTIMIZATION AND SIMULATION SOFTWARE

□ Problems or Models
- ➪ Linear vs. nonlinear; constrained vs. unconstrained; continuous vs. discrete

□ Platform and Programming
- ➪ Command-line vs. Windows; Editor-oriented vs. Object-oriented

□ Programming Skill
- ➪ Solver package vs. Program coding => Compiling vs. Running of Program codes

● COMPONENTS OF AN OPTIMIZATION MODEL

□ Objective Function(s)
- ➪ Single vs. Multiple
- ➪ Linear vs. Nonlinear
- ➪ Convex (Concave) vs. Non-convex

□ Constraints
- ➪ Constrained vs. Un-constrained
- ➪ Linear vs. Nonlinear
- ➪ Convex vs. Non-convex Feasible Regions

□ Decision Variables
- ➪ Continuous vs. Discrete
- ➪ Deterministic vs. Stochastic

□ System Parameters (Coefficients)
- ➪ Deterministic vs. Stochastic
- ➪ Division into Sub-Models

- □ Fuzzy Sets and Grey Information (Interval Variables)
  - ⇨ 'Probability,' 'Likelihood,' and 'Possibility'
  - ⇨ Uncertainties about Decision Variables and Parameters are Incorporated
  - ⇨ Division into Sub-Models
- □ Formulation of Optimization Models
  - ⇨ Plain Form: Straightforward but not suitable for large-scaled or complex problems
  - ⇨ Algebraic Formulations => Vectors and Matrices
  - ⇨ Algebraic Formulations with text description of variables and parameters
  - ⇨ Sets and Indices

- ● PROPERTIES OF AN LP
  - □ Proportionality, Additivity, Divisibility, Certainty , and Non-Negativity
  - □ Non-negative Decision Variables => What if negative values are needed?
  - □ A "Convex Programming" Model
  - □ Additional Terminology
    - ⇨ Feasible Region or Solution Space
    - ⇨ Vertex, Extreme Points or Corner Points
    - ⇨ Decision Space or Objective Space

- ● SOLUTION PROCEDURE OF AN LP
  - □ Pre-Optimal Analysis, Optimization (Solution) and Post-Optimization Analysis
  - □ Graphical, Simplex, Dual Simplex, Interior Point and Other Methods
  - □ Infeasible, Un-bounded and Degenerate Solutions
  - □ A "Convex Programming" Model: Feasible Region and Extreme Points
    - ⇨ Characteristics of Feasible Region for the LP: Convex, Compact, and Continuous
    - ⇨ Extreme Points (Corner Points) vs. Interior Points

- ● THE SIMPLEX METHOD
  - □ Augmented Form of the LP Models
    - ⇨ "Less-than-and-equal-to" Inequality constraints => Slack variables
    - ⇨ "Greater-than-and-equal-to" Inequality constraints => Surplus & Artificial Variables
    - ⇨ Equality constraints => Artificial variables => 'Big-M Treatment'
  - □ Terminology and Procedure of the Simplex Method
    - ⇨ Basic vs. non-basic variables
    - ⇨ Feasible basic solution => "Adjacent"
    - ⇨ Ratio test for Pivoting
    - ⇨ "Optimality"
  - □ Simplex Tableaus and An Animated Presentation

- ● EXAMPLES OF LINEAR PROGRAMMING
  - □ Glass Production at Wyndor Glass Co.
    - ⇨ Objective Function: Maximizing the profit
    - ⇨ Decision Variables: Production rate of the two types of products
    - ⇨ Constraints: Production time limits at the three plants

TABLE 3.1 Data for the Wyndor Glass Co. problem

Plant	Production Time per Batch, Hours — Product 1	2	Production Time Available per Week, Hours
1	1	0	4
2	0	2	12
3	3	2	18
Profit per batch	\$3,000	\$5,000	

$$\text{Maximize} \quad Z = 3x_1 + 5x_2,$$

subject to the restrictions

$$\begin{aligned} x_1 \quad &\le 4 \\ 2x_2 &\le 12 \\ 3x_1 + 2x_2 &\le 18 \end{aligned}$$

and

$$x_1 \ge 0, \qquad x_2 \ge 0.$$

- □ Homewood Masonry -- A Material Production Problem
  - ⇨ Objective Function: Maximizing the production profit
  - ⇨ Decision Variables: Two building products to be produced
  - ⇨ Constraints: Resource availability, work hours, and curing vat capacity

Resource	HYDIT	FILIT	Availability
Wahash Red Clay	2 m^3/ton	4 m^3/ton	28 m^3/wk
Blending time	5 hr/ton	5 hr/ton	50 hr/wk
Curing vat capacity	8 tons	6 tons	
Profit	\$140/ton	\$160/ton	

- SENSITIVITY ANALYSIS
  - □ Overview and Post-Optimality Analysis
  - □ Sensitivity Analysis on RHS (Resource) Coefficients
    - ⇨ Shadow price, marginal value of a resource and economic interpretation
    - ⇨ Dual price (?)
  - □ Sensitivity Analysis on Objective Function Coefficients
  - □ Graphical Illustration
  - □ Outputs from Optimization Packages and Analytical Interpretation (?)
  - □ Parametric Programming

- DUALITY THEORY
  - □ Model Formulations
  - □ Dual-Primal Relationships
    - ⇨ Implementation from Production Problem
    - ⇨ Implementation from Resource Allocation Problem
  - □ Shadow Price
  - □ Primal-Dual Methods for Optimization (Lagrange Algorithms)

- HOMEWORK #1 (2011/03/15 Due): *Solve* the example of Homewood Masonry in ReVelle's textbook (p.44) by using AIMMS, LINGO, What'sBest, and GAMS.