# 國立臺北大學自然資源與環境管理研究所
## 九十九學年度第二學期
## 『環境系統分析』課程講義（五、六）

進度：不確定性分析與隨機規劃

Uncertainty Analysis and Stochastic Programming

- PROBABILITY THEORY, STOCHASTIC PROCESS AND RANDOM FIELD
  - □ Deterministic vs. Stochastic Systems
    - ⇨ Vagueness, Uncertainty and 'Stochasticity'
    - ⇨ Possibility, Likelihood, and Probability
  - □ Probability Theory
    - ⇨ The Axioms of Probability
    - ⇨ Random Variables: Discrete and Continuous
    - ⇨ Statistics (Moments) of a Random Variable: Expected Value, Variance, …etc.
    - ⇨ Multiple Random Variables: Multivariate Statistics => Covariances
    - ⇨ Distribution: Probability Density Function, Cumulated Distribution Function
    - ⇨ Conditional Probability and Baye's Theorem => Bayesian Decision Analysis
  - □ Normal Distribution
    - ⇨ Two-Parameter Distribution: Location and Dispersion => Mean and Variance
    - ⇨ Standardization and *t*-Distribution
    - ⇨ Confidence Interval and Standard Deviation
    - ⇨ Multivariate Gaussian Distribution
  - □ Stochastic Process
    - ⇨ Serial Random Variables: Temporal, Spatial, Spatio-Temporal Stochastic Processes
    - ⇨ Serial Correlation => Deterministic Term (Trend) + Disturbance (Noise)
    - ⇨ Poisson Process, Markov's Chains, and Random Walks
  - □ Random Field
    - ⇨ Random Variables Distributed ('Regionalized') in Space
    - ⇨ Spatial Variability (Correlation) => Trend + Disturbance
    - ⇨ Geostatistics: Kriging (Simple, Ordinary, Universal…) => GIS

- STOCHASTIC PROGRAMMING
  - □ Uncertainty Analysis
    - ⇨ Mathematical (Quantitative) Analyses Related to the Uncertainties about 'Systems'
    - ⇨ System Uncertainties: Uncertainties about Measurement, Modeling, and Parameters
  - □ Uncertainties Related to Mathematical Programming Systems
    - ⇨ Modeling Uncertainties: Assumptions, Objective Functions, and Constraints
      - ●Mathematical Program with Recourse: Multi-Stage Stochastic Programming
    - ⇨ Uncertainties 'Embedded' in Decision Variables: Fuzziness, Grey Information…
      - ●Intervals or Specified Ranges => Grey Numbers => Grey Programming
      - ●Degree of Set Membership => Fuzzy Set => Fuzzy Programming
    - ⇨ Uncertainties about Model Parameters: Coefficients of Objective Function, RHS, $A_{ij}$

●Parameters (Coefficients) of the Optimization Model are Random Variables
●Treat Decision Variables as 'Deterministic Variables' to be determined
●Probabilistic Constraints => Chance-Constrained Programming

□ Other Considerations

⇨ Stochastic Dynamic Programming and Markov Decision Process

⇨ Optimal Control and System Dynamics

- TWO-STAGE STOCHASTIC PROGRAMMING WITH RECOURSE

□ What is "recourse"? "Wait-and-See"?

⇨ Recourse is the ability to take corrective action after a random event has taken place.

□ Scenarios and Stages

⇨ Deterministic Equivalent => Expected-Value Formulation

□ An Example (http://wiki.mcs.anl.gov/NEOS/index.php/Stochastic_Programming)

- CHANCE CONSTRAINED PROGRAMMING

□ What are Chance Constraints?

□ Significance Level => System Reliability

□ Row Independence => Independently and Identically Distributed (iid)

□ Right-Hand-Side Random => Univariate Normal Distribution

□ Technical Coefficients Random => Multivariate Normal Distribution

□ Row Dependence => Joint Chance Constraint (relatively complicated!)

Chance Constraints: $p\left(\sum_{j=1}^{n} a_{ij} \cdot x_j \propto b_i\right) \geq 1-\alpha_i; \quad \forall i = 1,\cdots m$

(1)RHS $b_i$ Random: Univariate probability distribution of $b_i$

i. $\propto \equiv \geq$

$$p\left(\sum_{j=1}^{n} a_{ij} \cdot x_j \geq b_i\right) \geq 1-\alpha_i \Rightarrow p\left(b_i \leq \sum_{j=1}^{n} a_{ij} \cdot x_j\right) \geq 1-\alpha_i \Rightarrow F(b_i = \sum a_{ij}x_j) \geq 1-\alpha_i$$

ii. $\propto \equiv \leq$

$$p\left(\sum_{j=1}^{n} a_{ij} \cdot x_j \leq b_i\right) \geq 1-\alpha_i \Rightarrow p\left(b_i \geq \sum_{j=1}^{n} a_{ij} \cdot x_j\right) \geq 1-\alpha_i \Rightarrow 1-F(b_i = \sum a_{ij}x_j) \geq 1-\alpha_i$$

(2)Technical Coefficients $a_{ij}$ Random: Multivariate probability distribution of $\sum a_{ij}x_j$

⇨ Variance-Covariance Matrix: Positively definite (symmetric) matrix

- HOMEWORK #3 (2011/04/13 Due)

1. Suppose that the third constraint of the Prototype Example (the Glass Production at Wyndor Co.) in Hillier and Lieberman (2003) is a chance constraint (RHS random), please construct the deterministic equivalent of the stochastic programming model.
2. Suppose that the RHS of the third constraint is a random number with normal distribution of $b_3 \rightarrow N(18, 3^2)$. Please use What'sBest and AIMMS to solve the deterministic equivalent model and compare the objective values with respect to the variations of significance levels of 0.5%, 1%, 5%, and 10%.