國立臺北大學自然資源與環境管理研究所
101 學年度第二學期『環境系統分析專題』

課程講義(三)：線性規劃回顧與進階主題
Review of Linear Programming and Advanced Topics

W.: What'sBest! Version 11.0 User's Manual
L.: Lindo Systems LINGO Version 13.0 User's Manual
K.: GLPK – GNU Linear Programming Kit: The GLPK documentation consists of the Reference Manual and the description of the GNU MathProg modeling language. => AMPL
G.: A GAMS Tutorial and GAMS - A User's Guide

- COMPONENTS OF AN OPTIMIZATION MODEL
  - □ Objective Function(s)
    - ⇨ Single vs. Multiple
    - ⇨ Linear vs. Nonlinear
    - ⇨ Convex (Concave) vs. Non-convex
  - □ Constraints
    - ⇨ Constrained vs. Un-constrained
    - ⇨ Linear vs. Nonlinear
    - ⇨ Convex vs. Non-convex Feasible Regions
  - □ Decision Variables
    - ⇨ Continuous vs. Discrete
    - ⇨ Deterministic vs. Stochastic
  - □ System Parameters (Coefficients)
    - ⇨ Deterministic vs. Stochastic
    - ⇨ Division into Sub-Models
  - □ Formulation of Optimization Models
    - ⇨ Plain Form: Straightforward but not suitable for large-scaled or complex problems
    - ⇨ Algebraic Formulations => Parameters (Scalars), Vectors, and Matrices (Tables)
    - ⇨ Algebraic Formulations with text description of variables and parameters
    - ⇨ Sets and Indices

- PROPERTIES OF AN LP
  - □ Proportionality, Additivity, Divisibility, Certainty , and Non-Negativity
  - □ Non-negative Decision Variables => What if negative values are needed?
  - □ A "Convex Programming" Model
  - □ Additional Terminology
    - ⇨ Feasible Region or Solution Space
    - ⇨ Vertex, Extreme Points or Corner Points
    - ⇨ Decision Space or Objective Space

- SOLUTION PROCEDURE OF AN LP
  - □ Pre-Optimal Analysis, Optimization (Solution) and Post-Optimization Analysis
  - □ Graphical, Simplex, Dual Simplex, Interior Point and Other Methods
  - □ Infeasible, Un-bounded and Degenerate Solutions
  - □ A "Convex Programming" Model: Feasible Region and Extreme Points

- ⇨ Characteristics of Feasible Region for the LP: Convex, Compact, and Continuous
- ⇨ Extreme Points (Corner Points) vs. Interior Points

- THE SIMPLEX METHOD
  - □ Augmented Form of the LP Models
    - ⇨ "Less-than-and-equal-to" Inequality constraints => Slack variables
    - ⇨ "Greater-than-and-equal-to" Inequality constraints => Surplus & Artificial Variables
    - ⇨ Equality constraints => Artificial variables => 'Big-M Treatment'
  - □ Terminology and Procedure of the Simplex Method
    - ⇨ Basic vs. non-basic variables
    - ⇨ Feasible basic solution => "Adjacent"
    - ⇨ Ratio test for Pivoting
    - ⇨ "Optimality"
  - □ Simplex Tableaus and An Animated Presentation

- EXAMPLES OF LINEAR PROGRAMMING
  - □ Homewood Masonry -- A Material Production Problem
    - ⇨ Objective Function: Maximizing the production profit
    - ⇨ Decision Variables: Two building products to be produced
    - ⇨ Constraints: Resource availability, work hours, and curing vat capacity

Resource	HYDIT	FILIT	Availability
Wahash Red Clay	2 m^3/ton	4 m^3/ton	28 m^3/wk
Blending time	5 hr/ton	5 hr/ton	50 hr/wk
Curing vat capacity	8 tons	6 tons	
Profit	$140/ton	$160/ton	

- SENSITIVITY ANALYSIS
  - □ Overview and Post-Optimality Analysis
  - □ Sensitivity Analysis on RHS (Resource) Coefficients
    - ⇨ Shadow price, marginal value of a resource and economic interpretation
    - ⇨ Dual price (?)
  - □ Sensitivity Analysis on Objective Function Coefficients
  - □ Graphical Illustration
  - □ Outputs from Optimization Packages and Analytical Interpretation (?)
  - □ Parametric Programming

- DUALITY THEORY
  - □ Model Formulations
  - □ Dual-Primal Relationships
    - ⇨ Implementation from Production Problem
    - ⇨ Implementation from Resource Allocation Problem
  - □ Shadow Price
  - □ Primal-Dual Methods for Optimization (Lagrange Algorithms)

- HOMEWORK #2 (2013/03/12 Due)：*Solve* the example problem of Homewood Masonry (ReVelle et al., 2004) by using What'sBest, LINGO, GLPK, and GAMS.