國立臺北大學自然資源與環境管理研究所
101 學年度第二學期『環境系統分析專題』

課程講義(四、五)：效率評估與資料包絡分析
Efficiency Evaluation and Data Envelopment Analysis

黃鏡如、傅祖壇、黃美瑛，2008，績效評估：效率與生產力之理論與應用，新陸書局，台北。
吳濟華、何柏正，2008，組織效率與生產力評估：資料包絡分析法，前程文化，台北。
Cooper, W.W., L.M. Seiford, and J. Zhu (editors), 2011, *Handbook on Data Envelopment Analysis*, 2nd ed., Springer, New York.
MaxDEA package and manual => http://www.maxdea.cn/download/maxdea.zip
DEAP: A Data Envelopment Analysis (Computer) Program => DEAP
Data Envelopment Analysis. 資料包絡分析. 長榮大學 國企系教授 劉春初 (**Liu**)
資料包絡分析法. 黃嘉彥教授. 勤益科技大學 研發與科技管理研究所

- FORMULATIONS OF LINEAR PROGRAMMING
  - □ Homewood Masonry -- A Material Production Problem

Resource	HYDIT	FILIT	Availability
Wahash Red Clay	2 m^3/ton	4 m^3/ton	28 m^3/wk
Blending time	5 hr/ton	5 hr/ton	50 hr/wk
Curing vat capacity	1	0	8 tons
	0	1	6 tons
Profit	$140/ton	$160/ton	

⇨ Plain Form:

Maximize $z = 140x_1 + 160x_2$

subject to
$$2x_1 + 4x_2 \le 28$$
$$5x_1 + 5x_2 \le 50$$
$$x_1 \le 8$$
$$x_2 \le 6$$

Minimize $w = 28y_1 + 50y_2 + 8y_3 + 6y_4$

subject to
$$2y_1 + 5y_2 + y_3 \ge 140$$
$$4y_1 + 5y_2 + y_4 \ge 160$$

⇨ Algebraic Form:

Maximize $z = \sum_{j=1}^{2} c_j x_j$

subject to $\sum_{j=1}^{2} a_{ij} x_j \le b_i \quad i = 1,2,\ldots 4$

Minimize $w = \sum_{i=1}^{4} b_i y_i$

subject to $\sum_{i=1}^{4} a_{ji} y_i \ge c_j \quad j = 1,2$

⇨ Matrix Form:

Maximize $z = [140 \quad 160] \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

subject to $\begin{bmatrix} 2 & 4 \\ 5 & 5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \begin{bmatrix} 28 \\ 50 \\ 8 \\ 6 \end{bmatrix}$

Minimize $w = [28 \quad 50 \quad 8 \quad 6] \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$

subject to $\begin{bmatrix} 2 & 5 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \ge \begin{bmatrix} 140 \\ 160 \end{bmatrix}$

- DATA ENVELOPMENT ANALYSIS
  - □ Production vs. Performance
  - □ Efficiency vs. Productivity
  - □ Total Factor Productivity (TFP) vs. Production Possibility Set (PPS)
  - □ Key Components of the Data Envelopment Analysis (DEA)
    - ⇨ Decision Making Unit (DMU)
    - ⇨ Inputs and Outputs
    - ⇨ Returns to Scale
    - ⇨ Orientations
    - ⇨ Reference (Peer) DMU
    - ⇨ Technical Efficiency and Scale Efficiency
  - □ Formulations of the Charnes, Cooper, and Rhodes (CCR) DEA Model

Table 1.1 CCR DEA model

Envelopment model		Multiplier model
Input-oriented		
$\min \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$		$\max z = \sum_{r=1}^{s} \mu_r y_{ro}$
subject to		subject to
$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta x_{io}$	$i = 1, 2, \ldots, m;$	$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0$
$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro}$	$r = 1, 2, \ldots, s;$	$\sum_{i=1}^{m} v_i x_{io} = 1$
$\lambda_j \ge 0$	$j = 1, 2, \ldots, n$	$\mu_r, v_i \ge \varepsilon > 0$
Output-oriented		
$\max \varphi + \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$		$\min q = \sum_{i=1}^{m} v_i x_{io}$
subject to		subject to
$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{io}$	$i = 1, 2, \ldots, m;$	$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} \ge 0$
$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = \varphi y_{ro}$	$r = 1, 2, \ldots, s;$	$\sum_{r=1}^{s} \mu_r y_{ro} = 1$
$\lambda_j \ge 0$	$j = 1, 2, \ldots, n.$	$\mu_r, v_i \ge \varepsilon > 0$

  - □ Variable Return to Scale (VRS) => Banker, Charnes, and Cooper (BBC) Model
  - □ Malmquist Productivity Indexes and DEA

- HOMEWORK #3 (2013/03/26 Due)：*Solve* the second example (2 inputs) presented by **Liu** using What'sBest, DEAP and MaxDEA.