

國立臺北大學自然資源與環境管理研究所  
101 學年度第二學期 『環境系統分析專題』

課程講義(十一、十二)：不確定性分析與隨機規劃  
Uncertainty Analysis and Stochastic Programming

Modelling Uncertainty ([http://dSPACE.library.cornell.edu/bitstream/1813/2804/13/08\\_chapter08.pdf](http://dSPACE.library.cornell.edu/bitstream/1813/2804/13/08_chapter08.pdf))  
Model Sensitivity and Uncertainty Analysis  
([http://dSPACE.library.cornell.edu/bitstream/1813/2804/12/09\\_chapter09.pdf](http://dSPACE.library.cornell.edu/bitstream/1813/2804/12/09_chapter09.pdf))  
Uncertainty analysis in *Encyclopedia of Environmetrics*  
([http://www.web-e.stat.vt.edu/vining/smith/u001-\\_o.pdf](http://www.web-e.stat.vt.edu/vining/smith/u001-_o.pdf))

● INTRODUCTION TO UNCERTAINTY ANALYSIS

- Types of Uncertainty
  - ⇒ Parametric Uncertainty
  - ⇒ Model or Structural Uncertainty
  - ⇒ Surprise/Indeterminacy
- Uncertainty Analysis
  - ⇒ Mathematical (Quantitative) Analyses Related to the Uncertainties about ‘Systems’
  - ⇒ System Uncertainties: Uncertainties about Measurement, Modeling, and Parameters
- Modeling Uncertainty
  - ⇒ Generating Values From Known Probability Distributions
  - ⇒ Monte Carlo Simulation
  - ⇒ Chance Constrained Models
  - ⇒ Markov Processes and Transition Probabilities
  - ⇒ Stochastic Optimization

● PROBABILITY THEORY, STOCHASTIC PROCESS AND RANDOM FIELD

- Deterministic vs. Stochastic Systems
  - ⇒ Vagueness, Uncertainty and ‘Stochasticity’
  - ⇒ Possibility, Likelihood, and Probability
- Probability Theory
  - ⇒ The Axioms of Probability
  - ⇒ Random Variables: Discrete and Continuous
  - ⇒ Statistics (Moments) of a Random Variable: Expected Value, Variance ...etc.
  - ⇒ Multiple Random Variables: Multivariate Statistics => Covariances
  - ⇒ Distribution: Probability Density Function, Cumulated Distribution Function
  - ⇒ Conditional Probability and Baye’s Theorem => Bayesian Decision Analysis
- Normal Distribution
  - ⇒ Two-Parameter Distribution: Location and Dispersion => Mean and Variance
  - ⇒ Standardization and *t*-Distribution
  - ⇒ Confidence Interval and Standard Deviation
  - ⇒ Multivariate Gaussian Distribution
- Stochastic Process
  - ⇒ Serial Random Variables: Temporal, Spatial, Spatio-temporal Stochastic Processes

⇒ Serial Correlation ⇒ Deterministic Term (Trend) + Disturbance (Noise)

⇒ Poisson Process, Markov's Chains, and Random Walks

□ Random Field

⇒ Random Variables Distributed ('Regionalized') in Space

⇒ Spatial Variability (Correlation) ⇒ Trend + Disturbance

⇒ Geostatistics: Kriging (Simple, Ordinary, Universal...) ⇒ GIS

• STOCHASTIC PROGRAMMING

□ Uncertainties Related to Mathematical Programming Systems

⇒ Modeling Uncertainties: Assumptions, Objective Functions, and Constraints

Mathematical Program with Recourse: Multi-Stage Stochastic Programming

⇒ Uncertainties 'Embedded' in Decision Variables: Fuzziness, Grey Information...

(1) Intervals or Specified Ranges ⇒ Grey Numbers ⇒ Grey Programming

(2) Degree of Set Membership ⇒ Fuzzy Set ⇒ Fuzzy Programming

⇒ Uncertainties about Model Parameters: Coefficients of Objective Function, RHS,  $A_{ij}$

(1) Parameters (Coefficients) of the Optimization Model are Random Variables

(2) Treat Decision Variables as 'Deterministic Variables' to be determined

(3) Probabilistic Constraints ⇒ Chance-Constrained Programming

• CHANCE CONSTRAINED PROGRAMMING

□ What are Chance Constraints?

□ Significance Level ⇒ System Reliability

□ Row Independence ⇒ Independently and Identically Distributed (i.i.d.)

□ Right-Hand-Side Random ⇒ Univariate Normal Distribution

□ Technical Coefficients Random ⇒ Multivariate Normal Distribution

□ Row Dependence ⇒ Joint Chance Constraint (relatively complicated!)

Chance Constraints:  $p\left(\sum_{j=1}^n a_{ij} \cdot x_j \leq b_i\right) \geq 1 - \alpha_i; \quad \forall i = 1, \dots, m$

(1) RHS  $b_i$  Random: Univariate probability distribution of  $b_i$

i.  $\alpha \equiv \geq$

$$p\left(\sum_{j=1}^n a_{ij} \cdot x_j \geq b_i\right) \geq 1 - \alpha_i \Rightarrow p\left(b_i \leq \sum_{j=1}^n a_{ij} \cdot x_j\right) \geq 1 - \alpha_i \Rightarrow F(b_i = \sum a_{ij} x_j) \geq 1 - \alpha_i$$

ii.  $\alpha \equiv \leq$

$$p\left(\sum_{j=1}^n a_{ij} \cdot x_j \leq b_i\right) \geq 1 - \alpha_i \Rightarrow p\left(b_i \geq \sum_{j=1}^n a_{ij} \cdot x_j\right) \geq 1 - \alpha_i \Rightarrow 1 - F(b_i = \sum a_{ij} x_j) \geq 1 - \alpha_i$$

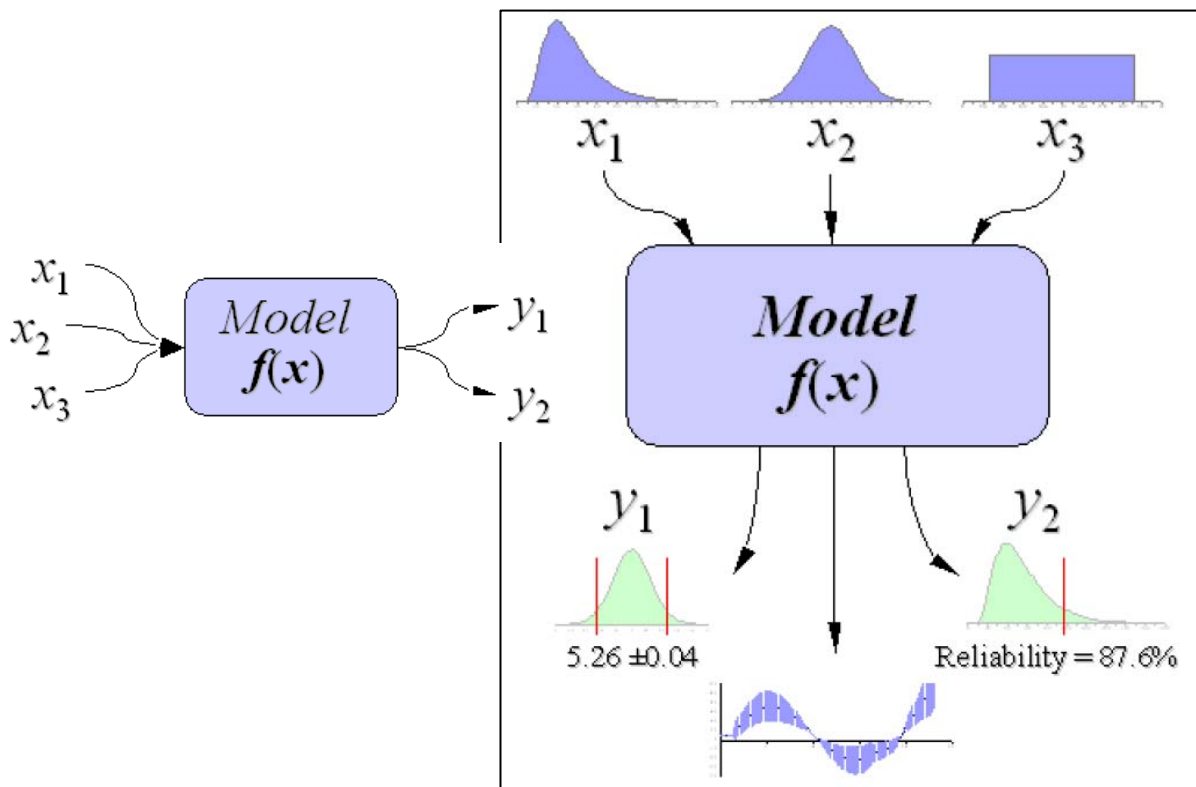
(2) Technical Coefficients  $a_{ij}$  Random: Multivariate probability distribution of  $\sum a_{ij} x_j$

⇒ Variance-Covariance Matrix: Positively definite (symmetric) matrix

• MONTE CARLO SIMULATION

□ Characteristics of Monte Carlo Simulation

- ⇒ Quantitative Risk Analysis
- ⇒ Simulation and then Optimization
- Monte Carlo Simulation Steps
  - Step 1: Create a parametric model,  $y = f(x_1, x_2, \dots, x_q)$ .
  - Step 2: Generate a set of random inputs,  $x_1^i, x_2^i, \dots, x_q^i$ .
  - Step 3: Evaluate the model and store the results as  $y^i$ .
  - Step 4: Repeat steps 2 and 3 for  $i = 1 \dots n$ .
  - Step 5: Analyze the results using histograms, statistics, confidence intervals, etc.
- Stages involved in Producing a Monte Carlo Risk Analysis Model
  - ⇒ Designing the structure of the risk analysis model
  - ⇒ Defining distributions that describe the uncertainty of the problem
  - ⇒ Modeling dependencies between model uncertainties
  - ⇒ Presenting and interpreting the risk analysis results
- Software Packages that can do Monte Carlo Risk Analysis
  - ⇒ Palisade @RISK; Oracle Crystal Ball



(<http://www.vertex42.com/ExcelArticles/mc/MonteCarloSimulation.html>)

- HOMEWORK #6 (2013/05/14 Due)
  - Please install Oracle Crystal Ball and try out the examples of:
    1. Risk Assessment at a Toxic Waste Site (Toxic Waste Site.xls)
    2. Groundwater Cleanup at a Toxic Waste Site (Groundwater Cleanup.xls)