

國立臺北大學自然資源與環境管理研究所
103 學年度第二學期 『環境系統分析專題』

課程講義(03)：線性規劃回顧與進階主題
Review of Linear Programming and Advanced Topics

W.: [What'sBest! Version 12.0 User's Manual](#)
L.: [Lindo Systems LINGO Version 14.0 User's Manual](#)
G.: [A GAMS Tutorial](#) and [GAMS - A User's Guide](#)
F.: [線性規劃 \(Linear Programming\)－方述誠](#)
D.: [Linear Programming and Extensions by George Bernard Dantzig](#)

● COMPONENTS OF AN OPTIMIZATION MODEL

- Objective Function(s)
 - ⇒ Single vs. Multiple
 - ⇒ Linear vs. Nonlinear
 - ⇒ Convex (Concave) vs. Non-convex
- Constraints
 - ⇒ Constrained vs. Un-constrained
 - ⇒ Linear vs. Nonlinear
 - ⇒ Convex vs. Non-convex Feasible Regions
- Decision Variables
 - ⇒ Continuous vs. Discrete
 - ⇒ Deterministic vs. Stochastic
- System Parameters (Coefficients)
 - ⇒ Deterministic vs. Stochastic
 - ⇒ Division into Sub-Models
- Formulation of Optimization Models
 - ⇒ Plain Form: Straightforward but not suitable for large-scaled or complex problems
 - ⇒ Algebraic Formulations => Parameters (Scalars), Vectors, and Matrices (Tables)
 - ⇒ Algebraic Formulations with text description of variables and parameters
 - ⇒ Sets and Indices

● PROPERTIES OF AN LP

- Proportionality, Additivity, Divisibility, Certainty, and Non-Negativity
- Non-negative Decision Variables => What if negative values are needed?
- A “Convex Programming” Model
- Additional Terminology
 - ⇒ Feasible Region or Solution Space
 - ⇒ Vertex, Extreme Points or Corner Points
 - ⇒ Decision Space or Objective Space

● SOLUTION PROCEDURE OF AN LP

- Pre-Optimal Analysis, Optimization (Solution) and Post-Optimization Analysis
- Graphical, Simplex, Dual Simplex, Interior Point and Other Methods
- Infeasible, Un-bounded and Degenerate Solutions
- A “Convex Programming” Model: Feasible Region and Extreme Points

- ⇒ Characteristics of Feasible Region for the LP: Convex, Compact, and Continuous
- ⇒ Extreme Points (Corner Points) vs. Interior Points

- **THE SIMPLEX METHOD**

- Augmented Form of the LP Models
 - ⇒ “Less-than-and-equal-to” Inequality constraints => Slack variables
 - ⇒ “Greater-than-and-equal-to” Inequality constraints => Surplus & Artificial Variables
 - ⇒ Equality constraints => Artificial variables => ‘Big-M Treatment’
- Terminology and Procedure of the Simplex Method
 - ⇒ Basic vs. non-basic variables
 - ⇒ Feasible basic solution => “Adjacent”
 - ⇒ Ratio test for Pivoting
 - ⇒ “Optimality”
- Simplex Tableaus and An Animated Presentation

- **EXAMPLES OF LINEAR PROGRAMMING**

- Homewood Masonry -- A Material Production Problem
 - ⇒ Objective Function: Maximizing the production profit
 - ⇒ Decision Variables: Two building products to be produced
 - ⇒ Constraints: Resource availability, work hours, and curing vat capacity

Resource	HYDIT	FILIT	Availability
Wahash Red Clay	2 m ³ /ton	4 m ³ /ton	28 m ³ /wk
Blending time	5 hr/ton	5 hr/ton	50 hr/wk
Curing vat capacity	8 tons	6 tons	
Profit	\$140/ton	\$160/ton	

- **SENSITIVITY ANALYSIS**

- Overview and Post-Optimality Analysis
- Sensitivity Analysis on RHS (Resource) Coefficients
 - ⇒ Shadow price, marginal value of a resource and economic interpretation
 - ⇒ Dual price (?)
- Sensitivity Analysis on Objective Function Coefficients
- Graphical Illustration
- Outputs from Optimization Packages and Analytical Interpretation (?)
- Parametric Programming

- **DUALITY THEORY**

- Model Formulations
- Dual-Primal Relationships
 - ⇒ Implementation from Production Problem
 - ⇒ Implementation from Resource Allocation Problem
- Shadow Price
- Primal-Dual Methods for Optimization (Lagrange Algorithms)

- **HOMEWORK #2 (2015/03/17 Due) :** *Solve* the example problem of Homewood Masonry (ReVelle et al., 2004) by using What’sBest, LINGO, and GAMS.