

國立臺北大學自然資源與環境管理研究所

103 學年度第二學期 『環境系統分析專題』

課程講義(05-06)：績效評估與資料包絡分析
Performance Evaluation and Data Envelopment Analysis

黃鏡如、傅祖壇、黃美瑛，2008，績效評估：效率與生產力之理論與應用，新陸書局，台北。
 吳濟華、何柏正，2008，組織效率與生產力評估：資料包絡分析法，前程文化，台北。
 Cooper, W.W., L.M. Seiford, and J. Zhu (editors), 2011, *Handbook on Data Envelopment Analysis*,
 2nd ed., Springer, New York.
 MaxDEA package and manual => <http://www.maxdea.cn/>
 DEAP: A Data Envelopment Analysis (Computer) Program => [DEAP](#)
[Data Envelopment Analysis](#). 資料包絡分析. 長榮大學 國企系教授 劉春初 (Liu)
[資料包絡分析法](#). 黃嘉彥教授. 勤益科技大學 研發與科技管理研究所

● FORMULATIONS OF LINEAR PROGRAMMING

□ Homewood Masonry -- A Material Production Problem

Resource	HYDIT	FILIT	Availability
Wahash Red Clay	2 m ³ /ton	4 m ³ /ton	28 m ³ /wk
Blending time	5 hr/ton	5 hr/ton	50 hr/wk
Curing vat capacity	1	0	8 tons
	0	1	6 tons
Profit	\$140/ton	\$160/ton	

⇒ Plain Form:

$$\begin{aligned} \text{Maximize } z &= 140x_1 + 160x_2 \\ \text{subject to } 2x_1 + 4x_2 &\leq 28 \\ 5x_1 + 5x_2 &\leq 50 \\ x_1 &\leq 8 \\ x_2 &\leq 6 \end{aligned}$$

$$\begin{aligned} \text{Minimize } w &= 28y_1 + 50y_2 + 8y_3 + 6y_4 \\ \text{subject to } 2y_1 + 5y_2 + y_3 &\geq 140 \\ 4y_1 + 5y_2 + y_4 &\geq 160 \end{aligned}$$

⇒ Algebraic Form:

$$\begin{aligned} \text{Maximize } z &= \sum_{j=1}^2 c_j x_j \\ \text{subject to } \sum_{j=1}^2 a_{ij} x_j &\leq b_i \quad i = 1, 2, \dots, 4 \end{aligned}$$

$$\begin{aligned} \text{Minimize } w &= \sum_{i=1}^4 b_i y_i \\ \text{subject to } \sum_{i=1}^4 a_{ji} y_i &\geq c_j \quad j = 1, 2 \end{aligned}$$

⇒ Matrix Form:

$$\begin{aligned} \text{Maximize } z &= [140 \quad 160] \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ \text{subject to } \begin{bmatrix} 2 & 4 \\ 5 & 5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &\leq \begin{bmatrix} 28 \\ 50 \\ 8 \\ 6 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{Minimize } w &= [28 \quad 50 \quad 8 \quad 6] \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \\ \text{subject to } \begin{bmatrix} 2 & 5 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} &\geq \begin{bmatrix} 140 \\ 160 \end{bmatrix} \end{aligned}$$

• DATA ENVELOPMENT ANALYSIS

- Production vs. Performance; Efficiency vs. Productivity
 - ⇒ 績效的評量方法應符合以下五種原則：明確性 (Specific)、可衡量的 (Measurable)、可達成的 (Attainable)、具挑戰性的 (Realistic)、切合時宜的 (Timely) (劉偉業, 1999)
 - ⇒ 效率評估方法：平衡計分卡 (Balance Score Card, BSC)、比例分析法 (Ratio Analysis)、迴歸分析法 (Regression Analysis)、財務比率分析法、多準則評估法、生產力及生產效率分析法、資料包絡分析法 (<http://www.wunan.com.tw/www2/download/preview/1FQF.PDF>)
- Total Factor Productivity (TFP) vs. Production Possibility Set (PPS)
- Key Components of the Data Envelopment Analysis (DEA)
 - ⇒ Decision Making Unit (DMU) => Technical vs. Scale Efficiencies; Reference (Peer) DMU
 - ⇒ Inputs and Outputs => Formulation Orientations => Returns to Scale
- Formulations of the Charnes, Cooper, and Rhodes (CCR) DEA Model

Table 1.1 CCR DEA model

Input-oriented	
Envelopment model	Multiplier model
$\min \theta - \varepsilon \left(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right)$	$\max z = \sum_{r=1}^s \mu_r y_{ro}$
subject to	subject to
$\sum_{j=1}^n x_{ij} \lambda_j + s_i^- = \theta x_{io} \quad i = 1, 2, \dots, m;$	$\sum_{r=1}^s \mu_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0$
$\sum_{j=1}^n y_{rj} \lambda_j - s_r^+ = y_{ro} \quad r = 1, 2, \dots, s;$	$\sum_{i=1}^m v_i x_{io} = 1$
$\lambda_j \geq 0 \quad j = 1, 2, \dots, n$	$\mu_r, v_i \geq \varepsilon > 0$
Output-oriented	
Envelopment model	Multiplier model
$\max \varphi + \varepsilon \left(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right)$	$\min q = \sum_{i=1}^m v_i x_{io}$
subject to	subject to
$\sum_{j=1}^n x_{ij} \lambda_j + s_i^- = x_{io} \quad i = 1, 2, \dots, m;$	$\sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s \mu_r y_{rj} \geq 0$
$\sum_{j=1}^n y_{rj} \lambda_j - s_r^+ = \varphi y_{ro} \quad r = 1, 2, \dots, s;$	$\sum_{r=1}^s \mu_r y_{ro} = 1$
$\lambda_j \geq 0 \quad j = 1, 2, \dots, n.$	$\mu_r, v_i \geq \varepsilon > 0$

- Variable Return to Scale (VRS) => Banker, Charnes, and Cooper (BBC) Model
- Malmquist Productivity Indexes and DEA

• HOMEWORK #3 (2015/04/08 Due) : Solve the second example (2 inputs) presented by Liu using What'sBest, DEAP and MaxDEA.