國立臺北大學自然資源與環境管理研究所
103 學年度第二學期『環境系統分析專題』

課程講義(15)：多目標規劃
Multiobjective Programming

International Doctoral School Algorithmic Decision Theory: MCDA and MOO Lecture 2: Multiobjective Linear Programming
Applied Mathematical Programming using Algebraic Systems Chapter XI Multi-Objective Programming
Multiobjective programming and planning - Google Books
(http://books.google.com.tw/books/about/Multiobjective_programming_and_planning.html?id=GFtwGswaMKYC)

- MULTIOBJECTIVE PROGRAMMING
  - □ Conflicting between Objectives (Goals) => Trade-off among objectives
  - □ Non-dominance, Non-inferiority, "Efficiency," or "Pareto Optimality"
  - □ Terminology
    - ⇨ Feasible Solution (Feasible Region)
    - ⇨ Decision Space vs. Objective Space
    - ⇨ Tradeoff 抵換 vs. Pay-off 償付
    - ⇨ Noninferior Solution or "Best-Compromise Solution"非劣解
  - □ Categories of MOP Solution Methods
    - ⇨ Information Flow: Bottom-Up or Top-Down
    - ⇨ Techniques that Incorporate Preferences
  - □ Generating Techniques: Evaluating Alternatives, Decision Support
    - ⇨ Weighting method, Constraint method
    - ⇨ NISE algorithm for two-objective problems
    - ⇨ Multiobjective simplex method, and others
  - □ Number of Decision Makers

- NONINFERIOR SOLUTION GENERATING TECHNIQUES
  - □ Weighting Method
    - ⇨ Indifference Curve (Linear)
    - ⇨ Extreme Points (in Objective Space)
    - ⇨ Computing Procedure:
      1. Specify the weights (positive, normalized)
      2. Rearrange the objectives
      3. Find the optimal solutions
      4. Illustrate the solutions as points (extreme points) in decision space
      5. "Interpolate" the noninferior sets
  - □ Constraint Method
    - ⇨ Range of the Objectives
    - ⇨ Computing Procedure
      1. Find the ranges of the objectives (construct the payoff table)
      2. Specify number of intervals (constraints)
      3. Rearrange the programming model and find the optimal solutions
      4. Plotting the solutions in decision space

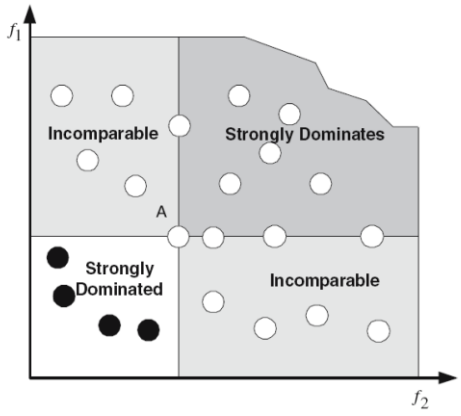
□ The NISE (Non-Inferior Set Estimation) Method (Cohon, 1978)

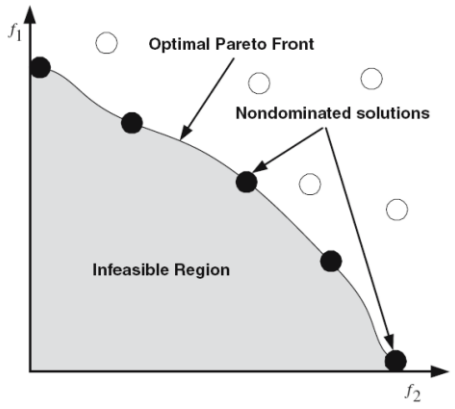
⇨ Working on the Objective Space

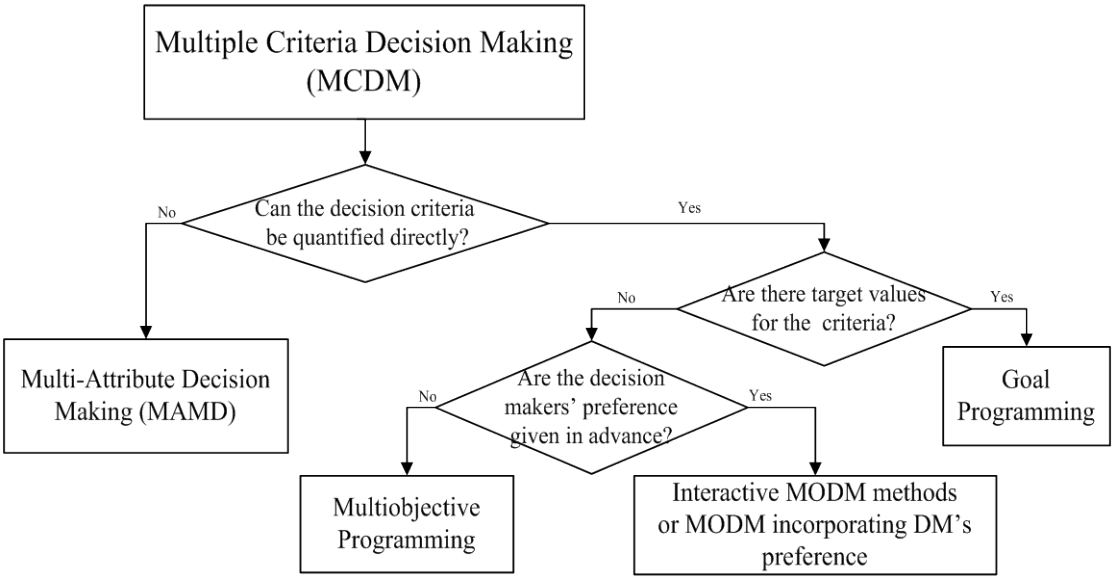
⇨ Convexity of the feasible region

⇨ The Algorithm (Calculation Procedure and Termination Criteria)

1. Optimize the objectives individually (construct the payoff table)
2. Find the weighted objective, optimize it and calculate the termination criterion
3. Decide whether stop or continue; repeat 2 if not stop







- HOMEWORK #8 (2015/06/16 due): Please use What'sBest and apply both the weighting method and the constraint method to solve the example illustrated in Cohon (1978). The model can be formulated as the following.

$$\text{Maximize } \mathbf{Z}(x_1, x_2) = [Z_1(x_1, x_2), Z_2(x_1, x_2)]$$

where

$$Z_1(x_1, x_2) = 5x_1 - 2x_2$$
$$Z_2(x_1, x_2) = -x_1 + 4x_2$$
$$\text{s.t.} \quad -x_1 + x_2 \le 3, \quad x_1 + x_2 \le 8$$
$$x_1 \le 6, \qquad x_2 \le 4$$