國立臺北大學自然資源與環境管理研究所
105 學年度第二學期『環境系統分析專題』

課程講義(12~13)：不確定性分析與隨機規劃
Uncertainty Analysis and Stochastic Programming

Chapter 8. Modelling Uncertainty and Chapter 9. Model Sensitivity and Uncertainty Analysis in *Water Resources Systems Planning and Management: An Introduction to Methods, Models and Applications* by Loucks, D.P., E. van Beek, J.R.Stedinger, J.P.M. Dijkman, and M.T. Villars, Paris: UNESCO (2005)

Uncertainty analysis in *Encyclopedia of Environmetrics* (http://www.web-e.stat.vt.edu/vining/smith/u001-_o.pdf)

- INTRODUCTION TO UNCERTAINTY ANALYSIS
  - □ Types of Uncertainty (in the fields of modeling)
    - ⇨ Parametric Uncertainty
    - ⇨ Model or Structural Uncertainty
    - ⇨ Surprise/Indeterminacy
  - □ Uncertainty Analysis
    - ⇨ Mathematical (Quantitative) Analyses Related to the Uncertainties about 'Systems'
    - ⇨ System Uncertainties: Uncertainties about Measurement, Modeling, and Parameters
  - □ Modeling Uncertainty
    - ⇨ Generating Values From Known Probability Distributions
    - ⇨ Monte Carlo Simulation
    - ⇨ Chance Constrained Models
    - ⇨ Markov Processes and Transition Probabilities
    - ⇨ Stochastic Optimization
  - □ Sensitivity Analysis: Sensitivity analysis is used to determine the importance of different parameters and components of the model on the output of the model.

- PROBABILITY THEORY, STOCHASTIC PROCESS AND RANDOM FIELD
  - □ Deterministic vs. Stochastic Systems
    - ⇨ Vagueness, Uncertainty and 'Stochasticity'
    - ⇨ Possibility, Likelihood, and Probability
    - ⇨ Response = Deterministic component + Stochastic component + Error
  - □ Probability Theory
    - ⇨ The Axioms of Probability
    - ⇨ Random Variables: Discrete and Continuous
    - ⇨ Statistics (Moments) of a Random Variable: Expected Value, Variance …etc.
    - ⇨ Multiple Random Variables: Multivariate Statistics => Covariance
    - ⇨ Distribution: Probability Density Function, Cumulated Distribution Function
    - ⇨ Conditional Probability and Bayes' Theorem => Bayesian Decision Analysis
  - □ Normal Distribution
    - ⇨ Two-Parameter Distribution: Location and Dispersion => Mean and Variance
    - ⇨ Standardization and *t*-Distribution
    - ⇨ Confidence Interval and Standard Deviation
    - ⇨ Multivariate Gaussian Distribution
  - □ Stochastic Process
    - ⇨ Serial Random Variables: Temporal, Spatial, Spatio-temporal Stochastic Processes

- ⇨ Serial Correlation => Deterministic Term (Trend) + Disturbance (Noise)
- ⇨ Poisson Process, Markov's Chains, and Random Walks

- □ Random Field
  - ⇨ Random Variables Distributed ('Regionalized') in Space
  - ⇨ Spatial Variability (Correlation) => Trend + Disturbance
  - ⇨ Geostatistics: Kriging (Simple, Ordinary, Universal…) => GIS

- STOCHASTIC PROGRAMMING
  - □ Uncertainties Related to Mathematical Programming Systems
    - ⇨ Modeling Uncertainties: Assumptions, Objective Functions, and Constraints
      Mathematical Program with Recourse: Multi-Stage Stochastic Programming
    - ⇨ Uncertainties 'Embedded' in Decision Variables: Fuzziness, Grey Information…
      (1) Intervals or Specified Ranges => Grey Numbers => Grey Programming
      (2) Degree of Set Membership => Fuzzy Set => Fuzzy Programming
    - ⇨ Uncertainties about Model Parameters: Coefficients of Objective Function, RHS, A_{ij}
      (1) Parameters (Coefficients) of the Optimization Model are Random Variables
      (2) Treat Decision Variables as 'Deterministic Variables' to be determined
      (3) Probabilistic Constraints => Chance-Constrained Programming

- CHANCE CONSTRAINED PROGRAMMING
  - □ What are Chance Constraints?
  - □ Significance Level => System Reliability
  - □ Row Independence => Independently and Identically Distributed (i.i.d.)
  - □ Right-Hand-Side Random => Univariate Normal Distribution
  - □ Technical Coefficients Random => Multivariate Normal Distribution
  - □ Row Dependence => Joint Chance Constraint (relatively complicated!)

Chance Constraints: $p\left(\sum_{j=1}^{n} a_{ij}\cdot x_j \propto b_i\right) \ge 1-\alpha_i;\quad \forall i = 1,\cdots m$

(1) RHS b_i Random: Univariate probability distribution of b_i

i. $\propto \equiv \ge$

$$p\left(\sum_{j=1}^{n} a_{ij}\cdot x_j \ge b_i\right) \ge 1-\alpha_i \Rightarrow p\left(b_i \le \sum_{j=1}^{n} a_{ij}\cdot x_j\right) \ge 1-\alpha_i \Rightarrow F(b_i = \sum a_{ij}x_j) \ge 1-\alpha_i$$

$$(\sum a_{ij}x_j - \mu_{b_i})/\sigma_{b_i} \ge F_z^{-1}(1-\alpha_i) \Rightarrow \sum a_{ij}x_j \ge \mu_{b_i} + F_z^{-1}(1-\alpha_i)\cdot\sigma_{b_i}$$

ii. $\propto \equiv \le$

$$p\left(\sum_{j=1}^{n} a_{ij}\cdot x_j \le b_i\right) \ge 1-\alpha_i \Rightarrow p\left(b_i \ge \sum_{j=1}^{n} a_{ij}\cdot x_j\right) \ge 1-\alpha_i \Rightarrow 1-F(b_i = \sum a_{ij}x_j) \ge 1-\alpha_i$$

$$F(b_i = \sum a_{ij}x_j) \le \alpha_i \Rightarrow (\sum a_{ij}x_j - \mu_{b_i})/\sigma_{b_i} \le F_z^{-1}(\alpha_i) \Rightarrow \sum a_{ij}x_j \le \mu_{b_i} + F_z^{-1}(\alpha_i)\cdot\sigma_{b_i}$$

(2) Technical Coefficients a_{ij} Random: Multivariate probability distribution of $\sum a_{ij}x_j$

- ⇨ Variance-Covariance Matrix: Positively definite (symmetric) matrix

- MONTE CARLO SIMULATION
  - Characteristics of Monte Carlo Simulation
    - ⇨ Quantitative Risk Analysis
    - ⇨ Simulation and then Optimization
  - Monte Carlo Simulation Steps

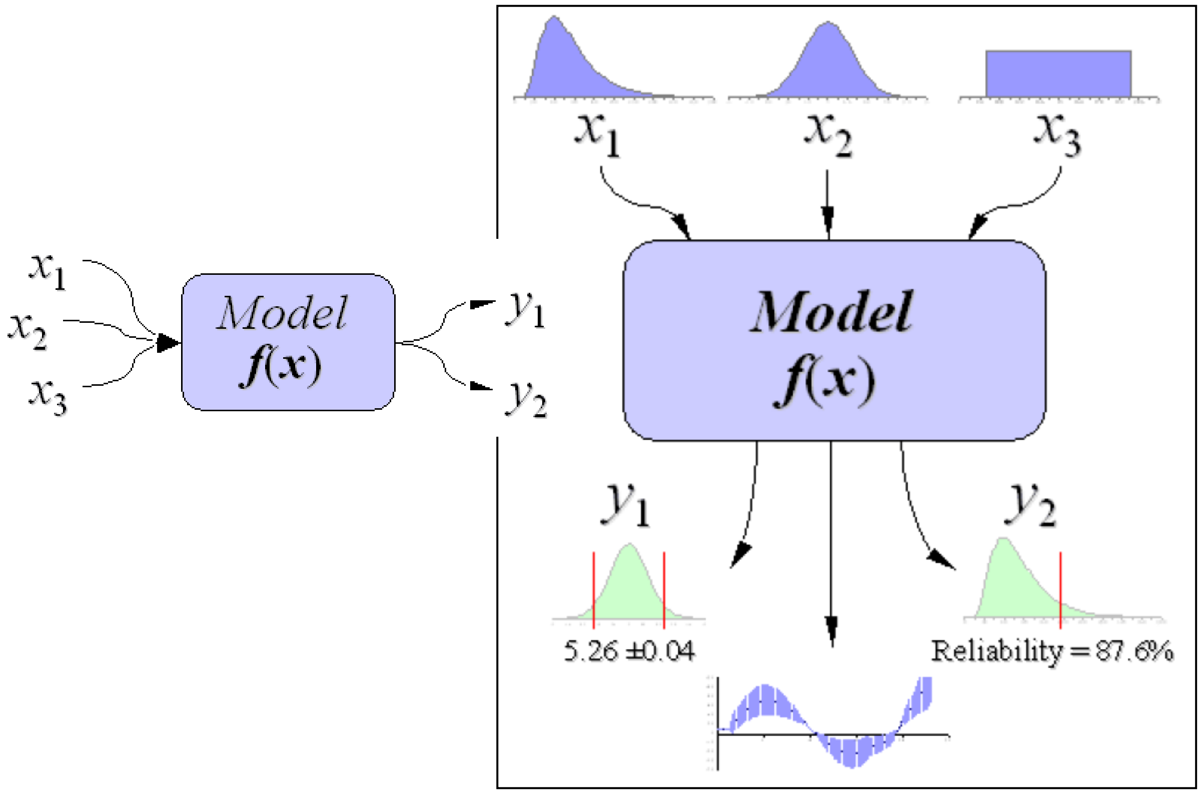
    Step 1: Create a parametric model, $y = f(x_1, x_2, \cdots, x_q)$.

    Step 2: Generate a set of random inputs, $x_1^i, x_2^i, \cdots, x_q^i$.

    Step 3: Evaluate the model and store the results as y^i .

    Step 4: Repeat steps 2 and 3 for $i = 1 \cdots n$.

    Step 5: Analyze the results using histograms, statistics, confidence intervals, etc.
  - Stages involved in Producing a Monte Carlo Risk Analysis Model
    - ⇨ Designing the structure of the risk analysis model
    - ⇨ Defining distributions that describe the uncertainty of the problem
    - ⇨ Modeling dependencies between model uncertainties
    - ⇨ Presenting and interpreting the risk analysis results
    - ⇨ Software Packages for Monte Carlo Simulation: Palisade @RISK; Oracle Crystal Ball



(http://www.vertex42.com/ExcelArticles/mc/MonteCarloSimulation.html)

- HOMEWORK #6: MONTE CARLO SIMULATION (Practice by yourself)

  Please install Oracle Crystal Ball and practice the example of: Risk Assessment at a Toxic Waste Site (Toxic Waste Site.xls)