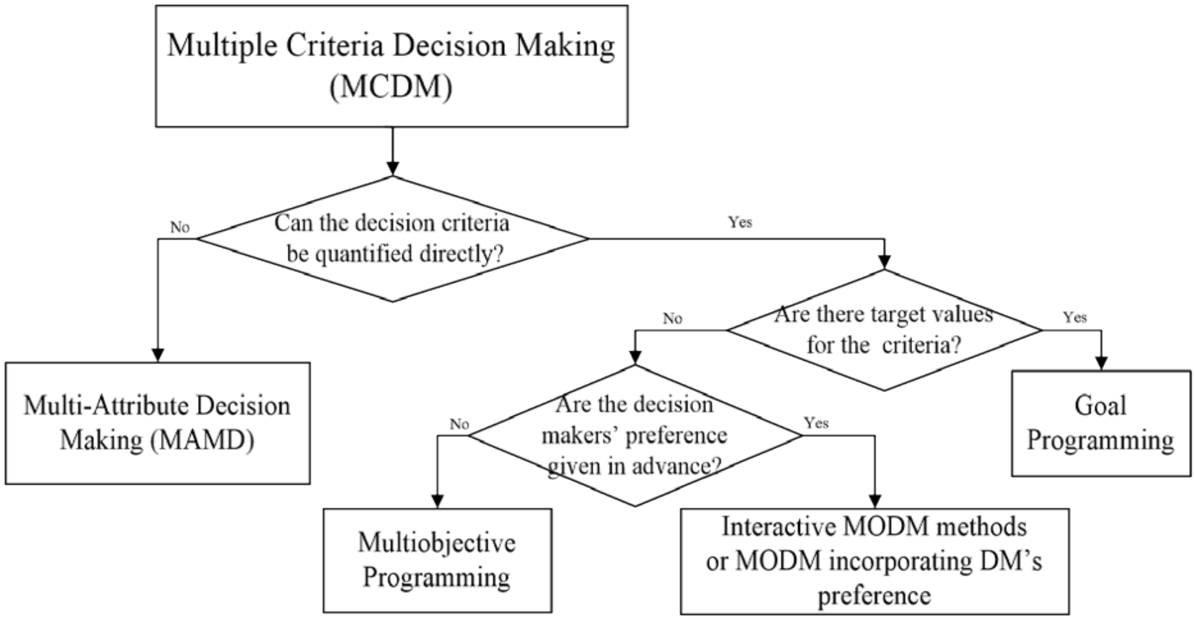
國立臺北大學自然資源與環境管理研究所
105 學年度第二學期『環境系統分析專題』

課程講義(16)：多目標規劃與多評準決策分析
Multiobjective Programming and Multi-Criteria Decision Making

International Doctoral School Algorithmic Decision Theory: Multiobjective Linear Programming
Applied Mathematical Programming using Algebraic Systems Chapter XI Multi-Objective Programming
Multiobjective programming and planning - Google Books
(http://books.google.com.tw/books/about/Multiobjective_programming_and_planning.html?id=GFtwGswaMKYC)
資料共享的「多評準決策理論與應用研究」網站介紹
MCDM Methods in http://en.wikipedia.org/wiki/Multi-criteria_decision_analysis

- MULTIOBJECTIVE PROGRAMMING
  - □ Conflicting between Objectives (Goals) => Trade-off among objectives
  - □ Non-dominance, Non-inferiority, "Efficiency," or "Pareto Optimality"
  - □ Terminology
    - ⇨ Decision Space vs. Objective Space
    - ⇨ Tradeoff 抵換 vs. Pay-off 償付
    - ⇨ Noninferior Solution or "Best-Compromise Solution"非劣解
  - □ Categories of MOP Solution Methods
    - ⇨ Information Flow: Bottom-Up or Top-Down
    - ⇨ Techniques that Incorporate Preferences
  - □ Generating Techniques: Evaluating Alternatives, Decision Support
    - ⇨ Weighting method, Constraint method
    - ⇨ NISE algorithm for two-objective problems
    - ⇨ Multiobjective simplex method, and others
  - □ Number of Decision Makers

- NONINFERIOR SOLUTION GENERATING TECHNIQUES
  - □ Weighting Method
    - ⇨ Indifference Curve (Linear)
    - ⇨ Extreme Points (in Objective Space)
    - ⇨ Computing Procedure:
      1. Specify the weights (positive, normalized)
      2. Rearrange the objectives
      3. Find the optimal solutions
      4. Illustrate the solutions as points (extreme points) in decision space
      5. "Interpolate" the noninferior sets
  - □ Constraint Method
    - ⇨ Range of the Objectives
    - ⇨ Computing Procedure
      1. Find the ranges of the objectives (construct the payoff table)
      2. Specify number of intervals (constraints)
      3. Rearrange the programming model and find the optimal solutions
      4. Plotting the solutions in decision space



- INTRODUCTION TO MULTICRITERIA DECISION MAKING
(Triantaphyllou, E., B. Shu, S.N. Sanchez, and T. Ray, 1998,"Multi-Criteria Decision Making: An Operations Research Approach", in *Encyclopedia of Electrical and Electronics Engineering*, 15:175-186)
  - Multiattribute Decision Making: A General Overview
    Multiattribute Decision Making is the most well known branch of decision making. It is a branch of a general class of Operations Research (or OR) models which deal with decision problems under the presence of a number of decision criteria. This super class of models is very often called multi-criteria decision making (or MCDM). According to many authors (see, e.g., [Zimmermann, 1991]) MCDM is divided into Multi-Objective Decision Making (or MODM) and Multiattribute Decision Making (or MADM). MODM studies decision problems in which the decision space is continuous. …. On the other hand, MADM concentrates on problems with discrete decision spaces. In these problems the set of decision alternatives has been predetermined.
  - MADM Methods (Malczewski, J.,1999. *GIS and Multicriteria Decision Analysis*. Wiley, N.Y.)
    - ⇨ Scoring (Weighted Sum)
    - ⇨ Multiattribute Value (MAVT)
    - ⇨ Multiattribute Utility (MAUT)
    - ⇨ Analytic Hierarchy Process (AHP)
    - ⇨ Ideal Point (TOPSIS)
    - ⇨ Concordance (ELECTRE)
    - ⇨ Ordered Weighted Averaging (Fuzzy TOPSIS?)

MADM Method Description (http://cost356.inrets.fr/pub/reference/reports/Ortega_aggregation_Turin07.pdf)

Scoring: They are based on the concept of a weighted average. The decision maker directly assigns weights of "relative importance" to each attribute. A total score is then obtained for each alternative by multiplying the importance weight assigned for each attribute by the scaled value given to the alternative on that attribute, and summing the products over all attributes. When the overall scores are calculated for all the alternatives, the alternative with the highest overall score is chosen. The decision rule evaluates each alternative, A_i, by the following formula:

$$A_i = \sum_j w_j \cdot x_{ij}$$

where x_{ij} is the score of the *i*th alternative with respect to the *j*th attribute, and the weight w_j is a normalized weight, so that $\sum w_j = 1$.

Multiattribute value: The value function approach is applicable in the decision situations under certainty (deterministic approach). This approach assumes that the decision maker is relatively "risk neutral" or that the attributes are known with certainty. Formally, the value function model is similar to "scoring method", except that the score x_{ij} is replaced by a value v_{ij} derived from the value function. The value function model can be written:

$$V_i = \sum_j w_j \cdot v_{ij}$$

where V_i is the overall value of the *i*th alternative, v_{ij} is the value of the *i*th alternative with respect to the *j*th attribute measured by means of the value function, and the weight w_j is a normalized weight or scaling constant for attribute *j*, so that $\sum w_j = 1$.

Multiattribute utility: In the utility function procedure, the decision's maker attitude toward risk is incorporated into assessment of a single-attribute utility function (Keeney, 1980). Thus utility is a convenient method of including uncertainty (risk preference) into decision-making process. The concept of a utility function is inherently probabilistic in nature. Formally, the utility function model is similar to "scoring method", except that the score x_{ij} is replaced by a utility u_{ij} derived from the utility function. The utility function model can be written:

$$U_i = \sum_j w_j \cdot u_{ij}$$

where U_i is the overall value of the *i*th alternative, u_{ij} is the utility of the *i*th alternative with respect to the *j*th attribute measured by means of the utility function, and the weight w_j is a normalized weight or scaling constant for attribute *j*, so that $\sum w_j = 1$.

Analytic hierarchy process: The analytical hierarchy process (AHP) method, developed by Saaty (1980), is based on tree principles: decomposition, comparative judgment and synthesis of priorities. The *decomposition principle* requires that the decision problem be decomposed into a hierarchy that captures the essential elements of the problem, the principle of *comparative judgment* requires assessment of pairwise comparisons of the elements within a given level of the hierarchical structure, with respect to their parent in the next-higher level, and the *synthesis principle* takes each of the derived ratio-scale local priorities in the various levels of the hierarchy and constructs a composite set of priorities for the elements at the lowest level of the hierarchy. In this final step, the goal is aggregate the relative weights of the levels obtained in the previous step to produce composite weights, this is done by means of a sequence of multiplications of the matrices of relative weights at each level of the hierarchy.

Ideal point methods: Ideal point methods order a set of alternatives on the basis of their separation from the ideal point. This point represents a hypothetical alternative that consists of the most deliverable weighted standardized levels of each criterion across the alternatives under consideration. The alternative that is closed to the ideal point is the best alternative. The separation is measured in terms of a distance metric. The ideal point decision rule is:

$$s_{i+} = \left[\sum_j w_j^p \left(v_{ij} - v_{+j} \right)^p \right]^{1/p}$$

where s_{i+} is the separation of the *i*th alternative from the ideal point, w_j is a weight assigned to the *j* criterion, v_{ij} is the standardized criterion value of the *i*th alternative, v_{+j} is the ideal value for the *j*th criterion, and *p* is a power parameter ranging from 1 to ∞.

Concordance methods: Concordance methods are based on a pairwise comparison of alternatives. They provide an ordinal ranking of the alternatives; that is, when two alternatives are compared, these methods can only express that alternative A is preferred to alternative B, but cannot indicate by how much. The most known concordance approach is the ELECTRE method and its modifications.

Ordered weighted averaging: Ordered weighted averaging is an aggregation technique based on the generalization of three basic types of aggregation functions, which are: (1) operators for the intersection of fuzzy set, (2) operators for the union of fuzzy sets, and (3) averaging operators. It provides continuous fuzzy aggregation operations between the fuzzy intersection and union, with a weighted-average combination falling midway in between.

- DECISION ANALYSIS AND MULTI-CRITERIA DECISION ANALYSIS METHODS
  - □ Decision Tree; Laplace, Maximin, Minimax, Hurwicz, Minimax Regret
  - □ Weighted Sum and Weighted Product
  - □ ELECTRE (ELimination Et Choix Traduisant la Réalité or Elimination and Choice Translating Reality)
  - □ TOPSIS (Technique for Order Preference by Similarity to Ideal Solution)
  - □ The Example Problem

	1.Criterion	**2.Criterion**	**3.Criterion**	**4.Criterion**
1.Alternative	0.120	0.129	0.119	0.456
2.Alternative	0.065	0.185	0.064	0.071
3.Alternative	0.569	0.068	0.484	0.170
4.Alternative	0.200	0.067	0.223	0.100
5.Alternative	0.045	0.551	0.109	0.203
Weights	0.137	0.347	0.065	0.452

    - ⇨ Test of Dominance => Alternative 4 is "dominated"!
    - ⇨ Weights of Criteria and Weighted Decision Matrix

- THE ANALYTIC HIERARCHY PROCESS (AHP)
  - □ Top Objective, Criteria, Sub-Criteria, Sub...-Criteria, Alternatives
  - □ Complete Hierarchy and Partial Hierarchy
  - □ Mathematical Fundamentals: Properties a Positive Reciprocal Matrix
  - □ Priority (Weighting) Vectors and Eigenvector
  - □ Inconsistency Index and Eigenvalues
    - ⇨ The Maximum Eigenvalue and Random Index
    - ⇨ Consistency Index or Consistency Ratio
  - □ Software Packages: Solving Linear Algebra Systems vs. ExpertChoice
  - □ Variations of AHP: Fuzzy AHP and Grey AHP (Preference Programming)
  - □ Analytical Network Process

- HOMEWORK #7 (2017/06/13 due): Please use What'sBest and apply both the weighting method and the constraint method to solve the example illustrated in Cohon (1978). The model can be formulated as the following.

$$\text{Maximize } \mathbf{Z}(x_1, x_2) = [Z_1(x_1, x_2), Z_2(x_1, x_2)]$$

where

$$Z_1(x_1, x_2) = 5x_1 - 2x_2$$
$$Z_2(x_1, x_2) = -x_1 + 4x_2$$
$$\text{s.t.} \quad -x_1 + x_2 \le 3, \quad x_1 + x_2 \le 8$$
$$x_1 \le 6, \qquad x_2 \le 4$$