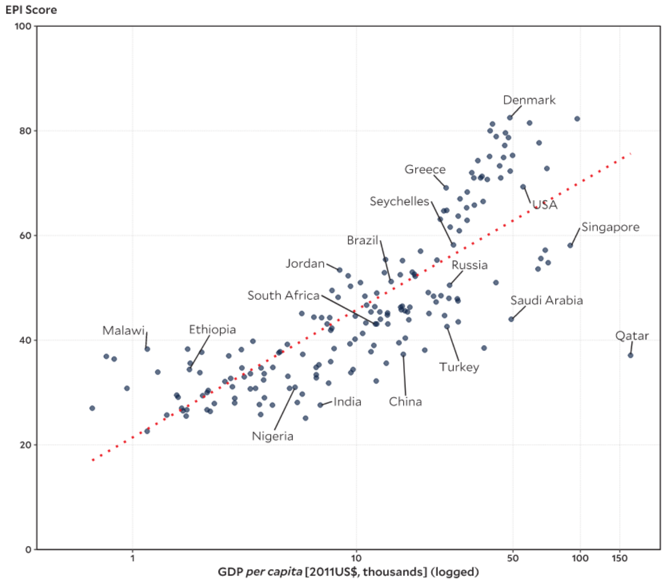
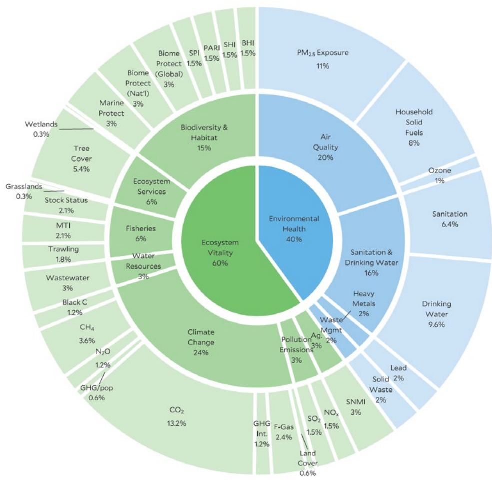
國立臺北大學自然資源與環境管理研究所
109 學年度第二學期『環境系統分析專題』

課程講義(03)：線性規劃回顧與進階主題

Review of Linear Programming and Advanced Topics

- **More on Composite Indicators**
  - □ Composite Indicators or Indices for Environmental Quality
    - ⇨ AQI (PSI); RPI (QWI); CTSI
    - ⇨ Risk Assessment => Cancerogenic Risk 10^{-6}; Hazard Index for Acute Toxicity
  - □ Environmental Performance Index (EPI)
    - ⇨ "Using 32 performance indicators across 11 issue categories, the EPI ranks 180 countries on environmental health and ecosystem vitality."
    - ⇨ Indicators => Transformation, Standardization, Normalization
    - ⇨ EPI => Weighted sum





The 2020 EPI Framework – Weights

  - □ The Global Risks Report 2021 (16th Edition)
    - ⇨ "Survey respondents (650 members) were asked to assess the likelihood and impact of the individual global risk on a scale of 1 to 5."

- COMPONENTS OF AN OPTIMIZATION MODEL
  - □ Objective Function(s)
    - ⇨ Single vs. Multiple
    - ⇨ Linear vs. Nonlinear
    - ⇨ Convex (Concave) vs. Non-convex
  - □ Constraints
    - ⇨ Constrained vs. Un-constrained
    - ⇨ Linear vs. Nonlinear
    - ⇨ Convex vs. Non-convex Feasible Regions
  - □ Decision Variables
    - ⇨ Continuous vs. Discrete
    - ⇨ Deterministic vs. Stochastic
  - □ System Parameters (Coefficients)
    - ⇨ Deterministic vs. Stochastic
    - ⇨ Division into Sub-Models
  - □ Formulation of Optimization Models
    - ⇨ Plain Form: Straightforward but not suitable for large-scaled or complex problems
    - ⇨ Algebraic Formulations => Parameters (Scalars), Vectors, and Matrices (Tables)
    - ⇨ Algebraic Formulations with text description of variables and parameters
    - ⇨ Sets and Indices => Equation Editor

- PROPERTIES OF AN LP
  - □ Proportionality, Additivity, Divisibility, Certainty, and Non-Negativity
  - □ Non-negative Decision Variables => What if negative values are needed?
  - □ A "Convex Programming" Model
  - □ Additional Terminology
    - ⇨ Feasible Region or Solution Space
    - ⇨ Vertex, Extreme Points or Corner Points
    - ⇨ Decision Space or Objective Space

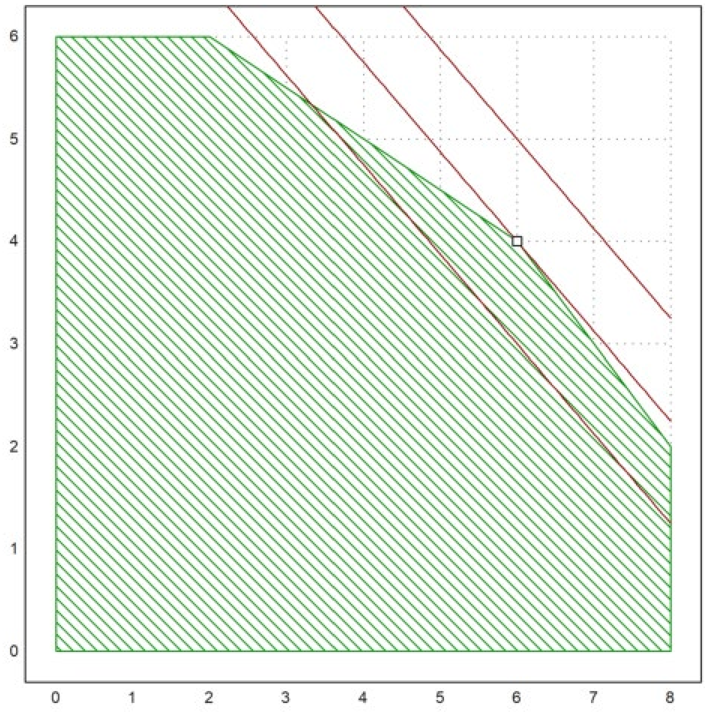
- SOLUTION PROCEDURE OF AN LP
  - □ Pre-Optimal Analysis, Optimization (Solution) and Post-Optimization Analysis
  - □ Graphical, Simplex, Dual Simplex, Interior Point and Other Methods
  - □ Infeasible, Un-bounded and Degenerate Solutions
  - □ A "Convex Programming" Model: Feasible Region and Extreme Points
    - ⇨ Characteristics of Feasible Region for the LP: Convex, Compact, and Continuous
    - ⇨ Extreme Points (Corner Points) vs. Interior Points

- THE SIMPLEX METHOD
  - □ Augmented Form of the LP Models
    - ⇨ "Less-than-and-equal-to" Inequality constraints => Slack variables
    - ⇨ "Greater-than-and-equal-to" Inequality constraints => Surplus & Artificial Variables
    - ⇨ Equality constraints => Artificial variables => 'Big-M Treatment'
  - □ Terminology and Procedure of the Simplex Method
    - ⇨ Basic vs. non-basic variables
    - ⇨ Feasible basic solution => "Adjacent"

- ⇨ Ratio test for Pivoting
- ⇨ "Optimality"
- □ Simplex Tableaus and [An Animated Presentation]

- **EXAMPLES OF LINEAR PROGRAMMING**
  - □ Homewood Masonry -- A Material Production Problem
    - ⇨ Objective Function: Maximizing the production profit
    - ⇨ Decision Variables: Two building products to be produced
    - ⇨ Constraints: Resource availability, work hours, and curing vat capacity

Resource	HYDIT	FILIT	Availability
Wahash Red Clay	2 m^3/ton	4 m^3/ton	28 m^3/wk
Blending time	5 hr/ton	5 hr/ton	50 hr/wk
Curing vat capacity	8 tons	6 tons	
Profit	$140/ton	$160/ton	



1. Algebraic formulation with numerical coefficients

$$\text{Maximize Profit } z = 140x_1 + 160x_2$$

Subject to

$$2x_1 + 4x_2 \leq 28$$
$$5x_1 + 5x_2 \leq 50$$
$$x_1 \leq 8$$
$$x_2 \leq 6$$

2. Algebraic formulation with symbolic coefficients

$$\text{Maximize Profit } z = \sum_{j=1}^{2} c_j x_j$$

Subject to

$$\sum_{j=1}^{2} a_{ij} x_j \le b_i; \quad i = 1, \cdots, 4$$

$$\boldsymbol{c} = \{c_j\} = [140, 160] \quad \boldsymbol{b} = \{b_i\} = \begin{bmatrix} 28 \\ 50 \\ 8 \\ 6 \end{bmatrix} \quad \boldsymbol{A} = \{a_{ij}\} = \begin{bmatrix} 2 & 4 \\ 5 & 5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3. Matrix formulation

$$\text{Maximize Profit } z = \boldsymbol{c}' \cdot \boldsymbol{x}$$

$$z = [140, 160] \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Subject to

$$\boldsymbol{A} \cdot \boldsymbol{x} \le \boldsymbol{b}$$

$$\begin{bmatrix} 2 & 4 \\ 5 & 5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \begin{bmatrix} 28 \\ 50 \\ 8 \\ 6 \end{bmatrix}$$

- SENSITIVITY ANALYSIS
  - □ Overview and Post-Optimality Analysis
  - □ Sensitivity Analysis on RHS (Resource) Coefficients
    - ⇨ Shadow price, marginal value of a resource and economic interpretation
  - □ Sensitivity Analysis on Objective Function Coefficients
  - □ Graphical Illustration
  - □ Parametric Programming

- DUALITY THEORY
  - □ Model Formulations
  - □ Dual-Primal Relationships
    - ⇨ Implementation from Production Problem
    - ⇨ Implementation from Resource Allocation Problem
  - □ Primal-Dual Methods for Optimization (Lagrange Algorithms)

- HOMEWORK #2 (2021/03/1 Due)：*Formulate* and *Solve* the example problem of Homewood Masonry (ReVelle et al., 2004) by using R and Euler Math Toolbox.