國立臺北大學自然資源與環境管理研究所
109 學年度第二學期『環境系統分析專題』

課程講義(15)：非線性規劃與演算法
Nonlinear Programming and Algorithms

- NONLINEAR PROGRAMMING -- INTRODUCTION
  - □ Local Optima vs. Global Optima
  - □ Convexity and Convex Programming
    - ⇨ Convexity of a Function: Convex, Concave and Un-determinant
    - ⇨ Convex Region vs. Non-convex Region
  - □ Analytical Solutions vs. Numerical Solutions
    - ⇨ Linearization of Nonlinear Objective Function => May not be necessary nowadays!
    - ⇨ Software Packages => Lingo, Euler Math Toolbox, Excel Solver Add-in, etc.
  - □ Nonlinear Programming (Chapter 13) in Applied Mathematical Programming (http://web.mit.edu/15.053/www/AMP-Chapter-13.pdf)

Portfolio Selection An investor has \$5000 and two potential investments. Let x_j for $j = 1$ and $j = 2$ denote his allocation to investment j in thousands of dollars. From historical data, investments 1 and 2 have an expected annual return of 20 and 16 percent, respectively. Also, the total risk involved with investments 1 and 2, as measured by the variance of total return, is given by $2x_1^2 + x_2^2 + (x_1 + x_2)^2$, so that risk increases with total investment and with the amount of each individual investment. The investor would like to maximize his expected return and at the same time minimize his risk. Clearly, both of these objectives cannot, in general, be satisfied simultaneously. There are several possible approaches. For example, he can minimize risk subject to a constraint imposing a lower bound on expected return. Alternatively, expected return and risk can be combined in an objective function, to give the model:

$$\text{Maximize } f(x) = 20x_1 + 16x_2 - \theta[2x_1^2 + x_2^2 + (x_1 + x_2)^2],$$

subject to:

$$g_1(x) = x_1 + x_2 \le 5,$$

$$x_1 \ge 0, \quad x_2 \ge 0, \qquad (\text{that is, } g_2(x) = -x_1, \qquad g_3(x) = -x_2).$$

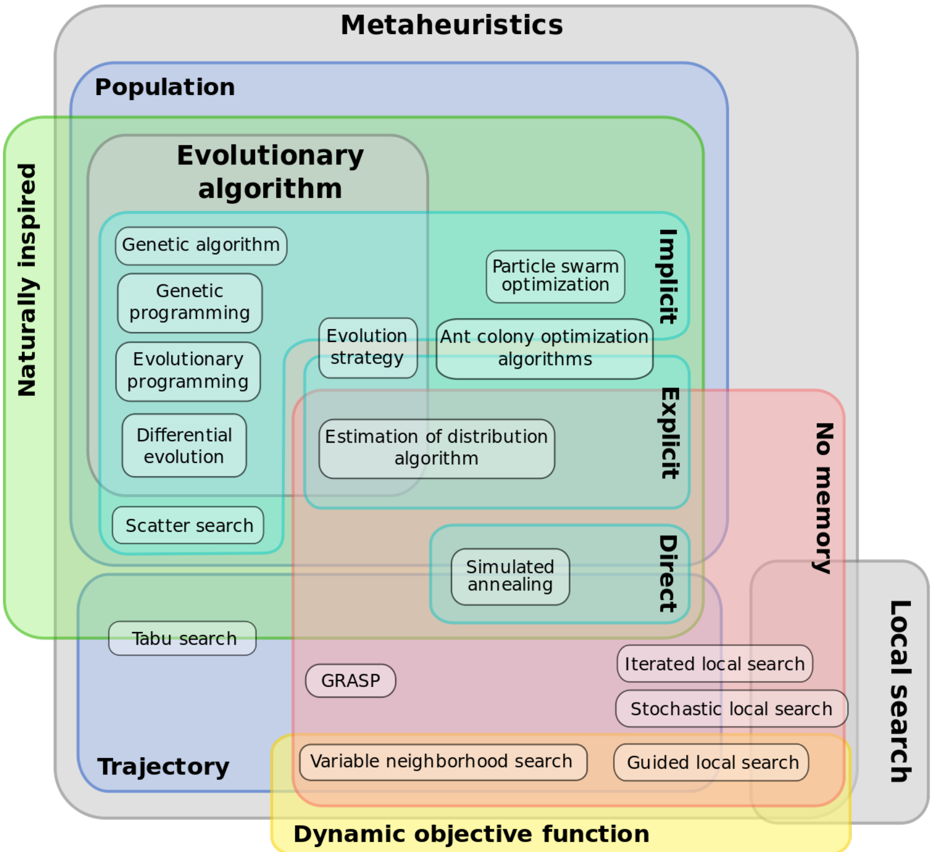
The nonnegative constant θ reflects his tradeoff between risk and return. If $\theta = 0$, the model is a linear program, and he will invest completely in the investment with greatest expected return. For very large θ, the objective contribution due to expected return becomes negligible and he is essentially minimizing his risk.

- UNCONSTRAINED OPTIMIZATION
  - □ Minima, Maxima and Saddle Points
    - ⇨ Necessary Conditions and Sufficient Conditions
  - □ Gradient of a Function (First Derivatives)
  - □ Hessian Matrix (Second Derivatives)
    - ⇨ Positively Definite: All the Eigenvalues are Positive

- LAGRANGE MULTIPLIERS AND OTHER METHODS
  - □ Lagrange Multiplier Method
    - ⇨ Constraints with All Equalities
    - ⇨ Properties of the Lagrange Multipliers

- □ Kuhn-Tucker Conditions: Constraints with Inequalities
- □ Gradient Search Procedure (Greedy) => Danger of Being Trapped at Local Optima
- □ Applying Maximum Entropy Principle to Solving the Unfair Dice Problem

- ALGORITHMS FOR NONLINEAR PROGRAMS
  - □ Numerical Methods (Chang, 2002, Chap.5)
    - ➩ Newton Method, Conjunctive Direction and Conjunctive Gradient Methods
  - □ Heuristic Algorithms => Soft Computation => Emulation of Natural Phenomena
    - ➩ Artificial Neural Network; Genetic Algorithms
    - ➩ Simulated Annealing; Tabu Search
    - ➩ Ant Search, Ant Colony Algorithm, Swarm Intelligence, etc.
  - □ Optimization Algorithms (Wikipedia)
    - ➩ Simplex Algorithm of George Dantzig, designed for linear programming
    - ➩ Interactive Methods: Newton's method, Sequential quadratic programming…
    - ➩ Heuristic (Metaheuristic) Algorithms
      - Mayfly Optimization Algorithm
      - Memetic algorithm
      - Differential evolution
      - Evolutionary algorithms
      - Dynamic relaxation
      - Genetic algorithms
      - Hill climbing with random restart
      - Nelder-Mead simplicial heuristic
      - Particle swarm optimization
      - Gravitational search algorithm
      - Simulated annealing
      - Stochastic tunneling
      - Tabu search
      - Forest Optimization Algorithm



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