國立臺北大學自然資源與環境管理研究所
111 學年度第二學期『資源管理與環境系統分析』

課程講義(10)：整數規劃與網路模式

Integer Programming and Network Models

- INTEGER PROGRAMMING
  - □ Characteristics of Integer Programming: Integer Decision Variables
    - ➪ General Integer vs. Binary Integer
    - ➪ Mixed Integer Programming and Binary Integer Programming
  - □ An Example in Introduction to Management Science, 13th ed. by Bernard Taylor III (https://media.pearsoncmg.com/ph/bp/bridgepages/bp_taylor_bridgepage/taylor_13e/online_modules/Taylor13_Mod_C.pdf)

The *branch and bound method* is not a solution technique specifically limited to integer programming problems. It is a *solution approach* that can be applied to a number of different types of problems. The branch and bound approach is based on the principle that the total set of feasible solutions can be partitioned into smaller subsets of solutions. These smaller subsets can then be evaluated systematically until the best solution is found. When the branch and bound approach is applied to an integer programming problem, it is used in conjunction with the normal noninteger solution approach. We will demonstrate the branch and bound method using the following example.

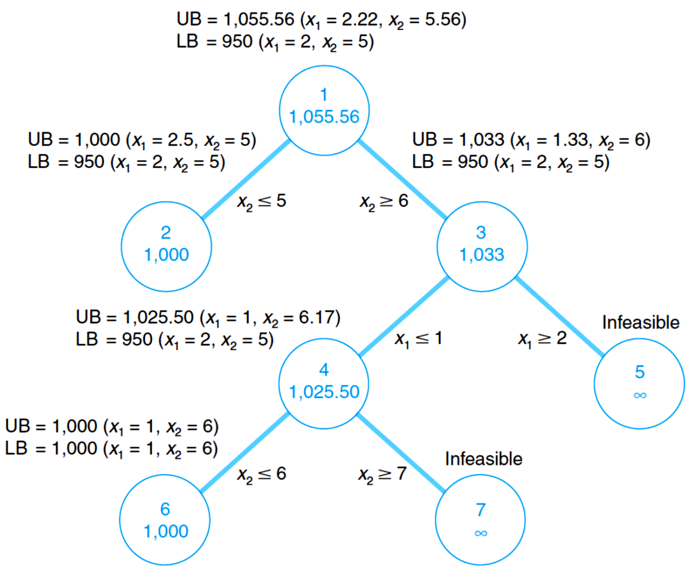
The owner of a machine shop is planning to expand by purchasing some new machines—presses and lathes. The owner has estimated that each press purchased will increase profit by \$100 per day and each lathe will increase profit by \$150 daily. The number of machines the owner can purchase is limited by the cost of the machines and the available floor space in the shop. The machine purchase prices and space requirements are as follows.

Machine	Required Floor Space (ft^2)	Purchase Price
Press	15	\$8,000
Lathe	30	4,000

maximize $Z = \$100x_1 + 150x_2$

subject to

$$8{,}000x_1 + 4{,}000x_2 \le \$40{,}000$$
$$15x_1 + 30x_2 \le 200 \text{ ft.}^2$$
$$x_1, x_2 \ge 0 \text{ and integer}$$

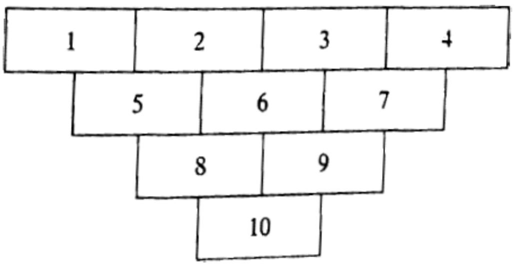


□ Solution Techniques for Integer Programming

⇨ The Cutting Plane Method and Branch-and-Bound Method

⇨ Relaxed Linear Programming and Integer Friendly Formulation => The mining problem

A mining problem.
A mining operation has identified an area where the ore is rich enough to excavate. The excavation proceeds in distinct blocks from the surface downwards. Because of the angle of slip, block 5 cannot be mined unless both blocks 1 and 2 are mined. How to formulated the constraints that reflect the situation?



- NETWORK MODEL

□ Introduction => *c.f.*: Continuous Mathematical Programming

□ Terminology: Node (Vertex), Arc (Link), Path, and Graph (Tree); Flow and Direction

□ Network Models with Linear Programming Formulation

⇨ The Transportation, Transshipment, and Assignment Problems (Chap.6, Taylor III, 2019)

□ Classical Network Programming Models (Chap.7 Network Flow Models, Taylor III, 2019)

⇨ Shortest Route Problem (Shortest-Path Problem -- Integer Friendly)

⇨ Minimum/Minimal Spanning Tree Problem

⇨ Maximum/Maximal Flow Problem

⇨ Minimum/Minimal Cost Flow Problem

□ Other Models: Traveling Salesman Problem

□ Facility Location (Siting) Theory

⇨ P-Median, P-Center, Set Covering, and Maximal Covering Problems

⇨ (To be assigned as the final project!)

□ Examples: The Shortest-Path Problem

⇨ The Seervada Park Shortest-Path Problem
(http://ocw.nctu.edu.tw/course/operation_research/or_lecturenotes/orchap9.pdf)

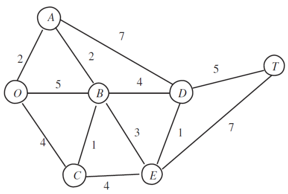


TABLE 10.2 Applying the shortest-path algorithm to the Seervada Park problem

n	Solved Nodes Directly Connected to Unsolved Nodes	Closest Connected Unsolved Node	Total Distance Involved	*n*th Nearest Node	Minimum Distance	Last Connection
1	*O*	*A*	2	*A*	2	*OA*
2, 3	*O* *A*	*C* *B*	4 2 + 2 = 4	*C* *B*	4 4	*OC* *AB*
4	*A* *B* *C*	*D* *E* *E*	2 + 7 = 9 4 + 3 = 7 4 + 4 = 8	*E*	7	*BE*
5	*A* *B* *E*	*D* *D* *D*	2 + 7 = 9 4 + 4 = 8 7 + 1 = 8	*D* *D*	8 8	*BD* *ED*
6	*D* *E*	*T* *T*	8 + 5 = 13 7 + 7 = 14	*T*	13	*DT*

⇨ Apply Dynamic Programming to solve the Shortest-Path Problem (has been covered!)
(http://ocw.nctu.edu.tw/course/operation_research/or_lecturenotes/orchap10.pdf)