

國立臺北大學自然資源與環境管理研究所  
111 學年度第二學期『資源管理與環境系統分析』

課程講義(13)：不確定性分析與隨機規劃  
Uncertainty Analysis and Stochastic Programming

● INTRODUCTION TO UNCERTAINTY ANALYSIS

- Types of Uncertainty (in the fields of modeling)
  - ⇒ Parametric Uncertainty
  - ⇒ Model or Structural Uncertainty
  - ⇒ Surprise/Indeterminacy => Climate Change? Pandemic? => Structural Uncertainty
  - ⇒ Uncertainty/Vagueness => Probability, Likelihood, Possibility => Fuzziness and Greyness
- Uncertainty Analysis
  - ⇒ Mathematical (Quantitative) Analyses Related to the Uncertainties about ‘Systems’
  - ⇒ System Uncertainties: Uncertainties about Measurement, Modeling, and Parameters
- Modeling Uncertainty
  - ⇒ Generating Values from Known Probability Distributions => Monte Carlo Simulation
  - ⇒ Markov Processes and Transition Probabilities => Markov Chain and Bayesian Analysis
  - ⇒ Stochastic Programming => Chance Constrained Models
  - ⇒ Sensitivity Analysis: Sensitivity analysis is used to determine the importance of different parameters and components of the model on the output of the model.

● PROBABILITY THEORY, STOCHASTIC PROCESS AND RANDOM FIELD

- Deterministic vs. Stochastic Systems
  - ⇒ Vagueness, Uncertainty, and ‘Stochasticity’
  - ⇒ Possibility, Likelihood, and Probability
  - ⇒ Response = Deterministic component + Stochastic component + Error
- Probability Theory
  - ⇒ The Axioms of Probability
  - ⇒ Random Variables: Discrete and Continuous
  - ⇒ Statistics (Moments) of a Random Variable: Expected Value, Variance ...etc.
  - ⇒ Multiple Random Variables: Multivariate Statistics => Covariance
  - ⇒ Distribution: Probability Density Function, Cumulated Distribution Function
  - ⇒ Conditional Probability and Bayes’ Theorem => Bayesian Decision Analysis
- Normal Distribution
  - ⇒ Two-Parameter Distribution: Location and Dispersion => Mean and Variance
  - ⇒ Standardization and  $t$ -Distribution
  - ⇒ Confidence Interval and Standard Deviation
  - ⇒ Multivariate Gaussian Distribution
- Stochastic Process
  - ⇒ Serial Random Variables: Temporal, Spatial, Spatio-temporal Stochastic Processes
  - ⇒ Serial Correlation => Deterministic Term (Trend) + Disturbance (Noise)
  - ⇒ Poisson Process, Markov’s Chains, and Random Walks
- Random Field

- ⇒ Random Variables Distributed ('Regionalized') in Space
- ⇒ Spatial Variability (Correlation) ⇒ Trend + Disturbance
- ⇒ Geostatistics: Kriging (Simple, Ordinary, Universal...) ⇒ GIS

## • STOCHASTIC PROGRAMMING

- Uncertainties Related to Mathematical Programming Systems
  - ⇒ Modeling Uncertainties: Assumptions, Objective Functions, and Constraints
    - Mathematical Program with Recourse: Multi-Stage Stochastic Programming
  - ⇒ Uncertainties 'Embedded' in Decision Variables: Fuzziness, Grey Information...
    - (1) Intervals or Specified Ranges ⇒ Grey Numbers ⇒ Grey Programming
    - (2) Degree of Set Membership ⇒ Fuzzy Set ⇒ Fuzzy Programming
  - ⇒ Uncertainties about Model Parameters: Coefficients of Objective Function, RHS,  $A_{ij}$ 
    - (1) Parameters (Coefficients) of the Optimization Model are Random Variables
    - (2) Treat Decision Variables as 'Deterministic Variables' to be determined
    - (3) Probabilistic Constraints ⇒ Chance-Constrained Programming

## • CHANCE CONSTRAINED PROGRAMMING

- What are Chance Constraints?
- Significance Level ⇒ System Reliability
- Row Independence ⇒ Independently and Identically Distributed (i.i.d.)
- Right-Hand-Side Random ⇒ Univariate Normal Distribution
- Technical Coefficients Random ⇒ Multivariate Normal Distribution
- Row Dependence ⇒ Joint Chance Constraint (relatively complicated!)

Chance Constraints:  $p\left(\sum_{j=1}^n a_{ij} \cdot x_j \in b_i\right) \geq 1 - \alpha_i; \quad \forall i = 1, \dots, m$

(1) RHS  $b_i$  Random: Univariate probability distribution of  $b_i$

i.  $\in \equiv \geq$

$$p\left(\sum_{j=1}^n a_{ij} \cdot x_j \geq b_i\right) \geq 1 - \alpha_i \Rightarrow p\left(b_i \leq \sum_{j=1}^n a_{ij} \cdot x_j\right) \geq 1 - \alpha_i \Rightarrow F(b_i = \sum a_{ij} x_j) \geq 1 - \alpha_i$$

$$\left(\sum a_{ij} x_j - \mu_{b_i}\right) / \sigma_{b_i} \geq F_z^{-1}(1 - \alpha_i) \Rightarrow \sum a_{ij} x_j \geq \mu_{b_i} + F_z^{-1}(1 - \alpha_i) \cdot \sigma_{b_i}$$

ii.  $\in \equiv \leq$

$$p\left(\sum_{j=1}^n a_{ij} \cdot x_j \leq b_i\right) \geq 1 - \alpha_i \Rightarrow p\left(b_i \geq \sum_{j=1}^n a_{ij} \cdot x_j\right) \geq 1 - \alpha_i \Rightarrow 1 - F(b_i = \sum a_{ij} x_j) \geq 1 - \alpha_i$$

$$F(b_i = \sum a_{ij} x_j) \leq \alpha_i \Rightarrow \left(\sum a_{ij} x_j - \mu_{b_i}\right) / \sigma_{b_i} \leq F_z^{-1}(\alpha_i) \Rightarrow \sum a_{ij} x_j \leq \mu_{b_i} + F_z^{-1}(\alpha_i) \cdot \sigma_{b_i}$$

(2) Technical Coefficients  $a_{ij}$  Random: Multivariate probability distribution of  $\sum a_{ij} x_j$

- ⇒ Variance-Covariance Matrix: Positively definite (symmetric) matrix

- MONTE CARLO SIMULATION

- Characteristics of Monte Carlo Simulation

- ⇒ Quantitative Risk Analysis
    - ⇒ Simulation and then Optimization

- Monte Carlo Simulation Steps

- Step 1: Create a parametric model,  $y = f(x_1, x_2, \dots, x_q)$ .

- Step 2: Generate a set of random inputs,  $x_1^i, x_2^i, \dots, x_q^i$ .

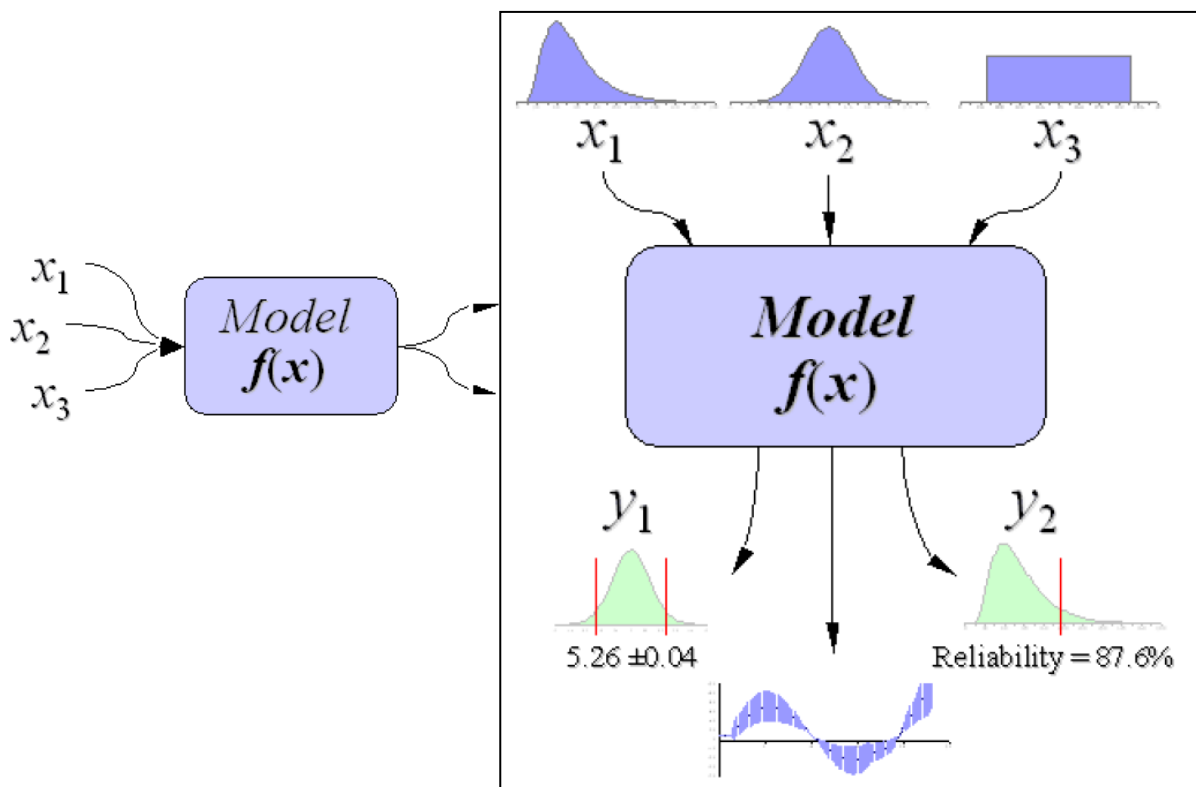
- Step 3: Evaluate the model and store the results as  $y^i$ .

- Step 4: Repeat steps 2 and 3 for  $i = 1 \dots n$ .

- Step 5: Analyze the results using histograms, statistics, confidence intervals, etc.

- Stages involved in Producing a Monte Carlo Risk Analysis Model

- ⇒ Designing the structure of the risk analysis model
    - ⇒ Defining distributions that describe the uncertainty of the problem
    - ⇒ Modeling dependencies between model uncertainties
    - ⇒ Presenting and interpreting the risk analysis results
    - ⇒ Software Packages for Monte Carlo Simulation: Palisade @RISK; Oracle Crystal Ball



(<http://www.vertex42.com/ExcelArticles/mc/MonteCarloSimulation.html>)

- HOMEWORK #6: MONTE CARLO SIMULATION (to be demonstrated in class)

Please install Oracle Crystal Ball and practice the examples of: “Toxic Waste Site” and “Groundwater Cleanup” \*  
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