國立臺北大學自然資源與環境管理研究所
113 學年度第二學期『資源管理與環境系統分析』

課程講義(02&03)：線性規劃
Linear Programming

- COMPONENTS OF AN OPTIMIZATION MODEL
  - □ Objective Function(s)
    - ⇨ Single vs. Multiple; Linear vs. Nonlinear
    - ⇨ Convex (Concave) vs. Non-convex
  - □ Constraints
    - ⇨ Constrained vs. Unconstrained; Linear vs. Nonlinear
    - ⇨ Convex vs. Non-convex Feasible Regions
  - □ Decision Variables
    - ⇨ Continuous vs. Discrete; Deterministic vs. Stochastic
  - □ System Parameters (Coefficients)
    - ⇨ Deterministic vs. Stochastic => Division into Sub-Models
  - □ Formulation of Optimization Models
    - ⇨ Plain Form: Straightforward but not suitable for large-scaled or complex problems
    - ⇨ Algebraic Formulations => Parameters (Scalars), Vectors, and Matrices (Tables)
    - ⇨ Algebraic Formulations with text (symbolic) description of variables and parameters
    - ⇨ Sets and Indices => Equation Editor for symbolic expressions

- PROPERTIES OF AN LP AND ITS SOLUTION
  - □ Proportionality, Additivity, Divisibility, Certainty, and Non-Negativity
  - □ Non-negative Decision Variables => What if negative values are needed?
  - □ A "Convex Programming" Model
    - ⇨ Linear (either Concave or Convex) Objective Function
    - ⇨ Convex, Compact, and Continuous Feasible Region
  - □ Solution Procedure of an LP
    - ⇨ Pre-Optimal Analysis, Optimization (Solution) and Post-Optimization Analysis
    - ⇨ Graphical, Simplex, Dual Simplex, Interior Point and Other Methods
    - ⇨ Optimal Solution => Infeasible, Un-bounded and Degenerate Solutions

- EXAMPLES OF LINEAR PROGRAMMING
  - □ Homewood Masonry -- A Material Production Problem
    - ⇨ Objective Function: Maximizing the production profit.
    - ⇨ Decision Variables: Two building products to be produced.
    - ⇨ Constraints: Resource availability, work hours, and curing vat capacity

Resource	HYDIT	FILIT	Availability
Wahash Red Clay	2 m^3/ton	4 m^3/ton	28 m^3/wk
Blending time	5 hr/ton	5 hr/ton	50 hr/wk
Curing vat capacity	8 tons	6 tons	
Profit	$140/ton	$160/ton	

1. Algebraic formulation with numerical coefficients

$$\text{Maximize Profit } z = 140x_1 + 160x_2$$

Subject to

$$2x_1 + 4x_2 \leq 28$$
$$5x_1 + 5x_2 \leq 50$$
$$x_1 \leq 8$$
$$x_2 \leq 6$$

2. Algebraic formulation with symbolic coefficients

$$\text{Maximize Profit } z = \sum_{j=1}^{2} c_j\, x_j$$

Subject to

$$\sum_{j=1}^{2} a_{ij} x_j \leq b_i; \qquad i = 1, \cdots, 4$$

$$\boldsymbol{c} = \{c_j\} = [140, 160] \quad \boldsymbol{b} = \{b_i\} = \begin{bmatrix} 28 \\ 50 \\ 8 \\ 6 \end{bmatrix} \quad \boldsymbol{A} = \{a_{ij}\} = \begin{bmatrix} 2 & 4 \\ 5 & 5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3. Matrix formulation

$$\text{Maximize Profit } z = \boldsymbol{c}' \cdot \boldsymbol{x}$$

$$z = [140, 160] \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Subject to

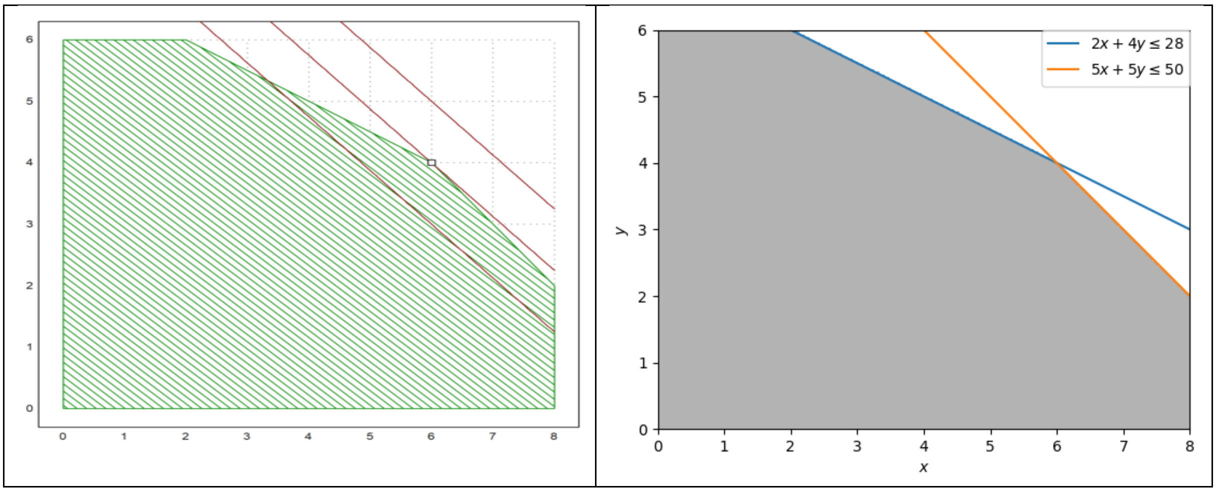
$$\boldsymbol{A} \cdot \boldsymbol{x} \leq \boldsymbol{b}$$

$$\begin{bmatrix} 2 & 4 \\ 5 & 5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 28 \\ 50 \\ 8 \\ 6 \end{bmatrix}$$

- THE SIMPLEX METHOD
  - □ Augmented Form of the LP Models
    - ⇨ "Less-than-and-equal-to" Inequality constraints => Slack variables
    - ⇨ "Greater-than-and-equal-to" Inequality constraints => Surplus & Artificial Variables
    - ⇨ Equality constraints => Artificial variables => 'Big-M Treatment'
  - □ Terminology and Procedure of the Simplex Method
    - ⇨ Basic vs. non-basic variables

- ⇨ Feasible basic solution => "Adjacent"
- ⇨ Ratio test for Pivoting
- ⇨ Check of "Optimality"

- □ Simplex Tableaus and An Animated Presentation => Copilot Answer



- **SENSITIVITY ANALYSIS**
  - □ Overview and Post-Optimality Analysis
  - □ Sensitivity Analysis on RHS (Resource) Coefficients
    - ⇨ Shadow price, marginal value of a resource and economic interpretation
  - □ Sensitivity Analysis on Objective Function Coefficients
  - □ Graphical Illustration
  - □ Parametric Programming

- **DUALITY THEORY**
  - □ Model Formulations
  - □ Dual-Primal Relationships
    - ⇨ Implementation from Production Problem
    - ⇨ Implementation from Resource Allocation Problem
  - □ Primal-Dual Methods for Optimization (Lagrange Algorithms)

- **DEMONSTRATION CODES**
  - □ Please install R, Rstudio, and Euler Math Toolbox in advance!
  - □ Euler Math Toolbox with built-in functions package and 'lpsolve'
  - □ R with packages of 'lpsolve' and 'gMOIP'
  - □ Excel with Solver (規劃求解) Add-in
  - □ lpsolveIDE?
  - □ GNU Octave with functions of 'glpk' and 'plot' => ChatGPT?
  - □ Python with packages of 'gilp' and 'lpsolve' (`pip install --only-binary :all: gilp`)

- **HOMEWORK #2** (2025/03/10 Due): *Formulate* and *Solve* the example problem of Homewood Masonry (ReVelle et al., 2004) by using Excel, R, and Euler Math Toolbox.