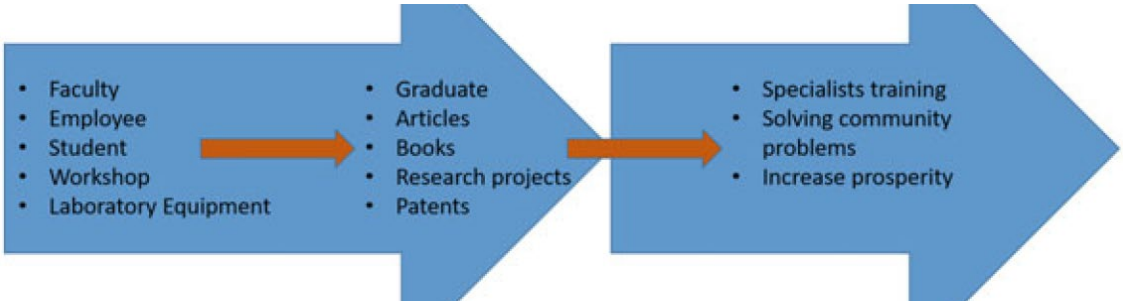
國立臺北大學自然資源與環境管理研究所
113 學年度第二學期『資源管理與環境系統分析』

課程講義(04-05)：績效評估與資料包絡分析
Performance Evaluation and Data Envelopment Analysis

- DATA ENVELOPMENT ANALYSIS
  - □ Performance, Efficiency, Effectiveness, and Productivity => Intensity
    - ⇨ CP Ratio (Cost-Performance ratio; Price-Performance ratio); BC Ratio (Benefit/Cost); 資源/能源生產力; Eco-Efficiency; Circularity
    - ⇨ 效率評估方法：平衡計分卡（Balance Score Card, BSC）、比例分析法（Ratio Analysis）、迴歸分析法（Regression Analysis）、多準則評估法、生產力及生產效率分析法、資料包絡分析法 (https://www.wunan.com.tw/bookdetail?NO=9296) (http://www.wunan.com.tw/www2/download/preview/1FQF.PDF) (https://ir.lib.nycu.edu.tw/bitstream/11536/77253/4/652504.pdf)
    - ⇨ Effectiveness => Cost Effectiveness => Goal Achievement
    - ⇨ Performance (Productivity) = Efficiency + Effectiveness
      Efficiency = Outputs / Inputs; Effectiveness = f (products, objectives)



(https://www.springer.com/gp/book/9783030242763)

  - □ Key Components of the Data Envelopment Analysis (DEA)
    - ⇨ Decision Making Unit (DMU) => Reference (Peer) DMU
    - ⇨ Technical and Scale Efficiencies => Technical, Allocative and Overall Efficiencies
    - ⇨ Inputs and Outputs => Formulation Orientations => Returns to Scale
  - □ Software or Developers' Packages for Data Envelopment Analysis (DEA)
    - ⇨ pyDEA (https://araith.github.io/pyDEA/)
    - ⇨ DEAFrontier Free Version (https://www.deafrontier.net/deafree.html)
    - ⇨ R Packages: Benchmarking, deaR, …
    - ⇨ Python => GitHub - wurmen/DEA: Data Envelopment Analysis 資料包絡分析
  - □ Formulations of the Charnes, Cooper, and Rhodes (CCR) DEA Model
    - ⇨ Constant Return to Scale (CRS)
    - ⇨ Envelopment Model vs. Multiplier Models

$$\text{Min } \sum_i v_i x_{io} \,/\, \sum_r u_r y_{ro}$$
$$\text{Subject to}$$
$$\sum_i v_i x_{ij} \,/\, \sum_r u_r y_{rj} \geq 1 \text{ for } j = 1, \ldots, n,$$
$$u_r, v_i \geq \varepsilon > 0 \text{ for all } i \text{ and } r.$$

Table 1-1. CCR DEA Model

Input-oriented	
Envelopment model	Multiplier model
$\min \theta - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)$ subject to $\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta x_{io} \quad i = 1,2,...,m;$ $\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro} \quad r = 1,2,...,s;$ $\lambda_j \geq 0 \qquad j = 1,2,...,n.$	$\max z = \sum_{r=1}^{s} \mu_r y_{ro}$ subject to $\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij} \leq 0$ $\sum_{i=1}^{m} \nu_i x_{io} = 1$ $\mu_r, \nu_i \geq \varepsilon > 0$
Output-oriented	
Envelopment model	Multiplier model
$\max \phi + \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)$ subject to $\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{io} \quad i = 1,2,...,m;$ $\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = \phi y_{ro} \quad r = 1,2,...,s;$ $\lambda_j \geq 0 \qquad j = 1,2,...,n.$	$\min q = \sum_{i=1}^{m} \nu_i x_{io}$ subject to $\sum_{i=1}^{m} \nu_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} \geq 0$ $\sum_{r=1}^{s} \mu_r y_{ro} = 1$ $\mu_r, \nu_i \geq \varepsilon > 0$

([(PDF) Data Envelopment Analysis: History, Models, and Interpretations])

□ Extensions to the CCR Model

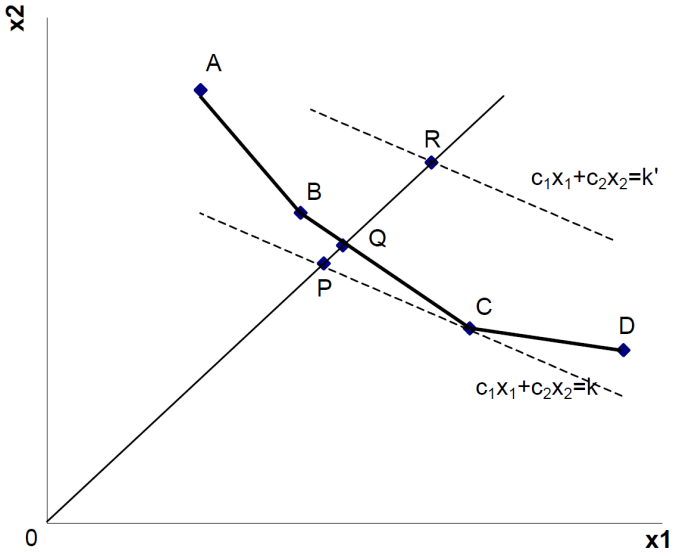
- ⇨ Nondiscretionary Inputs and Outputs => 不易調整投入
- ⇨ Undesirable Factors (Outputs) => 非意欲產出(可拋性)
- ⇨ Categorical Inputs and Outputs
- ⇨ Incorporating Judgment or A Priori Knowledge
- ⇨ Window Analysis

□ Allocative and Overall Efficiency

- ⇨ Technical efficiency: $\frac{OQ}{OR}$
- ⇨ Price efficiency
  Allocative efficiency $\frac{OP}{OQ}$
- ⇨ Overall efficiency $\frac{OP}{OQ}\frac{OQ}{OR} = \frac{OP}{OR}$

□ Profit Efficiency

□ Variable Return to Scale (VRS) => Banker, Charnes, and Cooper (BBC) Model



□ Malmquist Productivity Indexes and DEA => Change of efficiencies over time

□ Ehrlich and Holdren (1971) Impact Equation (I=PAT) and Kaya Identity

⇨ Human impact on the environment (I): Population (P), Affluence (A) and Technology (T)

⇨ Affluence (A): Per capita GDP; Technology (T): Emissions/GDP=> Energy Intensity

⇨ Simplified Kaya Identity: $CO_2 = GDP \times (Energy/GDP) \times (CO_2/Energy)$ => Emission factor

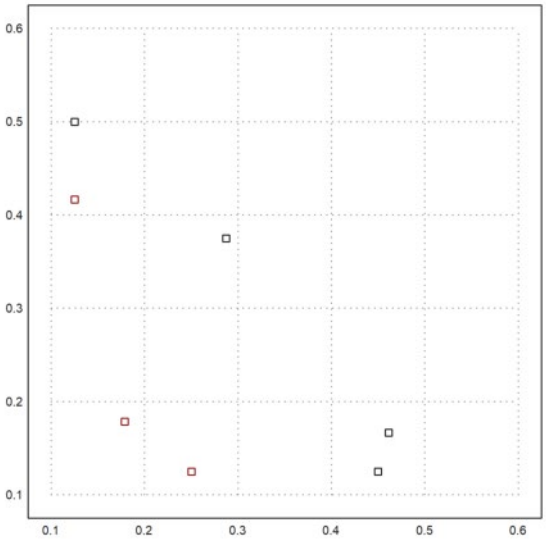
□ 企業溫室氣體管理績效分析=>臺灣能源期刊 第 7 卷 第 2 期 pp.181-195

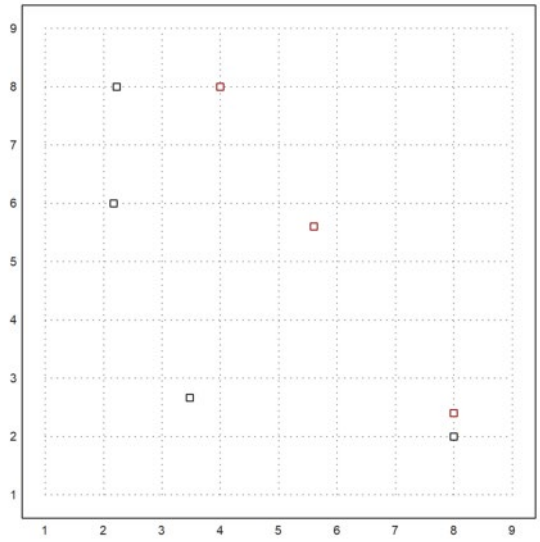
□ A Simple Example: 1 output; 2 inputs; and 7 DMU

決策單位	A	B	C	D	E	F	G
投入一	9.0	8.3	4.6	1.75	1.5	2.5	4.0
投入二	2.5	3.0	6.0	7.0	5.0	2.5	2.0
產出	20.0	18.0	16.0	14.0	12.0	14.0	16.0

(https://www.slideserve.com/vaughn/tel-06-2785123-2162-e-mail-lccmail-cjcu-tw)

⇨ Production Frontier





⇨ Solutions by pyDEA

DMU	Envelopment Model				Multiplier Model			
	I-CRS	I-VRS	O-CRS	O-VRS	I-CRS	I-VRS	O-CRS	O-VRS
A	1	1	1	1	1	1	1	1
B	0.7500	0.7831	0.7500	0.9259	0.7500	0.7831	0.7500	0.9259
C	0.5882	0.8696	0.5882	0.9709	0.5882	0.8696	0.5882	0.9709
D	1	1	1	1	1	1	1	1
E	1	1	1	1	1	1	1	1
F	1	1	1	1	1	1	1	1
G	1	1	1	1	1	1	1	1

- HOMEWORK #3 (No hand-in required)：*Practice* the DEA example (7 DMUs with 1 input and 2 inputs) presented in the class by using Excel with DEAFrontier or pyDEA.