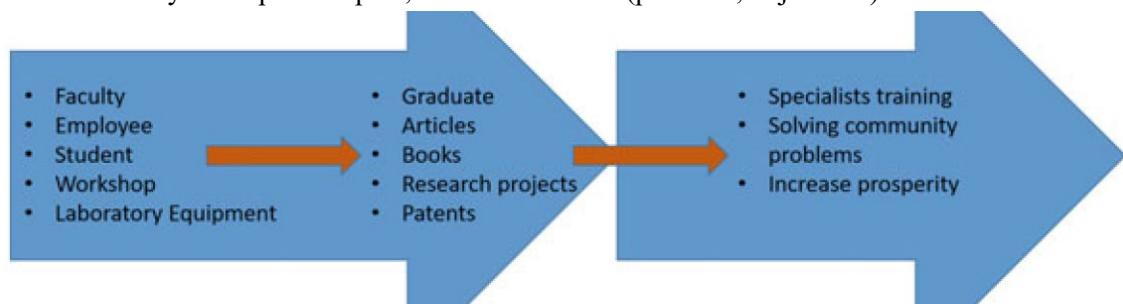


國立臺北大學自然資源與環境管理研究所
113 學年度第二學期『資源管理與環境系統分析』

課程講義(04-05)：績效評估與資料包絡分析
Performance Evaluation and Data Envelopment Analysis

• DATA ENVELOPMENT ANALYSIS

- Performance, Efficiency, Effectiveness, and Productivity => Intensity
 - ⇒ CP Ratio (Cost-Performance ratio; Price-Performance ratio); BC Ratio (Benefit/Cost); 資源/能源生產力; Eco-Efficiency; Circularity
 - ⇒ 效率評估方法：平衡計分卡（Balance Score Card, BSC）、比例分析法（Ratio Analysis）、迴歸分析法（Regression Analysis）、多準則評估法、生產力及生產效率分析法、資料包絡分析法 (<https://www.wunan.com.tw/bookdetail?NO=9296>) (<http://www.wunan.com.tw/www2/download/preview/1FQF.PDF>) (<https://ir.lib.nycu.edu.tw/bitstream/11536/77253/4/652504.pdf>)
 - ⇒ Effectiveness => Cost Effectiveness => Goal Achievement
 - ⇒ Performance (Productivity) = Efficiency + Effectiveness
Efficiency = Outputs / Inputs; Effectiveness = f (products, objectives)



(<https://www.springer.com/gp/book/9783030242763>)

- Key Components of the Data Envelopment Analysis (DEA)
 - ⇒ Decision Making Unit (DMU) => Reference (Peer) DMU
 - ⇒ Technical and Scale Efficiencies => Technical, Allocative and Overall Efficiencies
 - ⇒ Inputs and Outputs => Formulation Orientations => Returns to Scale
- Software or Developers' Packages for Data Envelopment Analysis (DEA)
 - ⇒ pyDEA (<https://araith.github.io/pyDEA/>)
 - ⇒ DEAFrontier Free Version (<https://www.deafrontier.net/deafree.html>)
 - ⇒ R Packages: [Benchmarking](#), [deaR](#), ...
 - ⇒ Python => [GitHub - wurmen/DEA: Data Envelopment Analysis](#) 資料包絡分析
- Formulations of the Charnes, Cooper, and Rhodes (CCR) DEA Model
 - ⇒ Constant Return to Scale (CRS)
 - ⇒ Envelopment Model vs. Multiplier Models

$$\begin{aligned} & \text{Min } \sum_i v_i x_{io} / \sum_r u_r y_{ro} \\ & \text{Subject to} \\ & \sum_i v_i x_{ij} / \sum_r u_r y_{rj} \geq 1 \text{ for } j = 1, \dots, n, \\ & u_r, v_i \geq \varepsilon > 0 \text{ for all } i \text{ and } r. \end{aligned}$$

Table 1-1. CCR DEA Model

Input-oriented	
Envelopment model	Multiplier model
$\min \theta - \varepsilon \left(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right)$ subject to $\sum_{j=1}^n x_{ij} \lambda_j + s_i^- = \theta x_{io} \quad i = 1, 2, \dots, m;$ $\sum_{j=1}^n y_{rj} \lambda_j - s_r^+ = y_{ro} \quad r = 1, 2, \dots, s;$ $\lambda_j \geq 0 \quad j = 1, 2, \dots, n.$	$\max z = \sum_{r=1}^s \mu_r y_{ro}$ subject to $\sum_{r=1}^s \mu_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0$ $\sum_{i=1}^m v_i x_{io} = 1$ $\mu_r, v_i \geq \varepsilon > 0$
Output-oriented	
Envelopment model	Multiplier model
$\max \phi + \varepsilon \left(\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right)$ subject to $\sum_{j=1}^n x_{ij} \lambda_j + s_i^- = x_{io} \quad i = 1, 2, \dots, m;$ $\sum_{j=1}^n y_{rj} \lambda_j - s_r^+ = \phi y_{ro} \quad r = 1, 2, \dots, s;$ $\lambda_j \geq 0 \quad j = 1, 2, \dots, n.$	$\min q = \sum_{i=1}^m v_i x_{io}$ subject to $\sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s \mu_r y_{rj} \geq 0$ $\sum_{r=1}^s \mu_r y_{ro} = 1$ $\mu_r, v_i \geq \varepsilon > 0$

[\(\(PDF\) Data Envelopment Analysis: History, Models, and Interpretations\)](#)

□ Extensions to the CCR Model

- ⇒ Nondiscretionary Inputs and Outputs => 不易調整投入
- ⇒ Undesirable Factors (Outputs) => 非意欲產出(可拋性)
- ⇒ Categorical Inputs and Outputs
- ⇒ Incorporating Judgment or A Priori Knowledge
- ⇒ Window Analysis

□ Allocative and Overall Efficiency

- ⇒ Technical efficiency:

$$\frac{OQ}{OR}$$

- ⇒ Price efficiency

Allocative efficiency

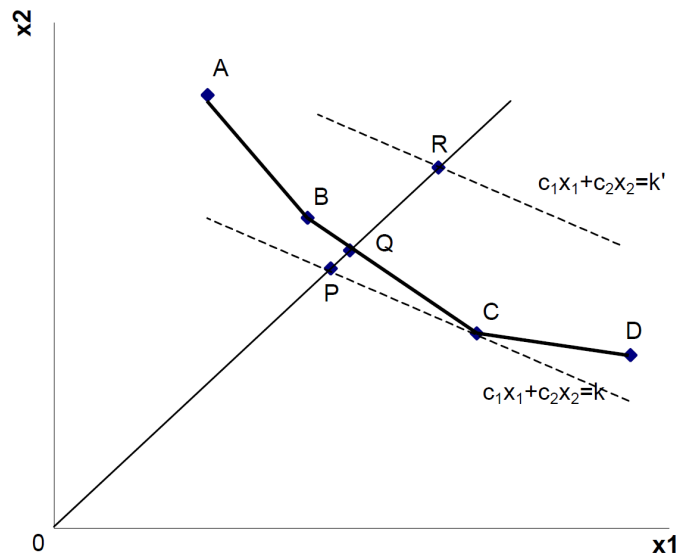
$$\frac{OP}{OQ}$$

- ⇒ Overall efficiency

$$\frac{OP}{OQ} \frac{OQ}{OR} = \frac{OP}{OR}$$

□ Profit Efficiency

- Variable Return to Scale (VRS) => Banker, Charnes, and Cooper (BBC) Model



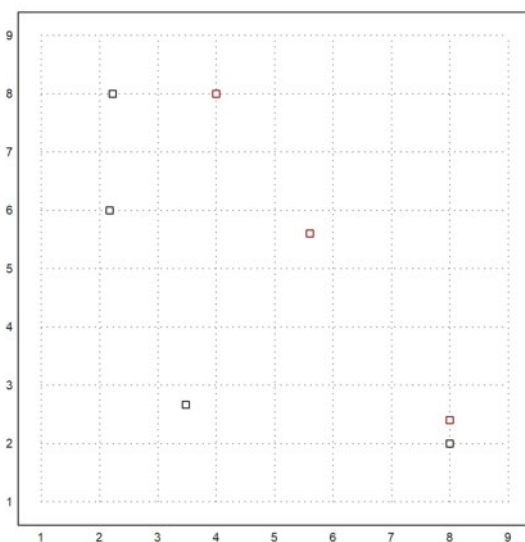
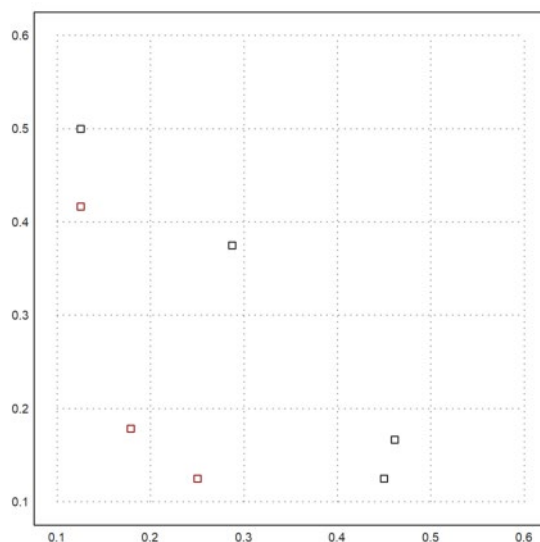
- Malmquist Productivity Indexes and DEA => Change of efficiencies over time
- [Ehrlich and Holdren \(1971\)](#) Impact Equation (I=PAT) and Kaya Identity
 - ⇒ Human impact on the environment (I): Population (P), Affluence (A) and Technology (T)
 - ⇒ Affluence (A): Per capita GDP; Technology (T): Emissions/GDP=> Energy Intensity
 - ⇒ [Simplified Kaya Identity: \$CO_2 = GDP \times \(Energy/GDP\) \times \(CO_2/Energy\)\$](#) => Emission factor
- 企業溫室氣體管理績效分析=>[臺灣能源期刊 第7卷 第2期 pp.181-195](#)

- A Simple Example: 1 output; 2 inputs; and 7 DMU

決策單位	A	B	C	D	E	F	G
投入一	9.0	8.3	4.6	1.75	1.5	2.5	4.0
投入二	2.5	3.0	6.0	7.0	5.0	2.5	2.0
產出	20.0	18.0	16.0	14.0	12.0	14.0	16.0

(<https://www.slideserve.com/vaughn/tel-06-2785123-2162-e-mail-lccmail-cjcu-tw>)

⇒ Production Frontier



⇒ Solutions by pyDEA

DMU	Envelopment Model				Multiplier Model			
	I-CRS	I-VRS	O-CRS	O-VRS	I-CRS	I-VRS	O-CRS	O-VRS
A	1	1	1	1	1	1	1	1
B	0.7500	0.7831	0.7500	0.9259	0.7500	0.7831	0.7500	0.9259
C	0.5882	0.8696	0.5882	0.9709	0.5882	0.8696	0.5882	0.9709
D	1	1	1	1	1	1	1	1
E	1	1	1	1	1	1	1	1
F	1	1	1	1	1	1	1	1
G	1	1	1	1	1	1	1	1

- HOMEWORK #3 (No hand-in required) : *Practice* the DEA example (7 DMUs with 1 input and 2 inputs) presented in the class by using Excel with DEA Frontier or pyDEA.