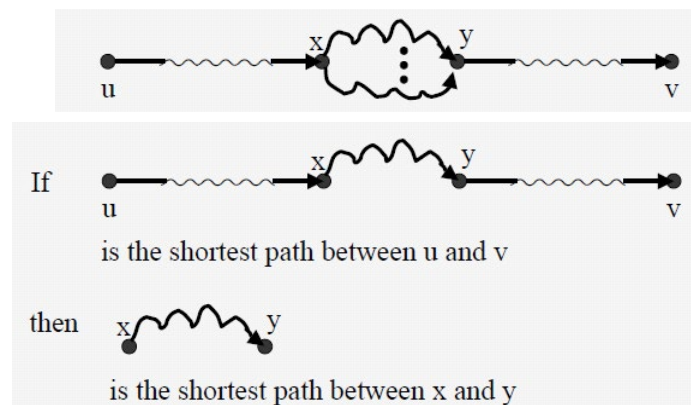


國立臺北大學自然資源與環境管理研究所  
113 學年度第二學期『資源管理與環境系統分析』

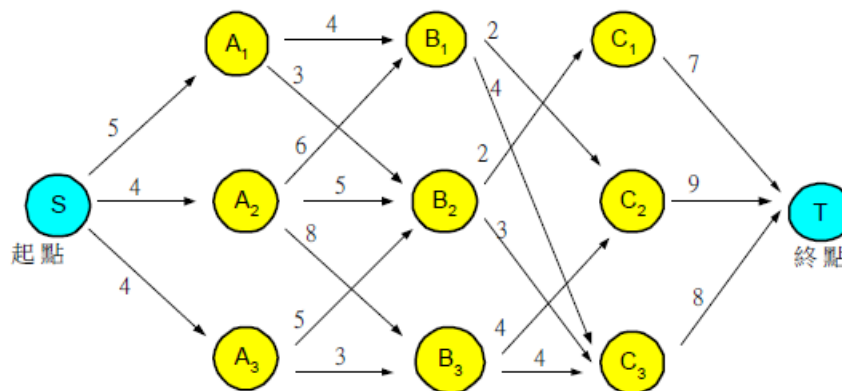
課程講義(07)：動態規劃與目標規劃  
Dynamic Programming and Goal Programming

• INTRODUCTION TO DYNAMIC PROGRAMMING

- Dynamic Programming = Divide and Conquer + Memorization
- No Specific Forms or Formulations=> Principle of Optimality
- Terminology: Stage, State, Decision, Return, Recursive Equation
- Dynamic programming is a technique for solving problems with a recursive structure with the following characteristics:
  - ⇒ Optimal substructure (principle of optimality): An optimal solution to a problem can be decomposed into optimal solutions for sub-problems.
  - ⇒ A small number of sub-problems: The total number of sub-instances to be solved is small.
  - ⇒ Overlapping sub-problems: During the computation same instances are referred to repeatedly.



- An Example of Dynamic Programming: The Shortest Path Problem
  - ⇒ Divide the problem into 4 subproblems (Stages)
  - ⇒ Find the optimal solution in the stage  $i$  and 'pass' into stage  $i+1$ .
  - ⇒ Formulate the recursive equation between stages.
  - ⇒ Backward vs. Forward => Can find the same optimal solution under deterministic conditions



## ● GOAL PROGRAMMING

- Criteria for Decision-Making: Attribute, Objective, Target, and Goal
  - ⇒ The UN SDGs: Goals, Targets, and Indicators
- Multiple Criteria Decision Making: Multiple Attribute and Multiobjective
- Classification of Goal Programming: Non-Preemptive vs. Preemptive
- Non-Preemptive Goal Programming
  - ⇒ Complementary relationship
  - ⇒ One-sided vs. Two-sided
- Preemptive Goal Programming or Lexicographic GP
- Drawbacks: Normalization and Weighting; Pareto Optimality?
- An Example of Goal Programming: Expansion of Production Lines
  - ⇒ Deviation variables  $\Rightarrow$  slack vs. surplus  $\Rightarrow$  Minimize penalty weighted deviations
  - ⇒ Maximize the profit  $\Rightarrow$  allow overshooting and penalty on undershooting
  - ⇒ Remain employee level  $\Rightarrow$  penalty on both sides (may have different weights)
  - ⇒ Minimize the investment  $\Rightarrow$  penalty on overshooting and allow undershooting

### 例題 3-1：

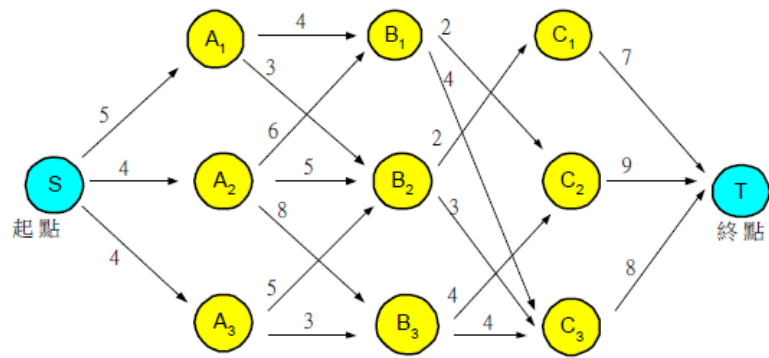
某一家公司考慮製造三種產品取代目前的產品，決策者優先考慮了三個主要因子：長期利潤、勞動力之穩定性和投資資本額。目標如下：

- (1) 三種產品之長期利潤至少 125,000,000 元。
- (2) 儘量維持現有雇用水準為員工 4,000 人。
- (3) 固定投資資本額少於 55,000,000 元。

所定的標準如下表 3.1：

表 3.1 例題 3-1 中之系統規劃條件

| 因子                       | 產品之生產參數 |   |    | 標的 (單位)            | 懲罰權重<br>(penalty weight) |
|--------------------------|---------|---|----|--------------------|--------------------------|
|                          | 1       | 2 | 3  |                    |                          |
| 長期利潤<br>( $10^3$ \$/產品)  | 12      | 9 | 15 | $\geq 125$ (百萬元/年) | 5                        |
| 雇用水準<br>(人)              | 5       | 3 | 4  | $= 40$ (百名員工/年)    | 2(+), 4(-)               |
| 投資資本額<br>( $10^3$ \$/產品) | 5       | 7 | 8  | $\leq 55$ (百萬元/年)  | 3                        |



階段1 (Stage 1):

| $S_1$          | $d_1$ | $f(S_1, d_1)$ | $f(S_1, d_1) + g^*(S_0)$<br>$S_0 = T, g^*(S_0) = g^*(T) = 0$ | $g^*(S_1)$ | $d_1^*$ |
|----------------|-------|---------------|--------------------------------------------------------------|------------|---------|
| C <sub>1</sub> | T     | 7             | 7 + 0                                                        | 7          | T       |
| C <sub>2</sub> | T     | 9             | 9 + 0                                                        | 9          | T       |
| C <sub>3</sub> | T     | 8             | 8 + 0                                                        | 8          | T       |

階段2 (Stage 2):

| $S_2$          | $d_2$          | $f(S_2, d_2)$ | $f(S_2, d_2) + g^*(S_1)$<br>$S_1 = d_2$ | $g^*(S_2)$ | $d_2^*$        |
|----------------|----------------|---------------|-----------------------------------------|------------|----------------|
| B <sub>1</sub> | C <sub>2</sub> | 2             | 2 + 9                                   | 11         | C <sub>2</sub> |
|                | C <sub>3</sub> | 4             | 4 + 8                                   |            |                |
| B <sub>2</sub> | C <sub>1</sub> | 2             | 2 + 7                                   | 9          | C <sub>1</sub> |
|                | C <sub>3</sub> | 3             | 3 + 8                                   |            |                |
| B <sub>3</sub> | C <sub>2</sub> | 4             | 4 + 9                                   | 12         | C <sub>3</sub> |
|                | C <sub>3</sub> | 5             | 4 + 8                                   |            |                |

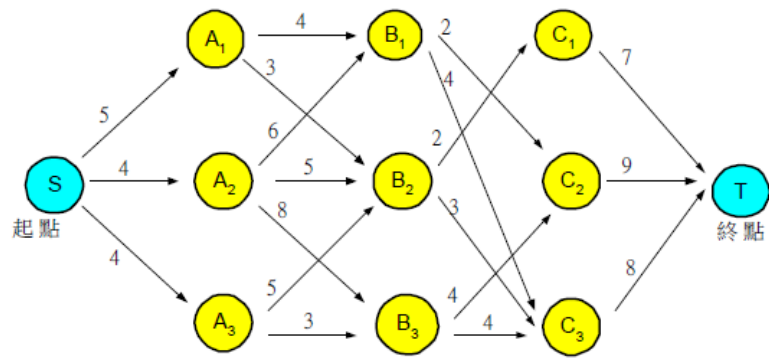
階段3 (Stage 3):

| $S_3$          | $d_3$          | $f(S_3, d_3)$ | $f(S_3, d_3) + g^*(S_2)$<br>$S_2 = d_3$ | $g^*(S_3)$ | $d_3^*$        |
|----------------|----------------|---------------|-----------------------------------------|------------|----------------|
| A <sub>1</sub> | B <sub>1</sub> | 4             | 4 + 11                                  | 12         | B <sub>2</sub> |
|                | B <sub>2</sub> | 3             | 3 + 9                                   |            |                |
| A <sub>2</sub> | B <sub>1</sub> | 6             | 6 + 11                                  | 14         | B <sub>2</sub> |
|                | B <sub>2</sub> | 5             | 5 + 9                                   |            |                |
|                | B <sub>3</sub> | 8             | 8 + 12                                  |            |                |
| A <sub>3</sub> | B <sub>2</sub> | 5             | 5 + 9                                   | 14         | B <sub>2</sub> |
|                | B <sub>3</sub> | 3             | 3 + 12                                  |            |                |

階段4 (Stage 4):

| $S_4$ | $d_4$          | $f(S_4, d_4)$ | $f(S_4, d_4) + g^*(S_3)$<br>$S_3 = d_4$ | $g^*(S_4)$ | $d_4^*$        |
|-------|----------------|---------------|-----------------------------------------|------------|----------------|
| S     | A <sub>1</sub> | 5             | 5 + 12                                  | 17         | A <sub>1</sub> |
|       | A <sub>2</sub> | 4             | 4 + 14                                  |            |                |
|       | A <sub>3</sub> | 4             | 4 + 14                                  |            |                |

$S \rightarrow A_1 \rightarrow B_2 \rightarrow C_1 \rightarrow T$



階段1 (Stage 1):

| $S_1$ | $d_1$          | $f(S_1, d_1)$ | $g^*(S_0) + f(S_1, d_1)$<br>$S_0 = S, g^*(S_0) = g^*(S) = 0$ | $g^*(S_1)$ | $d_1^*$        |
|-------|----------------|---------------|--------------------------------------------------------------|------------|----------------|
| S     | A <sub>1</sub> | 5             | 0 + 5                                                        | 5          | A <sub>1</sub> |
|       | A <sub>2</sub> | 4             | 0 + 4                                                        | 4          | A <sub>2</sub> |
|       | A <sub>3</sub> | 4             | 0 + 4                                                        | 4          | A <sub>3</sub> |

階段2 (Stage 2):

| $S_2$          | $d_2$          | $f(S_2, d_2)$ | $g^*(S_1) + f(S_2, d_2)$<br>$S_1 = d_1$ | $g^*(S_2)$ | $d_2^*$        |
|----------------|----------------|---------------|-----------------------------------------|------------|----------------|
| A <sub>1</sub> | B <sub>1</sub> | 4             | 5 + 4                                   |            |                |
|                | B <sub>2</sub> | 3             | 5 + 3                                   | 8          | B <sub>2</sub> |
| A <sub>2</sub> | B <sub>1</sub> | 6             | 4 + 6                                   |            |                |
|                | B <sub>2</sub> | 5             | 4 + 5                                   | 9          | --             |
|                | B <sub>3</sub> | 8             | 4 + 8                                   |            |                |
| A <sub>3</sub> | B <sub>2</sub> | 5             | 4 + 5                                   |            |                |
|                | B <sub>3</sub> | 3             | 4 + 3                                   | 7          | B <sub>3</sub> |

階段3 (Stage 3):

| $S_3$          | $d_3$          | $f(S_3, d_3)$ | $g^*(S_2) + f(S_3, d_3)$<br>$S_2 = d_2$ | $g^*(S_3)$ | $d_3^*$        |
|----------------|----------------|---------------|-----------------------------------------|------------|----------------|
| B <sub>1</sub> | C <sub>2</sub> | 2             | --                                      |            |                |
|                | C <sub>3</sub> | 4             | --                                      |            |                |
| B <sub>2</sub> | C <sub>1</sub> | 2             | 8 + 2                                   | 10         | C <sub>1</sub> |
|                | C <sub>3</sub> | 3             | 8 + 3                                   |            |                |
| B <sub>3</sub> | C <sub>2</sub> | 4             | 7 + 4                                   | 11         | C <sub>2</sub> |
|                | C <sub>3</sub> | 5             | 7 + 5                                   |            |                |

階段4 (Stage 4):

| $S_4$          | $d_4$ | $f(S_4, d_4)$ | $g^*(S_3) + f(S_4, d_4)$<br>$S_3 = d_3$ | $g^*(S_4)$ | $d_4^*$ |
|----------------|-------|---------------|-----------------------------------------|------------|---------|
| C <sub>1</sub> | T     | 7             | 10 + 7                                  | 17         | T       |
| C <sub>2</sub> | T     | 9             | 11 + 9                                  | 20         | T       |
| C <sub>3</sub> | T     | 8             | --                                      | --         | --      |

T → C<sub>1</sub> → B<sub>2</sub> → A<sub>1</sub> → S