

# Scheduling Algorithms for Minimizing Age of Information in Wireless Broadcast Networks with Random Arrivals

Yu-Pin Hsu\*, Eytan Modiano<sup>†</sup>, and Lingjie Duan<sup>‡</sup>

\*Department of Communication Engineering, National Taipei University

<sup>†</sup>Laboratory for Information and Decision Systems, Massachusetts Institute of Technology

<sup>‡</sup>Engineering Systems and Design Pillar, Singapore University of Technology and Design  
yupinhsu@mail.ntpu.edu.tw, modiano@mit.edu, lingjie\_duan@sutd.edu.sg

**Abstract**—Age of information is a newly proposed metric that captures the freshness of information at end-users. The age measures the amount of time that elapsed since the latest information was generated at a source. In this context, we study an age minimization problem over a wireless broadcast network to keep many users updated on timely information, where only one user can be served at a time. We formulate a Markov decision process (MDP) to find dynamic scheduling algorithms, with the purpose of minimizing the long-run average age. We show that an optimal scheduling algorithm for the MDP is a simple stationary switch type. However, the optimal scheduling algorithm is not easy to implement because of the MDP's infinite state space. Using a sequence of finite-state approximate MDPs, we successfully develop both optimal off-line and on-line scheduling algorithms. We validate the algorithms via numerical studies, and surprisingly show that the performance of no-buffer networks is very close to that of networks with buffers.

## I. INTRODUCTION

Traditional networks' designs have been focused on network throughput or delay. In addition to those performance metrics, in recent years there has been a growing interest in an age of information [2]. The age of information is motivated by a variety of networks requiring timely information. Examples range from information updates for network users, e.g., traffic, transportation, air quality, and weather, to status updates for cyber-physical systems or Internet of things, e.g., smart home systems, smart transportation systems, and smart grid systems.

Illustrated in Fig. 1 is a network, where network users  $u_1, \dots, u_N$  are running some applications that monitor time-varying information (e.g., user  $u_1$  is interested in traffic and transportation information for planning the best route), while at some epochs, snapshots of the information are generated at the sources and sent to the applications in the form of packets over wired or wireless networks. In other words, the applications are being updated and keep the latest information only. Once a packet is generated at its source, it starts to age as time elapses. The age of information is therefore defined to capture the freshness of the information in the applications; more precisely, it measures the time that elapsed since the generation of the information, and our goal is to minimize the long-run average age. In addition to the timely information

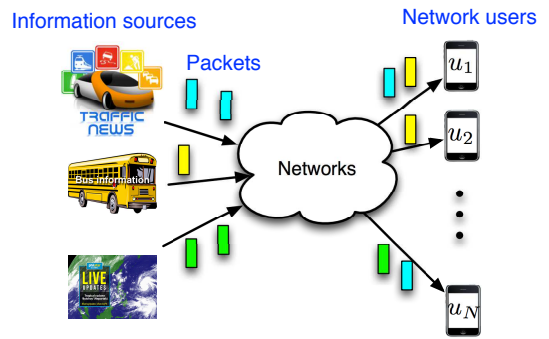


Fig. 1. Timely information updates for network users.

for the network users, the cyber-physical systems or Internet of things also need timely information (e.g., locations and velocities in smart transportation systems) to accomplish some tasks (e.g., collision-free smart transportation systems). As such, the age of information is a good metric to characterize performance of these age-sensitive networks.

While the packet delay is usually referred to as the elapsed time from the generation to its delivery, the age includes not only the packet delay but also the inter-delivery time because the age of information increases until the information is updated at the end-users. We hence need to jointly consider the two parameters so as to design an age-optimal network. Moreover, while traditional relays (i.e., intermediate nodes) need to keep all packets that are not served yet, the relays in the network of Fig. 1 for timely information only store the latest information and discard out-of-date packets. That is, a new arrival always replaces the old packet in a buffer.

In resource-constrained networks, scheduling is a critical issue to optimize network performance. In this paper, we consider a wireless broadcast network consisting of a base-station (BS) and many network users, with time-varying packet arrivals at the BS. Because of a transmission capacity, we assume that the BS can serve at most one user for each transmission opportunity. We note that transmission scheduling schemes for maximizing throughput have been extensively studied using the Lyapunov theory (e.g., [3]). However, these Lyapunov-based algorithms might result in poor delay [4].

As the scheduling design in this paper is driven by the age including the packet delay, we will design our dynamic scheduling algorithms based on Markov decision processes (MDPs) and reinforcement learning.

We formulate our problem as an MDP, with the objective of determining an optimal decision for each state for minimizing the long-run average total age. Although there have been several classic works (e.g., [5–8]) on generic MDPs’ problems, there is no clear methodology to find optimal decisions for the problems that possess the proprieties of infinite horizon and average cost optimization, as well as have a countably infinite state space. In fact, [6] concludes that such problems are difficult to analyze and obtain optimal algorithms.

### A. Contributions and outlines

We start with a noiseless broadcast network *without* buffers at the BS in Section II, i.e., all arriving packets that are not served in the current slot are discarded. By formulating an MDP, in Section III we show that an optimal dynamic scheduling algorithm is *stationary* and *deterministic*. In particular, it is a simple *switch type*, i.e., given the ages of other users, an optimal decision for a user is a *threshold type*, where the BS updates the user if its age is larger than the threshold.

We then propose a *sequence of finite-state approximations* and rigorously show its convergence in Section IV-A, since no practical algorithm can work on infinite-state MDPs like ours. In Section IV-B we proposed an *optimal off-line* scheduling algorithm based on the finite-state approximate MDPs, and in Section IV-C we propose an *optimal on-line* scheduling algorithm by applying reinforcement learning techniques.

We then successfully extend our results and algorithms in Section V to the case where buffers are available at the BS to store the latest information for each user. Finally, in Section VI, we compare these scheduling algorithms via numerical studies, and surprisingly find that the buffers improve performance marginally.

### B. Related works

The general idea of *age* was proposed in [9] to study how to refresh a local copy of an autonomous information source to maintain the copy up-to-date. This paper considers a *pull-based* replication (i.e., the local copy is updated on request), and assumes no communication time between the data source and the local caches. On the contrary, the *age of information* recently proposed in [2] considers communication time while focusing on the *push-based* operation, i.e., it considers not only a network to deliver updates but a communication system over there. Moreover, the age defined in [9] is associated with *discrete* events at the information source, where the age is zero until the source is updated. Differently, the age of information in [2] measures the age of a sample of *continuous* events; therefore, the sample immediately becomes old after generated. Our work will contribute to the theory of the age of information.

The previous works [2, 10–17] study the age of information for a single link. The papers [2, 10–13] consider queues to store all unserved packets (i.e., out-of-date packets are also

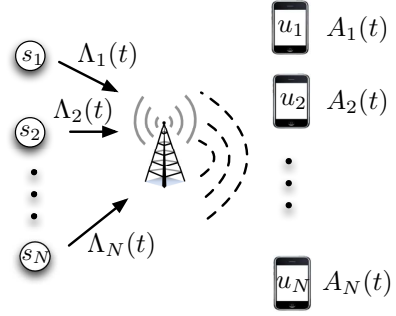


Fig. 2. A BS updates  $N$  users  $u_1, \dots, u_N$  on information of sources  $s_1, \dots, s_N$ , respectively.

stored) and analyze the long-run average age, based on various queuing models. They show that the optimal sampling rate for the average age is not consistent with the throughput optimum or delay optimum. The paper [14] considers a *smart* update and shows the *always update* scheme cannot minimize the average age. Moreover, [15, 16] develop power-efficient updating algorithms for minimizing the average age. The model in [17] uses a small buffer to store the latest information.

Of the most relevant works on scheduling multiple users are [18–22]. The works [18–20] consider queues at a BS to store all unsent packets, different from ours. The paper [21] considers a buffer to store the latest information with *periodic* arrivals, while information updates in [22] can be generated *at will*. Our work is the first to consider *random* arrivals and to develop optimal off-line and on-line scheduling algorithms for both the no-buffer network and the buffer-network.

## II. SYSTEM OVERVIEW

We consider a network consisting of a base-station (BS), and  $N$  wireless users  $u_1, \dots, u_N$  as showed in Fig. 2, where each user  $u_i$  is interested in information generated by an associated source<sup>1</sup>  $s_i$ . The information is transmitted through the BS in the form of packets. The network is a fundamental model to investigate centralized scheduling schemes. We focus on noiseless channels between the BS and users, where all transmissions from the BS to each user are considered to be successful with a sufficiently high probability. Please see Appendix H for an extension to unreliable channels.

We consider a discrete-time system with slots  $t = 0, 1, \dots$ . The packets from the sources (if any) arrive at the BS at the *beginning* of each slot. We consider that the BS can transmit at most one packet during each slot, i.e., the BS can update at most one user in each slot. We assume that the arrivals at the BS for different users are independent of each other and also independent and identically distributed (i.i.d.) over slots, following a Bernoulli distribution. Precisely, by  $\Lambda_i(t)$ , we indicate if a packet from source  $s_i$  arrives at the BS in slot  $t$ , in which  $\Lambda_i(t) = 1$  if there is a packet; otherwise,  $\Lambda_i(t) = 0$ , where  $P[\Lambda_i(t) = 1] = p_i$ .

Moreover, the BS can buffer at most one packet for each user, i.e., an arriving packet always replaces the old one in the

<sup>1</sup>If many users have the same target information, we can group them as a super user.

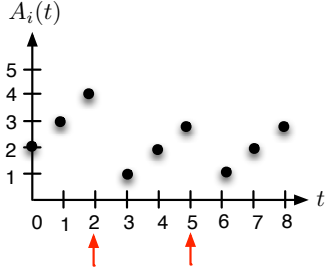


Fig. 3. Evolution of the age of information over slots.

buffer. We start with the scenario without any buffer, where the arrivals are discarded if not transmitted in the current slot. The *no-buffer network* is easy to implement for practical systems, and it was shown to achieve good performance in a single link (see [17]). In Section V, we will extend our results by considering the buffers. It turns out that the main results hold for the *buffer-network*, and the performance of the no-buffer network is very close to the buffer-network (see Sections V and VI).

#### A. Age of information model

The *age of information* implies the freshness of the information at the *users*. We initialize the ages of the *packets* to be zero when reaching the BS. On receiving a packet, the age of information for the user is reset to be one because of one slot of the transmission.

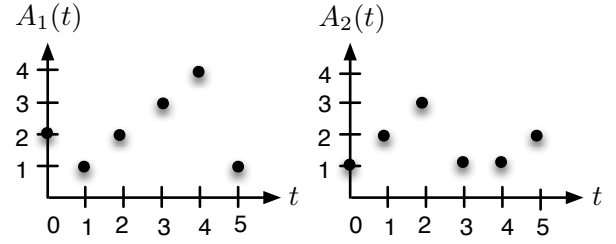
Let  $A_i(t)$  be the age for user  $u_i$  in slot  $t$  before the BS makes a decision. Considering a linearly increasing age over slots, the age of user  $u_i$  in slot  $t$  is  $A_i(t) = t - r_i(t)$ , where  $r_i(t)$  is the slot that the latest packet received by user  $u_i$  was generated. Take Fig. 3 for example, where the arrivals in slots  $t = 2, 5$  for user  $u_i$  are delivered in slots  $t = 3, 6$  and hence the dots represent the ages of information for user  $u_i$  for each slot.

We remark that since the BS can update at most one user for each slot,  $A_i(t) \geq 1$  for all  $i$ ,  $A_i(t) \neq A_j(t)$  for all  $i \neq j$ , and  $\sum_{i=1}^N A_i(t) \geq 1 + 2 + \dots + N$  for all  $t$ .

#### B. Markov decision process model

We use a Markov decision process (MDP) to develop scheduling algorithms for the BS to adaptively update a user at each transmission opportunity. According to [5], we describe the components of our MDP in detail below, followed by Example 1.

- **Decisions and decision epochs:** For each slot, the BS makes a decision immediately after receiving packets (if any). By  $D(t) \in \{0, 1, \dots, N\}$  we denote a decision of the MDP in slot  $t$ , where  $D(t) = 0$  if the BS does not transmit any packet and  $D(t) = i$  for  $i = 1, \dots, N$  if user  $u_i$  is scheduled to be updated in slot  $t$ . Note that the decision space is finite.
- **States:** We define the state  $\mathbf{S}(t)$  of the MDP in slot  $t$  as  $\mathbf{S}(t) = (A_1(t), \dots, A_N(t), \Lambda_1(t), \dots, \Lambda_N(t))$ . By  $\mathbf{S}$  we define the state space including all possible states. Note



$t$	$\Lambda_1(t)$	$\Lambda_2(t)$	$D(t)$	Cost( $t$ )
0	1	1	1	3
1	0	0	0	5
2	0	1	2	4
3	1	1	2	5
4	1	0	1	3

Fig. 4. Example of the age evolution for two users.

that  $\mathbf{S}$  is a *countable infinite* set because the ages are possibly unbounded.

- **Transition probabilities:** Under decision  $D(t) = d$ , as the transmission time is one slot, the age of information in the next slot is

$$A_i(t+1) = \begin{cases} 1 & \text{if } i = d \text{ and } \Lambda_i(t) = 1; \\ A_i(t) + 1 & \text{else.} \end{cases}$$

Let  $P_{\mathbf{s}, \mathbf{s}'}(d)$  be the transition probability from state  $\mathbf{s} = (a_1, \dots, a_N, \lambda_1, \dots, \lambda_N)$  to state  $\mathbf{s}' = (a'_1, \dots, a'_N, \lambda'_1, \dots, \lambda'_N)$  under decision  $D(t) = d$ , with the probability law: if  $a'_i = 1$  for  $i = d$ ,  $\lambda_i = 1$ , and  $a'_i = a_i + 1$  else, then

$$P_{\mathbf{s}, \mathbf{s}'}(d) = \prod_{i: \lambda'_i=1} p_i \prod_{i: \lambda'_i=0} (1 - p_i);$$

otherwise,  $P_{\mathbf{s}, \mathbf{s}'}(d) = 0$ .

- **Cost:** Let  $C(\mathbf{S}(t), D(t) = d)$  be the *immediate cost* if decision  $D(t) = d$  is taken in slot  $t$  under state  $\mathbf{S}(t)$ . We consider the *total age* after making a decision in slot  $t$ :

$$\begin{aligned} C(\mathbf{S}(t), D(t) = d) &\triangleq \sum_{i=1}^N A_i(t+1) \\ &= \sum_{i=1}^N (A_i(t) + 1) - A_d(t) \cdot \Lambda_d(t), \end{aligned} \quad (1)$$

where we define  $A_0(t) = 0$  and  $\Lambda_0(t) = 0$  for all  $t$  (for the case of  $d = 0$ ), while the last term indicates that user  $u_d$  is updated in slot  $t$ . We remark that we focus on the total age for the purpose of delivering clean results; whereas our analysis and design can also work perfectly for the *weighted sum* of the ages.

**Example 1.** In Fig. 4, we illustrate the age evolution (in the sub-figures) in terms of the arrivals and the decisions (in the table), where the initial ages are  $A_1(0) = 2$  and  $A_2(0) = 1$  for the two users. We also show the immediate cost for each slot in the table. As  $D(t) = 1$  in slot  $t = 0$ , the resulting

age in  $t = 1$  for  $u_1$  and  $u_2$  are  $A_1(1) = 1$  and  $A_2(1) = 2$ , respectively. As a result, the cost associated with the decision is  $A_1(1) + A_2(1) = 3$ . Similarly, we can calculate the cost for each slot in the table.

### C. Average-optimal scheduling algorithm design

A scheduling algorithm  $\theta = \{D(0), D(1), \dots\}$  specifies a decision for each slot. An algorithm is *history dependent* if  $D(t)$  depends on  $D(0), \dots, D(t-1)$  and  $\mathbf{S}(0), \dots, \mathbf{S}(t)$ . An algorithm is *stationary* if  $D(t_1) = D(t_2)$  when  $\mathbf{S}(t_1) = \mathbf{S}(t_2)$  for any  $t_1, t_2$ . Moreover, a *randomized* algorithm specifies a probability distribution on the set of decisions for each decision epoch, while a *deterministic* algorithm makes a decision with certainty. In general, an algorithm belongs to one of the following sets [5]:

- $\Pi^{\text{HR}}$ : a set of randomized history dependent algorithms;
- $\Pi^{\text{SR}}$ : a set of randomized stationary algorithms;
- $\Pi^{\text{SD}}$ : a set of deterministic stationary algorithms.

The *long-run average cost* under scheduling algorithm  $\theta \in \Pi^{\text{HR}}$  is given by

$$V(\theta) = \limsup_{T \rightarrow \infty} \frac{1}{T+1} E_{\theta} \left[ \sum_{t=0}^T C(\mathbf{S}(t), D(t)) | \mathbf{S}(0) \right],$$

where  $E_{\theta}$  represents the conditional expectation, given that scheduling algorithm  $\theta$  is employed.

**Definition 2.** A scheduling algorithm  $\theta$  (that belongs to  $\Pi^{\text{HR}}$ ) is *average-optimal* if it minimizes the long-run average cost  $V(\theta)$ .

Our goal is to characterize and obtain an *average-optimal scheduling algorithm*. However, we note (as shown in [5]) that  $\Pi^{\text{SD}} \subset \Pi^{\text{SR}} \subset \Pi^{\text{HR}}$ , while the complexity of the scheduling algorithms increases from left to right. According to [5], there may not exist a  $\Pi^{\text{SR}}$  or  $\Pi^{\text{SD}}$  algorithm that is average-optimal. Hence, we target at characterizing a regime such that an average-optimal scheduling lies in  $\Pi^{\text{SD}}$  and developing a simple associated scheduling algorithm.

## III. CHARACTERIZATION OF THE AVERAGE-OPTIMALITY

We first introduce an infinite horizon  $\alpha$ -discounted cost case, where  $0 < \alpha < 1$  is a discount factor; we then connect to the average cost case because structures of an average-optimal algorithm is usually associated with the discounted cost case.

### A. $\alpha$ -discounted MDP model

Given initial state  $\mathbf{S}(0) = \mathbf{s}$ , the *expected total  $\alpha$ -discounted cost* under scheduling algorithm  $\theta \in \Pi^{\text{HR}}$  is

$$V_{\alpha}(\mathbf{s}; \theta) = \lim_{T \rightarrow \infty} E_{\theta} \left[ \sum_{t=0}^T \alpha^t C(\mathbf{S}(t), D(t)) | \mathbf{S}(0) = \mathbf{s} \right].$$

**Definition 3.** A scheduling algorithm  $\theta$  is  *$\alpha$ -optimal* if it minimizes the expected total discounted cost  $V_{\alpha}(\mathbf{s}; \theta)$ . In particular, we define

$$V_{\alpha}(\mathbf{s}) = \min_{\theta} V_{\alpha}(\mathbf{s}; \theta).$$

Moreover, by  $h_{\alpha}(\mathbf{s}) = V_{\alpha}(\mathbf{s}) - V_{\alpha}(\mathbf{0})$  we define the *relative cost function*, which is the difference of the discounted cost between state  $\mathbf{s}$  and a reference state  $\mathbf{0}$ . We can arbitrarily choose the reference state, e.g.,  $\mathbf{0} = (1, 2, \dots, N, 1, \dots, 1)$ . We first introduce the *discounted cost optimality equation* of  $V_{\alpha}(\mathbf{s})$  below.

**Proposition 4.** The optimal expected total discounted cost  $V_{\alpha}(\mathbf{s})$ , for state  $\mathbf{s}$ , satisfies the following discounted cost optimality equation:

$$V_{\alpha}(\mathbf{s}) = \min_{d \in \{0, 1, \dots, N\}} C(\mathbf{s}, d) + \alpha E[V_{\alpha}(\mathbf{s}')], \quad (2)$$

where the expectation is taken over all possible next state  $\mathbf{s}'$  reachable from the state  $\mathbf{s}$ . A deterministic stationary algorithm that realizes the minimum of right hand side (RHS) of the discounted cost optimality equation in Eq. (2) will be an  $\alpha$ -optimal algorithm. Moreover, we define  $V_{\alpha, n}(\mathbf{s})$  by  $V_{\alpha, 0}(\mathbf{s}) = 0$  and for any  $n \geq 0$ ,

$$V_{\alpha, n+1}(\mathbf{s}) = \min_{d \in \{0, 1, \dots, N\}} C(\mathbf{s}, d) + \alpha E[V_{\alpha, n}(\mathbf{s}')]. \quad (3)$$

Then,  $V_{\alpha, n}(\mathbf{s}) \rightarrow V_{\alpha}(\mathbf{s})$  as  $n \rightarrow \infty$  for every  $\mathbf{s}$ , and  $\alpha$ .

*Proof:* Please see Appendix A. ■

The *value iteration* in Eq. (3) will be helpful for identifying properties of  $V_{\alpha}(\mathbf{s})$ , e.g., to prove that  $V_{\alpha}(\mathbf{s})$  is a non-decreasing function in the following.

**Proposition 5.**  $V_{\alpha}(a_i, \mathbf{a}_{-i}, \boldsymbol{\lambda})$  is a non-decreasing function in  $a_i$  given  $\mathbf{a}_{-i} = \{a_1, \dots, a_N\} - \{a_i\}$  and  $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_N)$ .

*Proof:* Please see Appendix B. ■

### B. Average-optimality of a deterministic stationary algorithm

Using the properties of  $\alpha$ -discounted MDP (Propositions 4 and 5), we show that the MDP defined in this paper has an average-optimal algorithm that is deterministic stationary.

**Lemma 6.** There exists a scheduling algorithm in  $\Pi^{\text{SD}}$  that is average-optimal. Moreover, there exists a finite constant  $V^* = \lim_{\alpha \rightarrow 1} (1 - \alpha) V_{\alpha}(\mathbf{s})$  for every state  $\mathbf{s}$  such that the average-optimal cost is  $V^*$ , independent of the initial state  $\mathbf{S}(0)$ .

*Proof:* Please see Appendix C. ■

We want to elaborate further on Lemma 6. First, note that there is no condition for the existence of an average-optimal scheduling algorithm in  $\Pi^{\text{SD}}$ . In general, we need some conditions to guarantee that the reduced Markov chain from applying a stationary scheduling algorithm is positive recurrent. We can think of the age of our problem as an age-queue, where the age-queueing system consists of an age-queue, input to the queue, and a server. The input rate is one per slot since the age increases by one for each slot, while the server can serve an *infinite* number of age-packets for each service opportunity. As such, we always can find a scheduling algorithm such that the average arrival rate is less than the service rate and thus the reduced Markov chain is positive recurrent. Second, we remark that even though the average-optimality of a deterministic stationary algorithm is shown in Lemma 6, we might not arrive at the average cost optimality

equation like Eq. (2) or the value iteration like Eq. (3) (see [8, 23] for details).

### C. Structural results

In addition to the average-optimality of a deterministic stationary algorithm, we show that an optimal scheduling algorithm has a nice structure facilitating the scheduling algorithm design in the next section.

**Definition 7.** A *switch-type* scheduling algorithm is a special deterministic stationary scheduling algorithm: for every user  $u_i$ , if the decision of the algorithm for state  $\mathbf{s} = (a_i, \mathbf{a}_{-i}, \boldsymbol{\lambda})$  is  $d_{\mathbf{s}} = i$ , then the decision for state  $\mathbf{s}' = (a_i + 1, \mathbf{a}_{-i}, \boldsymbol{\lambda})$  is  $d_{\mathbf{s}'} = i$  as well.

**Theorem 8.** An *average-optimal scheduling algorithm is the switch type*.

*Proof:* (Sketch) We first prove that an  $\alpha$ -optimal scheduling algorithm is the switch type by applying the value iteration in Eq. (3). Then, we show that the structure holds for the average-optimum by letting  $\alpha \rightarrow 1$ . Please see Appendix D for details. ■

In particular, when the arrival rates of all information sources are the same, we can obtain a nice *index algorithm* as follows.

**Corollary 9.** *If the arrival rates of all information sources are the same, i.e.,  $p_i = p_j$  for all  $i \neq j$ , then an optimal scheduling algorithm transmits a coming packet with the largest age of information, i.e.,  $D(t) = \arg \max_i A_i(t)\Lambda_i(t)$  for each slot  $t$ .*

*Proof:* Please see Appendix E. ■

We also notice that the index algorithm is indeed an on-line algorithm, without the prior knowledge of the arrival statistics. For general asymmetric arrivals, an average-optimal scheduling algorithm depends on both the arrival statistics and the current ages. However, it is not obvious to get an average-optimal scheduling (like Corollary 9) for the asymmetric arrivals. This key challenge hence motivates us to investigate both off-line and on-line scheduling algorithms in the next section.

## IV. SCHEDULING ALGORITHM DESIGN

We start with proposing finite-state approximations to the original MDP as in practice we can only work on a finite-state MDP to avoid formidably high computational complexity. We will rigorously show the convergence of the proposed truncation as in general an MDP truncation might not converge to the original MDP according to [8].

Based on the approximate finite-state MDPs, we first develop a structural value iteration algorithm in Section IV-B to pre-compute an optimal decision for each state, whose complexity would be lower than the conventional value iteration algorithm by leveraging the switch-type structure. Moreover, we develop an on-line algorithm using reinforcement learning techniques [24, 25] and stochastic approximations [26] in Section IV-C

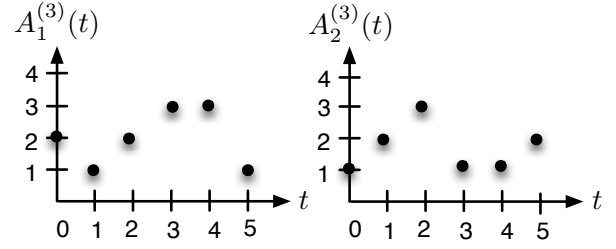


Fig. 5. Example of the age evolution for the finite-state approximation  $\Delta_3$ .

### A. Finite-state MDP approximations

Let  $\Delta$  be the Markov decision process defined in Section II-B. By  $\{\Delta_m\}_{m=1}^{\infty}$  we define a sequence of approximate MDPs for  $\Delta$  whose state space is  $\mathbf{S}_m = \{\mathbf{s} \in \mathbf{S} : a_i \leq m\}$  with the bounded *virtual ages*, while the decision space and cost definition (see Eq. (1)) are the same as  $\Delta$ .

Let  $A_i^{(m)}(t)$  be the age of information for user  $u_i$  in slot  $t$  for  $\Delta_m$ . Different from  $\Delta$ , under decision  $D(t) = d$  the age of the next slot for  $\Delta_m$  is

$$A_i^{(m)}(t+1) = \begin{cases} 1 & \text{if } i = d \text{ and } \Lambda_i(t) = 1; \\ \left[ A_i^{(m)}(t) + 1 \right]_m^+ & \text{else,} \end{cases}$$

where we define the notation  $[x]_m^+$  by  $[x]_m^+ = x$  if  $x \leq m$  and  $[x]_m^+ = m$  if  $x > m$ .

**Example 10.** In Fig. 5, we illustrate the age evolution for the finite-state approximation  $\Delta_3$ . Different from  $\Delta$  in Fig. 4, in slot 4 the age of the user  $u_1$  keeps three.

Next, we will show that the proposed finite-state approximation will be asymptotically average-optimal.

**Theorem 11.** *Let  $V^*$  and  $V_m^*$  be the average-optimal cost to  $\Delta$  and  $\Delta_m$ , respectively. Then,  $V_m^* \rightarrow V^*$  as  $m \rightarrow \infty$ .*

*Proof:* Please see Appendix F. ■

Now, we are ready to propose scheduling algorithms based on  $\Delta_m$ . In other words, our algorithms make decisions according to the *virtual age*  $A_i^{(m)}(t)$  on  $\mathbf{S}_m$ , instead of the *real age*  $A_i(t)$  on  $\mathbf{S}$ . The real age can increase beyond  $m$  but the virtual age will be smaller than  $m + 1$  (see Figs. 4 and 5 for example).

### B. Structural off-line scheduling algorithm

The traditional *relative value iteration algorithm* (RVIA), as follows, can be applied to get an optimal deterministic stationary algorithm on  $\Delta_m$ :

$$V_{n+1}(\mathbf{s}) = \min_{d \in \{0,1,\dots,N\}} C(\mathbf{s}, d) + E[V_n(\mathbf{s}')] - V_n(\mathbf{0}), \quad (4)$$

---

**Algorithm 1: Structural off-line scheduling algorithm**


---

```

1  $V(\mathbf{s}) \leftarrow 0$  for all states  $\mathbf{s} \in \mathbf{S}_m$ ;
2 while 1 do
3   forall  $\mathbf{s} \in \mathbf{S}_m$  do
4     if there exists  $\zeta > 0$  and  $i \in \{1, \dots, N\}$  such
       that  $d_{(a_i - \zeta, \mathbf{a}_{-i}, \lambda)}^* = i$  then
5       |  $d_{\mathbf{s}}^* \leftarrow i$ ;
6     else
7       |  $d_{\mathbf{s}}^* \leftarrow \arg \min_{d \in \{0, 1, \dots, N\}} C(\mathbf{s}, d) + E[V(\mathbf{s}')]$ ;
8     end
9      $V_{\text{tmp}}(\mathbf{s}) \leftarrow C(\mathbf{s}, d_{\mathbf{s}}^*) + E[V(\mathbf{s}')] - V(\mathbf{0})$ ;
10  end
11   $V(\mathbf{s}) \leftarrow V_{\text{tmp}}(\mathbf{s})$  for all  $\mathbf{s} \in \mathbf{S}_m$ .
12 end

```

---

for all  $\mathbf{s} \in \mathbf{S}_m$ . For each iteration  $n$ , we need to update decisions for *all* virtual states by minimizing the RHS of Eq. (4) as well as update  $V(\mathbf{s})$  for *all*  $\mathbf{s} \in \mathbf{S}_m$ . The complexity is very high<sup>2</sup> because many users result in the curse of dimensionality [27]. Therefore, we propose in Alg. 1 a structural off-line algorithm based on the RVIA along with the switch-type structure.

In Alg. 1, we seek an optimal decision  $d_{\mathbf{s}}^*$  for each virtual state  $\mathbf{s} \in \mathbf{S}_m$  by iteration. For each iteration, we update both the optimal decision  $d_{\mathbf{s}}^*$  and  $V(\mathbf{s})$  for all virtual states. If the switch property holds<sup>3</sup>, we can determine an optimal decision *immediately* in Line 5; otherwise we find an optimal decision according to Line 7. By  $V_{\text{tmp}}(\mathbf{s})$  in Line 9 we temporarily keep the updated value, which will replace  $V(\mathbf{s})$  in Line 11. Using the switch structure to prevent from the minimum operations on all virtual states in the RVIA, we can reduce the computational complexity greatly.

**Theorem 12.** *The limit point of  $d_{\mathbf{s}}^*$  in Alg. 1 is an average-optimal decision for every virtual state  $\mathbf{s} \in \mathbf{S}_M$ . In particular, Alg. 1 converges in a finite number of iterations.*

*Proof:* (Sketch) According to [5, Theorem 8.6.6], we only need to verify that the truncated MDPs are *unichain*. Please see Appendix G for details. ■

### C. On-line scheduling algorithm

We note that Alg. 1 needs the arrival statistics to pre-compute an optimal decision for each virtual state. We will develop an on-line scheduling algorithm if the statistics is unavailable. Instead of updating  $V(\mathbf{s})$  for all virtual states in each iteration, we update  $V(\mathbf{s})$  following a *sample path*, which is a set of outcomes of the arrivals over slots. It turns out that the sample-path updates will converge to the average-optimal solution.

To that end, we need a *stochastic version* of the RVIA. However, the RVIA in Eq. (4) is not suitable because the

<sup>2</sup>As the size of the state space is  $O(m^N)$ , the complexity to update all states in each iteration of Eq. (4) is higher than  $O(m^N)$ .

<sup>3</sup>The average-optimal scheduling algorithm for the truncated MDP is the threshold type as well by the same proof as Theorem 8.

expectation is inside the minimization (as in [27]). While minimizing the RHS of Eq. (4) for a given current state, we would need the transition probabilities to calculate the expectation. To tackle these challenges, we design post-decision states [27, 28] for our problem.

We define the post-decision state  $\tilde{\mathbf{s}}$  as the ages and the arrivals *after* a decision. The state we used before is referred to as the *pre-decision* state. If  $\mathbf{s} = (a_1, \dots, a_N, \lambda_1, \dots, \lambda_N) \in \mathbf{S}_m$  is a virtual state of the system, then the virtual post-decision state after decision  $d$  is  $\tilde{\mathbf{s}} = (\tilde{a}_1, \dots, \tilde{a}_N, \tilde{\lambda}_1, \dots, \tilde{\lambda}_N)$  with

$$\tilde{a}_i = \begin{cases} 1 & \text{if } i = d \text{ and } \lambda_i = 1; \\ [a_i + 1]_m^+ & \text{else,} \end{cases}$$

and  $\tilde{\lambda}_i = \lambda_i$  for all  $i$ .

Let  $\tilde{V}(\tilde{\mathbf{s}})$  be the value function based on the post-decision states defined by

$$\tilde{V}(\tilde{\mathbf{s}}) = E_{\mathbf{s}}[V(\mathbf{s})],$$

where the expectation  $E_{\mathbf{s}}$  is taken over all possible the pre-decision states reachable from the post-decision state. We can then write down the post-decision average cost optimality equation [27] for the virtual post-decision state  $\tilde{\mathbf{s}} = (\tilde{a}_1, \dots, \tilde{a}_N, \tilde{\lambda}_1, \dots, \tilde{\lambda}_N)$  with  $\tilde{\mathbf{s}} \in \mathbf{S}_m$ :

$$\begin{aligned} & \tilde{V}(\tilde{\mathbf{s}}) + V^* \\ &= E \left[ \min_{d \in \{0, 1, \dots, N\}} C((\tilde{\mathbf{a}}, \tilde{\lambda}'), d) + \tilde{V}([\tilde{\mathbf{a}} + \mathbf{1} - \tilde{\mathbf{a}}_d \tilde{\lambda}'_d]_m^+, \tilde{\lambda}') \right], \end{aligned}$$

where  $\tilde{\lambda}'$  summarizes the possible next arrivals. Moreover,  $\tilde{\mathbf{a}}_i = (0, \dots, \tilde{a}_i, \dots, 0)$  is the zero vector except for the  $i$ -th entry being substituted by  $\tilde{a}_i$ , and  $\tilde{\lambda}'_i$  is defined similarly. The vector  $\mathbf{1} = (1, \dots, 1)$  is the unit vector. We also recall that  $V^*$  here is the constant as in Lemma 6.

From above optimality equation, the RVIA is as follows:

$$\begin{aligned} & \tilde{V}_{n+1}(\tilde{\mathbf{s}}) \\ &= E \left[ \min_{d \in \{0, 1, \dots, N\}} C((\tilde{\mathbf{a}}, \tilde{\lambda}'), d) + \tilde{V}_n([\tilde{\mathbf{a}} + \mathbf{1} - \mathbf{a}_d \tilde{\lambda}'_d]_m^+, \tilde{\lambda}') \right] \\ & \quad - \tilde{V}_n(\mathbf{0}). \end{aligned} \quad (5)$$

Subsequently, we propose the on-line algorithm in Alg. 2 based on the stochastic version of the RVIA. In Lines 1-3, we initialize  $\tilde{V}(\tilde{\mathbf{s}})$  of all virtual post-decision states and start from the reference point. Moreover, by  $v$  we record  $\tilde{V}(\tilde{\mathbf{s}})$  of the current virtual post-decision state.

By observing the current arrivals  $\Lambda(t)$  and plugging in Eq. (5), in Line 5 we optimally update a user by minimizing Eq. (6); as such, the expectation in Eq. (5) is removed. Then, we update  $\tilde{V}(\tilde{\mathbf{s}})$  of the current virtual post-decision state in Line 7, where  $\gamma(t)$  is the *stochastic step-size* (see [27]) in slot  $t$  and hence  $\tilde{V}(\tilde{\mathbf{s}})$  is balanced between the previous  $\tilde{V}(\tilde{\mathbf{s}})$  and the current value  $v$ . Finally, the next virtual post-decision state is updated in Lines 8 and 9

**Theorem 13.** *If  $\sum_t \gamma(t) = \infty$  and  $\sum_t \gamma^2(t) < \infty$ , then Alg. 2 converges to the average-optimal value.*

*Proof:* According to [24, 25], we only need to verify that

---

**Algorithm 2:** On-line scheduling algorithm
 

---

```

/* Initialization */
1  $\tilde{V}(\tilde{s}) \leftarrow 0$  for all states  $\tilde{s} \in \mathbf{S}_m$ ;
2  $\tilde{s} \leftarrow \mathbf{0}$ ;
3  $v \leftarrow 0$ ;
4 while 1 do
   /* Decision in slot  $t$  */
5   We optimally make a decision  $D^*(t)$  in slot  $t$ 
   according to the current arrivals
    $\Lambda(t) = (\Lambda_1(t), \dots, \Lambda_N(t))$  in slot  $t$ :
       
$$D^*(t) = \underset{d \in \{0,1,\dots,N\}}{\operatorname{arg\,min}} C((\tilde{\mathbf{a}}, \Lambda(t)), d)
       + \tilde{V}([\tilde{\mathbf{a}} + \mathbf{1} - \tilde{\mathbf{a}}_d \Lambda_d(t)]_M^+, \Lambda(t)); \quad (6)$$

   /* Value update */
6    $v \leftarrow C((\tilde{\mathbf{a}}, \Lambda(t)), D^*(t)) + \tilde{V}([\tilde{\mathbf{a}} + \mathbf{1} - \tilde{\mathbf{a}}_{D^*(t)} \Lambda_{D^*(t)}(t)]_M^+, \Lambda(t)) - \tilde{V}(\mathbf{0})$ ;
7    $\tilde{V}(\tilde{s}) \leftarrow (1 - \gamma(t))\tilde{V}(\tilde{s}) + \gamma(t)v$ ;
   /* Post-decision state update */
8    $\tilde{\mathbf{a}} \leftarrow [\tilde{\mathbf{a}} + \mathbf{1} - \tilde{\mathbf{a}}_{D^*(t)} \Lambda_{D^*(t)}(t)]_M^+$ ;
9    $\tilde{\lambda} \leftarrow \Lambda(t)$ .
10 end

```

---

the truncated MDPs are unichain, which has been completed in Appendix G. ■

In the above theorem,  $\sum_t \gamma(t) = \infty$  implies that Alg. 2 needs an infinite number of iterations to learn the average-optimal solution, while Alg. 1 can converge to the optimal solution in a finite number of iterations. Moreover,  $\sum_t \gamma^2(t) < \infty$  means that the *noise* from measuring  $\tilde{V}(\tilde{s})$  can be controlled.

Finally, we want to emphasize that the proposed Algs. 1 and 2 have been proven to be asymptotically optimal, i.e., they converge to the optimal solution when the finite state space  $m$  and the slot  $t$  go to infinity. In Section VI, we will also numerically study the performance of our algorithms over finite slots.

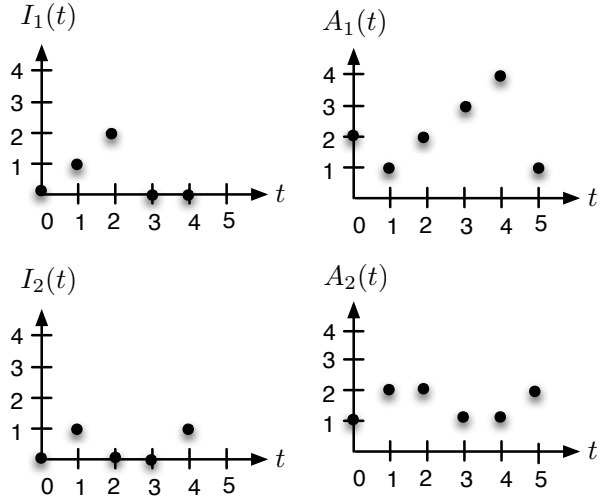
## V. SCHEDULING FOR THE BUFFER-NETWORK

Thus far, our design and analysis of scheduling focus on the no-buffer network. Intuitively, by storing the latest information, the buffers at the BS can reduce the average age in case of no arrivals in the next slot. To what extent can the buffers improve the performance at the expense of the deployed buffers?

Hence, we want to answer the question by considering the *buffer-network*, where for each user there is a buffer at the BS to store the latest information. That is, a new arrival will replace the old packet (if any) in a buffer.

The MDP for the buffer-network is to the no-buffer network. We emphasize the differences as follows.

- **States:** Because of the buffers at the BS, we need to redefine the states. In addition to the age  $A_i(t)$  of information in *user*  $u_i$ , by  $I_i(t)$  we define the *initial age*



$t$	$\Lambda_1(t)$	$\Lambda_2(t)$	$D(t)$	Cost( $t$ )
0	1	1	1	3
1	0	0	2	4
2	0	1	2	4
3	1	1	2	5
4	1	0	1	3

Fig. 6. Example of the age evolution for the buffer-network.

of the information at the buffer for user  $u_i$ ; precisely,

$$I_i(t) = \begin{cases} 0 & \text{if } \Lambda(t) = 1; \\ I_i(t-1) + 1 & \text{else.} \end{cases}$$

We note that the initial age at the BS keeps increasing even after the packet has been delivered. Then, we define the state by  $\mathbf{S}(t) = \{A_1(t), \dots, A_N(t), I_1(t), \dots, I_N(t)\}$ .

- **Transition probabilities:** Under decision  $D(t) = d$ , the ages are updated according to

$$A_i(t+1) = \begin{cases} I_d(t) + 1 & \text{if } i = d; \\ A_i(t) + 1 & \text{else,} \end{cases}$$

where the user  $u_d$  is updated on the packet in the buffer; as such, its age becomes  $I_d(t)$  plus one slot of the transmission. Similarly, we can write down the transition probability  $P_{\mathbf{s},\mathbf{s}'}(d)$  from state  $\mathbf{s} = (a_1, \dots, a_N, I_1, \dots, I_N)$  to state  $\mathbf{s}' = (a'_1, \dots, a'_N, I'_1, \dots, I'_N)$ : if  $I'_i = 0$  for  $\lambda'_i = 1$ ,  $I'_i = I_i + 1$  for  $\lambda'_i = 0$ ,  $a'_i = I_i + 1$  for  $i = d$ , and  $a'_i = a_i + 1$  for  $i \neq d$ , then

$$P_{\mathbf{s},\mathbf{s}'}(d) = \prod_{i:\lambda'_i=1} p_i \prod_{i:\lambda'_i=0} (1 - p_i);$$

otherwise,  $P_{\mathbf{s},\mathbf{s}'}(d) = 0$ .

- **Cost:** The immediate cost is redefined as

$$C(\mathbf{S}(t), D(t) = d) = \sum_{i=1}^N (A_i(t) + 1) - (A_d(t) - I_d(t)),$$

where we define  $I_0(t) = 0$  for all  $t$ .

**Example 14.** We investigate the advantage of the buffers through Fig. 6. Here, we illustrate the age evolution in Fig. 4 after deploying the buffers. We note that in slot 1, the user  $u_2$  can be updated using the packet in the buffer. By means of the buffers, the BS can update a user even without any arrival at present.

We then consider the same long-run average cost as before and intend to find average-optimal scheduling algorithms for the buffer-network. Using the same definition of the discounted cost, we can similarly show the average-optimality of a deterministic stationary algorithm.

**Theorem 15.** *For the buffer-network, there exists a deterministic stationary algorithm that is average-optimal. Moreover, there exists a finite constant  $V^* = \lim_{\alpha \rightarrow 1} (1 - \alpha)V_\alpha(\mathbf{s})$  for every state  $\mathbf{s}$  such that the average-optimal cost is  $V^*$ .*

Let  $\mathbf{I} = (I_1, \dots, I_N)$  be the vector including all initial ages. We then characterize a structure of an average-optimal algorithm as follows.

**Theorem 16.** *An average-optimal scheduling algorithm for the buffer-network is a switch type. For every user  $u_i$ , if an average-optimal decision at state  $\mathbf{s} = (a_i, \mathbf{a}_{-i}, \mathbf{I})$  is  $d_{(a_i, \mathbf{a}_{-i}, \mathbf{I})}^* = i$ , then  $d_{(a_{i+1}, \mathbf{a}_{-i}, \mathbf{I})}^* = i$ .*

#### A. Finite-state MDP approximations

Similar to Section IV-A, we define a sequence of finite-state approximations  $\{\Delta_m\}_{m=1}^\infty$  to the original MDP  $\Delta$  of the buffer-network, where the finite state space for  $\Delta_m$  is  $\mathbf{S}_m = \{\mathbf{s} \in \mathbf{S} : a_i \leq m, I_i \leq m\}$ , while the decision-space and cost definition are the same as  $\Delta$ .

Similarly, under decision  $D(t) = d$  the virtual age in the next slot for  $\Delta_m$  is

$$A_i^{(m)}(t+1) = \begin{cases} \left[ I_d^{(m)}(t) + 1 \right]_m^+ & \text{if } i = d; \\ \left[ A_i^{(m)}(t) + 1 \right]_m^+ & \text{else.} \end{cases}$$

In addition, the virtual initial age is

$$I_i^{(m)}(t) = \begin{cases} \left[ I_i^{(m)}(t-1) + 1 \right]_m^+ & \text{if } \Lambda_i(t) = 0; \\ 0 & \text{else.} \end{cases}$$

Similar to Theorem 11, we can show that the proposed finite-state approximation of the buffer-network will be asymptotically average-optimal, as follows.

**Theorem 17.** *Let  $V^*$  and  $V_m^*$  be the average-optimal cost to  $\Delta$  and  $\Delta_m$  of the buffer-network, respectively. Then,  $V_m^* \rightarrow V^*$  as  $m \rightarrow \infty$ .*

#### B. Structural off-line scheduling algorithm

Similar to Alg. 1, we propose the structural off-line scheduling algorithm for the buffer-network in Alg. 3 to pre-compute an optimal decision for each virtual state. Note that in Line 5 the switch structure for the buffer-network is applied. Similar to Theorem 12, we can get the following result.

---

#### Algorithm 3: Structural off-line scheduling algorithm for the buffer-network

---

```

1  $V(\mathbf{s}) \leftarrow 0$  for all states  $\mathbf{s} \in \mathbf{S}_m$ ;
2 while 1 do
3   forall  $\mathbf{s} \in \mathbf{S}_m$  do
4     if there exists  $\zeta > 0$  and  $i \in \{1, \dots, N\}$  such
       that  $d_{(a_i - \zeta, \mathbf{a}_{-i}, \mathbf{I})}^* = i$  then
5       |  $d_{\mathbf{s}}^* \leftarrow i$ ;
6     else
7       |  $d_{\mathbf{s}}^* \leftarrow \arg \min_{d \in \{0, 1, \dots, N\}} C(\mathbf{s}, d) + E[V(\mathbf{s}')]$ ;
8     end
9      $V_{\text{tmp}}(\mathbf{s}) \leftarrow C(\mathbf{s}, d_{\mathbf{s}}^*) + E[V(\mathbf{s}')] - V(\mathbf{0})$ ;
10  end
11   $V(\mathbf{s}) \leftarrow V_{\text{tmp}}(\mathbf{s})$  for all  $\mathbf{s} \in \mathbf{S}_m$ .
12 end

```

---

**Theorem 18.** *The limit point of  $d_{\mathbf{s}}^*$  in Alg. 3 is an average-optimal decision for every state  $\mathbf{s} \in \mathbf{S}_M$  of the buffer-network. In particular, Alg. 3 converges in a finite number of iterations.*

#### C. On-line scheduling algorithm

Similar to Alg. 2, we propose the on-line scheduling algorithm for the buffer-network in Alg. 4. In Line 5, we update the virtual pre-decision state  $\mathbf{I}$  by observing the current arrivals, while making a decision according to the current virtual initial age in Line 6, where  $\mathbf{I}_i = (0, \dots, I_i, \dots, 0)$  is the zero vector except for the  $i$ -th item being replaced by  $I_i$ .

**Theorem 19.** *For the buffer-network, if  $\sum_t \gamma(t) = \infty$  and  $\sum_t \gamma^2(t) < \infty$ , then Alg. 4 converges to the average-optimal cost.*

Similarly, the algorithms for the buffer-network are asymptotically optimal, while their performance over finite slots will be numerically studied in the next section.

## VI. NUMERICAL RESULTS

In this section, we conduct extensive simulations of the proposed scheduling algorithms. We will demonstrate the switch-type structure in Section VI-A. In Section VI-B we study the proposed off-line scheduling algorithms and compare the performance between the no-buffer network and buffer-network. While we have shown that the proposed on-line scheduling algorithms will converge to the optimal solution as the slot approaches infinity, in Section VI-C we show the performance of the on-line algorithms over finite slots.

#### A. Switch-type structure of Alg. 1

Figs. 7-(a) and 7-(b) show the switch-type structure of an average-optimal scheduling algorithm for two users in the no-buffer network. The experiment setting is as follows. We run Alg. 1 with the boundary  $m = 10$  over 100,000 slots to search an optimal decision for each virtual state. Moreover, we consider various arrival rates with  $(p_1, p_2)$  being  $(0.9, 0.9)$  and  $(0.9, 0.5)$ , respectively, in Figs. 7-(a) and 7-(b), where the



---

**Algorithm 4:** On-line scheduling algorithm for the buffer-network

---

```

/* Initialization */
1  $\tilde{V}(\tilde{\mathbf{s}}) \leftarrow 0$  for all states  $\tilde{\mathbf{s}} \in \mathbf{S}_m$ ;
2  $\tilde{\mathbf{s}} \leftarrow \mathbf{0}$ ;
3  $v \leftarrow 0$ ;
4 while 1 do
    /* Decision in slot  $t$  */
5 Update the initial ages according to the current
   arrivals  $\mathbf{\Lambda}(t) = (\Lambda_1(t), \dots, \Lambda_N(t))$  in slot  $t$ :
    $I_i \leftarrow \tilde{I}_i$  if  $\Lambda_i(t) = 0$ ; otherwise,  $I_i \leftarrow 0$ ;
6 We optimally make a decision  $D^*(t)$  in slot  $t$ :

   
$$D^*(t) = \underset{d \in \{0,1,\dots,N\}}{\operatorname{arg\,min}} C((\tilde{\mathbf{a}}, \mathbf{I}), d)
   + \tilde{V}([\tilde{\mathbf{a}} + \mathbf{1} - (\tilde{\mathbf{a}}_d - \mathbf{I}_d)]_m^+, [\mathbf{I} + \mathbf{1}]_m^+);$$


    /* Value update */
7  $v \leftarrow C((\tilde{\mathbf{a}}, \tilde{\mathbf{I}}), D^*(t)) + \tilde{V}([\tilde{\mathbf{a}} + \mathbf{1} - (\tilde{\mathbf{a}}_{D^*(t)} - \mathbf{I}_{D^*(t)})]_m^+, [\mathbf{I} + \mathbf{1}]_m^+) - \tilde{V}(\mathbf{0})$ ;
8  $\tilde{V}(\tilde{\mathbf{s}}) \leftarrow (1 - \gamma(t))\tilde{V}(\tilde{\mathbf{s}}) + \gamma(t)v$ ;
    /* Post-decision state update */
9  $\tilde{\mathbf{a}} \leftarrow [\tilde{\mathbf{a}} + \mathbf{1} - (\tilde{\mathbf{a}}_{D^*(t)} - \mathbf{I}_{D^*(t)})]_m^+$ ;
10  $\tilde{\mathbf{I}} \leftarrow [\mathbf{I} + \mathbf{1}]_m^+$ .
11 end

```

---

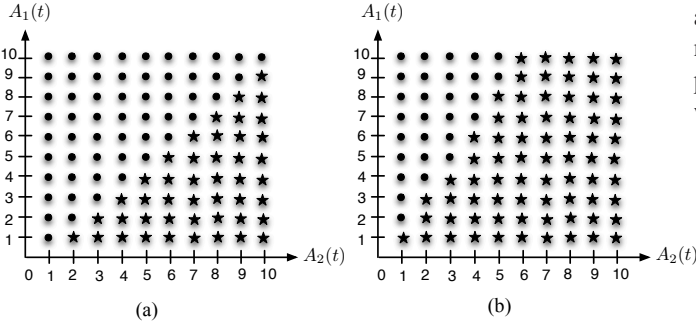


Fig. 7. (a) Switch structure for  $p_1 = p_2 = 0.9$ ; (b) switch structure for  $p_1 = 0.9$  and  $p_2 = 0.5$ .

dots represent  $D(t) = 1$  and the stars mean  $D(t) = 2$ . We observe the switch structure in the figures, while Fig. 7-(a) is consistent with the index algorithm by simply comparing the ages of the two users, as stated in Corollary 9. Moreover, after fixing the arrival rate  $p_1 = 0.9$  for the first user, the BS will give a higher priority to the second user as  $p_2$  decreases. That is because the second user takes more time to wait for the next arrival and becomes the bottleneck.

### B. Off-line algorithms with and without buffers: Algs. 1 and 3

Thus far, we obtain an optimal scheduling decision for each state, like Section VI-A. Now, we are ready to investigate the time-average age by employing the obtained scheduling decisions in the last subsection. The experiment setting is as follows. We consider the truncated MDP with the boundary

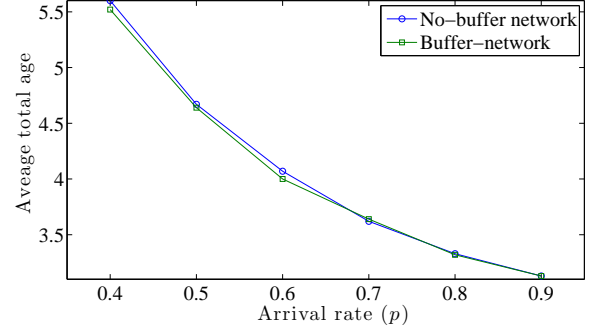


Fig. 8. Average total age for two users by running the off-line algorithms.

$m = 30$  and generate arrivals for each user according to the Bernoulli ( $p$ ) distribution, i.e.,  $p_1 = p_2 = p$ . The BS then decides to update a user according to the arrivals and the age of information for each user. After averaging the age over 100,000 slots, we obtain the average total age in Fig. 8 (blue curve with the circle markers) for various  $p$ . Moreover, we also show the average total age for the buffer-network in Fig. 8 (green curve with the square markers), where the BS updates a user based on the optimal decisions from Alg. 3 according to the arrivals, initial ages at the buffers, and ages of the users.

Intuitively, we can improve the average total age by exploiting the buffers to store unsent packets when both packets arrive in the same slot. However, it is quite interesting that the no-buffer network and the buffer-network result in the similar performance in Fig. 8. Let us discuss the following three cases when *both users have the arrivals in some slot*:

- *When both  $p_1$  and  $p_2$  are high*: That means the user who is not updated currently has a new arrival in the next slot with a high probability; as such, the old packet in the buffer seems not that effective.
- *When both  $p_1$  and  $p_2$  are low*: Then, the possibility of the two arrivals in the same slot is very low. Hence, this would be a trivial case.
- *When one of  $p_1$  and  $p_2$  are high and the other is low*: In this case, the BS will give the user with the lower arrival rate a higher update priority, while a packet for the other user will arrive shortly.

Moreover, since we aim at minimizing the average total age, the BS would prefer users with new arrivals to those with some old packets in the buffer. According to the above discussions, we observe that the buffers seem not that effective as expected; whereas, the no-buffer network is not only simple for practical implementation but also works pretty well.

We also compare the average age per user for different numbers of users in Fig. 9 by running the index algorithm (in Corollary 9), where the identical arrival rates are  $p = 0.5$  (red line), 0.7 (green line), and 0.9 (blue line), respectively.

### C. On-line algorithms with and without buffers: Algs. 2 and 4

Thus far, we analyze the average age of the optimal off-line algorithms. In this subsection, we will examine the proposed

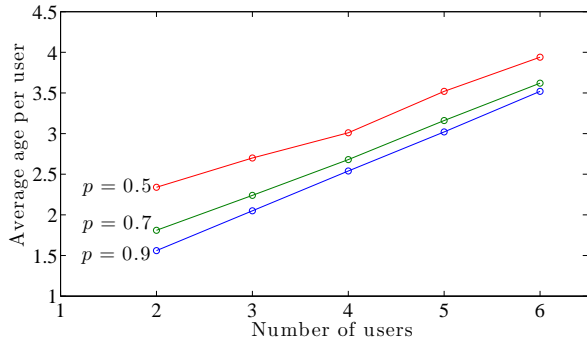


Fig. 9. Average age per user by running the index algorithm in Corollary 9.

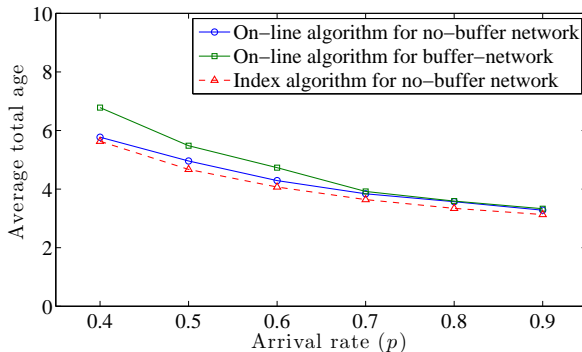


Fig. 10. Average total age for two users by running the on-line or off-line algorithms.

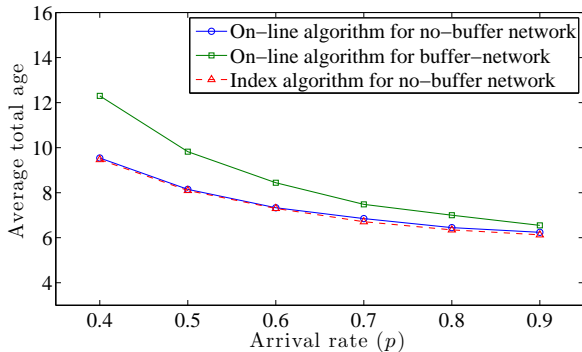


Fig. 11. Average total age for three users by running the on-line or off-line algorithms.

on-line algorithms, with the step size<sup>4</sup>  $\gamma(t) = 1/t$ .

First, we show the average total age of the index algorithm and on-line scheduling algorithms in Fig. 10, where we consider two users with  $p_1 = p_2 = p$ . The BS then decides to update a user according to the current ages and arrivals by running Algs. 2 and 4 for the no-buffer network and buffer-network, respectively. The solid curves indicate the on-line algorithms for the no-buffer network and the buffer-network, whose performance is slightly worse than the index algorithm in the dashed curve.

<sup>4</sup>The choice of the step size is similar to [28] and works well for the no-buffer network. How to choose the best step size is an interesting practical issue, but is not within the scope of this paper.

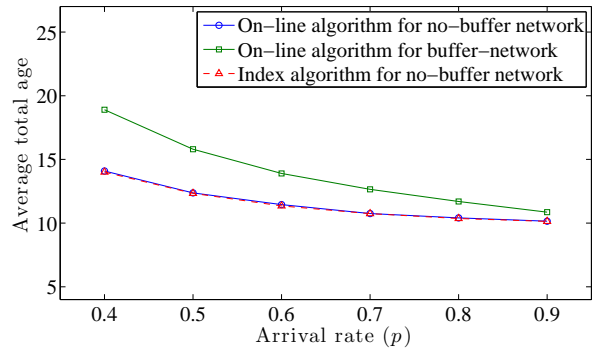


Fig. 12. Average total age for four users by running the on-line or off-line algorithms.

Intuitively, the on-line algorithms learn the optimal value for each state by updating the value of the current post-decision state during the *exploration* [27], and then the on-line algorithms can use an optimal decision by *exploiting* the optimal value for each state after the exploration. However, if the state space is huge, it is quite easy to become stuck in a *local* solution because we have poor estimates of the values of some states or we never visit some states during the simulation (see [27] for more discussions). Hence, we can theoretically guarantee the performance of the on-line algorithm for an infinite horizon; whereas, the optimal decisions are not available for problems with a large state space.

Furthermore, we show the average total age for three and four users in Figs. 11 and 12, respectively. We find that the on-line algorithm for the no-buffer network even outperforms that for the buffer-network. That is because the state space of the buffer-network is much larger than the no-buffer network. We note that for four users the state space of the on-line algorithm for the buffer-network is  $m^8$ , where  $m$  is the boundary of the truncated MDP, and it is  $30^8$  if  $m = 30$ . During the simulation time of 100,000 slots, at least  $30^8 - 10^5$  states are not visited; as such, it is much harder for the on-line algorithm to learn optimal decisions in the buffer-network.

To conclude, the optimal average age of the no-buffer network is close to that of the buffer-network (see Fig. 8). Given the option to deploy buffers for the network, we would also suggest implementing the on-line algorithm for the simple no-buffer network as a result of the smaller state space.

## VII. CONCLUSION

In this paper, we consider a wireless broadcast network, where many users are interested in different information that should be delivered by a base-station. We theoretically investigate the average age of information by designing and analyzing optimal scheduling algorithms. We show that an optimal scheduling algorithm is a simple stationary switch type. To tackle the infinite state-space Markov decision process (MDP), we propose a sequence of finite-state approximate MDPs. Based on the approximate MDPs, we propose both optimal off-line and on-line scheduling algorithms. The algorithms are further studied and compared by numerical results. It turns out that no-buffer networks are not only simple for practical

implementation but have good performance close to buffer-networks.

APPENDIX A  
PROOF OF PROPOSITION 4

According to [7, 23], it suffices to show that  $V_\alpha(\mathbf{s}) < \infty$  for every initial state  $\mathbf{s}$  and discount factor  $\alpha$ .

Let  $f$  be the stationary algorithm of always doing nothing (i.e.,  $D(t) = 0$  for all  $t$ ) for each slot. By definition of the optimality, if  $V_\alpha(\mathbf{s}; f) < \infty$ , then  $V_\alpha(\mathbf{s}) < \infty$ . Note that

$$\begin{aligned} V_\alpha(\mathbf{s}; f) &= \lim_{T \rightarrow \infty} E_f \left[ \sum_{t=0}^T \alpha^t C(\mathbf{S}(t), D(t)) | \mathbf{S}(0) = \mathbf{s} \right] \\ &= \sum_{t=0}^{\infty} \alpha^t [(a_1 + t) + \dots + (a_N + t)] \\ &= \frac{a_1 + \dots + a_N}{1 - \alpha} + \frac{\alpha N}{(1 - \alpha)^2} < \infty. \end{aligned}$$

APPENDIX B  
PROOF OF PROPOSITION 5

The proof is based on induction on  $n$  in Eq (3). The result clearly holds for  $V_{\alpha,0}(\mathbf{s})$ .

Now, we assume that  $V_{\alpha,n}(\mathbf{s})$  is non-decreasing in  $a_i$ . First, we note that the immediate cost  $C(\mathbf{s}, d) = \sum_{i=1}^N (a_i + 1) - a_d \lambda_d$  is a non-decreasing function in  $a_i$ . Next,  $E[V_{\alpha,n}(\mathbf{s}')] is also a non-decreasing function in  $a_i$  according to the induction assumption. Since the minimum operator (in Eq. (3)) holds the non-decreasing property, we conclude that  $V_{\alpha,n+1}(\mathbf{s})$  is a non-decreasing function as well.$

APPENDIX C  
PROOF OF LEMMA 6

According to [23], we need to prove that the following two conditions are satisfied.

- 1) *There exists a deterministic stationary algorithm  $f$  of our MDP such that the average total age is finite:* We can view the age of information for each user as an *age-queue*, where the age-queueing system consists of the age-queue, inputs to the queue, and a server. The input rate of the age-queue is one per slot because the age increases by one per slot, while the service rate is infinite as the age is set to be one once updated. It is then easy to establish the algorithm  $f$  (e.g., to update the user with the largest age for each slot) such that the average size of the age-queue is finite, i.e., the age-queue is stable [3], since the input rate is less than the service rate.
- 2) *There exists a nonnegative  $L$  such that the relative cost function  $h_\alpha(\mathbf{s}) \geq -L$  for all  $\mathbf{s}$  and  $\alpha$ :* Let  $C_{\mathbf{s},\mathbf{s}'}(\theta)$  be the expected cost of the first passage from state  $\mathbf{s}$  to state  $\mathbf{s}'$  using scheduling algorithm  $\theta$ . Then, using the deterministic stationary algorithm  $f$  in the first condition, we have  $C_{\mathbf{s},\mathbf{s}'}(f) < \infty$  (see [23, Proposition 4]) and  $h_\alpha(\mathbf{s}) \geq -C_{\mathbf{0},\mathbf{s}}$  (see [23, proof of Proposition 5] or [29, proof of Proposition 4.2]). Moreover, as  $V_\alpha(\mathbf{s})$  is a non-decreasing function in  $a_i$  (Proposition 5), only

state  $\mathbf{s}$  with  $a_i \leq N$  for all  $i$  can probably result in a lower value of  $V_\alpha(\mathbf{s})$  than  $V_\alpha(\mathbf{0})$ . We hence can choose  $L = \max_{\mathbf{s} \in \mathbf{S}: a_i \leq N, \forall i} C_{\mathbf{0},\mathbf{s}}$ .

By verifying the two conditions, the result immediately follows from [23].

APPENDIX D  
PROOF OF THEOREM 8

First, we show that an  $\alpha$ -optimal scheduling algorithm is the switch type. Let  $\nu_\alpha(\mathbf{s}; d) = C(\mathbf{s}, d) + \alpha E[V_\alpha(\mathbf{s}')]$ . Then,  $V_\alpha(\mathbf{s}) = \min_{d \in \{0,1,\dots,N\}} \nu_\alpha(\mathbf{s}; d)$ . Without loss of generality, we suppose that an  $\alpha$ -optimal decision at state  $\mathbf{s} = (a_1, \mathbf{a}_{-1}, \boldsymbol{\lambda})$  is to update the user  $u_1$  with  $\lambda_1 = 1$ . Then, according to the  $\alpha$ -optimality of  $d_{(\alpha_1, \mathbf{a}_{-1}, \boldsymbol{\lambda})}^* = 1$ ,

$$\nu_\alpha(a_1, \mathbf{a}_{-1}, \boldsymbol{\lambda}; 1) - \nu_\alpha(a_1, \mathbf{a}_{-1}, \boldsymbol{\lambda}; j) \leq 0,$$

for all  $j \neq 1$ .

Let  $\mathbf{1} = (1, \dots, 1)$  be the vector with all entries being one. Let  $\mathbf{a}_i = (0, \dots, a_i, \dots, 0)$  be the zero vector except for the  $i$ -th entry being replaced by  $a_i$ . To demonstrate the switch-type structure, we consider the two cases as follows.

- 1) *For any other user  $u_j$  with  $\lambda_j = 1$ :* Since  $V_\alpha(a_1, \mathbf{a}_{-1}, \boldsymbol{\lambda})$  is a non-decreasing function in  $a_1$  (Proposition 5), we get

$$\begin{aligned} &\nu_\alpha(a_1 + 1, \mathbf{a}_{-1}, \boldsymbol{\lambda}; 1) - \nu_\alpha(a_1 + 1, \mathbf{a}_{-1}, \boldsymbol{\lambda}; j) \\ &= a_j - (a_1 + 1) \\ &\quad + \alpha E[V_\alpha(1, \mathbf{a}_{-1} + \mathbf{1}, \boldsymbol{\lambda}') - V_\alpha(a_1 + 2, \mathbf{a}_{-1} + \mathbf{1} - \mathbf{a}_j, \boldsymbol{\lambda}')] \\ &\leq a_j - a_1 \\ &\quad + \alpha E[V_\alpha(1, \mathbf{a}_{-1} + \mathbf{1}, \boldsymbol{\lambda}') - V_\alpha(a_1 + 1, \mathbf{a}_{-1} + \mathbf{1} - \mathbf{a}_j, \boldsymbol{\lambda}')] \\ &= \nu_\alpha(a_1, \mathbf{a}_{-1}, \boldsymbol{\lambda}; 1) - \nu_\alpha(a_1, \mathbf{a}_{-1}, \boldsymbol{\lambda}; j) \leq 0, \end{aligned}$$

where  $\boldsymbol{\lambda}'$  summarizes the possible next arrivals.

- 2) *For any other user  $u_j$  with  $\lambda_j = 0$ :* Similarly, we have

$$\begin{aligned} &\nu_\alpha(a_1 + 1, \mathbf{a}_{-1}, \boldsymbol{\lambda}; 1) - \nu_\alpha(a_1 + 1, \mathbf{a}_{-1}, \boldsymbol{\lambda}; j) \\ &= - (a_1 + 1) \\ &\quad + \alpha E[V_\alpha(1, \mathbf{a}_{-1} + \mathbf{1}, \boldsymbol{\lambda}') - V_\alpha(a_1 + 2, \mathbf{a}_{-1} + \mathbf{1}, \boldsymbol{\lambda}')] \\ &\leq 0. \end{aligned}$$

Considering the two cases, an  $\alpha$ -optimal decision for state  $(a_1 + 1, \mathbf{a}_{-1}, \boldsymbol{\lambda})$  is still to update  $u_1$ , yielding the switch-type structure.

Then, let  $\{\alpha_n\}$  be a sequence of the discount factors. According to [23], there exists a subsequence  $\{\beta_n\}$  such that an average-optimal algorithm is the limit point of the  $\beta_n$ -optimal algorithms. Similar to the argument in [30, Theorem 18], an average-optimal algorithm holds such the switch-type structure.

APPENDIX E  
PROOF OF COROLLARY 9

Without loss of generality (according to Appendix D), we focus on an  $\alpha$ -optimal algorithm and assume that  $a_1 \geq \max(a_2, \dots, a_N)$ .

Let  $\mathbf{a}_{ij} = (0, \dots, a_j, \dots, 0)$  be the zero vector except for the  $i$ -th entry being replaced by  $a_j$ . By the symmetry of the

users, swap of the initial ages of the two users achieves the same expected total discounted cost, i.e.,

$$E[V_\alpha(a_1, \mathbf{a}_{-1}, \boldsymbol{\lambda})] = E[V_\alpha(a_j, \mathbf{a}_{-1} - \mathbf{a}_j + \mathbf{a}_{j1}, \boldsymbol{\lambda})],$$

for all  $j \neq 1$ . Similar to Appendix D, here we focus on the case that  $\lambda_1 = 1$  and  $\lambda_j = 1$  and the result follows from the non-decreasing function of  $V_\alpha(a_1, \mathbf{a}_{-1}, \boldsymbol{\lambda})$  and  $a_1 \geq a_j$  for all  $j \neq 1$ :

$$\begin{aligned} & \nu_\alpha(a_1, \mathbf{a}_{-1}, \boldsymbol{\lambda}; 1) - \nu_\alpha(a_1, \mathbf{a}_{-1}, \boldsymbol{\lambda}; j) \\ = & a_j - a_1 \\ & + \alpha E[V_\alpha(1, \mathbf{a}_{-1} + \mathbf{1}, \boldsymbol{\lambda}') - V_\alpha(a_1 + 1, \mathbf{a}_{-1} + \mathbf{1} - \mathbf{a}_j, \boldsymbol{\lambda}')] \\ = & a_j - a_1 \\ & + \alpha E[V_\alpha(a_j + 1, \mathbf{a}_{-1} + \mathbf{1} - \mathbf{a}_j, \boldsymbol{\lambda}') \\ & - V_\alpha(a_1 + 1, \mathbf{a}_{-1} + \mathbf{1} - \mathbf{a}_j, \boldsymbol{\lambda}')] \leq 0. \end{aligned}$$

#### APPENDIX F PROOF OF THEOREM 11

Let  $V_\alpha^{(m)}(\mathbf{s})$  and  $h_\alpha^{(m)}(\mathbf{s})$  be the  $\alpha$ -optimal cost and the relative cost function for  $\Delta_m$ , respectively. According to [29], we need to prove the following two conditions are satisfied.

- 1) *There exists a nonnegative  $L$ , a nonnegative finite function  $F(\cdot)$  on  $\mathbf{S}$  such that  $-L \leq h_\alpha^{(m)}(\mathbf{s}) \leq F(\mathbf{s})$  for all  $\mathbf{s} \in \mathbf{S}_m$ , where  $m = 1, 2, \dots$ , and  $0 < \alpha < 1$ : We consider a randomized stationary algorithm  $f$  that updates each user (with packet arrival) with equal probability for each slot. Similar to the proof of Lemma 6, let  $C_{\mathbf{s},0}(f)$  and  $C_{\mathbf{s},0}^{(m)}(f)$  be the expected cost from state  $\mathbf{s} \in \mathbf{S}_m$  to the reference state  $\mathbf{0}$  by applying the algorithm  $f$  to  $\Delta$  and  $\Delta_m$ , respectively. Then,  $h_\alpha^{(m)}(\mathbf{s}) \leq C_{\mathbf{s},0}^{(m)}(f)$  and  $C_{\mathbf{s},0}(f) < \infty$  similar to the argument in the proof of Lemma 6. In the following, we will show that  $C_{\mathbf{s},0}^{(m)}(f) \leq C_{\mathbf{s},0}(f)$  and then we can choose the function  $F(\mathbf{s}) = C_{\mathbf{s},0}(f)$ .*

Let  $P_{\mathbf{s},\mathbf{s}'}^{(m)}(d)$  be the transition probability for  $\Delta_m$ . Then,

$$P_{\mathbf{s},\mathbf{s}'}^{(m)}(d) = P_{\mathbf{s},\mathbf{s}'}(d) + \sum_{\mathbf{r} \in \mathbf{S} - \mathbf{S}_m} P_{\mathbf{s},\mathbf{r}}(d),$$

for some (or no) *excess probabilities* [29] on state  $\mathbf{r} \in \mathbf{S} - \mathbf{S}_m$ . Note that the algorithm  $f$  is independent of the age and then  $C_{\mathbf{r},0}(f) \geq C_{\mathbf{j},0}(f)$  for  $\mathbf{r} \geq \mathbf{j}$ ; as such, we have

$$\begin{aligned} & \sum_{\mathbf{s}' \in \mathbf{S}_m} P_{\mathbf{s},\mathbf{s}'}^{(m)}(d) C_{\mathbf{s}',0}(f) \\ \leq & \sum_{\mathbf{s}' \in \mathbf{S}_m} P_{\mathbf{s},\mathbf{s}'}(d) C_{\mathbf{s}',0}(f) + \sum_{\mathbf{r} \notin \mathbf{S}_m} P_{\mathbf{s},\mathbf{r}}(d) C_{\mathbf{r},0}(f) \\ = & \sum_{\mathbf{s}' \in \mathbf{S}} P_{\mathbf{s},\mathbf{s}'}(d) C_{\mathbf{s}',0}(f). \end{aligned}$$

Using the above inequality, we then can conclude

$C_{\mathbf{s},0}(f) \geq C_{\mathbf{s},0}^{(m)}(f)$  because

$$\begin{aligned} C_{\mathbf{s},0}(f) &= E_f[C(\mathbf{s}, d) + \sum_{\mathbf{s}' \in \mathbf{S}} P_{\mathbf{s},\mathbf{s}'}(d) C_{\mathbf{s}',0}(f)] \\ &\geq E_f[C(\mathbf{s}, d) + \sum_{\mathbf{s}' \in \mathbf{S}_m} P_{\mathbf{s},\mathbf{s}'}^{(m)}(d) C_{\mathbf{s}',0}(f)], \end{aligned}$$

along with the fact that  $C_{\mathbf{s},0}^{(m)}(f)$  is the solution to the equation:

$$C_{\mathbf{s},0}^{(m)}(f) = E_f[C(\mathbf{s}, d) + \sum_{\mathbf{s}' \in \mathbf{S}_M} P_{\mathbf{s},\mathbf{s}'}^{(m)}(d) C_{\mathbf{s}',0}^{(m)}(f)].$$

On the other hand, we can choose  $L = \max_{\mathbf{s} \in \mathbf{S}: a_i \leq N, \forall i} C_{\mathbf{0},\mathbf{s}}(f)$ , since  $h_\alpha^{(m)}(\mathbf{s}) \geq -C_{\mathbf{0},\mathbf{s}}^{(m)}(f)$  similar to the second condition in the proof of Lemma 6 and  $-C_{\mathbf{0},\mathbf{s}}^{(m)}(f) \geq -C_{\mathbf{0},\mathbf{s}}(f)$  similar to above.

- 2) *Let  $\limsup_{m \rightarrow \infty} V_m^* = V_\infty^*$  and then  $V_\infty^* \leq V^*$ : We claim that  $V_\alpha^{(m)}(\mathbf{s}) \leq V_\alpha(\mathbf{s})$  for all  $m$ , and then the condition holds as*

$$\begin{aligned} V_m^* &= \limsup_{\alpha \rightarrow 1} (1 - \alpha) V_\alpha^{(m)}(\mathbf{s}) \\ &\leq \limsup_{\alpha \rightarrow 1} (1 - \alpha) V_\alpha(\mathbf{s}) = V^*. \end{aligned}$$

To verify this claim, we first note that since  $V_\alpha(\mathbf{s})$  is a non-decreasing function in  $a_i$ , and hence we have

$$\sum_{\mathbf{s}' \in \mathbf{S}_m} P_{\mathbf{s},\mathbf{s}'}^{(m)}(d) V_\alpha(\mathbf{s}') \leq \sum_{\mathbf{s}' \in \mathbf{S}} P_{\mathbf{s},\mathbf{s}'}(d) V_\alpha(\mathbf{s}'). \quad (7)$$

We now prove the claim by induction on  $n$  in Eq. (3). It is obvious when  $n = 0$ . Suppose that  $V_{\alpha,n}^{(m)}(\mathbf{s}) \leq V_{\alpha,n}(\mathbf{s})$ , and then

$$\begin{aligned} & V_{\alpha,n+1}^{(m)}(\mathbf{s}) \\ = & \min_{d \in \{0,1,\dots,N\}} C(\mathbf{s}, d) + \alpha \sum_{\mathbf{s}' \in \mathbf{S}_m} P_{\mathbf{s},\mathbf{s}'}^{(m)}(d) V_{\alpha,n}^{(m)}(\mathbf{s}') \\ \stackrel{(a)}{\leq} & \min_{d \in \{0,1,\dots,N\}} C(\mathbf{s}, d) + \alpha \sum_{\mathbf{s}' \in \mathbf{S}_m} P_{\mathbf{s},\mathbf{s}'}^{(m)}(d) V_{\alpha,n}(\mathbf{s}') \\ \stackrel{(b)}{\leq} & \min_{d \in \{0,1,\dots,N\}} C(\mathbf{s}, d) + \alpha \sum_{\mathbf{s}' \in \mathbf{S}} P_{\mathbf{s},\mathbf{s}'}(d) V_{\alpha,n}(\mathbf{s}') \\ = & V_{\alpha,n+1}(\mathbf{s}), \end{aligned}$$

where (a) results from the induction assumption, and (b) is according to Eq. (7).

#### APPENDIX G PROOF OF THEOREM 12

According to [5, Theorem 8.6.6], the RVIA in Eq. (4) converges to the optimal solution in finite iterations if the truncated MDPs are *unichain*, i.e., the Markov chain corresponding to every deterministic stationary algorithm consists of a single recurrent class plus a possibly empty set of transient states. We notice that for every truncated MDP, there is only one recurrent class as the state  $(m, \dots, m, 0, \dots, 0)$  is reachable (e.g., there is no arrival in the next  $m$  slots) from all other states [31] (where remember that  $m$  is the boundary of the truncated MDP). Hence, the truncated MDPs are unichain and the theorem follows immediately.

APPENDIX H  
UNRELIABLE CHANNELS

Here, we discuss the extension of the proposed algorithms to fit unreliable networks. We focus on the ON-OFF channels, where by  $C_i(t) \in \{0, 1\}$  we indicate if the channel between the BS and the user  $u_i$  is ON or OFF in slot  $t$ .

A. *The no-buffer network*

For the no-buffer network, the state in slot  $t$  can be revised as  $\mathbf{S}(t) = (A_1(t), \dots, A_N(t), \Lambda_1(t)C_i(t), \dots, \Lambda_N(t)C_N(t))$ , where we can regard  $\Lambda_i(t)C_i(t)$  as a *virtual* arrival, i.e., there is an effective arrival for user  $u_i$  in slot  $t$  when both  $\Lambda_i(t) = 1$  and  $C_i(t) = 1$ . Then, the proposed off-line and on-line algorithms can be applied directly.

B. *The buffer-network*

By  $\mathbf{S}(t) = (A_1(t), \dots, A_N(t), I_1(t), \dots, I_N(t), C_1(t), \dots, C_N(t))$  we redefine the state for the buffer-network. Since the channel states are finite, the proposed finite-state approximations in Section V can work perfectly; meanwhile, we can use the proposed optimal off-line and on-line scheduling algorithms in Alg. 3 and Alg. 4 associated with the revised state.

REFERENCES

- [1] Y.-P. Hsu, E. Modiano, and L. Duan, "Age of Information: Design and Analysis of Optimal Scheduling Algorithms," *Proc. of IEEE ISIT*, pp. 561–565, 2017.
- [2] S. Kaul, R. D. Yates, and M. Gruteser, "Real-Time Status: How Often Should One Update?" *Proc. of IEEE INFOCOM*, pp. 2731–2735, 2012.
- [3] L. Georgiadis, M. J. Neely, and L. Tassiulas, *Resource Allocation and Cross-Layer Control in Wireless Networks*. Now Publishers Inc, 2006.
- [4] M. G. Markakis, E. Modiano, and J. N. Tsitsiklis, "Max-Weight Scheduling in Queueing Networks with Heavy-Tailed Traffic," *IEEE/ACM Trans. Netw.*, vol. 22, no. 1, pp. 257–270, 2014.
- [5] M. L. Puterman, *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. The MIT Press, 1994.
- [6] D. P. Bertsekas, *Dynamic Programming and Optimal Control Vol. I and II*. Athena Scientific, 2012.
- [7] S. M. Ross, *Introduction to Stochastic Dynamic Programming*. Academic Press, 1994.
- [8] L. I. Sennott, *Stochastic Dynamic Programming and the Control of Queueing Systems*. John Wiley & Sons, 1998.
- [9] J. Cho and H. Garcia-Molina, "Synchronizing a Database to Improve Freshness," *Proc. of ACM SIGMOD*, vol. 29, no. 2, pp. 117–128, 2000.
- [10] C. Kam, S. Kompella, and A. Ephremides, "Age of Information under Random Updates," *Proc. of IEEE ISIT*, pp. 66–70, 2013.
- [11] C. Kam, S. Kompella, G. D. Nguyen, and A. Ephremides, "Effect of Message Transmission Path Diversity on Status Age," *IEEE Trans. Inf. Theory*, vol. 62, no. 3, pp. 1360–1374, 2016.
- [12] R. D. Yates and S. Kaul, "Real-Time Status Updating: Multiple Sources," *Proc. of IEEE ISIT*, pp. 2666–2670, 2012.
- [13] L. Huang and E. Modiano, "Optimizing Age-of-Information in a Multi-Class Queueing System," *Proc. of IEEE ISIT*, pp. 1681–1685, 2015.
- [14] Y. Sun, E. Uysal-Biyikoglu, R. D. Yates, C. E. Koksal, and N. B. Shroff, "Update or Wait: How to Keep Your Data Fresh," *Proc. of IEEE INFOCOM*, pp. 1–9, 2016.
- [15] R. D. Yates, "Lazy is Timely: Status Updates by an Energy Harvesting Source," *Proc. of IEEE ISIT*, pp. 3008–3012, 2015.
- [16] B. T. Bacinoglu, E. T. Ceran, and E. Uysal-Biyikoglu, "Age of Information under Energy Replenishment Constraints," *Proc. of ITA*, pp. 25–31, 2015.
- [17] M. Costa, M. Codreanu, and A. Ephremides, "On the Age of Information in Status Update Systems with Packet Management," *IEEE Trans. Inf. Theory*, vol. 62, no. 4, pp. 1897–1910, 2016.
- [18] L. Golab, T. Johnson, and V. Shkapenyuk, "Scalable Scheduling of Updates in Streaming Data Warehouses," *IEEE Trans. Knowl. Data Eng.*, vol. 24, no. 6, pp. 1092–1105, 2012.
- [19] Q. He, D. Yuan, and A. Ephremides, "Optimal Link Scheduling for Age Minimization in Wireless Systems," *IEEE Trans. Inf. Theory*, 2017.
- [20] C. Joo and A. Eryilmaz, "Wireless Scheduling for Information Freshness and Synchrony: Drift-Based Design and Heavy-Traffic Analysis," *Proc. of IEEE WIOPT*, pp. 1–8, 2017.
- [21] I. Kadota, E. Uysal-Biyikoglu, R. Singh, and E. Modiano, "Minimizing Age of Information in Broadcast Wireless Networks," *Proc. of Allerton*, 2016.
- [22] R. D. Yates, P. Ciblat, A. Yener, and M. Wigger, "Age-Optimal Constrained Cache Updating," *Proc. of IEEE ISIT*, pp. 141–145, 2017.
- [23] L. I. Sennott, "Average Cost Optimal Stationary Policies in Infinite State Markov Decision Processes with Unbounded Costs," *Operations Research*, vol. 37, pp. 626–633, 1989.
- [24] V. S. Borkar and S. P. Meyn, "The ODE Method for Convergence of Stochastic Approximation and Reinforcement Learning," *SIAM Journal on Control and Optimization*, vol. 38, no. 2, pp. 447–469, 2000.
- [25] J. Abounadi, D. Bertsekas, and V. Borkar, "Learning Algorithms for Markov Decision Processes with Average Cost," *SIAM Journal on Control and Optimization*, vol. 40, no. 3, pp. 681–698, 2001.
- [26] V. S. Borkar, *Stochastic Approximation: A Dynamical Systems Viewpoint*. Cambridge University Press, 2008.
- [27] W. B. Powell, *Approximate Dynamic Programming: Solving the Curses of Dimensionality*. John Wiley & Sons, 2011.
- [28] N. Salodkar, A. Bhorkar, A. Karandikar, and V. S. Borkar, "An On-Line Learning Algorithm for Energy Efficient Delay Constrained Scheduling over a Fading Channel," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 4, pp. 732–742, 2008.
- [29] L. I. Sennott, "On Computing Average Cost Optimal Policies with Application to Routing to Parallel Queues," *Mathematical Methods of Operations Research*, vol. 45, pp. 45–62, 1997.
- [30] Y.-P. Hsu, N. Abedini, N. Gautam, A. Sprintson, and S. Shakkottai, "Opportunities for Network Coding: To Wait or Not to Wait," *IEEE/ACM Trans. Netw.*, vol. 23, no. 6, pp. 1876–1889, 2015.
- [31] R. G. Gallager, *Discrete Stochastic Processes*. Springer Science & Business Media, 2012, vol. 321.