Journal of Monetary Economics 31 (1993) 3–19. North-Holland

Asset bubbles and endogenous growth*

Gene M. Grossman
Princeton University, Princeton, NJ 08544, USA

Noriyuki Yanagawa
University of Tokyo, Tokyo, Japan

Received February 1992, final version received September 1992

We study the interaction between productive and nonproductive savings in economies with endogenous long-run growth. As in the neoclassical growth setting with overlapping generations, asset bubbles can exist in an economy with endogenous growth provided that they are not too large and that the growth rate in the equilibrium without bubbles exceeds the interest rate. Here, the existence conditions for bubbles reflect parameters of tastes and technology. We find that bubbles, when they exist, retard the growth of the economy, perhaps even in the long run, and reduce the welfare of all generations born after the bubble appears.

1. Introduction

Can the market price of an asset deviate from market fundamentals (i.e., the present discounted value of dividend payments) in a world populated by rational, farsighted investors? Tirole (1982) has shown that it cannot, if the economy comprises a finite number of infinitely-lived traders, while Wallace (1980) and Tirole (1985) have shown that the same is true in a non-growing economy no matter how long are investors' trading horizons. But Tirole (1985) and Weil (1987) have established that 'bubbles' sometimes can exist in the general equilibrium of a growing economy with overlapping generations.

A rational investor will only hold an asset priced differently than its fundamentals if she expects that the bubble component will yield at least a normal rate

Correspondence to: Gene M. Grossman, Woodrow Wilson School, Princeton University, Princeton, NJ 08544, USA.

*We are grateful to Ben Bernanke, Elhanan Helpman, Paolo Pesenti, Jean Tirole, Harald Uhlig, and an anonymous referee for comments on earlier drafts. Grossman thanks the National Science Foundation for financial support.

0304-3932/93/$05.00

of return, i.e., that it will grow at least at the real rate of interest. But if bubbles grow at the rate of interest in every period, eventually their value will exceed the income of the young generations who must purchase these assets from the old, unless the income of these generations is growing at least as fast. Tirole (1985) investigated the conditions under which a Diamond (1965) economy with an expanding population would grow fast enough to allow for the existence of bubbles in asset prices.[1] He related the existence condition to the intertemporal efficiency of the general equilibrium without bubbles.[2] Of course, in the Diamond economy with a neoclassical production function and no technological progress, per capita incomes stagnate in the long run.

In this paper, we extend Tirole's (1985) results to include economies that grow in the long run at an endogenous rate. As is well known by now, long-run growth can be sustained in an economy in which real returns to whatever capital goods are being accumulated (physical, knowledge, or human) are bounded from below by a number that exceeds the discount rate. In other words, there must be nondecreasing returns to accumulable factors in the long run. These nondecreasing returns may be inherent to the production technology [as in, e.g., Rebelo (1991) and Jones and Manuelli (1990)] or they may arise due to externalities generated in the process of capital accumulation [as in, e.g., Romer (1986, 1990) and Lucas (1988)]. We choose a simple specification that includes externalities from physical capital [following Romer (1986)] and investigate the existence conditions for bubbles and the effects that bubbles have on the growth rate of the economy and on the welfare of the various generations of agents.[3] We find that the conditions under which bubbles can exist are similar to those identified by Tirole (1985), but that bubbles are not so benign in this setting as they are in the Diamond economy with an exogenous growth rate.

2. A Diamond–Romer economy without bubbles

As in Diamond (1965), agents live for two periods. They work, consume, and save when they are young, and enjoy the fruits of their savings when they are old. Each period a new generation of young is born. The young are endowed with a fixed amount of potential working time, which they supply inelastically in the labor market. They use their labor income to buy output for consumption and investment purposes and to purchase the existing capital stock from the old. We

[1]Weil (1987) used a similar framework to study 'stochastically-bursting bubbles'

[2]See Tirole (1990) and Blanchard and Fischer (1989, ch. 5) for excellent introductions to this literature.

[3]We choose this specification with capital externalities initially to bring out the similarities with the Tirole (1985) analysis of the Diamond economy. But the existence conditions for bubbles are similar in economies with other sources of endogenous long-run growth; see section 5

assume for now that capital goods are the only store of value. For simplicity, we assume that the economy's population is constant through time and equal to $2L$.

A representative member of the generation born at time t consumes c_{yt} units of the homogeneous final good when young, and c_{ot+1} units of this good when old. She chooses her consumption profile to maximize a utility function, $U(c_{yt}, c_{ot+1})$, subject to an intertemporal budget constraint. Letting r_{t+1} be the rate of return (or real interest rate) on savings invested at time t, the constraint can be written as

$$c_{yt} + \frac{c_{ot+1}}{1 + r_{t+1}} = I_t, \tag{1}$$

where I_t is the individual's labor income earned at time t.

The consumer's optimization yields equality between the marginal rate of intertemporal substitution, U_1/U_2, and one plus the interest rate, $1 + r_{t+1}$, as usual. This equation generates an implicit savings function, $s_t = s(I_t, r_{t+1})$. We assume henceforth that individual preferences represented by $U(\cdot)$ are homothetic. Then $s(\lambda I_t, r_{t+1}) = \lambda s(I_t, r_{t+1})$.

Firms hire the available labor force, L (half the population, namely the young generation), and the available aggregate capital stock, K_t, and produce the homogeneous output, Y_t. A firm i that rents K_t^i units of capital from the old generation that owns it and that employs L_t^i young workers generates net output (after accounting for capital depreciation) of

$$Y_t^i = F[K_t^i, A(K_t)L_t^i],$$

where $A(\cdot)$ represents labor productivity, $A' > 0$. Here we have incorporated a positive spillover from the size of the aggregate capital stock to the productivity of workers in individual firms, in the manner suggested by Arrow (1962) and formalized by Sheshinski (1967) and Romer (1986).[4] We assume that $F(\cdot,\cdot)$ exhibits constant returns to scale and that firms behave competitively. In hiring capital, the individual firm ignores its tiny influence on the aggregate capital stock and thus on the productivity of its own workers. Thus, each firm hires capital up to the point where its (private) marginal product equals the rental rate, r_t, and it hires workers until their marginal product equals the wage rate. In

[4]As we noted in the introduction, we are not wedded to this specification of the technology. Alternative formulations that preserve long-run incentives for capital accumulation would serve equally well. For a general discussion of what is needed to sustain long-run growth in a model of capital accumulation, see Grossman and Helpman (1991, ch. 2). Section 5 shows how the results of the present analysis extend to economies with alternative engines of growth

view of the homogeneity of degree one of $F(\cdot,\cdot)$, this give the following relationships at the aggregate level:

$$r_t = F_1(K_t, A_tL) = f'(k_t), \tag{2}$$

$$w_t = F(K_t, A_tL) - K_tF_1(K_t, A_tL) = f(k_t) - k_tf'(k_t), \tag{3}$$

where $A_t = A(K_t)$, $k_t \equiv K_t/A_tL$ (capital per unit of efficiency labor), and $f(k_t) \equiv F(K_t/A_tL, 1)$. Combining (2) and (3) gives a relationship between equilibrium factor prices,

$$w_t = \phi(r_t). \tag{4}$$

Product market equilibrium obtains when aggregate investment equals aggregate savings, i.e., the sum of desired savings by the young and desired dissavings by the old. Since the old wish to dissave their entire holdings of capital, K_t, this implies $K_{t+1} - K_t = s(w_tA_tL, r_{t+1}) - K_t$, or

$$K_{t+1} = s(w_tA_tL, r_{t+1}). \tag{5}$$

Eqs. (3), (4), and (5) determine the dynamic evolution of the economy (factor prices and capital stock) from any initial stock of capital, K_0.

In order to ensure the existence of a steady state for this economy, we take a particular functional form for the capital externality, making it linear in the aggregate capital stock, i.e.,

$$A(K_t) = K_t/a. \tag{6}$$

Without further loss of generality, we normalize the size of the population to two, so that $L = 1$. Then $k_t = a$ for all t, and (3) and (4) imply

$$r_t = \rho \qquad \text{for all } t, \tag{3$'$}$$

$$w_t = \phi(\rho) \qquad \text{for all } t, \tag{4$'$}$$

where $\rho \equiv f'(a)$. Now since $s(\cdot,\cdot)$ is homogeneous of degree one in its first argument, $s(w_tA_tL, r_{t+1}) = A_tLs(w_t, r_{t+1})$. Then, after substituting (6), (3′), and (4′), eq. (5) becomes

$$K_{t+1} = K_ts[\phi(\rho), \rho]/a.$$

The capital stock grows at the rate

$$g_t = \bar{s}/a - 1, \tag{7}$$

where $\bar{s} \equiv s[\phi(\rho), \rho]$. By (6), labor productivity $A(\cdot)$ grows at this same rate, and since $F(\cdot, \cdot)$ has constant returns to scale, so does per capita income.

Before leaving this section, we note that the dynamic equilibrium without bubbles is not Pareto-efficient. For suppose that at time t the old were to consume as in the above equilibrium while the young saved an additional amount $\mathrm{d}s$. This would increase the capital stock at $t+1$ by $\mathrm{d}s$ and would generate additional output of $(\mathrm{d}Y_{t+1}/\mathrm{d}K_{t+1})\mathrm{d}s = (F_1 + F_2/a)\mathrm{d}s > r_{t+1}\mathrm{d}s$. If the entirety of this extra output in period $t+1$ were given to the (then) old, then the utility of this generation would rise (since it has set its marginal rate of intertemporal substitution equal to $1 + r_{t+1}$, the extra output in the second period of life yields more utility than the loss from the consumption foregone in the first period) while no generation would lose. Of course, the inefficiency of the market equilibrium reflects the fact that (small) individual agents have no incentive to incorporate the spillover effect from capital in their private investment decisions.

3. Existence of asset bubbles

We now assume that the generation that is old at time 0 possesses M paper assets that are intrinsically worthless. That is, the assets produce no real output and therefore generate no dividends. The old attempt to sell these assets to the young at a positive price p_0 (in terms of goods) for each piece of paper. Would a rational, foresighted, young investor be willing to purchase one of these assets? Only if she believed that she could resell the asset when old (i.e., in period 1) to a member of the next young generation for a price that includes a real rate of return comparable to that available on other assets. The real (gross) rate of return on alternative assets is $1 + r_1$ units of output in period 1. Therefore, the young investor in period 0 is willing to buy the intrinsically useless asset if she expects its price in period 1 to be at least $p_1 = (1 + r_1)p_0$. Similarly, the young generation in any period t must expect the price of the paper to be $p_t(1 + r_{t+1})$ in period $t+1$, if it is to acquire the asset from the old generation at that time at a price p_t. If all of these expectations for capital gains on the asset can be fulfilled, then the intrinsically useless paper can be traded indefinitely; that is, there can exist a *bubble*.

Let $B_t = p_t M$ be the aggregate value of the bubble at time t, and assume for the moment that the self-fulfilling prophecy can be realized. By the condition of no-arbitrage between bubbles and other assets, we have

$$B_{t+1} = (1 + r_{t+1})B_t. \tag{8}$$

We define $b_t \equiv B_t/A_tL$ as the aggregate value of the bubble per efficiency unit of labor.

The young generation must purchase the entirety of existing bubbles from the old generation in each period. The condition for goods market equilibrium becomes

$$K_{t+1} - K_t = A_tLs(w_t, r_{t+1}) - (B_t + K_t); \tag{9}$$

the left-hand side is net investment, while the right-hand side is the difference betwen savings by the young and dissavings by the old [the term in parentheses on the far right of (9)]. Note that (3′) and (4′) continue to describe factor prices when $A_t = K_t/a$ and $L = 1$. Substituting these expressions into (9), we derive $K_{t+1} = A_t(\bar{s} - b_t)$, or $K_{t+1}/K_t = (\bar{s} - b_t)/a$. Thus,

$$g_t = \frac{\bar{s} - b_t}{a} - 1. \tag{10}$$

We now discuss whether, given an initial bubble of size B_0 such that $b_0 = B_0/A(K_0)$, the dynamics described by (8) and (10) are sustainable. If they are, then an initial bubble of size B_0 can exist in an economy that has an initial capital stock of K_0. Note that labor productivity grows at rate g_t, so (8) and (3′) imply

$$b_{t+1} = \frac{1 + \rho}{1 + g_t} b_t. \tag{11}$$

Substituting (10) into (11) gives a single, recursive equation for the evolution of the value of the bubble per efficiency unit of labor,

$$b_{t+1} = a(1 + \rho)\frac{b_t}{\bar{s} - b_t}. \tag{12}$$

The curve labelled BB in the top part of fig. 1 depicts this relationship between the (normalized) size of the bubble in successive periods. Clearly, when this curve lies above the 45 degree line, the bubble is growing relative to the stock of efficiency labor (and, therefore, aggregate output), whereas when the curve lies below the 45 degree line, the bubble is shrinking relative to efficiency labor.

We see from the figure that, if the initial size of the bubble is such that $b_0 < b^*$, the normalized bubble shrinks monotonically over time. In this case, the assumed existence of the initial bubble does not lead to any contradiction. Asymptotically, the bubble becomes arbitrarily small in relation to the stock of efficiency labor, and the economy converges to the steady state described in section 2. If, alternatively, the initial size of the bubble is such that $b_0 > b^*$, then

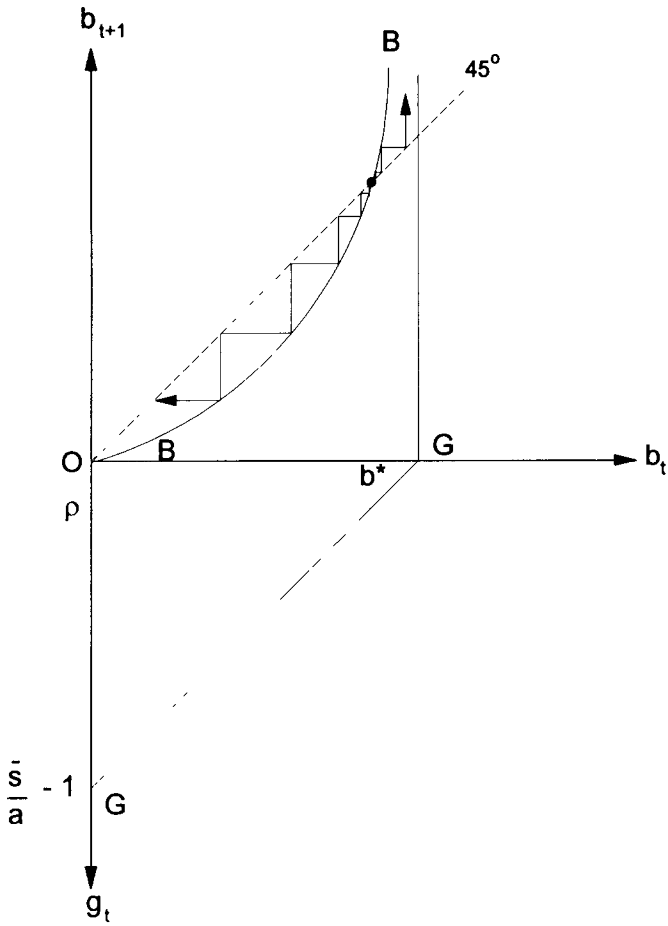

Fig. 1. Bubble dynamics.

the normalized bubble grows monotonically over time. Eventually, at some T, $b_T > \bar{s}$. But then the aggregate savings of the young do not suffice to allow them to acquire the bubble from the old generation at the required price. The old at T would have foreseen this eventuality at $T-1$, and so would not have purchased the bubble from the (then) old generation at that time. The bubble unravels back to time 0, i.e., an initial bubble of the assumed size cannot be sustained. The remaining possibility is that $b_0 = b^*$. Then the bubble and the economy remain in fixed proportion to one another and the economy immediately enters a steady state.

Evidently, bubbles can exist in this economy provided their initial size is no larger than b^*. Notice the similarity between the existence condition for bubbles here and that described by Tirole (1985) for the Diamond economy. No bubble of any size will be possible if the BB curve is steeper at the origin than the 45 degree line. The slope of the BB curve at the origin equals $a(1+\rho)/\bar{s}$. Thus bubbles can exist if and only if $\bar{s}/a - 1 > \rho$. But $\bar{s}/a - 1$ is the economy's growth rate in the absence of any bubbles, while ρ is the real interest rate, so here, as in Tirole, the existence of bubbles requires that the growth rate exceeds the interest rate in the equilibrium of the bubbleless economy.

Of course, in the model with exogenous growth, the bubble cannot affect the long-run growth rate. Here, that is not true. The bottom portion of the figure illustrates the effect that bubbles have on the growth rate. The line GG, depicting eq. (10), shows the relationship between the (normalized) size of the bubble at time t and the growth rate. If the initial bubble happens to be of size B^* such that $B^*/A(K_0)L = b^*$, then the bubble permanently lowers the rate of growth from $g_t = \bar{s}/a - 1$ to $g_t = \rho$. (Smaller initial bubbles reduce the rate of growth in every period, but the depression of the growth rate asymptotically approaches zero as t grows large.) The reason is straightforward. The existence of the asset bubble diverts savings away from productive investment in capital, and it is capital investment that drives long-run growth in the Diamond–Romer model.

4. Bubbles and intertemporal efficiency

Tirole (1985, 1990) has shown for the Diamond economy that the asymptotically bubbly equilibrium (in which the bubble remains in fixed proportion to the size of the economy in the long run) is intertemporally efficient. The bubble eliminates the overaccumulation of capital that can arise in the Diamond model because living generations cannot trade with the as yet unborn. In fact, the bubbles can exist in that setting if and only if the bubbleless equilibrium is inefficient.

One might expect a different result in our model of externality-based endogenous growth. As we have seen at the end of section 2, the economy without bubbles has less capital formation than is required for intertemporal efficiency. Bubbles divert savings from productive use in financing capital accumulation into an unproductive store of value and thereby exacerbate the existing distortion in the market equilibrium.

Suppose that a bubble of size B_0 first appears in the economy at time 0. What are the welfare implications? The generation that is old at time 0 benefits of course, as their sale of the new asset to the young enables them to consume more than otherwise. The labor income of the generation that is young at time 0 depends upon K_0 and A_0, both of which are predetermined at time 0. Their savings accrue interest at rate $r_{t+1} = \rho$ with or without the bubble. So this generation, which has (indirect) utility given by $V[\phi(\rho)A_0, \rho]$, is not affected by the bubble. All subsequent generations are harmed, however, as growth of labor productivity is reduced by the bubble and so each generation born after time 0 earns less labor income than it would have otherwise.

It is interesting to note that the generation that is born at time 1, by itself, suffers income losses from the bubble that are sufficiently large that this generation could more than compensate the initial old for their gain from the bubble, if only there were a way to effect this intergenerational transfer. The bubble allows the initial old to increase their consumption in period 0 by B_0. The slowdown in productivity growth causes the generation that is young at time 1 to lose wage income in period 1 of $(\hat{A}_1 - A_1)\phi(\rho) = A_0(\hat{g}_0 - g_0)\phi(\rho)$, where a circumflex indicates a variable in the equilibrium without bubbles. Noting the growth rates recorded in (7) and (10), we find $\hat{g}_0 - g_0 = b_0/a$. Thus, the income loss at time 1 equals $B_0\phi(\rho)/a$. Since this comes one period later than the gain to the initial old, the value of this income loss discounted back to time 0 is $B_0\phi(\rho)/a(1+\rho)$, where we have used the market interest rate (which equals also the intertemporal rate of substitution for every generation) to perform the discounting. Now we calculate the difference between the discounted loss to the young at time 1 and the gain to the initial old as

$$\frac{B_0\phi(\rho)}{a(1+\rho)} - B_0 = \frac{B_0}{a(1+\rho)}[\phi(\rho) - a(1+\rho)]. \tag{13}$$

The existence condition for a sustainable bubble requires $\bar{s}/a - 1 > \rho$, or $\bar{s} > a(1+\rho)$. Since the wealth constraint requires $w = \phi(\rho) > \bar{s}$, the term in square brackets on the far right-hand side of (13) is positive. This establishes our claim.[5]

But the fact that the young born at time 1 could bribe the old at time 0 to 'retire' the bubble asset does not mean that the bubble can be made to disappear. The problem is that these two generations do not trade with one another. The transfer from the young at time 1 to the old at time 0 must be effected through the generation that is young at time 0. When this generation pays the bribe to the initial old, it will want to collect $(1+\rho)$ times the amount that it has laid out

[5]In fact, it can be shown that *any* generation born after the period in which the bubble forms suffers a greater loss than the gain to the initial old generation (even after allowing for discounting). The loss to the generation born at time n, discounted to time 0, is

$$\frac{A_0\phi(\rho)}{a^n(1+\rho)^n}\left[\bar{s}^n - \prod_{i=0}^{i=n}(\bar{s} - b_i)\right].$$

Since $\phi(\rho) > \bar{s} > a(1+\rho)$, the difference between the loss to the generation born at time n and the gain to the old at time 0 exceeds

$$\frac{A_0}{\bar{s}^n}(\bar{s} - b_0)\left[\bar{s}^n - \prod_{i=1}^{i=n}(\bar{s} - b_i)\right] > 0.$$

from the young at time 1. In the interim, it has an 'IOU' that is exactly like the bubble asset.[6] The young at time 0 divert savings from capital formation in order to pay the bribe, with the result that the potential gain from the intergenerational transfer scheme disappears. The harm caused by the bubble to generations born after time 0 cannot be avoided by a simple tax/subsidy scheme that redistributes income across generations.[7]

5. Extensions

We have derived our results for a simple economy in which externalities from capital formation sustain long-run growth. In this section, we show that our results apply also to economies with other sources of sustained growth, such as endogenous innovation or human capital accumulation. Then we discuss the existence conditions for bubbles in economies with various types of rents.

5.1. Alternative engines of growth

We have used a simple model of growth based on capital externalities in order to highlight the similarities to the Tirole (1985) analysis. However, our results apply to a much broader class of economies with endogenous growth. Consider for example an economy like that described in Romer (1987), where endogenous innovation sustains growth. Suppose that final output is produced by firm i from labor and differentiated intermediate goods according to the production function

$$Y_t^i = (L_t^i)^\alpha \left[\sum_{j=1}^{n_t} (x_{jt}^i)^{1-\alpha} \right], \tag{14}$$

where L_t^i again is the labor input of firm i at time t (with an aggregate labor supply normalized to one) and x_{jt}^i is its input of variety j of the intermediate input. Only inputs numbered 1 through n_t can be used at time t, because only these inputs have previously been developed in the research lab. A research firm active in period t can invent a new variety of intermediate input by using a' units of final output.[8] Having done so, the owners of the research firm acquire an infinitely-lived, transferable patent that grants its holders the unique right to manufacture the new product. The production of one unit of any known intermediate good requires one unit of final output.

[6]This IOU is like a national debt O'Connell and Zeldes (1986) and Tirole (1990) have discussed the analogy between asset bubbles and public debt in overlapping generations models. The analogy remains apt in the present context. See Alogoskoufis and van der Ploeg (1990) for an analysis of the effects of public debt in an externalities-based model of endogenous growth.

[7]Welfare of all generations can be improved, however, by a policy that stimulates investment and causes individuals to internalize the externality associated with capital formation.

[8]Or, equivalently, the research process uses a technology similar to that described in (14), so that the inputs that can produce one unit of final output can also produce $1/a'$ new blueprints.

In this setting, the savings of the young are used (in the absence of bubbles) to acquire the existing blueprints and associated patents from the old generation and to finance new inventions. Each blueprint sells for a' units of the final good. The return on an existing blueprint consists of the monopoly rent that can be earned by a firm that manufactures the unique variety of intermediate good and sells its output to producers of final goods. As is well known, the producers of the intermediates face a constant elasticity of demand and maximize profits by pricing at a fixed mark-up over their (unit) marginal cost. Thus, the price of intermediate j is $p_{jt} = 1/(1-\alpha)$, and the firm that produces this good earns a profit of $\pi_{jt} = x_{jt}\alpha/(1-\alpha)$, where x_{jt} is its total sales at time t.

It is easy to verify that sales per variety are constant over time, and thus so are the profits of any intermediate good producer.[9] The cost of a blueprint also is constant, so the rate of return on an investment in a blueprint is constant; let this constant rate of return be ρ'. Savings of the young depend on labor income and ρ'. Using the properties of the Cobb–Douglas production function, we calculate the wage rate to be

$$w_t = \alpha n_t \bar{x}^{1-\alpha} \equiv \phi(\bar{x}) n_t,$$

where $\bar{x}$ is the (constant) output of a typical intermediate good. Then the homotheticity of preferences implies that aggregate savings of the young are equal to $n_t \bar{s}'$, where $\bar{s}' \equiv s[\phi(\bar{x}), \rho']$. Equating these savings to the cost of the new inventions plus the cost of the blueprints purchased from the old generation, we have

$$n_t \bar{s}' = a' n_t + a'(n_{t+1} - n_t), \tag{15}$$

or (after dividing by n_t and rearranging terms)

$$g_t = \bar{s}'/a' - 1. \tag{16}$$

Here g_t is the rate of growth in the number of blueprints at time t. It is also the growth rate of the economy (in the absence of bubbles), since final output is proportional to the number of blueprints, i.e., $Y_t = n_t \bar{x}^{1-\alpha}$.

Notice the similarity between (16) and (7). The existence conditions for bubbles and the effects of any such bubbles on the two economies with different

[9]Final good producers spend a fraction $(1-\alpha)$ of their total costs on intermediate goods. Since the final goods industry is competitive, costs equal total revenues. Therefore, $n_t x_t/(1-\alpha) = (1-\alpha) Y_t$, where x_t is the output of a typical intermediate good at time t and Y_t is aggregate output of the final good. From the production function (14) we have $Y_t = n_t x_t^{1-\alpha}$ (recall that $L_t = 1$). Equating the two expressions for Y_t shows that x_t is constant through time.

sources of growth also is similar. Eq. (8) describes the evolution of any bubble. In place of (9) we have

$$a'(n_{t+1} - n_t) = n_t s[\phi(\bar{x}), \rho'] - (B_t + a'n_t). \tag{17}$$

From this we can derive

$$g_t = \frac{s' - b'_t}{a'} - 1, \tag{18}$$

where $b'_t \equiv B_t/n_t$. Compare this with (10) above. Bubbles slow the pace of growth in the economy with innovations just as they do in the economy with capital externalities. Any bubble eventually becomes small in relation to the size of the economy, unless the ratio of the size of the initial bubble to the initial number of differentiated inputs happens to take on a certain value. Initial bubbles larger than this critical value cannot exist, because their continued growth would eventually require that the young turn over more than 100 percent of their labor income to the old to purchase the assets, which of course is impossible.

Bubbles will retard growth in a wide class of economies besides the two examples considered here. In all economies with sustained long-run growth, the returns to whatever is being accumulated (e.g., physical, human, or knowledge capital) must eventually become constant. Then the level of savings and the productivity of investment together determine the pace of economic expansion. A bubble that diverts savings away from investments in productive assets is bound to slow growth.

Our welfare results of section 4 also extend to a wider class of economies. Consider, for example, an initial bubble of size B'_0 in the economy with growth driven by innovation. The bubble causes the generation born at time 1 to lose wage income in period 1 equal to $(\hat{n}_1 - n_1)\phi(\bar{x})$, where $\hat{n}_1$ is the number of intermediate goods available in period 1 in the economy with the bubble and n_1 is the number that would have been available without the bubble. The income loss can be written as $B'_0\phi(\bar{x})/a'$, since $\hat{g}_0 - g_0 = b'_0/a'$ from (16) and (18) and $b'_0 = B'_0/n_0$ by definition. Now it is apparent that this loss in income for the generation born in period 1, discounted to time 0 using the market interest rate ρ', must exceed the gain to the initial old generation from the formation of the bubble.[10] The argument relies, as before, on the fact that wage income exceeds savings and that the bubble can exist only if the growth rate absent the bubble exceeds the interest rate (i.e., $g_0 > \rho'$). More generally, the same welfare argument can be made for *any* overlapping generations economy where there are constant returns to an accumulated asset and the labor share in national income is constant.

[10]It can also be shown that all future generations, not just the one born in period 1, suffer income losses that, when discounted back to period 0, exceed the initial value of the bubble.

5.2. Economies with rents

Until now, we have considered economies with only two types of assets: one type is productive and accumulable while the other is nonproductive and nonaccumulable. A third type of asset contributes to the production of output but cannot be accumulated and so cannot serve as an engine of growth. Such assets – of which land, natural resources, and paintings by old masters are examples – generate real *rents*. We now consider how the presence of such assets affects the conditions for the existence of bubbles in economies with endogenous growth.

We follow Tirole (1985) in distinguishing two cases. In the first case, the total amount of rents in the economy is fixed in units of the final good. In this case, endogenous growth makes the rents decreasingly important in relation to the size of the economy. The second case we consider is one in which rents expand to keep pace with the growth of output.

Consider once again a Diamond–Romer economy with capital externalities, but introduce now an asset that generates a fixed quantity of rents per unit time. In particular, let the aggregate production function be

$$Y_t = F[K_t, A(K_t)L] + qT, \tag{19}$$

where $A(K_t) = K_t/a$ and T is the fixed stock of an asset that produces q units of output per period. Let V_t be the market price of a unit of the asset T at time t. All units of T, like the bubble, are owned by the old generation at the beginning of any period. This generation collects the rents generated by the asset during the period and then sells the asset to the young generation after production has taken place. Then the accumulation eq. (9) must be replaced by

$$K_{t+1} - K_t = A_t Ls(w_t, r_{t+1}) - (B_t + K_t + V_t T). \tag{20}$$

Arbitrage ensures that the rent-generating asset yields a normal rate of return, or that

$$r_{t+1} V_t = V_{t+1} - V_t + q. \tag{21}$$

It is fairly clear that the existence of such a rent-generating asset does not change our main arguments. The capital externality fixes the marginal product of capital at ρ; so (3′) still applies. Then (4′) gives the wage rate. If the rent-generating asset is priced at its fundamental value, then $V_t = q/\rho$ for all t. We can define $v_t \equiv V_t/A_t$, so that $v_{t+1}/v_t = A_t/A_{t+1} = 1/(1+g_t)$. Clearly, v_t goes to zero if capital accumulation is sustained. From (20) we derive the growth rate at time t,

$$g_t = \frac{\bar{s} - b_t - v_t}{a} - 1. \tag{22}$$

Then substituting this expression into (11) gives

$$b_{t+1} = a(1+\rho)\frac{b_t}{\bar{s} - b_t - v_t}, \tag{23}$$

while

$$v_{t+1} = a\frac{v_t}{\bar{s} - b_t - v_t}. \tag{24}$$

From these dynamics for v_t and b_t we can conclude the following. First, a bubble that is not too large initially can exist here, just as in section 3. For almost all values of b_0 the bubble eventually becomes small in relation to the size of the economy. But now there may be an initial stage during which the bubble grows faster than output (i.e., a range of t such that $b_{t+1} > b_t$). As before, there does exist a unique value for b_0 (that depends on v_0 and therefore on q) such that the bubble comes to absorb a fixed share of savings in the long run. If the initial bubble is of this size, it grows faster than the economy at all t, but asymptotes to the same steady-state b^* as in the economy without rents.

Now consider the second case where total rents grow as the economy does. This would happen, for example, if the aggregate production function took the form

$$Y_t = F[K_t, A(K_t)G(L, T)], \tag{25}$$

for $A(K_t) = K_t/a$ and $G(\cdot)$ homogeneous of degree one. Here, the capital externality serves to raise not only the productivity of labor, but also the productivity of the rent-generating asset.[11] For concreteness, let us call this asset 'land', and let its market value once again be denoted by V_t. It is easy to show that (normalized) factor prices again will be constant in this case, with rents of (say) $\bar{q}$ units of final output per unit of effective land. Thus, total rents, $\bar{q}A_tT$, grow at same rate as final output; both grow at the rate g_t given in eq. (22).

We will now show that no bubble can exist in this economy. We do so by establishing a contradiction after first assuming that a bubble does exist. In the event, the size of the bubble must evolve according to eq. (23), while from (20),

[11] Tirole (1985) considers an economy where new rent-generating assets are created in every period. There, claims to future rents cannot be sold prior to the appearance of the rents in the economy. Tirole shows that bubbles can exist in such an economy where prices of existing assets do not capitalize the value of future rents. In our case, land prices at time t do capitalize the value of all rents that will be generated subsequently Were we to assume the opposite, we would find like Tirole that bubbles can exist in an economy where rents grow at the same rate as final output.

(3′), and (22) we derive a similar equation for the normalized value of a unit of land, namely

$$v_{t+1} = a(1+\rho)\frac{v_t}{\bar{s} - b_t - v_t} - \bar{q}. \tag{26}$$

We know from our previous discussions that b_t cannot grow without bound. Suppose b_t were to approach a constant $\bar{b} > 0$ as $t \to \infty$. Then, from (23) we would have $\bar{s} - \bar{b} - v_t = a(1+\rho)$ in the long run, so that (26) would imply $v_{t+1} = v_t - \bar{q}$. But this is impossible, because the value of land cannot become negative. The remaining possibility is that $b_t \to 0$ as $t \to \infty$. The normalized value of land must approach some constant, for if it were always increasing then land purchases eventually would absorb more than 100% of available savings and if it were always decreasing it would eventually become negative. Let $\bar{v}$ denote the long-run (normalized) value of land. Note that we can solve for this land value by setting $v_{t+1} = v_t$ and $b_t = 0$ in (26).

We have identified a steady state, which is the only possible long-run equilibrium for the growing economy. Our final task is to show that this steady state cannot be approached along a trajectory with $b_t > 0$ for some t. We note that, in the neighborhood of the steady state, $v_{t+1}/v_t \approx 1$. But notice from (23) and (26) that $b_{t+1}/b_t > v_{t+1}/v_t$ for all t. This means that, as the normalized value of land approaches $\bar{v}$, the bubble must be growing faster than output. This is impossible because the size of the *normalized* bubble must approach zero in the long run. It follows that the unique equilibrium trajectory has $b_t = 0$ and $v_t = \bar{v}$ for all t.

6. Conclusions

In settings where long-run growth is driven by investments in physical, human, or knowledge capital, the existence of an unproductive asset – one that yields a financial return but does not contribute to the production of real output – can be harmful to growth. The unproductive asset, or bubble, attracts savings away from more productive uses. Each new generation purchases the asset at least partly at the expense of investment in growth-promoting capital.

In this paper, we have examined the conditions under which asset bubbles can exist in economies with endogenous growth. As in the neoclassical growth setting, a bubble can survive only if the equilibrium growth rate exceeds the interest rate in the bubbleless economy. Here, however, the equilibrium growth rate, like the interest rate, is determined by parameters of tastes and technology. Bubbles are more likely to be possible when households are patient (i.e., savings propensities are high for a given interest rate) and when investments in accumulable assets are very productive. The existence in the economy of assets that produce rents does not rule out the formation of bubbles, unless the aggregate rents grow at the same rate as the economy and the price of the rent-generating assets in any period fully capitalizes the value of all subsequent dividends. When bubbles do exist, they retard economic growth along the transition path to the steady state and possibly even in the long run. The bubbles also harm all generations born after the period in which the asset first appears, and to an extent that exceeds the gain to the generation that benefits from the bubble.

In our models, bubbles can exist only on nonaccumulable useless assets. That is, there cannot be any bubble in the price of capital or blueprints. This is because new units of an asset must have the same price as old, and it is always possible to create new units of capital or new blueprints at a constant cost in terms of final output.[12] Thus, competition from potential new supply prevents exponential growth in the price of the accumulable assets.

This raises the fundamental question about asset bubbles: what determines their supply? Every individual is willing to exchange a worthless piece of paper for a positive amount of goods. If asset bubbles do appear in the economy, is there any way to predict ahead of time how many and which ones? Is there any way to prevent their formation in situations where their existence will retard growth? These questions remain to be answered.

References

Alogoskoufis, George S. and Frederick van der Ploeg, 1990, Endogenous growth and overlapping generations, Discussion paper no. 90/26 (Birkbeck College, University of London, London).

Arrow, Kenneth J., 1962, The economic implications of learning by doing, Review of Economic Studies 28, 155–173.

Blanchard, Olivier J. and Stanley Fischer, 1989, Lectures on macroeconomics (MIT Press, Cambridge, MA).

Diamond, Peter A., 1965, National debt in a neoclassical growth model, American Economic Review 55, 1126–1150

Grossman, Gene M. and Elhanan Helpman, 1991, Innovation and growth in the global economy (MIT Press, Cambridge, MA).

Jones, Larry E. and Rodolfo Manuelli, 1990, A convex model of equilibrium growth, Journal of Political Economy 98, 1008–1038.

Lucas, Robert E., Jr., 1988, On the mechanics of economic development, Journal of Monetary Economics 22, 2–42.

O'Connell, Stephen and Stephen Zeldes, 1988, Rational ponzi games, International Economic Review 29, 431–450.

Romer, Paul M., 1986, Increasing returns and long-run growth, Journal of Political Economy 94, 1002–1037.

Romer, Paul M., 1987, Growth based on increasing returns due to specialization, American Economic Review Papers and Proceedings 77, 56–63.

Romer, Paul M., 1990, Endogenous technological progress, Journal of Political Economy 98, S71–S102.

[12]This argument assumes that different units of capital (or different blueprints), which are economically indistinguishable in our model, also are physically indistinguishable. Otherwise, there could be bubbles in the prices of specific units of capital. One might say, in such a case, that the capital is priced at its fundamental value, but that the 'names' of specific pieces of equipment (which are intrinsically useless assets) acquire value as bubbles.

Rebelo, Sergio, 1991, Long run policy analysis and long run growth, Journal of Political Economy 99, 500–521.

Sheshinski, Eitan, 1967, Optimal accumulation with learning by doing, in: K. Shell, ed., Essays on the theory of optimal economic growth (MIT Press, Cambridge, MA).

Tirole, Jean, 1982, On the possibility of speculation under rational expectations, Econometrica 50, 1163–1181.

Tirole, Jean, 1985, Asset bubbles and overlapping generations, Econometrica 53, 1499–1528.

Tirole, Jean, 1990, Intertemporal efficiency, intergenerational transfers, and asset pricing: An introduction, in: P. Champsaur et al., eds., Essays in honor of Edmond Malinvaud, Vol. 1: Microeconomics (MIT Press, Cambridge, MA).

Wallace, Neil, 1980, The overlapping generations model of fiat money, in: J. Kareken and N. Wallace, eds., Models of monetary economics (Federal Reserve Bank of Minneapolis, Minneapolis, MN).

Weil, Phillippe, 1987, Confidence and the real value of money in overlapping generations models, Quarterly Journal of Economics 102, 1–22.