14.3. The equity premium puzzle

Table 14.3.1 depicts empirical first and second moments of yields on relatively riskless bonds and risky equity in the U.S. data over the 90-year period 1889–1978. The average real yield on the Standard & Poor's 500 index was 7 percent, while the average yield on short-term debt was only 1 percent. The equity premium puzzle is that with aggregate consumption data, it takes an extraordinarily large value of the coefficient of relative risk aversion to generate such a large gap between the returns on equities and risk-free securities.[2]

	Mean	Variance-Covariance		
		$1+r^s_{t+1}$	$1+r^b_{t+1}$	c_{t+1}/c_t
$1+r^s_{t+1}$	1.070	0.0274	0.00104	0.00219
$1+r^b_{t+1}$	1.010		0.00308	−0.000193
c_{t+1}/c_t	1.018			0.00127

Table 14.3.1: Summary statistics for U.S. annual data, 1889–1978. The quantity $1+r^s_{t+1}$ is the real return to stocks, $1+r^b_{t+1}$ is the real return to relatively riskless bonds, and c_{t+1}/c_t is the growth rate of per capita real consumption of nondurables and services. Source: Kocherlakota (1996a, Table 1), who uses the same data as Mehra and Prescott (1985).

We choose to proceed in the fashion of Hansen and Singleton (1983) and to illuminate the equity premium puzzle by studying unconditional averages of Euler equations under the assumptions that returns are log normal. Let the real rates of return on stocks and bonds between periods t and $t+1$ be denoted $1+r^s_{t+1}$ and $1+r^b_{t+1}$, respectively. In our Lucas tree model, these numbers would be given by $1+r^s_{t+1}=(y_{t+1}+p_{t+1})/p_t$ and $1+r^b_{t+1}=R_{1t}$. Concerning the real rate of return on bonds, we now use time subscript $t+1$ to allow for uncertainty at time t about its realization. Since the numbers in Table 14.3.1

[2] For insightful reviews and lists of possible resolutions of the equity premium puzzle, see Aiyagari (1993), Kocherlakota (1996a), and Cochrane (1997).

are computed on the basis of nominal bonds, real bond yields are subject to inflation uncertainty. To allow for such uncertainty and to switch notation, we rewrite Euler equations (13.2.4) and (13.2.5) as

$$1 = \beta E_t \left[\left(1 + r^i_{t+1}\right) \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} \right], \quad \text{for } i = s, b. \tag{14.3.1}$$

We posit exogenous stochastic processes for both endowments (consumption) and rates of return,

$$\frac{C_{t+1}}{C_t} = \bar{c}_\Delta \exp\left\{\varepsilon_{c,t+1} - \sigma_c^2/2\right\}, \tag{14.3.2}$$

$$1 + r^i_{t+1} = \left(1 + \bar{r}^i\right) \exp\left\{\varepsilon_{i,t+1} - \sigma_i^2/2\right\}, \text{ for } i = s, b, \tag{14.3.3}$$

where exp is the exponential function and $\{\varepsilon_{c,t+1}, \varepsilon_{s,t+1}, \varepsilon_{b,t+1}\}$ are jointly normally distributed with zero means and variances $\{\sigma_c^2, \sigma_s^2, \sigma_b^2\}$. Thus, the logarithm of consumption growth and the logarithms of rates of return are jointly normally distributed. When the logarithm of a random variable η is normally distributed with some mean μ and variance σ^2, the mean of η is $\exp(\mu + \sigma^2/2)$. Thus, the mean of consumption growth and the means of real yields on stocks and bonds are here equal to $\bar{c}_\Delta$, $1+\bar{r}^s$, and $1+\bar{r}^b$, respectively.

Assume the constant relative risk-aversion utility function $u(C_t) = (C_t^{1-\gamma} - 1)/(1-\gamma)$. After substituting this utility function and the stochastic processes (14.3.2) and (14.3.3) into equation (14.3.1), we take unconditional expectations of equation (14.3.1). By the law of iterated expectations, we obtain

$$\begin{aligned} 1 &= \beta E \left[\left(1 + r^i_{t+1}\right) \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \right], \\ &= \beta \left(1 + \bar{r}^i\right) \bar{c}_\Delta^{-\gamma} E \left\{ \exp\left[\varepsilon_{i,t+1} - \sigma_i^2/2 - \gamma\left(\varepsilon_{c,t+1} - \sigma_c^2/2\right)\right] \right\} \\ &= \beta \left(1 + \bar{r}^i\right) \bar{c}_\Delta^{-\gamma} \exp\left[(1+\gamma)\gamma\sigma_c^2/2 - \gamma \operatorname{cov}\left(\varepsilon_i, \varepsilon_c\right)\right], \\ &\quad \text{for } i = s, b, \end{aligned} \tag{14.3.4}$$

where the second equality follows from the expression in braces being log normally distributed. Taking logarithms of equation (14.3.4) yields

$$\log\left(1 + \bar{r}^i\right) = -\log(\beta) + \gamma \log\left(\bar{c}_\Delta\right) - (1+\gamma)\gamma\sigma_c^2/2 + \gamma \operatorname{cov}\left(\varepsilon_i, \varepsilon_c\right), \quad \text{for } i = s, b. \tag{14.3.5}$$

It is informative to interpret equation (14.3.5) for the risk-free interest rate in Bohn's model of section 13.10.2 under the auxiliary assumption of log normally distributed dividend growth, so that equilibrium consumption growth is given by equation (14.3.2). Since interest rates are time invariant, we have $\text{cov}(\varepsilon_b, \varepsilon_c) = 0$. In the case of risk-neutral agents ($\gamma = 0$), equation (14.3.5) has the familiar implication that the interest rate is equal to the inverse of the subjective discount factor β, regardless of any uncertainty. In the case of deterministic growth ($\sigma_c^2 = 0$), the second term of equation (14.3.5) says that the safe interest rate is positively related to the coefficient of relative risk aversion γ, as we also found in the example of Figure 13.10.1. Likewise, the downward pressure on the interest rate due to uncertainty in Figure 13.10.1 shows up as the third term of equation (14.3.5).[3] This downward pressure as σ_c^2 grows reflects the workings of a precautionary savings motive of the type to be discussed in chapter 17. At a given γ, a higher σ_c^2 induces people to want to save more. The risk-free rate must decline to prevent them from doing so.

We now turn to the equity premium by taking the difference between the expressions for the rates of return on stocks and bonds, as given by equation (14.3.5),

$$\log\left(1+\bar{r}^s\right) - \log\left(1+\bar{r}^b\right) = \gamma\left[\text{cov}\left(\varepsilon_s, \varepsilon_c\right) - \text{cov}\left(\varepsilon_b, \varepsilon_c\right)\right]. \tag{14.3.6}$$

Using the approximation $\log(1+r) \approx r$, and noting that the covariance between consumption growth and real yields on bonds in Table 14.3.1 is virtually zero, we can write the theory's interpretation of the historical equity premium as

$$\bar{r}^s - \bar{r}^b \approx \gamma\, \text{cov}\left(\varepsilon_s, \varepsilon_c\right). \tag{14.3.7}$$

After approximating $\text{cov}(\varepsilon_s, \varepsilon_c)$ with the covariance between consumption growth and real yields on stocks in Table 14.3.1, equation (14.3.7) states that an equity premium of 6 percent would require a γ of 27. Kocherlakota (1996a, p. 52) summarizes the prevailing view that "a vast majority of economists believe that values of [γ] above ten (or, for that matter, above five) imply highly implausible behavior on the part of individuals." That statement is a reference to the argument of Pratt, described in the preceding section. This constitutes the equity

[3] Since the term involves the square of γ, the safe interest rate must eventually be a decreasing function of the coefficient of relative risk aversion when $\sigma_c^2 > 0$, but only at very high and therefore uninteresting values for the coefficient of relative risk aversion.