

Appendix A

We define $\hat{\delta} = \frac{D_1(D_1-r)(2D_2-r)^2}{D_2(D_2-r)(2D_1-r)^2}$, $a_1 = \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1}$, $a_2 = \frac{-T_2 + \sqrt{T_2^2 - 4T_1T_3}}{2T_1}$, $\hat{a}_1 = \frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$, and $\hat{a}_2 = \frac{-\hat{T}_2 + \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$.

Condition set C1 in Theorem 2(ib):

(i) $\delta^2 < \hat{\delta}$, $J^2 \in (J_1, \hat{J}_1]$, $\tilde{a}_2 < a_1$, and $a \in (\max\{a_2, \tilde{a}_2\}, a_1)$, where

$$\tilde{a}_2 = \begin{cases} \frac{-\hat{T}_2 + \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} & \text{if } \hat{T}_1 > 0, \hat{T}_2 < 0, \text{ and } \hat{T}_3 < 0; \\ \sqrt{-\frac{\hat{T}_3}{\hat{T}_1}} > 0 & \text{if } \hat{T}_1 > 0, \hat{T}_2 = 0, \text{ and } \hat{T}_3 < 0; \\ -\frac{\hat{T}_3}{\hat{T}_2} > 0 & \text{if } \hat{T}_1 = 0, \hat{T}_2 > 0, \text{ and } \hat{T}_3 < 0. \end{cases}$$

(ii) $\delta^2 < \hat{\delta}$, $J^2 > \hat{J}_1$, and $a \in (\max\{a_2, \hat{a}_2\}, \min\{a_1, \hat{a}_1\})$ if $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$, $a_2 < \hat{a}_2 < \hat{a}_1 < a_1$, or $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$.

(iii) $\delta > \hat{\delta}$, $\delta^2 > \hat{\delta}$, and $J^2 \in (J_1, \hat{J}_3]$ with $a \in (\tilde{a}_2, a_1)$; with $a \in (a_2, \hat{a}_1) \cup (\hat{a}_2, a_1)$ if $a_2 < \hat{a}_1 < \hat{a}_2 < a_1$; or with $a \in (\max\{a_2, \hat{a}_1\}, a_1)$ if $\hat{a}_1 < a_1$, where

$$\tilde{a}_2 = \begin{cases} \frac{-\hat{T}_2 + \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} & \text{if } \hat{T}_1 > 0, \hat{T}_2 < 0, \text{ and } \hat{T}_3 > 0 \text{ with } \hat{a}_1 < \hat{a}_2 < a_2 < a_1, \\ & a_2 < \hat{a}_1 < a_1 < \hat{a}_2, \text{ or } a_2 < a_1 < \hat{a}_1 < \hat{a}_2; \\ \hat{a}_2 & \text{if } \hat{T}_1 > 0, \hat{T}_2 < 0, \text{ and } \hat{T}_3 > 0 \text{ with } \hat{a}_1 < a_2 < \hat{a}_2 < a_1. \end{cases}$$

(iv) $\delta > \hat{\delta}$ and $\delta^2 > \hat{\delta}$ with $J^2 \in (J_1, \hat{J}_1]$, $\tilde{a}_2 < a_1$, and $a \in (\max\{a_2, \tilde{a}_2\}, a_1)$; or with $J^2 > \hat{J}_1$ and $a \in (\max\{a_2, \hat{a}_2\}, \min\{a_1, \hat{a}_1\})$ if $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$, $a_2 < \hat{a}_2 < \hat{a}_1 < a_1$, or $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$, where

$$\tilde{a}_2 = \begin{cases} \frac{-\hat{T}_2 + \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} & \text{if } \hat{T}_1 > 0, \hat{T}_2 < 0, \text{ and } \hat{T}_3 < 0; \\ \sqrt{-\frac{\hat{T}_3}{\hat{T}_1}} > 0 & \text{if } \hat{T}_1 > 0, \hat{T}_2 = 0, \text{ and } \hat{T}_3 < 0. \end{cases}$$

Condition set C2 in Theorem 2(ic):

(i) $J^2 \in (J_1, \hat{J}_1]$ and $a \in (0, a_2]$.

(ii) $J^2 > \hat{J}_1$ and $a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2] \cup [a_1, \infty)$ if $\hat{a}_2 < \hat{a}_1 < a_2 < a_1$ or $a \in (0, \min\{a_2, \hat{a}_2\}] \cup [\max\{a_1, \hat{a}_1\}, \infty)$ otherwise.

- (iii) $\delta > \hat{\delta}$, $\delta^2 > \hat{\delta}$, and $J^2 \in (J_1, \hat{J}_3]$ with $a \in (0, \min\{\hat{a}_1, a_2\}]$; with $a \in [\hat{a}_1, \min\{a_2, \hat{a}_2\}]$ if $\hat{a}_1 < \hat{a}_2 < a_2 < a_1$ or $\hat{a}_1 < a_2 < \hat{a}_2 < a_1$; or with $a \in [\max\{a_1, \hat{a}_1\}, \hat{a}_2]$ if $a_2 < \hat{a}_1 < a_1 < \hat{a}_2$ or $a_2 < a_1 < \hat{a}_1 < \hat{a}_2$.

Condition set C3 in Theorem 2(ii):

- (i) $J^2 \in (J_2, J_1]$ (or $J^2 \in (\hat{J}_1, J_1]$), $\tilde{a}_2 < \hat{a}_1$, and $a \in (\max\{\hat{a}_2, \tilde{a}_2\}, \hat{a}_1)$, where $\tilde{a}_2$ is defined in C1(i).
- (ii) $\delta > \frac{1}{\delta}$, $J^2 \in (\hat{J}_1, J_2]$ (or $J^2 \in (J_3, J_2]$), $\tilde{a}_2 < \hat{a}_1$, and $a \in (\max\{\hat{a}_2, \tilde{a}_2\}, \hat{a}_1)$, where $\tilde{a}_2$ is defined in C1(i).
- (iii) $J^2 > J_1$ and $a \in (\max\{a_2, \hat{a}_2\}, \min\{a_1, \hat{a}_1\})$ if $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$, $a_2 < \hat{a}_2 < \hat{a}_1 < a_1$, or $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$.
- (iv) $\delta > \frac{1}{\delta}$, $\delta^2 \geq \frac{1}{\delta}$, $J^2 \in (J_1, J_3]$, and

$$a \in \begin{cases} (\hat{a}_2, \hat{a}_1) & \text{if } \hat{a}_2 < \hat{a}_1 < a_1 < a_2 \text{ or } a_1 < a_2 < \hat{a}_2 < \hat{a}_1; \\ (\hat{a}_2, a_1) & \text{if } \hat{a}_2 < a_1 < \hat{a}_1 < a_2; \\ (a_2, \hat{a}_1) & \text{if } a_1 < \hat{a}_2 < a_2 < \hat{a}_1. \end{cases}$$

Condition set C4 in Theorem 2(ii):

- (i) $J^2 \in (J_2, J_1]$ (or $J^2 \in (\hat{J}_1, J_1]$), $\tilde{a}_2 < \hat{a}_1$, and $a \in (0, \min\{a_2, \tilde{a}_2\}]$, where $\tilde{a}_2$ is defined in C1(i).
- (ii) $J^2 \in (\hat{J}_1, J_1]$ ($J^2 \in (\hat{J}_1, J_2]$ or $J^2 \in (J_2, J_1]$), $\tilde{a}_2 > \hat{a}_1$, and $a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2]$, where $\tilde{a}_2$ is defined in C1(i).
- (iii) $\delta > \frac{1}{\delta}$, $J^2 \in (\hat{J}_1, J_2]$ (or $J^2 \in (J_3, J_2]$), $\tilde{a}_2 < \hat{a}_1$, and $a \in (0, \min\{a_2, \hat{a}_2\}]$, where $\tilde{a}_2$ is defined in C1(i).
- (iv) $J^2 > J_1$ with $a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2] \cup [a_1, \infty)$ if $\hat{a}_2 < \hat{a}_1 < a_2 < a_1$; or with $a \in (0, \min\{a_2, \hat{a}_2\}] \cup [\max\{a_1, \hat{a}_1\}, \infty)$ if $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$, $a_2 < \hat{a}_2 < \hat{a}_1 < a_1$, or $a_2 < a_1 < \hat{a}_2 < \hat{a}_1$.
- (v) $\delta > \frac{1}{\delta}$, $\delta^2 < \frac{1}{\delta}$, $J^2 \in (J_2, J_1]$, $\tilde{a}_2 > \hat{a}_1$, and $a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2]$, where $\tilde{a}_2$ is defined in C1(i).
- (vi) $\delta > \frac{1}{\delta}$ and $\delta^2 \geq \frac{1}{\delta}$ with $J^2 \in (J_3, J_2]$, $\tilde{a}_2 > \hat{a}_1$, and $a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2]$; or with

$J^2 \in (J_1, J_3]$, $\tilde{a}_2 > \hat{a}_1$, and

$$a \in \begin{cases} [a_1, a_2] & \text{if } \hat{a}_2 < \hat{a}_1 < a_1 < a_2, \\ [\hat{a}_1, a_2] & \text{if } \hat{a}_2 < a_1 < \hat{a}_1 < a_2, \\ [a_1, \hat{a}_2] \cup [\hat{a}_1, \infty) & \text{if } a_1 < \hat{a}_2 < \hat{a}_1 < a_2, \\ [a_1, \hat{a}_2] & \text{if } a_1 < \hat{a}_2 < a_2 < \hat{a}_1, \\ [a_1, a_2] & \text{if } a_1 < a_2 < \hat{a}_2 < \hat{a}_1, \end{cases}$$

where $\tilde{a}_2$ is defined in condition set C1(i).

Condition set C1 in Theorem 2(iii):

(i) $a \in (0, \min\{a', \hat{a}_1\})$ if $a' < \hat{a}_1 < \hat{a}_2$ or $\hat{a}_1 < a' < \hat{a}_2$; or $a \in (0, \hat{a}_1) \cup (\hat{a}_2, a')$ if

$\hat{a}_1 < \hat{a}_2 < a'$ with one of the following conditions.

(ia) $J_1 < \hat{J}_1 < J_2 < \hat{J}_2 < J_3 < \hat{J}_3$ and $J^2 \in (J_1, \hat{J}_1]$.

(ib) $J_1 < J_2 < J_3 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 \in (J_1, \hat{J}_2]$ or $J^2 \in (J_2, J_3]$

Here

$$a' = \begin{cases} \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} & \text{if (S7) and (S1) are taken;} \\ \sqrt{-\frac{T_3}{T_1}} > 0 & \text{if (S9) and (S1) are taken;} \\ -\frac{T_2}{T_1} > 0 & \text{if (S13) and (S1) are taken.} \end{cases}$$

(ii) $a \in (0, \min\{a', a''\})$ with one of the following conditions.

(iia) $J_1 < \hat{J}_1 < J_2 < \hat{J}_2 < J_3 < \hat{J}_3$ with $J^2 \in (J_1, \hat{J}_1]$ or with $J^2 \in (\hat{J}_1, J_2]$.

(iib) $J_1 < J_2 < J_3 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 \in (J_1, J_3]$.

Here

$$(a', a'') = \begin{cases} \left(\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1}, -\frac{\hat{T}_3}{\hat{T}_2} \right) & \text{if (S7) and (S10) are taken;} \\ \left(\sqrt{-\frac{T_3}{T_1}}, -\frac{\hat{T}_3}{\hat{T}_2} \right) & \text{if (S9) and (S10) are taken;} \\ \left(\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1}, \frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} \right) & \text{if (S7) and (S11) or (S7) and (S13) are taken;} \\ \left(\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1}, -\frac{\sqrt{-4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} \right) & \text{if (S7) and (S12) are taken;} \\ \left(\sqrt{-\frac{T_3}{T_1}}, \frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} \right) & \text{if (S9) and (S11) or (S9) and (S12) are taken;} \\ \left(-\frac{T_2}{T_1}, \frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} \right) & \text{if (S13) and (S11) or (S13) and (S12) are taken;} \\ \left(\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1}, -\frac{\hat{T}_2}{\hat{T}_1} \right) & \text{if (S7) and (S14) are taken;} \\ \left(-\frac{T_2}{T_1}, \frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} \right) & \text{if (S13) and (S13) are taken;} \\ \left(-\frac{T_2}{T_1}, \frac{\hat{T}_2}{\hat{T}_1} \right) & \text{if (S13) and (S10) or (S13) and (S14) are taken;} \end{cases}$$

Condition set C2 in Theorem 2(iii):

- (i) $a \in [\max\{a_1, \hat{a}_1\}, \hat{a}_2]$ if $a_1 < \hat{a}_1 < \hat{a}_2$ or $\hat{a}_1 < a_1 < \hat{a}_2$ with one of the conditions given in parts (ia)-(ib) of condition set C1.
- (ii) $a \geq \max\{a', a''\}$ with one of the conditions given in parts (iia)-(iib) of condition set C1.

Condition set C3 in Theorem 2(iv):

- (i) $a \in (0, \min\{a', a''\})$ if $a'' < a' < a_2$ or $a' < a'' < a' < a_2$; or $a \in (0, a') \cup (a_2, a'')$ otherwise; and each of them is with $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1 < J_2 < J_3$ and $J^2 \in (\hat{J}_1, \hat{J}_2]$, where

$$(a', a'') = \begin{cases} \left(\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1}, \frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} \right) & \text{if (S5) and (S11) or (S5) and (S13) are taken;} \\ \left(\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1}, -\sqrt{\frac{-4\hat{T}_1\hat{T}_3}{2\hat{T}_1}} \right) & \text{if (S5) and (S12) are taken;} \\ \left(\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1}, \frac{-\hat{T}_2}{\hat{T}_1} \right) & \text{if (S5) and (S14) are taken.} \end{cases}$$

- (ii) $a \in (\max\{a_2, a''\}, a')$ if $a'' < a'$ with $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1 < J_2 < J_3$ and $J^2 \in (\hat{J}_2, \hat{J}_3]$, where

$$(a', a'') = \begin{cases} \left(\frac{-T_3}{T_2}, \frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} \right) & \text{if (S6) and (S13) are taken;} \\ \left(\frac{-T_3}{T_2}, \frac{-\hat{T}_2}{\hat{T}_1} \right) & \text{if (S6) and (S14) are taken.} \end{cases}$$

(iii) $a \in (\hat{a}_2, a')$ if $\hat{a}_2 < a' < \hat{a}_1$ or $a \in (\hat{a}_2, a_1)$ if $\hat{a}_2 < \hat{a}_1 < a'$, where

$$a' = \begin{cases} \frac{-T_3}{T_2} & \text{if (S6) and (S}\hat{6}\text{) are taken;} \\ \sqrt{\frac{-T_3}{T_1}} & \text{if (S9) and (S}\hat{6}\text{) are taken;} \\ \frac{-T_2}{T_1} & \text{if (S13) and (S}\hat{6}\text{) are taken;} \end{cases}$$

(iv) $a \in (\hat{a}_2, \hat{a}_1)$ if $\hat{a}_2 < \hat{a}_1 < a_1 < a_2$ or $a_1 < a_2 < \hat{a}_2 < \hat{a}_1$; $a \in (\hat{a}_2, a_1)$ if $\hat{a}_2 < a_1 < \hat{a}_1 < a_2$; or $a \in (a_2, \hat{a}_1)$ if $a_1 < \hat{a}_2 < a_2 < \hat{a}_1$; and each of them is with $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1 < J_2 < J_3$ and $J^2 \in (\hat{J}_3, J_1]$, $J^2 \in (J_1, J_2]$, or $J^2 \in (J_2, J_3]$.

(v) $a \in (0, \min\{a_1, a''\})$ with $\hat{J}_1 < J_1 < \hat{J}_2 < J_2 < \hat{J}_3 < J_3$ and $J^2 \in (\hat{J}_2, J_2]$ or $J^2 \in (J_2, \hat{J}_3]$, where

$$a'' = \begin{cases} \frac{-\hat{T}_3}{\hat{T}_2} & \text{if (S7) and (S}\hat{13}\text{) are taken;} \\ \frac{-\hat{T}_2}{\hat{T}_1} & \text{if (S7) and (S}\hat{14}\text{) are taken.} \end{cases}$$

Condition set C $\hat{4}$ in Theorem 2(iv):

(i) $a \in [\max\{a_1, \hat{a}_2\}, \hat{a}_2]$ if $\hat{a}_2 < a_1 < a_2$ or $a_1 < \hat{a}_2 < a_2$ with one of the conditions given in part (ia) of condition set C $\hat{3}$.

(ii) $a \in (0, \min\{a_2, a''\})$ with one of the conditions given in part (ii) of condition set C $\hat{3}$, and a'' is defined in condition set C $\hat{3}$.

(iii) $a \in [a', \frac{-\hat{T}_2 + \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}] \cup [\frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}, \infty)$ if $a' < \hat{a}_2 < \hat{a}_1$ or $a \geq \max\{a', \frac{-\hat{T}_2 + \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}\}$, where a' is defined in part (iii) of condition set C $\hat{3}$.

(iv) $a \in [a_1, a_2]$ if $\hat{a}_2 < \hat{a}_1 < a_1 < a_2$; $a \in [\hat{a}_1, a_2]$ if $\hat{a}_2 < a_1 < \hat{a}_1 < a_2$; $a \in [a_1, \hat{a}_2] \cup [\hat{a}_1, a_2]$ if $a_1 < \hat{a}_2 < \hat{a}_1 < a_2$; $a \in [a_1, \hat{a}_2] \cup [\hat{a}_1, a_2]$ if $a_1 < \hat{a}_2 < a_2 < \hat{a}_1$; $a \in [a_1, a_2]$ if $a_1 < a_2 < \hat{a}_2 < \hat{a}_1$; and each of them is with $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1 < J_2 < J_3$ and $J^2 \in (J_1, J_2]$ or $J^2 \in (J_2, J_3]$.

(v) $a \geq \max\{a_1, a''\}$ with one of the conditions given in part (v) of condition set C $\hat{3}$ and a'' is also defined in part (v) of condition set C $\hat{3}$.

Condition set C5 in Theorem 3(ib):

Here $\delta_1 = \frac{\delta_m(2D_2-r)[rc_1+rc_2-2c_1D_2]}{2(D_2-r)(rc_2-2c_1D_2)}$ and $\delta_3 = \frac{2\delta_m c_1(2D_2-r)(c_1D_2-rc_2)}{(2c_1D_2-rc_2)[2c_1D_2-rc_1-rc_2]}$.

(i) $2c_1D \in (rc_2, 2rc_2]$, $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, $\tilde{a} > \hat{a}_1$, and $a \in (\hat{a}_2, \hat{a}_1)$, where

$$\tilde{a} = \begin{cases} \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} & \text{if } T_1 < 0, T_2 > 0, \text{ and } T_3 > 0; \\ -\frac{T_2}{T_1} & \text{if } T_1 < 0, T_2 < 0, \text{ and } T_3 = 0. \end{cases}$$

(ii) $2c_1D > 2rc_2$, $\delta \leq \delta_3$, $T_2^2 > 4T_1T_3$, $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, and $a \in (\max\{a_2, \hat{a}_2\}, \min\{a_1, \hat{a}_1\})$.

(iii) $2c_1D > 2rc_2$, $\delta \in (\delta_3, \delta_1]$, and $J_2 < J_3 < J_1$ with $a \in (a_2, \hat{a}_1)$ if $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$;

or with $a \in (\hat{a}_2, a_1)$ if $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$.¹

(iv) $2c_1D > 2rc_2$, $\delta \in (\delta_3, \delta_1]$, $J_1 = J_2 < J_3$, $J^2 \in (J_1, J_3]$, $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, $\tilde{a} > \hat{a}_2$, and $a \in (\hat{a}_2, \hat{a}_1)$, where $\tilde{a}$ is defined in part (i).

(v) $2c_1D > 2rc_2$, $\delta > \delta_1$, $J^2 > J_3$, $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$, and $a \in (\hat{a}_2, a_1)$.²

Condition set C6 in Theorem 3(ic):

(i) $2c_1D \in (rc_2, 2rc_2]$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$ with $\tilde{a} \leq \hat{a}_2$ and $a \in [\tilde{a}, \hat{a}_2] \cup [\hat{a}_1, \infty)$; with $\hat{a}_2 < \tilde{a} \leq \hat{a}_1$ and $a \in [\hat{a}_1, \infty)$; or with $\tilde{a} > \hat{a}_1$ and $a \in [\tilde{a}, \infty)$, where $\tilde{a}$ is defined in part (i) of condition set C5.

(ii) $2c_1D \in (rc_2, 2rc_2]$, $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, and $a \in [\hat{a}_1, \infty)$.

(iii) $2c_1D > 2rc_2$ and $\delta \leq \delta_3$ with one of the following:

(iiia) $T_2^2 > 4T_1T_3$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$ with $\hat{a}_2 < \hat{a}_1 < a_2 < a_1$ and $a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2] \cup [a_1, \infty)$; with $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$ and $a \in (0, \hat{a}_2] \cup [a_1, \infty)$; with $a_2 < \hat{a}_2 < \hat{a}_1 < a_1$ and $a \in (0, a_2] \cup [a_1, \infty)$; with $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$ and $a \in (0, a_2] \cup [\hat{a}_1, \infty)$;

or with $a_2 < a_1 < \hat{a}_2 < \hat{a}_1$ and $a \in (0, a_2] \cup [a_1, \hat{a}_2] \cup [\hat{a}_1, \infty)$.

(iiib) $T_2^2 > 4T_1T_3$, $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, and $a \in (0, a_2] \cup [a_1, \infty)$.

(iiic) $T_2^2 \leq 4T_1T_3$, $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, and $a \in (0, \hat{a}_2] \cup [\hat{a}_1, \infty)$.

(iiid) $T_2^2 \leq 4T_1T_3$, $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, and for all $a \geq 0$.

(iv) $2c_1D > 2rc_2$, $\delta \in (\delta_3, \delta_1]$, and $J_2 < J_3 < J_1$ with one of the following:

¹Similar conclusions hold if $J_2 < J_3 < J_1$ is replaced with $J_1 = J_2 < J_3$ and $J^2 > J_3$.

²Similar conclusions hold if $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$ is replaced with $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$.

- (iva) $T_2^2 > 4T_1T_3$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$ with $\hat{a}_2 < \hat{a}_1 < a_2 < a_1$ and $a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2] \cup [a_1, \infty)$; with $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$ and $a \in (0, \hat{a}_2] \cup [a_1, \infty)$; with $a_2 < \hat{a}_2 < \hat{a}_1 < a_1$ and $a \in (0, a_2] \cup [a_1, \infty)$; with $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$ and $a \in (0, a_2] \cup [\hat{a}_1, \infty)$; or with $a_2 < a_1 < \hat{a}_2 < \hat{a}_1$ and $a \in (0, a_2] \cup [a_1, \infty)$.
- (ivb) $T_2^2 > 4T_1T_3$, $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, and $a \in (0, a_2] \cup [a_1, \infty)$,
- (ivc) $T_2^2 \leq 4T_1T_3$, $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, and $a \in (0, \hat{a}_2] \cup [\hat{a}_1, \infty)$, and
- (ivd) $T_2^2 \leq 4T_1T_3$, $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, and for all $a \geq 0$.

Conditions (iva)-(ivd) remain valid if $\delta \in (\delta_3, \delta_1]$ and $J_2 < J_3 < J_1$ are replaced with $\delta \in (\delta_3, \delta_1]$, $J_1 = J_2 < J_3$, and $J^2 > J_3$, or with $\delta > \delta_1$ and $J^2 > J_3$.

Condition set C $\hat{5}$ in Theorem 3(ii):

- (i) $a \in (\max\{a_2, \hat{a}_2\}, \min\{a_1, \hat{a}_1\})$ if $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$, $a_2 < \hat{a}_2 < \hat{a}_1 < a_1$, or $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$ with one of the following conditions.
- (ia) $\hat{J}_2 < J_2 < \hat{J}_3 < \hat{J}_1 = J_1 < J_3$ with $J^2 > J_3$.
- (ib) $\hat{J}_2 < \hat{J}_3 < J_2 < \hat{J}_1 = J_1 < J_3$ with $J^2 > J_3$.
- (ic) $\hat{J}_2 < \hat{J}_3 < \hat{J}_1 = J_1 < J_2 < J_3$ with $J^2 > J_3$.
- (ii) $a \in (\hat{a}_2, a')$ if $\hat{a}_2 < a' < \hat{a}_1$ or $\hat{a}_2 < \hat{a}_1 < a'$ with $\hat{J}_2 < \hat{J}_3 < \hat{J}_1 = J_1 < J_2 < J_3$ and $J^2 \in (J_1, J_2]$, where

$$a' = \begin{cases} \frac{-\hat{T}_2 + \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} & \text{if (S7) and (S}\hat{6}\text{) are taken;} \\ \sqrt{\frac{-\hat{T}_3}{\hat{T}_1}} & \text{if (S7) and (S}\hat{6}\text{) are taken;} \\ \frac{-\hat{T}_2}{\hat{T}_1} & \text{if (S13) and (S}\hat{6}\text{) are taken.} \end{cases}$$

Condition set C $\hat{6}$ in Theorem 3(ii):

- (i) $a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2] \cup [a_1, \infty)$ if $\hat{a}_2 < \hat{a}_1 < a_1$; or $a \in (0, \min\{a_2, \hat{a}_2\}] \cup [\max\{a_1, \hat{a}_1\}, \infty)$ otherwise; and each of them is with one of the conditions given in parts (ia)-(ic) of condition set C $\hat{5}$.
- (ii) $a \in [a', \hat{a}_2] \cup [\hat{a}_1, \infty)$ if $a' < \hat{a}_2 < \hat{a}_1$; or $a \geq \max\{a', \hat{a}_1\}$; and each of them is with the conditions given in part (ii) of condition set C $\hat{5}$.

Condition set C $\hat{5}$ in Theorem 3(iii):

- (i) The conditions are those given in part (iv) or (v) of condition set C $\hat{3}$.
- (ii) The conditions are those given in part (i) or (ii) of condition set C $\hat{5}$.

Condition set C $\hat{6}$ in Theorem 3(iii):

- (i) The conditions are those given in part (iii), (iv), or (v) of condition set C $\hat{4}$.
- (ii) The conditions are those given in part (i) of condition set C $\hat{6}$.

Condition set C7 in Theorem 4(ib):

- (i) $2c_1D_2 \in (rc_2, rc_1 + rc_2]$ and $J^2 \in (\hat{J}_1, J_1]$ with one of the following:³

- (ia) $T_2^2 > 4T_1T_3$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$ with $\hat{a}_2 < \hat{a}_1 < a_1 < a_2$ and $a \in (\hat{a}_2, \hat{a}_1)$; with $\hat{a}_2 < a_1 < \hat{a}_1 < a_2$ and $a \in (\hat{a}_2, a_1)$; with $a_1 < \hat{a}_2 < a_2 < \hat{a}_1$ and $a \in (a_2, \hat{a}_1)$; or with $a_1 < a_2 < \hat{a}_2 < \hat{a}_1$ and $a \in (\hat{a}_2, \hat{a}_1)$.
- (ib) $T_2^2 \leq 4T_1T_3$, $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, and $a \in (\hat{a}_2, \hat{a}_1)$.

- (ii) $2c_1D_2 \in (rc_2, rc_1 + rc_2]$, $J^2 \geq J_1$, and $a \in (\hat{a}_2, \hat{a}_1)$ if $\tilde{a}_1 > \hat{a}_1$, where⁴

$$\tilde{a}_1 = \begin{cases} \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} & \text{if } T_1 < 0, T_2 < 0, \text{ and } T_3 > 0, \\ -\frac{T_3}{T_2} & \text{if } T_1 = 0, T_2 < 0, \text{ and } T_3 > 0, \\ \sqrt{-\frac{T_3}{T_1}} & \text{if } T_1 < 0, T_2 = 0, \text{ and } T_3 > 0, \\ -\frac{T_3}{T_1} & \text{if } T_1 < 0, T_2 > 0, \text{ and } T_3 = 0. \end{cases}$$

- (iii) $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$, $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, and $a \in (\hat{a}_2, \hat{a}_1)$.⁵

- (iv) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \leq \delta_3$, $J^2 \in (\hat{J}_1, J_1]$, $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, and $a \in (\max\{\tilde{a}_2, \hat{a}_2\}, \hat{a}_1)$,

³Parts (ia)-(ib) remain unchanged if we replace $2c_1D_2 \in (rc_2, rc_1 + rc_2]$ and $J^2 \in (\hat{J}_1, J_1]$ with (a) $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$, $\delta < \delta_1$, and $J^2 \in (\hat{J}_1, J_2]$; (b) $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$, $\delta \geq \delta_1$, and $J^2 \in (\hat{J}_1, J_1]$; or (c) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta > \delta_3$, and $J^2 \in (\hat{J}_1, \min\{J_1, J_2, J_3\}]$, $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$.

⁴Similar conclusions hold if we replace $2c_1D_2 \in (rc_2, rc_1 + rc_2]$ and $J^2 \geq J_1$ with (a) $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$ and $J^2 > J_1$; (b) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta > \delta_3$, $J^2 \in (\hat{J}_1, J_1]$, $T_1 = 0$, $T_2 < 0$, and $T_3 > 0$; or (c) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta > \delta_3$, and $J^2 \in (J_1, J_3]$.

⁵Similar conclusions hold if we replace $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$ with (a) $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$, $\delta < \delta_1$, and $J^2 \in (J_2, J_1]$; (b) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \in (\delta_3, \delta_1]$, $J_2 < J_3 < J_1$, $J^2 \leq J_1$, $T_1 > 0$, $T_2 \geq 0$, and $T_3 \geq 0$; (c) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \leq \delta_3$, $J^2 \in [J_3, J_2]$, $T_1 > 0$, $T_2 = 0$,

where⁶

$$\tilde{a}_2 = \begin{cases} \frac{-T_2 + \sqrt{T_2^2 - 4T_1T_3}}{2T_1} & \text{if } T_1 > 0, T_2 > 0, T_3 < 0, \text{ or } T_1 > 0, T_2 \leq 0, T_3 \leq 0, \\ -\frac{T_3}{T_2} & \text{if } T_1 = 0, T_2 > 0, \text{ and } T_3 < 0. \end{cases}$$

- (v) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \leq \delta_3$, $J_3 \leq J_2 < J_1$, $J^2 > J_1$, $T_2^2 > 4T_1T_3$, $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, and $a \in (\max\{a_2, \hat{a}_2\}, \min\{a_1, \hat{a}_1\})$ if $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$, $a_2 < \hat{a}_2 < \hat{a}_1 < a_1$, or $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$.⁷

Condition set C8 in Theorem 4(ic):

- (i) $2c_1D_2 \in (rc_2, rc_1 + rc_2]$ and $J^2 \in (\hat{J}_1, J_1]$ with one of the following:⁸

(ia) $T_2^2 > 4T_1T_3$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$ with $\hat{a}_2 < \hat{a}_1 < a_1 < a_2$ and $a \in [a_1, a_2]$; with $\hat{a}_2 < a_1 < \hat{a}_1 < a_2$ and $a \in [\hat{a}_1, a_2]$; with $a_1 < \hat{a}_2 < \hat{a}_1 < a_2$ and $a \in [a_1, \hat{a}_2] \cup [\hat{a}_1, a_2]$; with $a_1 < \hat{a}_2 < a_2 < \hat{a}_1$ and $a \in [a_1, \hat{a}_2]$; or with $a_1 < a_2 < \hat{a}_2 < \hat{a}_1$ and $a \in [a_1, \hat{a}_2]$.

(ib) $T_2^2 > 4T_1T_3$, $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, and $a \in [a_1, a_2]$.

- (ii) $2c_1D_2 \in (rc_2, rc_1 + rc_2]$, $J^2 \geq J_1$, and $a \in [\tilde{a}_1, \infty)$, where⁹

$$\tilde{a}_1 = \begin{cases} \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} & \text{if } T_1 < 0, T_2 < 0 \text{ (or } T_2 > 0), \text{ and } T_3 > 0, \\ -\frac{T_3}{T_2} & \text{if } T_1 = 0, T_2 < 0, \text{ and } T_3 > 0, \\ \sqrt{-\frac{T_3}{T_1}} & \text{if } T_1 < 0, T_2 = 0, \text{ and } T_3 > 0, \text{ and} \\ -\frac{T_3}{T_1} & \text{if } T_1 < 0, T_2 > 0, \text{ and } T_3 = 0. \end{cases}$$

and $T_3 = 0$; or (d) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \in (\delta_3, \delta_1]$, $J_1 = J_2 < J_3$, $J^2 \in (\hat{J}_1, J_1]$, $T_1 > 0$, $T_2 = 0$, and $T_3 > 0$.

⁶Similar conclusions hold if we replace $2c_1D_2 \in (2rc_2, 2c_2D_1]$ and $\delta \leq \delta_3$ with $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \in (\delta_3, \delta_1]$, $J_2 < J_3 < J_1$, $J^2 \in (\hat{J}_1, J_1]$, $T_1 \geq 0$, $T_2 > 0$, and $T_3 < 0$.

⁷Similar conclusions hold if $\delta \leq \delta_3$, $J_3 \leq J_2 < J_1$, and $J^2 > J_1$ are replaced with (a) $\delta \in (\delta_3, \delta_1]$, $J_2 < J_3 < J_1$, and $J^2 > J_1$; (b) $\delta \in (\delta_3, \delta_1]$, $J_1 = J_2 < J_3$, and $J^2 > J_3$; or (c) $\delta > \delta_1$, $J_1 < J_2 < J_3$, and $J^2 > J_3$.

⁸Similar conclusions hold if we replace $2c_1D_2 \in (rc_2, rc_1 + rc_2]$ and $J^2 \in (\hat{J}_1, J_1]$ with (a) $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$, $\delta < \delta_1$, and $J^2 < J_2$; (b) $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$, $\delta \geq \delta_1$, and $J^2 \leq J_1$; or (c) $2c_1D_2 \in (2rc_2, 2c_2D_1]$ and $J^2 \leq \min\{J_1, J_2, J_3\}$.

⁹Similar conclusions hold if $2c_1D_2 \in (rc_2, rc_1 + rc_2]$ and $J^2 \geq J_1$ are replaced with (a) $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$ and $J^2 > J_1$; or (b) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta > \delta_3$, and $J^2 \leq J_3$.

(iii) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \leq \delta_3$, and $J^2 \in (\hat{J}_1, J_1]$ with one of the following:¹⁰

$$\begin{cases} a \in (0, \max\{a_2, \hat{a}_2\}] & \text{if } \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, \\ a \in (0, a_2] & \text{if } \hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3. \end{cases}$$

(iv) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \leq \delta_3$, and $J^2 > J_1$ with one of the following:¹¹

(iva) $T_2^2 > 4T_1T_3$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$ with $a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2] \cup [a_1, \infty)$ if $\hat{a}_2 < \hat{a}_1 < a_2 < a_1$; or with $a \in (0, \min\{a_2, \hat{a}_2\}] \cup [\max\{\hat{a}_1, a_1\}, \infty)$ if $a_2 < \hat{a}_2 < \hat{a}_1 < a_1$, $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$, or $a_2 < a_1 < \hat{a}_2 < \hat{a}_1$.

(ivb) $T_2^2 > 4T_1T_3$, $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, and $a \in (0, a_2] \cup [a_1, \infty)$.

(ivc) $T_2^2 \leq 4T_1T_3$, $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, and $a \in (0, \hat{a}_2] \cup [\hat{a}_1, \infty)$.

(ivd) $T_2^2 \leq 4T_1T_3$, $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, and $a > 0$.

Condition set C9 in Theorem 5(ib):

(i) $2c_1D_2 \in (rc_2, 2rc_2]$, $J^2 \in (J_1, \hat{J}_1]$, and $a \in (\tilde{a}_1, \tilde{a}_2)$ for $\tilde{a}_1 < \hat{a}_2$, where¹²

$$(\tilde{a}_1, \tilde{a}_2) = \begin{cases} (\hat{a}_2, a_1) & \text{if } T_1 < 0, T_3 > 0, \hat{T}_1 > 0, \text{ and } \hat{T}_3 < 0, \\ (\hat{a}_2, -\frac{T_3}{T_2}) & \text{if } T_1 = 0, T_2 < 0, T_3 > 0, \hat{T}_1 > 0, \text{ and } \hat{T}_3 < 0, \\ (-\frac{\hat{T}_3}{\hat{T}_2}, -\frac{T_3}{T_2}) & \text{if } T_1 \leq 0, T_2 \leq 0, T_3 > 0, \hat{T}_1 = 0, \hat{T}_2 > 0, \text{ and } \hat{T}_3 < 0, \\ (-\frac{\hat{T}_3}{\hat{T}_2}, a_1) & \text{if } T_1 < 0, T_3 > 0, \hat{T}_1 = 0, \hat{T}_2 > 0, \text{ and } \hat{T}_3 < 0, \\ (\hat{a}_2, \sqrt{-\frac{T_3}{T_1}}) & \text{if } T_1 < 0, T_2 = 0, T_3 > 0, \hat{T}_1 > 0, \text{ and } \hat{T}_3 < 0, \\ (\hat{a}_2, -\frac{T_2}{T_1}) & \text{if } T_1 < 0, T_2 > 0, T_3 = 0, \hat{T}_1 = 0, \text{ and } \hat{T}_3 < 0, \text{ and} \\ (-\frac{\hat{T}_3}{\hat{T}_2}, -\frac{T_2}{T_1}) & \text{if } T_1 < 0, T_2 > 0, T_3 = 0, \hat{T}_1 = 0, \hat{T}_2 > 0, \text{ and } \hat{T}_3 < 0. \end{cases}$$

(ii) $2c_1D_2 \in (rc_2, 2rc_2]$, $J^2 > \hat{J}_1$, and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$ with one of the following:¹³

¹⁰Similar conclusions hold if we replace $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \leq \delta_3$, and $J^2 \in (\hat{J}_1, J_1]$ with $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \in (\delta_3, \delta_1]$, $J_2 < J_3 < J_1$, and $J^2 \in [J_3, J_1]$.

¹¹These conclusions remain true if $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \leq \delta_3$, and $J^2 > J_1$ are replaced with (a) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta \in (\delta_3, \delta_1]$, $J_2 < J_3 < J_1$, and $J^2 > J_1$; or (b) $2c_1D_2 \in (2rc_2, 2c_2D_1]$, $\delta > \delta_3$, and $J^2 > J_3$.

¹²These conditions remain unchanged if $2c_1D_2 \in (rc_2, 2rc_2]$ and $J^2 \in (J_1, \hat{J}_1]$ are replaced with (a) $2c_1D_2 > 2rc_2$, $\delta \in (\delta_3, \delta_1]$, and $J^2 \in (J_1, \min\{\hat{J}_1, J_3\}]$; or (b) $2c_1D_2 > 2rc_2$, $\delta > \delta_1$, and $J^2 \in (J_1, \min\{\hat{J}_1, J_3\}]$.

¹³Similar results hold if $2c_1D_2 \in (rc_2, 2rc_2]$ and $J^2 > J_1$ are replaced with (a) $2c_1D_2 \in (rc_1 +$

(iia) $a \in (\hat{a}_2, \check{a})$ if $\hat{a}_2 < \check{a} < \hat{a}_1$.

(iib) $a \in (\hat{a}_2, \hat{a}_1)$ if $\hat{a}_1 < \check{a}$.

Here

$$\check{a} = \begin{cases} a_1 & \text{if } T_1 < 0, T_3 > 0, \hat{T}_1 < 0, \hat{T}_2 > 0, \text{ and } \hat{T}_3 < 0, \\ -\frac{T_3}{T_2} & \text{if } T_1 = 0, T_2 < 0, T_3 > 0, \hat{T}_1 < 0, \hat{T}_2 > 0, \text{ and } \hat{T}_3 < 0, \\ \sqrt{-\frac{T_3}{T_2}} & \text{if } T_1 < 0, T_2 = 0, T_3 > 0, \hat{T}_1 < 0, \hat{T}_2 > 0, \text{ and } \hat{T}_3 < 0, \text{ and} \\ -\frac{T_2}{T_1} & \text{if } T_1 < 0, T_2 > 0, T_3 = 0, \hat{T}_1 < 0, \hat{T}_2 > 0, \text{ and } \hat{T}_3 < 0. \end{cases}$$

(iii) $2c_1D_2 > 2rc_2$, $\delta \leq \delta_3$, $J^2 \in (J_1, \hat{J}_1]$, $T_2^2 > 4T_1T_3$, and $a \in (\max\{a_2, \dot{a}\}, a_1)$,
where¹⁴

$$\dot{a} = \begin{cases} \hat{a}_2 & \text{if } T_1 < 0, T_2 > 0, T_3 < 0, \hat{T}_1 > 0, \text{ and } \hat{T}_3 < 0, \\ -\frac{\hat{T}_3}{\hat{T}_2} & \text{if } T_1 < 0, T_2 > 0, T_3 < 0, \hat{T}_1 = 0, \hat{T}_2 > 0, \text{ and } \hat{T}_3 < 0. \end{cases}$$

(iv) $2c_1D_2 > 2rc_2$, $J^2 > \max\{\hat{J}_1, J_3\}$, $T_2^2 > 4T_1T_3$, and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$ with one of the following.

(iva) $a \in (a_2, \hat{a}_1)$ if $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$.

(ivb) $a \in (\hat{a}_2, \hat{a}_1)$ if $a_2 < \hat{a}_2 < \hat{a}_1 < a_1$.

(ivc) $a \in (\hat{a}_2, a_1)$ if $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$.

Condition set C10 in Theorem 5(ic):

(i) $2c_1D_2 \in (rc_2, 2rc_2]$, $J^2 \in (J_1, \hat{J}_1]$, and $a \in [\tilde{a}_1, \hat{a}_2]$ for $\hat{a}_2 > \tilde{a}_1$, where $\tilde{a}_1$ is defined in C9(i). Moreover, the other situations supporting this part are identical to those specified in C9(i).

(ii) $2c_1D_2 \in (rc_2, 2rc_2]$ and $J^2 > \hat{J}_1$ with one of the following.

(iia) $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$ with $\check{a} \leq \hat{a}_2$ and $a \in [\check{a}, \hat{a}_2] \cup [\hat{a}_1, \infty)$; with $\hat{a}_2 < \check{a} < \hat{a}_1$ and $a \in [\hat{a}_1, \infty)$; or with $\check{a} > \hat{a}_1$ and $a \in [\check{a}, \infty)$.

(iib) $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$ and $a \in [\check{a}, \infty)$.

$rc_2, 2rc_2]$ and $J^2 \in (\hat{J}_1, J_2]$, or (b) $2c_1D_2 > 2rc_2$, $\delta > \delta_3$, $\hat{J}_1 < J_3$, and $J^2 \in (\max\{\hat{J}_1, J_2\}, J_3]$.

¹⁴Similar conclusions hold if $2c_1D_2 > 2rc_2$, $\delta \leq \delta_3$, $J^2 \in (J_1, \hat{J}_1]$ are replaced with $2c_1D_2 > 2rc_2$, $\delta > \delta_3$, $J_3 < \hat{J}_1$, and $J^2 \in (J_3, \hat{J}_1]$.

Here $\check{a}$ is defined in C9(ii). And the other situations supporting these conditions are identical to those specified in C9(ii).

(iii) $2c_1D_2 > 2rc_2$, $\delta \leq \delta_3$, and $J^2 \in (\max\{J_1, J_3\}, \hat{J}_1]$ with one of the following:¹⁵

(iiia) $a \in (0, \min\{a_2, \hat{a}_2\}]$ if $T_2^2 > 4T_1T_3$.

(iiib) $a \in (0, a_2] \cup [a_1, \infty)$ if $T_2^2 \leq 4T_1T_3$.

(iv) $2c_1D_2 > 2rc_2$ and $J^2 > \max\{\hat{J}_1, J_3\}$ with one of the following:

(iva) $T_2^2 > 4T_1T_3$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$ with $\hat{a}_2 < \hat{a}_1 < a_2 < a_1$ and $a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2] \cup [a_1, \infty)$; with $\hat{a}_2 < a_2 < \hat{a}_1 < a_1$ and $a \in (0, \hat{a}_2] \cup [a_1, \infty)$; with $a_2 < \hat{a}_2 < \hat{a}_1 < a_1$ and $a \in (0, a_2] \cup [\hat{a}_1, \infty)$; with $a_2 < \hat{a}_2 < a_1 < \hat{a}_1$ and $a \in (0, a_2] \cup [\hat{a}_1, \infty)$; or with $a_2 < a_1 < \hat{a}_2 < \hat{a}_1$ and $a \in (0, a_2] \cup [a_1, \hat{a}_2] \cup [\hat{a}_1, \infty)$.

(ivb) $a \in (0, a_2] \cup [a_1, \infty)$ if $T_2^2 > 4T_1T_3$ and $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$.

(ivc) $a \in (0, \hat{a}_2] \cup [\hat{a}_1, \infty)$ if $T_2^2 \leq 4T_1T_3$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$.

(ivd) $a \geq 0$ if $T_2^2 \leq 4T_1T_3$ and $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$.

¹⁵Similar conclusions hold if $2c_1D_2 > 2rc_2$, $\delta > \delta_3$, $J_3 < \hat{J}_1$, and $J^2 \in (J_3, \hat{J}_1]$.