\textbf{Appendix of the Manuscript "International Environmental Agreements Under Different Evolutionary Imitation Mechanisms"}

\textit{Proof of Proposition 1:} After some calculations, \( u \) is the minimum of \( k \) satisfying
\[
k(d + b - \frac{c}{2}) + \alpha - bn - \frac{c}{2} \geq 0, \quad (1)
\]
and \( v \) is the minimum of \( k \) satisfying
\[
k(d + b + \frac{c}{2}) - dn - \alpha + \frac{c(n + 1)}{2} \geq 0. \quad (2)
\]
According to relative sizes of \((d + b)\) and \(\frac{\epsilon}{2}\), we have three cases below.

\textbf{Case 1:} Suppose \((d + b) < \frac{\epsilon}{2}\). We then have \(bn + \frac{\epsilon}{2} - (1 - n)d - b - \frac{c(n + 2)}{2} = (n - 1)(d + b) - \frac{c(n + 1)}{2} < \frac{c(n - 1)}{2} - \frac{c(n + 1)}{2} = -c < 0\), which implies \(bn + \frac{\epsilon}{2} < l_0 \equiv (1 - n)d + b + \frac{c(n + 2)}{2}\).

Accordingly, there are three sub-cases.

First, if \(\alpha > l_0\), then \((22)\) fails at \(k = 1\), and hence \(v = \lceil \frac{dn + \alpha - \frac{c(n+1)}{2}}{d + b + \frac{\epsilon}{2}} \rceil \geq 2\). On the other hand, \(\alpha > l_0\) implies \(\alpha > bn + \frac{\epsilon}{2}\), which suggests \(u = \lceil \frac{\alpha - bn - \frac{\epsilon}{2}}{d - b} \rceil \geq 1\) by \((21)\).

Thus, \(S_* = \{\bar{C}\}\) and \(E(T_*^e) = \epsilon^{-u}\) if \(u < v\), \(S_* = \{\bar{C}, \bar{D}\}\) and \(E(T_*^e) = \epsilon^0\) if \(u = v\), and \(S_* = \{\bar{D}\}\) and \(E(T_*^e) = \epsilon^{-v}\) if \(u > v\). These prove Proposition 1(ia).

Second, if \(\alpha \in (bn + \frac{\epsilon}{2}, l_0]\), then \(u = \lceil \frac{\alpha - bn - \frac{\epsilon}{2}}{d - b} \rceil \geq 1\) by \((21)\), and \(v = 1\) by \(\alpha < l_0\) and \((22)\). Thus, \(S_* = \{\bar{C}, \bar{D}\}\) and \(E(T_*^e) = \epsilon^0\) if \(u = 1\) and \(S_* = \{\bar{D}\}\) and \(E(T_*^e) = \epsilon^{-1}\) if \(u > 1\). These prove Proposition 1(ib).

Third, if \(\alpha \leq bn + \frac{\epsilon}{2}\), then \((21)\) fails for all \(k\), and hence \(u = n\). On the other hand, since \(\alpha \leq bn + \frac{\epsilon}{2}\), we have \(\alpha < l_0\), which implies that \((22)\) holds at \(k = 1\) and \(v = 1\). Since \(u = n > v = 1\), \(S_* = \{\bar{D}\}\) and \(E(T_*^e) = \epsilon^{-1}\). These prove Proposition 1(ic).

\textbf{Case 2:} Suppose \((d + b) = \frac{\epsilon}{2}\). Under the circumstance, we have \(bn + \frac{\epsilon}{2} - (1 - n)d - b - \frac{c(n + 2)}{2} < 0\). Thus, there are two sub-cases.

First, if \(\alpha > l_0\), then \(v \geq 2\) because \((22)\) fails at \(k = 1\). We have \(\alpha > bn + \frac{\epsilon}{2}\) by \(\alpha > l_0\), which implies \(u = 1\). Thus, \(S_* = \{\bar{C}\}\) and \(E(T_*^e) = \epsilon^{-1}\) as shown by Proposition
1(iiia). Second, if $\alpha \in [bn + \frac{c}{2}, l_0]$, then $v = 1$ by $\alpha < l_0$ and $u = 1$ by $\alpha > bn + \frac{c}{2}$. Thus, $S_* = \{\vec{C}, \vec{D}\}$ and $E(T_\epsilon) = \epsilon^0$ as shown by Proposition 1(iib). Third, if $\alpha < bn + \frac{c}{2}$, then (21) fails for all $k$, and hence $u = n$. But $\alpha < l_0$ implies $v = 1$. Thus, $S_* = \{\vec{D}\}$ and $E(T_\epsilon) = \epsilon^{-1}$ as shown by Proposition 1(iic).

Case 3: Suppose $(d + b) > \frac{c}{2}$. Then, relative sizes of $bn + \frac{c}{2}$ and $l_0$ become unsure, and there are two sub-cases. First, suppose $\alpha \geq bn + \frac{c}{2}$. Then (21) holds at $k = 1$, and hence $u = 1$. However, if $\alpha > l_0$, then $v \geq 2$, and hence $S_* = \{\vec{C}\}$ and $E(T_\epsilon) = \epsilon^{-1}$. In contrast, if $\alpha \leq l_0$, then $v = 1$, and hence $S_* = \{\vec{C}, \vec{D}\}$ and $E(T_\epsilon) = \epsilon^0$. These are the content of 1(iiia).

Second, if $\alpha < bn + \frac{c}{2}$, then (21) implies $u = \left\lceil \frac{bn + \frac{c}{2} - \alpha}{d + b + \frac{c}{2}} \right\rceil \geq 1$ due to $\frac{bn + \frac{c}{2} - \alpha}{d + b + \frac{c}{2}} > 0$. If $\alpha \leq l_0$, then (22) implies $v = 1$. Accordingly, $S_* = \{\vec{C}, \vec{D}\}$ and $E(T_\epsilon) = \epsilon^0$ if $u = 1$, and $S_* = \{\vec{D}\}$ and $E(T_\epsilon) = \epsilon^{-1}$ if $u > 1$. These prove Proposition 1(iiiia).

In contrast, if $\alpha > l_0$, then (22) fails at $k = 1$. Thus, $v = \left\lceil \frac{dn + \alpha - \alpha}{d + b + \frac{c}{2}} \right\rceil \geq 2$. Accordingly, $S_* = \{\vec{C}\}$ and $E(T_\epsilon) = \epsilon^{-u}$ if $u < v$, $S_* = \{\vec{C}, \vec{D}\}$ and $E(T_\epsilon) = \epsilon^0$ if $u = v$, and $S_* = \{\vec{D}\}$ and $E(T_\epsilon) = \epsilon^{-v}$ if $u > v$. These prove Proposition 1(iiiic).

Proof of Theorem 1: Before comparing Propositions 1 and C1, we need to know relative
sizes of the following variables:

\[ 2(\alpha - bn) - 2\alpha = -2bn < 0, \quad (3) \]

\[ 2(\alpha - bn) - 2(d - b) = 2(\alpha - d) + 2b(1 - n) < 0 \quad \text{by } d > \alpha \text{ and } n \geq 2, \quad (4) \]

\[ \frac{2[\alpha - (1 - n)d - b]}{n+2} - \frac{2[\alpha + (d - b)(n-1)]}{n+2} = \frac{2b(n-2)}{n+2} > 0 \quad \text{by } n \geq 3, \quad (5) \]

\[ 2\alpha - \frac{2[\alpha - (1 - n)d - b]}{n+2} = \frac{2[\alpha(n+1) - d(n-1) + b]}{n+2} \]

\[ \geq (\leq) 0 \quad \text{iff } d \leq (\geq) \frac{b + \alpha(n+1)}{n-1}, \quad (6) \]

\[ 2\alpha - \frac{2[\alpha + (d - b)(n-1)]}{n+2} = \frac{2[\alpha(n+1) + b(n-1) - d(n-1)]}{n+2} \]

\[ \geq (\leq) 0 \quad \text{iff } d \leq (\geq) \frac{\alpha(n+1)}{n-1}, \quad (7) \]

\[ 2(d - b) - \frac{2[\alpha - (1 - n)d - b]}{n+2} = \frac{2[3d - b(n+1) - \alpha]}{n+2} \]

\[ \geq (\leq) 0 \quad \text{iff } d \geq (\leq) \frac{b(n+1) + \alpha}{3}, \quad (8) \]

\[ 2(d - b) - \frac{2[\alpha + (d - b)(n-1)]}{n+2} = \frac{2[3(d - b) - \alpha]}{n+2} \]

\[ \geq (\leq) 0 \quad \text{iff } d \geq (\leq) \frac{b + \alpha}{3}, \quad (9) \]

\[ 2(d + b) - \frac{2[\alpha - (1 - n)d - b]}{n+2} = \frac{2[3d + n + 3b - \alpha]}{n+2} > 0, \quad (10) \]

\[ 2(\alpha - bn) - \frac{2[\alpha + (d - b)(n-1)]}{n+2} = -\frac{2[d(n-1) + b(n^2 + 3n - 1) - \alpha(n+1)]}{n+2} \]

\[ < 0 \quad \text{by } n \geq 2, \quad (11) \]

\[ (d - b + \alpha) - \frac{2[\alpha - (1 - n)d - b]}{n+2} = -d(n-4) - bn + n\alpha \quad \frac{n+2}{n+2} < 0 \quad \text{by } n \geq 4, \quad (12) \]

\[ (d - b + \alpha) - \frac{2[\alpha + (d - b)(n-1)]}{n+2} = \frac{(b - d)(n-4) + \alpha n}{n+2} > 0 \quad \text{if } d < b + \frac{\alpha}{3}. \quad (13) \]

Next, we need to know relative sizes of the thresholds given in (23)-(33). Some calculations yield

\[ b + \frac{\alpha}{3} < \frac{b + \alpha(n+1)}{n-1} < b + \alpha < \frac{\alpha(n+1)}{n-1} < \frac{b(n+1) + \alpha}{3} \]

for \( n \geq 4 \). These inequalities divide the values of \( d \) into six mutually exclusive ranges as discussed below.
Case 1: Suppose $d \geq \frac{b(n+1) - \alpha}{3}$. We then have
\[
2(\alpha - bn) < 2\alpha < \frac{2(\alpha + (d-b)(n-1))}{n+2} < \frac{2(\alpha - (1-n)d - b)}{n+2} < 2(d - b) < 2(d + b)
\]
by (23), (25), (27), and (28). These inequalities divide the values of $c$ into seven mutually exclusive intervals. At each interval, we can derive the LREs under the two dynamics by Propositions 1 and C1. Under the imitating-the-best-total dynamic; $\vec{D}$ is the LRE for $c > 2(d+b)$ by Proposition 1(iic), $c > 2(d+b)$, $c \geq 2(\alpha - bn)$; $\vec{D}$ is the LRE at $c = 2(d+b)$ by Proposition 1(iic), $c = 2(d+b)$, and $c > 2(\alpha - bn)$; $\vec{D}$ is the LRE for $c \in \left(\frac{2(\alpha - (1-n)d - b)}{n+2}, 2(d+b)\right)$ by Proposition 1(iiib), $c < 2(d+b)$, $\alpha \leq (1-n)d + b + \frac{c(n+2)}{2}$, and $c > d + \alpha - b(n-1)$; $\vec{D}$ is the LRE for $c \in (\hat{c}, \frac{2(\alpha - (1-n)d - b)}{n+2})$ by Proposition 1(iic), $c > 2(\alpha - bn)$, $c < \frac{2(\alpha - (1-n)d - b)}{n+2}$, and $c > \hat{c}$; $\{\vec{C}, \vec{D}\}$ is the LRE at $c = \hat{c}$ by Proposition 1(iic), $c > 2(\alpha - bn)$, and $c < \frac{2(\alpha - (1-n)d - b)}{n+2}$; $\vec{C}$ is the LRE for $c \in (2(\alpha - bn), \hat{c})$ by Proposition 1(iiic), $c > 2(\alpha - bn)$, $c < \frac{2(\alpha - (1-n)d - b)}{n+2}$, and $c > \hat{c}$; and $\vec{C}$ is the LRE for $c \in (0, 2(\alpha - bn))$ by Proposition 1(iiiia), $c \leq \frac{\alpha - bn}{2}$, and $\alpha > (1-n)d + b + \frac{c(n+2)}{2}$.
Here $\hat{c} \in (2(\alpha - bn), \frac{2(\alpha - (1-n)d - b)}{n+2})$ satisfies condition
\[
\frac{bn + \hat{c} - \alpha}{d + b - \hat{c}} = \frac{dn + \alpha - \frac{\hat{c}(n+1)}{2}}{d + b + \frac{\hat{c}}{2}}.
\]

Under the imitating-the-best-average dynamic; $\vec{D}$ is the LRE for $c > 2(d - b)$ by Proposition C1(ib) and $c \geq 2\alpha$; $\vec{D}$ is the LRE at $c = 2(d - b)$ by Proposition C1(iic) and $c \geq 2\alpha$; $\vec{D}$ is the LRE for $c \in (\hat{c}_c, 2(d - b))$ by Proposition C1(iiib), $c > 2\alpha$, and $c > \hat{c}_c$; $\{\vec{C}, \vec{D}\}$ is the LRE at $c = \hat{c}_c$ by Proposition C1(iic), $c > 2\alpha$, and $c = \hat{c}_c$; $\vec{C}$ is the LRE for $c \in (2\alpha, \hat{c}_c)$ by Proposition C1(iiiib), $c > 2\alpha$, and $c < \hat{c}_c$; $\vec{C}$ is the LRE at $c = 2\alpha$ by Proposition C1(iiiia), $c < 2(d - b)$, and $c = 2\alpha$; and $\vec{C}$ is the LRE for $c \in (0, 2\alpha)$ by Proposition C1(iiiia), $c < 2\alpha$, and $\alpha > (1-n)d + b + \frac{c(n+2)}{2}$. Here $\hat{c}_c \in (2\alpha, 2(d - b))$ satisfies condition
\[
\frac{\hat{c}_c - \alpha}{d - b - \frac{\hat{c}_c}{2}} = \frac{(d - b - \frac{\hat{c}_c}{2})n + \alpha - \frac{\hat{c}_c}{2}}{d - b + \frac{\hat{c}_c}{2}}.
\]
In summary, the \( C \) interval making \( \vec{C} \) the LRE under the imitating-the-best-average dynamic is \((0, \hat{c}_c]\) and the associated interval under the imitating-the-best-total dynamic is \((0, \hat{c}]\). Thus, if \( \hat{c}_c > \hat{c} \), then \( \vec{C} \) is more likely to be the LRE under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic. The converse will hold if \( \hat{c}_c < \hat{c} \), and both dynamics will make \( \vec{C} \) emerge equally likely if \( \hat{c}_c = \hat{c} \).

Case 2: Suppose \( b + \frac{a(n+1)}{n-1} \leq d < \frac{b(n-1)+\alpha}{3} \). We then have
\[
2(\alpha - bn) < 2\alpha < \frac{2[\alpha + (d-b)(n-1)]}{n+2} < 2(d-b) < \frac{2[\alpha - (1-n)d-b]}{n+2} < 2(d+b)
\]
by (23), (27), (28), (29), and (30). These inequalities divide the values of \( c \) into seven mutually exclusive intervals. Proposition 1 implies that the \( C \) interval making \( \vec{C} \) the LRE under the imitating-the-best-average dynamic is \((0, \hat{c}_c]\), and the associated interval under the imitating-the-best-total dynamic is \((0, \hat{c}]\) by Proposition C1. Thus, the conclusions are same as Case 1’s.

Case 3: Suppose \( (b + \alpha) \leq d < b + \frac{a(n+1)}{n-1} \). We then have
\[
2(\alpha - bn) < 2\alpha < \frac{2[\alpha + (d-b)(n-1)]}{n+2} < 2(d-b) < \frac{2[\alpha - (1-n)d-b]}{n+2} < 2(d+b)
\]
by (27), (28), (30), (31), and \( d \geq b + \alpha \). These inequalities divide the values of \( c \) into seven mutually exclusive intervals. According to Propositions 1 and C1, the \( C \) interval making \( \vec{C} \) the LRE under the imitating-the-best-average dynamic is \((0, \hat{c}_c]\), and the associated interval under the imitating-the-best-total dynamic is \((0, \hat{c}]\). Again, the results are the same as Case 1’s.

Case 4: Suppose \( \frac{b+\alpha(n-1)}{n-1} \leq d < (b + \alpha) \). We then have
\[
2(\alpha - bn) < 2\alpha < \frac{2[\alpha + (d-b)(n-1)]}{n+2} < 2(d-b) < (d-b + \alpha) < 2\alpha < \frac{2[\alpha - (1-n)d-b]}{n+2} < 2(d+b)
\]
by (26), (29), (30), (31), and \( (d-b) < \alpha \). These inequalities divide the values of \( c \) into eight mutually exclusive intervals. At each interval, we can derive the LREs under
both dynamics by Propositions 1 and C1. Under the imitating-the-best-total dynamic; \(\tilde{D}\) is the LRE for \(c > 2(d + b)\) by Proposition 1(ic); \(\tilde{D}\) is the LRE at \(c = 2(d + b)\) by Proposition 1(iic); \(\tilde{D}\) is the LRE for \(c \in [\frac{2|\alpha - (1-n)d-b|}{n+2}, 2(d + b)]\) by Proposition 1(iiiib), \(c > 2(\alpha - bn)\), \(\alpha \leq (1 - n)d + b + \frac{c(n+2)}{2}\), and \([\frac{bn+c/2-\alpha}{d+b-c/2}] > 1;\) \(\tilde{D}\) is the LRE for \(c \in (\hat{c}, \frac{2|\alpha - (1-n)d-b|}{n+2})\) by Proposition 1(iiiic), \(c > 2(\alpha - bn)\), \(c < \frac{2|\alpha - (1-n)d-b|}{n+2}\), and \(c > \hat{c};\) \(\tilde{C}, \tilde{D}\) is the LRE at \(c = \hat{c}\) by Proposition 1(iic), \(c > 2(\alpha - bn)\), and \(c < \frac{2|\alpha - (1-n)d-b|}{n+2}\); \(\tilde{C}\) is the LRE for \(c \in (2(\alpha - bn), \hat{c})\) by Proposition 1(iic), \(c > 2(\alpha - bn)\), \(c < \frac{2|\alpha - (1-n)d-b|}{n+2}\), and \(c < \hat{c};\) and \(\tilde{C}\) is the LRE for \(c \in (0, 2(\alpha - bn)]\) by Proposition 1(iiiia), \(c \leq \frac{\alpha - bn}{2}\), and \(\alpha > (1 - n)d + b + \frac{c(n+2)}{2}\). Here \(\hat{c} \in (2(\alpha - bn), \frac{2|\alpha - (1-n)d-b|}{n+2})\) satisfies (34).

Under the imitating-the-best-average dynamic; \(\tilde{D}\) is the LRE for \(c \geq 2\alpha\) by Proposition C1(ib), \(c > 2(d - b)\), and \(c \geq 2\alpha;\) \(\tilde{D}\) is the LRE for \(c \in ((d - b + \alpha), 2\alpha)\) by Proposition C1(ia), \(c > 2(d - b)\), \(c < 2\alpha\), and \(c > (d - b + \alpha);\) \(\tilde{C}, \tilde{D}\) is the LRE for \(c \in (2(d - b), (d - b + \alpha)]\) by Proposition C1(ia), \(c > 2(d - b)\), \(c \leq (d - b + \alpha)\), and \(\alpha < l_0^2;\) \(\tilde{C}\) is the LRE at \(c = 2(d - b)\) by Proposition C1(iia); \(\tilde{C}\) is the LRE for \(c \in [\frac{2(\alpha+(d-b)(n-1)}{n+2}, 2(d - b))\) by Proposition C1(iiiia), \(c < 2(d - b)\), and \(c < 2\alpha;\) and \(\tilde{C}\) is the LRE for \(c \in (0, \frac{2[\alpha+(d-b)(n-1)]}{n+2})\) by Proposition C1(iiiia), \(c < 2(d - b)\), \(c \leq 2\alpha\), and \(\alpha > l_0^2.

In summary, the \(c\) interval making \(\tilde{C}\) the LRE under the imitating-the-best-average dynamic is \((0, (d - b + \alpha)],\) and the associated interval under the imitating-the-best-total dynamic is \((0, \hat{c}].\) Thus, if \((d - b + \alpha) > \hat{c}\), then \(\tilde{C}\) is more likely to be the LRE under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic. The converse will hold if \((d - b + \alpha) < \hat{c}\), and both dynamics will make \(\tilde{C}\) emerge equally likely if \((d - b + \alpha) = \hat{c}\).

\footnote{That is because \(bn+c/2-\alpha-[d+b-c/2] = -d+b(n-1)+c-\alpha > b(n-2) > 0\) by \(c > d-b+\alpha\) and \(n \geq 4.\)}
Case 5: Suppose $b + \frac{a}{3} \leq d < \frac{a(n-1)+b}{n-1}$. We then have

$$2(\alpha - bn) < \frac{2[a + (d-b)(n-1)]}{n+2} < 2(d-b) < (d-b+\alpha) < \frac{2[a - (1-n)d-b]}{n+2}$$

$$< 2\alpha < 2(d+b)$$

by (26), (29), (31), and (32). These inequalities divide the values of $c$ into eight mutually exclusive intervals. As in Case 4, the $c$ interval making $\vec{C}$ the LRE under the imitating-the-best-average dynamic is $(0, (d-b+\alpha)]$, and the associated interval under the imitating-the-best-total dynamic is $(0, \hat{c}]$. Thus, the conclusions are the same as Case 4’s.

Case 6: Suppose $d \leq b + \frac{a}{3}$. We then have

$$2(\alpha - bn) < 2(d-b) < \frac{2[a + (d-b)(n-1)]}{n+2} < d-b+\alpha < \frac{2[a - (1-n)d-b]}{n+2}$$

$$< 2\alpha < 2(d+b)$$

by (24), (28), (29), (32), and (33). These inequalities divide the values of $c$ into eight mutually exclusive intervals. As in Case 4, the $c$ interval making $\vec{C}$ the LRE under the imitating-the-best-average dynamic is $(0, (d-b+\alpha)]$, and the associated interval under the imitating-the-best-total dynamic is $(0, \hat{c}]$. Again, the conclusions are the same as Case 4’s.

In summary, Cases 1-3 show that for $d \geq b+\alpha$, $\vec{C}$ is more likely to emerge when the imitating-the-best-average rule is adopted if $\hat{c} < \hat{c}_c$. For $d < b+\alpha$, the same conclusions can be drawn if $\hat{c} < (d-b+\alpha)$. The two conditions will hold as displayed below, and hence Theorem 1 is proved.

Claim 1. Suppose $d \geq b+\alpha$. We then have $\hat{c} < \hat{c}_c$ with $\hat{c} \in (2(\alpha - bn), \frac{2[a - (1-n)d-b]}{n+2})$ and $\hat{c}_c \in (2\alpha, 2(d-b))$.

Proof. To simplify the notations, we define $x = \frac{\hat{c}}{2}$ and $y = \frac{\hat{c}_c}{2}$. Solving (34) and (35)
yields
\[ x = \frac{(d + b)(n + 1) - \sqrt{(d + b)^2(n + 1)^2 - n(d + b)[2\alpha + n(d - b)]}}{n} \]
and
\[ y = \frac{(d - b)(n + 1) - \sqrt{(d - b)^2(n + 1)^2 - n(d - b)[2\alpha + n(d - b)]}}{n} \]

Accordingly, we have
\[ (x - y) = \frac{[2b(n + 1) - \sqrt{A} + \sqrt{B}]}{n} \]
where \( A = (d + b)^2(n + 1)^2 - n(d + b)[2\alpha + n(d - b)] \geq 0 \) and \( B = (d - b)^2(n + 1)^2 - n(d - b)[2\alpha + n(d - b)] > 0 \) with \( A > B \) by \( d > b \). To show \( x < y \), it is enough to show \( A > 4(n + 1)^2b^2 + B + 4b(n + 1)\sqrt{B} \). Note that \( \sqrt{\alpha - \beta} \leq \sqrt{\alpha} - \sqrt{\beta} \) if \( \alpha > \beta > 0 \).

Thus,
\[
A - 4(n + 1)^2b^2 - B - 4b(n + 1)\sqrt{B} \\
= (n + 1)^2(4bd - 4b^2) - 2bn[(d - b)n + 2\alpha] - 4(n + 1)b\sqrt{B} \\
> (n + 1)^24b(d - b) - 2bn[(d - b)n + 2\alpha] - 4b(n + 1)\sqrt{(d - b)^2(n + 1)^2} \\
\quad + 4b(n + 1)\sqrt{n(d - b)[(d - b)n + 2\alpha]} \\
= 2b\{2(n + 1)\sqrt{n(d - b)[(d - b)n + 2\alpha]} - n[(d - b)n + 2\alpha]\} \\
> 0,
\]

where the first inequality is implied by \( \sqrt{B} < \sqrt{(d - b)^2(n + 1)^2 - n(d - b)[(d - b)n + 2\alpha]} \) and the second inequality is because
\[
4(n + 1)^2n(d - b)[(d - b)n + 2\alpha] - n^2[(d - b)n + 2\alpha]^2 \\
= n[(d - b)n + 2\alpha][4(n + 1)^2(d - b) - n^2(d - b) - 2\alpha n] \\
> n[(d - b)n + 2\alpha][4(n + 1)^2(d - b) - n^2(d - b) - 2n(d - b)] \\
= n(d - b)[(d - b)n + 2\alpha][3n^2 + 6n + 4] > 0
\]
by \( -\alpha > -(d - b) \). Thus, we have \( x < y \) and \( \hat{c} < \hat{c}_c \), which prove Claim 1.

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To meet the range requirements, we take the negative roots.
Claim 2. Suppose \( d < b + \alpha \) and \( \hat{c} \in (2(\alpha - bn), \frac{2(\alpha - (1-n)d - b)}{n+2}) \) with \( \alpha > bn \). We then have \( \hat{c} < (d - b + \alpha) \).

Proof. Plugging \( \hat{c} = d - b + \alpha \) into (34) yields

\[
L \equiv \frac{bn + \frac{d-b+b+\alpha}{2} - \alpha}{d + b - \frac{d-b+b+\alpha}{2}} = \frac{d + b(2n - 1) - \alpha}{d + 3b - \alpha} \geq 1 \text{ by } n \geq 2, \text{ and} \\
R \equiv \frac{dn + \alpha - \frac{(n+1)(d-b+b+\alpha)}{2}}{d + b + \frac{d-b+b+\alpha}{2}} = \frac{d(n-1) + b(n-1) - \alpha(n+1)}{3d + b + \alpha}.
\]

Thus

\[
L - R = \frac{-d^2(n - 4) - b^2(n + 4) - \alpha^2(n + 2) + 2nb + 2d\alpha(n - 1) + 2b\alpha(3n + 1)}{[d + 3b - \alpha][3d + b + \alpha]}.
\]

Denote \( N \equiv -d^2(n - 4) - b^2(n + 4) - \alpha^2(n + 2) + 2nb + 2d\alpha(n - 1) + 2b\alpha(3n + 1) \).

We then have

\[
N > -(n - 4)(b + \alpha)^2 - b^2(n + 4) - \alpha^2(n + 2) + 2nb + 2\alpha(n - 1) + 2\alpha(3n + 1) \\
= \alpha^2(-2n + 2) + b\alpha(6n + 8) \\
> \alpha^2(-2n + 2) + \alpha^2(6n + 8) \\
= \alpha^2(4n + 10) \\
> 0.
\]

The first inequality is due to \( d < b + \alpha \) and the second inequality is by \( \alpha > bn \). These imply \( L > R \). Thus, to make \( \hat{c} \) satisfy (34), we must have \( \hat{c} < (d - b + \alpha) \) to lower \( L \) and raise \( R \). Claim 2 is then proved.

Proof of Proposition 2: After some calculations, \( u \) is the minimum of \( k \) satisfying

\[
(dk + \alpha) - b(n - k) \geq \frac{c[2k^2 + 3k + 1]}{6}, \tag{16}
\]

and \( v \) is the minimum of \( k \) satisfying

\[
(d + b)k - \alpha - dn \geq \frac{-c[2(n^2 + nk + k^2) + 3(n + k) + 1]}{6}. \tag{17}
\]

Define \( g(k) \equiv 2k^2 + 3k + 1 - \frac{6[(d+b)k+\alpha-bn]}{c} \) with \( g'(k) = 4k + 3 - \frac{6(d+b)}{c} \) and \( g''(k) = 4 > 0 \) for all \( k \). These imply that \( g(k) \) is a strictly convex function of \( k \) with \( g'(k) \geq (\leq) 0 \)
iff \( k \geq (\leq) k \equiv \frac{6(d+b)-3c}{4c} \), \( g'(1) = 7 - \frac{6(d+b)}{c} \geq (\leq) 0 \) iff \( c \geq (\leq) \frac{6(d+b)}{c} \), \( k \geq (\leq) 1 \) iff \( c \leq (\geq) \frac{6(d+b)}{c} \), and \( g(1) \geq (\leq) 0 \) iff \( c \geq (\leq) [d + b(1 - n) + \alpha] \). Since \( u \) is the minimum of \( k \) satisfying \( g(k) \leq 0 \) by (36), it depends on the values of \( g'(1) \) and \( g(1) \). Thus, if \( c < \frac{6(d+b)}{c} \), we have \( g'(1) < 0 \) and \( k > 1 \), which suggest

\[
u = \begin{cases} 
1 & \text{if } g(1) \leq 0, \\
\lceil k_g \rceil & \text{if } g(1) > 0 \text{ and } g(k) < 0, \\
n & \text{if } g(1) > 0 \text{ and } g(k) > 0,
\end{cases}
\]  

(18)

where \( k_g \) satisfies \( g(k_g) = 0 \). For \( c = \frac{6(d+b)}{c} \), we have \( g'(1) = 0 \) and \( k = 1 \), which imply

\[
u = \begin{cases} 
1 & \text{if } g(1) \leq 0, \\
n & \text{if } g(1) > 0.
\end{cases}
\]  

(19)

In contrast, if \( c > \frac{6(d+b)}{c} \), we have \( g'(1) > 0 \) and \( k < 1 \), which suggest

\[
u = \begin{cases} 
1 & \text{if } g(1) \leq 0, \\
n & \text{if } g(1) > 0.
\end{cases}
\]  

(20)

On the other hand, define \( h(k) \equiv 2(n^2 + nk + k^2) + 3(n + k) + 1 + \frac{6}{c}[(d+b)k - \alpha - dn] \) based on \( k \geq 1 \) with \( h'(k) = 2n + 4k + 3 + \frac{6(d+b)}{c} > 0 \) for all \( k \geq 1 \), \( h''(k) = 4 > 0 \) for all \( k \geq 1 \), \( h'(1) = 2n + 7 + \frac{6(d+b)}{c} > 0 \), and \( h(1) = 2n^2 + 5n + 6 + \frac{6(d+b)}{c} - \frac{6(\alpha + dn)}{c} \geq (\leq) 0 \) iff \( c \geq (\leq) \frac{6(d(n-1)+\alpha-b)}{2n^2+5n+6} \). These suggest that \( h(k) \) is a strictly convex function with minimum value \( h(1) \). Accordingly, by (37), we have

\[
u = \begin{cases} 
1 & \text{if } h(1) \geq 0, \\
\lceil k_h \rceil & \geq 2 \text{ if } h(1) < 0,
\end{cases}
\]  

(21)

where \( k_h \) satisfies \( h(k_h) = 0 \).

Now we are ready to get relative sizes of \( u \) and \( v \) by comparing \( g(k) \), \( h(k) \), and (38)-(41). First, we need to know relative sizes of \( \frac{6(d+b)}{c} \), \( \frac{6(d(n-1)+\alpha-b)}{2n^2+5n+6} \), and \( [d + \alpha + (1 - n)b] \).  

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Some calculations show
\[
\frac{6(d+b)}{7} - [d + \alpha + (1-n)b] = -\frac{[d+7\alpha -b(7n-1)]}{7} \geq (\leq) 0 \text{ iff } d \leq (\geq) b(7n-1) - 7\alpha \tag{22}
\]
\[
\frac{6(d+b)}{7} - \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} = \frac{6[d(2n^2-2n+13)+b(2n^2+5n+13)-7\alpha]}{7(2n^2+5n+6)} > 0 \text{ by } d > b > \alpha, \tag{23}
\]
\[
[d + \alpha + (1-n)b] - \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} = \frac{6[(2n^2-n+12)-b(2n^3+3n^2+n-12)+\alpha(2n^2+5n)]}{2n^2+5n+6}
\geq (\leq) 0 \text{ iff } d \geq (\leq) \frac{b(2n^3+3n^2+n-12)-\alpha(2n^2+5n)}{2n^2-n+12}. \tag{24}
\]
Moreover, since \([b(7n-1)-7\alpha] > \frac{b(2n^3+3n^2+n-12)-\alpha(2n^2+5n)}{2n^2-n+12}\), (42)-(44) divide the values of \(d\) into three mutually exclusive ranges as stated below.

**Case 1:** Suppose \(d \geq [b(7n-1)-7\alpha]\). We then have
\[
l_1 \equiv \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d+b)}{7} < [d + \alpha + (1-n)b].
\]
These inequalities divide the values of \(c\) into four mutually exclusive intervals.

**Case 1a:** Suppose \(c \geq [d + \alpha + (1-n)b]\). We have \(h(1) > 0\) by \(c > l_1\) and \(v = 1\) by (41). Then \(g(1) > 0\) by \(c \geq d + \alpha + b(1-n)\) and \(g'(1) > 0\) by \(c > \frac{6(d+b)}{7}\), which imply \(u = n\) by (40). Thus, \(S_* = \{\vec{D}\}\) and \(E(T_e) = \epsilon^{-1}\).

**Case 1b:** Suppose \(\frac{6(d+b)}{7} \leq c < [d + \alpha + (1-n)b]\). We have \(h(1) > 0\) by \(c > l_1\) and \(v = 1\) by (41). Then \(g(1) < 0\) by \(c < d + \alpha + b(1-n)\) and \(g'(1) \geq 0\) by \(c \geq \frac{6(d+b)}{7}\), which imply \(u = 1\) by (39)-(40). Thus, \(S_* = \{\vec{C}, \vec{D}\}\) and \(E(T_e) = \epsilon^0\).

**Case 1c:** Suppose \(l_1 \leq c < \frac{6(d+b)}{7}\). We have \(h(1) \geq 0\) by \(c \geq l_1\) and \(v = 1\) by (41). Then \(g(1) < 0\) by \(c < [d + \alpha + b(1-n)]\) and \(g'(1) < 0\) by \(c < \frac{6(d+b)}{7}\), which imply \(u = 1\) by (38). Thus, \(S_* = \{\vec{C}, \vec{D}\}\) and \(E(T_e) = \epsilon^0\).

**Case 1d:** Suppose \(c < l_1\). We have \(h(1) < 0\) by \(c < l_1\) and \(v = \lceil k_h \rceil \geq 2\) by (41). Then \(g(1) < 0\) by \(c < [d + \alpha + b(1-n)]\) and \(g'(1) < 0\) by \(c < \frac{6(d+b)}{7}\), which imply \(u = 1\) by (38). Thus, \(S_* = \{\vec{C}\}\) and \(E(T_e) = \epsilon^{-1}\).

Propositions 2(ia), 2(ib) and 2(ic) are proved by the results of Case 1a, Cases 1b-1c and Case 1d, respectively.
Case 2: Suppose \( \frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12} \leq d < [b(7n - 1) - 7\alpha] \). We then have
\[
l_1 \equiv \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6} < d + \alpha + (1-n)b < \frac{6(d + b)}{7}.
\]
These inequalities divide the values of \( c \) into four mutually exclusive intervals.

Case 2a: Suppose \( c \geq \frac{6(d+b)}{7} \). The results here are the same as Case 1a’s.

Case 2b: Suppose \( [d + \alpha + (1-n)b] \leq c < \frac{6(d+b)}{7} \). We have \( h(1) > 0 \) by \( c > l_1 \) and \( v = 1 \) by (41). Then \( g(1) > 0 \) by \( c > [d + \alpha + b(1-n)] \) and \( g'(1) < 0 \) by \( c < \frac{6(d+b)}{7} \), which imply \( u = n \) or \( \lceil k_g \rceil \) by (38). Thus, \( S_* = \{ \vec{D} \} \) and \( E(T_\epsilon) = \epsilon^{-1} \).

Case 2c: Suppose \( l_1 \leq c < [d + \alpha + (1-n)b] \). The results here are the same as Case 1c’s.

Case 2d: Suppose \( c < l_1 \). The results here are the same as Case 1d’s.

Propositions 2(iia), 2(iib) and 2(iic) are proved by the results of Cases 2a-2b, Case 2c and Case 2d, respectively.

Case 3: Suppose \( d \leq \frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12} \). We then have
\[
[d + \alpha + (1-n)b] < l_1 \equiv \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6} < \frac{6(d + b)}{7}.
\]
These inequalities divide the values of \( c \) into four mutually exclusive intervals.

Case 3a: Suppose \( c \geq \frac{6(d+b)}{7} \). The results here are the same as Case 1a’s.

Case 3b: Suppose \( l_1 \leq c < \frac{6(d+b)}{7} \). We have \( h(1) \geq 0 \) by \( c \geq l_1 \) and \( v = 1 \) by (41). Then \( g(1) > 0 \) by \( c > [d + \alpha + b(1-n)] \) and \( g'(1) < 0 \) by \( c < \frac{6(d+b)}{7} \), which imply \( u = n \) or \( \lceil k_g \rceil \) by (38). Thus, \( S_* = \{ \vec{D} \} \) and \( E(T_\epsilon) = \epsilon^{-1} \).

Case 3c: Suppose \( [d + \alpha + (1-n)b] \leq c < l_1 \). We have \( h(1) < 0 \) by \( c < l_1 \) and \( v = \lceil k_h \rceil \geq 2 \) by (41). Then \( g(1) > 0 \) by \( c > [d + \alpha + b(1-n)] \) and \( g'(1) < 0 \) by \( c < \frac{6(d+b)}{7} \), which imply \( u = n \) or \( \lceil k_g \rceil \geq 2 \) by (38). We can show \( \lceil k_g \rceil < \lceil k_h \rceil \) below, and it implies \( S_* = \{ \vec{D} \} \) and \( E(T_\epsilon) = \epsilon^{-\lceil k_h \rceil} \).
**Claim 3.** We have \([k_g] > [k_h]\).

*Proof.* Since \(k_g\) satisfies \(g(k_g) = 0\), we have

\[
 k_g = -3 + \frac{6(d+b)}{c} + \sqrt{[3 - \frac{6(d+b)}{c}]^2 - 8[1 - \frac{6(\alpha-bn)}{c}]}.
\]

Similarly, since \(k_h\) satisfies \(h(k_g) = 0\), we have

\[
 k_h = -\frac{2n + 3 + \frac{6(\alpha+bn)}{c}}{4} - \sqrt{[2n + 3 + \frac{6(d+b)}{c}]^2 - 8[2n^2 + 3n + 1 - \frac{6(\alpha+dn)}{c}]}.
\]

Define \(A = [3 - \frac{6(d+b)}{c}]^2 - 8[1 - \frac{6(\alpha-bn)}{c}]\) and \(B = [2n + 3 + \frac{6(d+b)}{c}]^2 - 8[2n^2 + 3n + 1 - \frac{6(\alpha+dn)}{c}]\). Then,

\[
 (k_g - k_h) = \frac{2n + 12(d+b)}{c} + \sqrt{A} - \sqrt{B}.
\]

To show \(k_g > k_h\), it is enough to prove \(B < A + 2\sqrt{A}[2n + \frac{12(d+b)}{c}] + 4n^2 + \frac{48(d+b)}{c} + \frac{144(d+b)^2}{c^2}\). Some calculations reveal

\[
 B - A - 2\sqrt{A}[2n + \frac{12(d+b)}{c}] - 4n^2 - \frac{48(d+b)}{c} - \frac{144(d+b)^2}{c^2}
 = -\left(\frac{12(d+b)}{c} - (3n + 1)^2 - 7n^2 - 6n + 1 - 4\sqrt{A}[n + \frac{d+b}{c}]\right)
 < 0.
\]

These suggest \(k_g > k_h\). Moreover, the above inequality remains true if we replace \(n\) with \((n-4)\). It means \(k_g > k_h + 1\). Thus, we will have Claim 3, \([k_g] > [k_h]\).

*Case 3d:* Suppose \(c < [d + \alpha + (1-n)b]\). The results here are the same as Case 1d’s.

Propositions 2(iiiia), 2(iiib) and 2(iiic) are proved by the results of Cases 3a-3b, Case 3c and Case 3d, respectively.

*Proof of Theorem 2:* Before comparing Propositions 2 and C2, we need to know relative

\footnote{To have \(k_g > 0\) and \(k_h > 0\), we take the positive roots.}
sizes of the following variables:

\[
\frac{6(d-b)}{7} - [d + \alpha + (1-n)b] = -\frac{1}{7} [d + 7\alpha - b(7n - 13)] \\
\geq (\leq) 0 \text{ iff } d \leq (\geq) \ (7bn - 13b - 7\alpha), \quad (25)
\]

\[
\alpha - \frac{(d-b)(2n^2 - 2n + 13)}{7} \geq (\leq) 0 \text{ iff } d \leq (\geq) \frac{b}{2n^2 - 2n + 13}, \quad (26)
\]

\[
\frac{6(d-b)}{7} - \frac{6(d(n-1) + \alpha - b)}{2n^2 + 5n + 6} = \frac{6[d(2n^2 + 4n + 7) - b(2n^2 + 5n + 5) - \alpha]}{7(2n^2 + 5n + 6)} \\
\geq (\leq) 0 \text{ iff } d \geq (\leq) \frac{b(2n^2 + 5n + 5) + \alpha}{2n^2 + 4n + 7}, \quad (27)
\]

\[
d - b + \alpha - [d + \alpha + (1-n)b] = b(n - 2) > 0 \text{ by } n \geq 3, \quad (28)
\]

\[
\frac{6[d(n-1) + \alpha - b]}{2n^2 + 5n + 6} - \frac{6[(d-b)(n-1) + \alpha]}{2n^2 + 5n + 6} = \frac{6b(n-2)}{2n^2 + 5n + 6} > 0 \text{ by } n \geq 3, \quad (29)
\]

\[
\frac{6(d+b)}{7} - [d - b + \alpha] = \frac{-(d-13b + 7\alpha)}{7} \geq (\leq) 0 \text{ iff } d \leq (\geq) (13b - 7\alpha), \quad (30)
\]

\[
\frac{6(d-b)}{7} - \frac{6[(d-b)(n-1) + \alpha]}{2n^2 + 5n + 6} = \frac{6(2n^2 - 12n + 13)[d - b - \frac{7\alpha}{2n^2 - 12n + 13}]}{7(2n^2 + 5n + 6)} \\
\geq (\leq) 0 \text{ iff } d \geq (\leq) \frac{b}{2n^2 - 2n + 13} + \frac{7\alpha}{2n^2 - 2n + 13}, \quad (31)
\]

\[
[d + \alpha + (1-n)b] - \frac{6[(d-b)(n-1) + \alpha]}{2n^2 + 5n + 6} = \frac{d(2n^2 - n + 12) - b(2n^3 + 3n^2 - 5n) + \alpha(2n^2 + 5n)}{2n^2 + 5n + 6} \\
\geq (\leq) 0 \text{ iff } d \geq (\leq) \frac{b(2n^3 + 3n^2 - 5n) - \alpha(2n^2 + 5n)}{2n^2 - n + 12}. \quad (32)
\]

Next, according to relative sizes of the thresholds of \( d \) specified in Proposition 2, there are three cases below.

**Case 1:** Suppose \( d \geq [b(7n - 1) - 7\alpha] \). We then have

\[
\frac{6[(d-b)(n-1) + \alpha]}{2n^2 + 5n + 6} < \frac{6[d(n-1) + \alpha - b]}{2n^2 + 5n + 6} < \frac{6(d-b)}{7} < \frac{6(d+b)}{7} \\
< [d + \alpha + (1-n)b] < (d - b + \alpha)
\]

by (42), (47), (48), (49), and \( d \geq [b(7n - 1) - 7\alpha] > \frac{b(2n^2 + 5n + 5) + \alpha}{2n^2 + 4n + 7} \). These inequalities divide the values of \( c \) into seven exclusive ranges. At each range, we can derive the LREs in both dynamics. Under the imitating-the-best-total dynamic; \( \bar{D} \) is the LRE for
c ≥ [d + α + (1 − n)b] by Proposition 2(ia); \( \{\vec{C}, \vec{D}\} \) is the LRE for \( c \in [\frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6}, [d+\alpha+(1-n)b)] \) by Proposition 2(ib); and \( \vec{C} \) is the LRE for \( c < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} \) by Proposition 2(ic).

Under the imitating-the-best-average dynamic; \( \vec{D} \) is the LRE for \( c > (d - b + \alpha) \) by Proposition C2(i); \( \{\vec{C}, \vec{D}\} \) is the LRE for \( c \in [\frac{6(d-b)}{7}, (d - b + \alpha)] \) by Proposition C2(iiia) and \( \alpha \leq \frac{(d-b)[2n^2-2n+13]}{7} \); \( \{\vec{C}, \vec{D}\} \) is the LRE for \( c \in [\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}, \frac{6(d-b)}{7}] \) by Proposition C2(iiib), \( c < \frac{6(d-b)}{7} \), \( \alpha \leq \frac{(d-b)[2n^2-2n+13]}{7} \) implied by (46) and \( d \geq [b(7n - 1) - 7\alpha] \), and \( c \in [\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}, \frac{6(d-b)}{7}] \); and \( \vec{C} \) is the LRE for \( c \in (0, \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6}) \) implied by (46) and \( d \geq [b(7n - 1) - 7\alpha] \), and \( c < \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} \).

In summary, the c interval making \( \vec{C} \) the LRE under the imitating-the-best-average dynamic is \((0, (d - b + \alpha)]\), and the associated interval under the imitating-the-best-total dynamic is \((0, (d + \alpha + (1 - n)b))\). Since \((0, (d + \alpha + (1 - n)b)) \subset (0, (d - b + \alpha)]\) by \((d - b + \alpha) > [d + \alpha + (1 - n)b] \) due to \( n \geq 3 \), \( \vec{C} \) is more likely to be the LRE under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic.

**Case 2:** Suppose \((7bn - 7\alpha - 13b) \leq d < [b(7n - 1) - 7\alpha] \). Under the circumstance, we need to know relative sizes of the thresholds of \( d \) specified in (45)-(51). Some calculations reveal

\[
    b + \frac{7\alpha}{2n^2 - 2n + 13} < \frac{b[2n^2 + 5n + 5] + \alpha}{2n^2 + 4n + 7} < 13b - 7\alpha < 7bn - 7\alpha - 13b < b(7n - 1) - 7\alpha.
\]

These inequalities divide the values of \( d \) into five mutually exclusive intervals.

**Case 2a:** Suppose \((7bn - 7\alpha - 13b) \leq d < [b(7n - 1) - 7\alpha] \). We then have

\[
    \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d-b)}{7} < [d+\alpha+(1-n)b] < \frac{6(d+b)}{7} < (d-b+\alpha)
\]

by (42), (45), (47), (49), and (50). These inequalities divide the values of \( c \) into seven
mutually exclusive intervals. The results here are the same as Case 1’s by $(d - b + \alpha) > [d + \alpha + (1 - n)b]$.

**Case 2b:** Suppose $(13b - 7\alpha) \leq d < (7bn - 7\alpha - 13b)$. We then have

$$\frac{6[(d - b)(n - 1) + \alpha]}{2n^2 + 5n + 6} < \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6} < [d + \alpha + (1 - n)b] < \frac{6(d - b)}{7}$$

$$< \frac{6(d + b)}{7} < (d - b + \alpha)$$

by (44), (45), (49), and (50). These inequalities divide the values of $c$ into seven mutually exclusive intervals. Again, the conclusions here are the same as Case 1’s due to $(d - b + \alpha) > [d + \alpha + (1 - n)b]$.

**Case 2c:** Suppose $\frac{b(2n^2 + 5n + 5) + \alpha}{2n^2 + 4n + 7} \leq d < 13b - 7\alpha$. We then have

$$\frac{6[(d - b)(n - 1) + \alpha]}{2n^2 + 5n + 6} < \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6} < [d + \alpha + (1 - n)b] < \frac{6(d - b)}{7}$$

$$< (d - b + \alpha) < \frac{6(d + b)}{7}$$

by (44), (45), (49), and (50). These inequalities divide the values of $c$ into seven mutually exclusive intervals. Similarly, Propositions 2 and C2 imply that $\vec{C}$ is more likely to emerge when countries adopt the imitating-the-best-average rule by $(d - b + \alpha) > [d + \alpha + (1 - n)b]$.

**Case 2d:** Suppose $b + \frac{7\alpha}{2n^2 - 2n + 13} \leq d < \frac{b(2n^2 + 5n + 5) + \alpha}{2n^2 + 4n + 7}$. We then have

$$\frac{6[(d - b)(n - 1) + \alpha]}{2n^2 + 5n + 6} < \frac{6(d - b)}{7} < \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6} < [d + \alpha + (1 - n)b]$$

$$< (d - b + \alpha) < \frac{6(d + b)}{7}$$

by (44), (47), (48), (50), and (51). These inequalities divide the values of $c$ into seven mutually exclusive intervals. The results obtained here are the same as Case 1’s by Propositions 2 and C2 due to $(d - b + \alpha) > [d + \alpha + (1 - n)b]$. 

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Case 2c: Suppose \( d \leq b + \frac{7a}{2n^2-2n+13} \). We then have
\[
\frac{6(d-b)}{7} < \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < [d+\alpha+(1-n)b]
\]
\[
< (d-b+\alpha) < \frac{6(d+b)}{7}
\]
by (44), (48), (49), (50), and (51). These inequalities divide the values of \( c \) into seven mutually exclusive intervals. Because \( (d-b+\alpha) > [d+\alpha+(1-n)b] \), \( \bar{C} \) is more likely to emerge in the long run under the imitating-the-best-average rule by Propositions 2 and C2.

Case 3: Suppose \( d < \frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12} \). We can show
\[
\frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12} < [7bn-7\alpha-13b] \text{ by } n \geq 4. \text{ On the other hand, we have }
\]
\[
\frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12} - (13b-7\alpha) < 0 \text{ if } n < 13, \text{ and } > 0 \text{ if } n \geq 13. \text{ Thus, the situation of } d \in [7bn-7\alpha-13b, b(7n-1)-7\alpha] \text{ discussed in Case 2a does not exist here. Accordingly, we will start with the case of } d \in [13b-7\alpha, 7bn-7\alpha-13b] \text{ as follows. In addition, we have }
\]
\[
\frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12} < \frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12}
\]
by \( n \geq 3 \).

Case 3a: Suppose \( (13b-7\alpha) \leq d < (7bn-7\alpha-13b) \). We then have
\[
\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < d+\alpha+(1-n)b < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d-b)}{7}
\]
\[
< \frac{6(d+b)}{7} < (d-b+\alpha)
\]
by (44), (47), (50), and (52) with \( d > \frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12} \) assumed.\(^6\) These inequalities divide the values of \( c \) into seven mutually exclusive intervals. At each range, we can derive the LREs in both dynamics. Under the imitating-the-best-total dynamic; \( \bar{D} \) is the LRE for \( c \geq \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} \) by Proposition 2(iii); \( \bar{D} \) is the LRE for

\(^6\)If \( d \leq \frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12} \), then we have \([d+\alpha+(1-n)b] < \frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d-b)}{7} < \frac{6(d+b)}{7} < (d-b+\alpha). \) Under the circumstance, the results of Case 3a remain true.
\[c \in [d + \alpha + (1 - n)b, \frac{6d(n-1)+\alpha-b}{2n^2+5n+6}]\] by Proposition 2(iiib); and \(\vec{C}\) is the LRE for \(c < [d + \alpha + (1 - n)b]\) by Proposition 2(iic).

Under the imitating-the-best-average dynamic; \(\vec{D}\) is the LRE for \(c > (d - b + \alpha)\) by Proposition C2(i); \(\{\vec{C}, \vec{D}\}\) is the LRE for \(c \in \left[\frac{6(d-b)}{7}, (d - b + \alpha)\right]\) by Proposition C2(iiia) and \(\alpha \leq \frac{(d-b)[2n^2-2n+13]}{7}\) implied by (46) and \(d > b + \frac{2n^2+5n+6}{b}\); \(\{\vec{C}, \vec{D}\}\) is the LRE for \(c \in \left[\frac{6(d-b)(n-1)+\alpha}{2n^2+5n+6}, \frac{6(d-b)}{7}\right]\) by Proposition C2(iiib), \(\alpha \leq \frac{(d-b)[2n^2-2n+13]}{7}\) implied by (46) and \(d > b + \frac{2n^2+5n+6}{b}\); and \(\vec{C}\) is the LRE for \(c \in (0, \frac{6(d-b)(n-1)+\alpha}{2n^2+5n+6})\) by Proposition C2(iiiia), \(\alpha \leq \frac{(d-b)[2n^2-2n+13]}{7}\) implied by (46) and \(d > b + \frac{2n^2+5n+6}{b}\), and \(c < \frac{6(d-b)(n-1)+\alpha}{2n^2+5n+6}\).

In summary, the \(c\) interval making \(\vec{C}\) the LRE under the imitating-the-best-average dynamic is \((0, (d - b + \alpha)\)], and the associated interval under the imitating-the-best-total dynamic is \((0, (d + (1 - n)b + \alpha)\)). Since \((0, (d + (1 - n)b + \alpha)\) \(\subset (0, (d - b + \alpha)\]) by \((d - b + \alpha) > (d + (1 - n)b + \alpha)\), \(\vec{C}\) is more likely to be the LRE under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic.

**Case 3b:** Suppose \(\frac{b[2n^2+5n+6]+\alpha}{2n^2+4n+7} \leq d < (13b - 7\alpha)\). Some calculations show

\[
\frac{b[2n^3+3n^2+n-12]-\alpha[2n^2+5n]}{2n^2-n+12} > \frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12} > \frac{b[2n^2+5n+5]+\alpha}{2n^2+4n+7}.
\]

We then have

\[
\frac{6[(d-b)(n-1)+\alpha]}{2n^2+5n+6} < d + \alpha + (1 - n)b < \frac{6[d(n-1)+\alpha-b]}{2n^2+5n+6} < \frac{6(d-b)}{7}
\]

by (44), (47), (50), and (52) with \(d > \frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12}\) assumed.\(^7\) These inequalities divide the values of \(c\) into seven mutually exclusive intervals. As in Case 3b, we obtain that \(\vec{C}\) is more likely to emerge under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic due to \((0, (d+(1-n)b+\alpha)) \subset (0, (d-b+\alpha)]\) by Propositions 2 and C2.

\(^7\)As in Case 3a, our results remain true if \(d \leq \frac{b[2n^3+3n^2-5n]-\alpha[2n^2+5n]}{2n^2-n+12}\).
Case 3c: Suppose \( b + \frac{7\alpha}{2n^2-2n+13} \leq d < \frac{b(2n^2 + 5n + 5) + \alpha}{2n^2 + 4n + 7} \). We will always have \( d < \frac{b(2n^2 + 3n^2 - 5n) - \alpha(2n^2 + 5n)}{2n^2 - n + 12} < \frac{b(2n^3 + 3n^2 + n - 12) - \alpha(2n^2 + 5n)}{2n^2 - n + 12} \). Then

\[
d + \alpha + (1 - n)b < \frac{6(d - b)(n - 1) + \alpha}{2n^2 + 5n + 6} < \frac{6(d - b)}{7} < \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6}
\]

\[
< (d - b + \alpha) < \frac{6(d + b)}{7}
\]

by (47), (50), (51), and (53) by \( d > b + \frac{7\alpha}{2n^2-2n+13} > \frac{(b - \alpha)(2n^2 + 5n)}{2n^2 - 2n + 13} \). These inequalities divide the values of \( c \) into seven mutually exclusive intervals. Again, the results here are the same as Case 3b’s due to \( (d - b + \alpha) > [d + (1 - n)b + \alpha] \).

Case 3d: Suppose \( d < b + \frac{7\alpha}{2n^2-2n+13} \). Under the circumstance, we need to know relative sizes of \( (d - b + \alpha) \), \( \frac{6(d(n - 1) + \alpha - b)}{2n^2 + 5n + 6} \), and \( \frac{6(d - b)(n - 1) + \alpha}{2n^2 + 5n + 6} \). Some calculations show

\[
(d - b + \alpha) - \frac{6(d(n - 1) + \alpha - b)}{2n^2 + 5n + 6} = \frac{[d(2n^2 - n + 12) - (b - \alpha)(2n^2 + 5n)]}{2n^2 + 5n + 6}
\]

\[
\geq (\leq) \text{ iff } d \geq (\leq) \frac{(b - \alpha)(2n^2 + 5n)}{2n^2 - n + 12}, \quad (33)
\]

\[
(d - b + \alpha) - \frac{6[(d - b)(n - 1) + \alpha]}{2n^2 + 5n + 6} = \frac{(2n^2 - n + 12)[d - b + \frac{\alpha(2n^2 + 5n)}{2n^2 - n + 12}]}{2n^2 + 5n + 6}
\]

\[
\geq (\leq) \text{ iff } d \geq (\leq) b - \frac{\alpha(2n^2 + 5n)}{2n^2 - n + 12}, \quad (34)
\]

\[
b - \frac{\alpha(2n^2 + 5n)}{2n^2 - n + 12} - \frac{(b - \alpha)(2n^2 + 5n)}{2n^2 - n + 12} = \frac{b(2n^2 - n + 11)}{2n^2 - n + 12} > 0 \text{ by } n \geq 2, \quad (35)
\]

\[
b - \frac{\alpha(2n^2 + 5n)}{2n^2 - n + 12} < b + \frac{7\alpha}{2n^2 - n + 12}. \quad (36)
\]

These suggest

\[
\frac{(b - \alpha)(2n^2 + 5n)}{2n^2 - n + 12} < b - \frac{\alpha(2n^2 + 5n)}{2n^2 - n + 12} < b + \frac{7\alpha}{2n^2 - n + 12},
\]

and the inequalities imply that there are three sub-cases below.

Case 3d-1: Suppose \( b - \frac{\alpha(2n^2 + 5n)}{2n^2 - n + 12} \leq d < b + \frac{7\alpha}{2n^2 - n + 12} \). We then have

\[
d + \alpha + (1 - n)b < \frac{6(d - b)}{7} < \frac{6[(d - b)(n - 1) + \alpha]}{2n^2 + 5n + 6} < \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6}
\]

\[
< (d - b + \alpha) < \frac{6(d + b)}{7}
\]

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by (45), (49), (50), (51), and (53). These inequalities divide the values of \( c \) into seven mutually exclusive intervals. As in Case 3b, \( \tilde{C} \) is more likely to emerge under the imitating-the-best-average dynamic than under the imitating-the-best-total dynamic due to \( (d - b + \alpha) > \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6} \) by Propositions 2 and C2.

**Case 3d-2:** Suppose \( \frac{(b - \alpha)(2n^2 + 5n)}{2n^2 + n + 12} \leq d < b - \frac{\alpha(2n^2 + 5n)}{2n^2 + n + 12} \). We then have \( (d - b + \alpha) > \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6} \) by (53), and \( (d - b + \alpha) < \frac{6[(d - b)(n - 1) + \alpha]}{2n^2 + 5n + 6} \) by (54). However, (49) implies \( \frac{6[(d - b)(n - 1) + \alpha]}{2n^2 + 5n + 6} < \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6} \), which leads to a contradiction. Thus, no solution exists in this case.

**Case 3d-3:** Suppose \( d \leq \frac{(b - \alpha)(2n^2 + 5n)}{2n^2 + n + 12} \). We then have

\[
[d + \alpha + (1 - n)b] < \frac{6(d - b)}{7} < (d - b + \alpha) < \frac{6[(d - b)(n - 1) + \alpha]}{2n^2 + 5n + 6}
\]

\[
< \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6} < \frac{6(d + b)}{7}
\]

by (43), (45), (49), and (54). These inequalities divide the values of \( c \) into seven mutually exclusive intervals. At each range, we can derive the LREs in both dynamics. Under the imitating-the-best-total dynamic; \( \tilde{D} \) is the LRE for \( c \geq \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6} \) by Proposition 2(iiiia); \( \tilde{D} \) can be the LRE for \( c \in [d + \alpha + (1 - n)b, \frac{6[d(n - 1) + \alpha - b]}{2n^2 + 5n + 6}] \) by Proposition 2(iiiib); and \( \tilde{C} \) is the LRE for \( c \in (0, [d + \alpha + (1 - n)b]) \) by Proposition 2(iiiic).

Under the imitating-the-best-average dynamic; \( \tilde{D} \) is the LRE for \( c > (d - b + \alpha) \) by Proposition C2(i); \( \tilde{C} \) is the LRE for \( c \in \left[ \frac{6(d - b)}{7}, (d - b + \alpha) \right] \) by Proposition C2(iib) and \( \alpha > \frac{(d - b)[2n^2 - 2n + 13]}{7} \) and \( c \in \left[ \frac{6(d - b)}{7}, \frac{6[(d - b)(n - 1) + \alpha]}{2n^2 + 5n + 6} \right] \); and \( \tilde{C} \) is the LRE for \( c \in (0, \frac{6(d - b)}{7}) \) by Proposition C2(iiiic), \( c < \frac{6(d - b)}{7} \), and \( \alpha > \frac{(d - b)[2n^2 - 2n + 13]}{7} \).

Accordingly, the \( c \) interval making \( \tilde{C} \) the LRE under the imitating-the-best-average dynamic is \( (0, (d - b + \alpha)] \), and the associated interval under the imitating-the-best-total dynamic is \( (0, (d + (1 - n)b + \alpha)) \). Since \( (0, (d + (1 - n)b + \alpha)) \subset (0, (d - b + \alpha)) \), \( \tilde{C} \) is more likely to be the LRE under the imitating-the-best-total dynamic than under
the imitating-the-best-average dynamic.

In summary, the results of Cases 1-3 prove Theorem 2.

**Proof of Proposition 3:** If \( [d + \alpha - b(n - 1)] < c_1 \), then \( [d + \alpha - b(n - 1)] < \frac{\sum_{i=1}^{k} c_i}{k} \). That is, (8) fails at \( k = 1 \), and hence \( u \geq 2 \). On the other hand, \( [d(n - 1) + \alpha - b] \leq \frac{\sum_{i=2}^{n} c_i}{n-1} \) implies that (11) holds at \( k = 1 \), and hence \( v = 1 \). We have \( S_* = \{ \tilde{D} \} \) and \( E(T_\epsilon) = \epsilon^{-1} \) as shown by Proposition 3(i). In contrast, if \( [d + \alpha - b(n - 1)] \geq c_1 \), then \( u = 1 \) due to (8) holding at \( k = 1 \), and \( v = 1 \) by \( [d(n - 1) + \alpha - b] \leq \frac{\sum_{i=2}^{n} c_i}{n-1} \) due to (11) holding at \( k = 1 \). Thus, \( S_* = \{ \tilde{C}, \tilde{D} \} \) and \( E(T_\epsilon) = \epsilon^0 \) as shown by Proposition 3(ii). Finally, if \( [d + \alpha - b(n - 1)] > c_n \), then we have \( [d + \alpha - b(n - 1)] > c_n > \frac{\sum_{i=1}^{k} c_i}{k} \), which implies \( u = 1 \) by (8). Moreover, we have \( d(n - 1) + \alpha - b - \frac{\sum_{i=2}^{n} c_i}{n-1} > d(n - 1) + \alpha - b - c_n > (n - 1)[c_n + b(n - 1) - \alpha] + \alpha - b - c_n = (n - 2)c_n + b(n - 1)^2 - \alpha(n - 2) > 0 \) by \( c_n > \frac{\sum_{i=2}^{n} c_i}{(n-1)} \) and \( d > c_n + b(n - 1) - \alpha \). This suggests that (11) fails at \( k = 1 \), and hence \( v \geq 2 \). Thus, \( S_* = \{ \tilde{C} \} \) and \( E(T_\epsilon) = \epsilon^{-1} \) as shown by Proposition 3(iii).