

Appendix B

Proof of Proposition 1: For notational simplicity, we define the following.

$$\begin{aligned}
J &\equiv 4D_1D_2 - r^2 > 0 \text{ by } r \in (-1, 1); \\
D_i &\equiv 1 + \frac{d_i}{K_i} \text{ for } i = 1, 2; \\
T_1 &\equiv 4D_1(D_1D_2 - r^2)(2D_2 - r)^2 - J^2(D_2 - r); \\
T_2 &\equiv 8\delta D_1(2D_2 - r)(D_1D_2 - r^2)(rc_2 - 2c_1D_2) + J^2\delta_m[2c_1D_2 - rc_1 - rc_2]; \\
T_3 &\equiv 4\delta^2 D_1(D_1D_2 - r^2)[rc_2 - 2c_1D_2]^2 + J^2\delta_m^2 c_1(rc_2 - c_1D_2); \\
J_1 &\equiv \frac{4D_1(D_1D_2 - r^2)(2D_2 - r)^2}{D_2 - r} > 0; \\
J_2 &\equiv \frac{8\delta D_1(D_1D_2 - r^2)(2D_2 - r)(rc_2 - 2c_1D_2)}{\delta_m[rc_1 + rc_2 - 2c_1D_2]}; \\
J_3 &\equiv \frac{4\delta^2 D_1(D_1D_2 - r^2)[rc_2 - 2c_1D_2]^2}{\delta_m^2 c_1(c_1D_2 - rc_2)}; \\
\delta_1 &\equiv \frac{\delta_m(2D_2 - r)[rc_1 + rc_2 - 2c_1D_2]}{2(D_2 - r)(rc_2 - 2c_1D_2)}; \\
\delta_2 &\equiv \frac{\delta_m^2 c_1(c_1D_2 - rc_2)(2D_2 - r)^2}{(D_2 - r)[rc_2 - 2c_1D_2]^2}; \text{ and} \\
\delta_3 &\equiv \frac{2\delta_m c_1(2D_2 - r)(c_1D_2 - rc_2)}{(2c_1D_2 - rc_2)[2c_1D_2 - rc_1 - rc_2]}.
\end{aligned}$$

Moreover, some calculations yield

$$T_1 \geq (\leq) 0 \text{ iff } J^2 \leq (\geq) J_1; \quad (A1)$$

$$T_2 \geq (\leq) 0 \text{ iff } J^2 \leq (\geq) J_2 \text{ for } 2c_1D_2 \in (rc_1, rc_2); \quad (A2)$$

$$T_2 < 0 \text{ for } 2c_1D_2 \in [rc_2, rc_1 + rc_2]; \quad (A3)$$

$$T_2 \geq (\leq) 0 \text{ iff } J^2 \geq (\leq) J_2 \text{ for } 2c_1D_2 > (rc_1 + rc_2); \quad (A4)$$

$$T_3 > 0 \text{ for } 2c_1D_2 \in (rc_1, 2rc_2]; \quad (A5)$$

$$T_3 \geq (\leq) 0 \text{ iff } J^2 \leq (\geq) J_3 \text{ for } 2c_1D_2 > 2rc_2; \quad (A6)$$

$$rc_1 \leq rc_2 < rc_1 + rc_2 < 2rc_2 < 2c_2D_1 < 2c_1D_2 \text{ by } c_2 \geq c_1; \quad (A7)$$

$$J_1 \geq (\leq) J_2 \text{ iff } \delta \leq (\geq) \delta_1 \text{ if } \delta_1 > 0; \quad (A8)$$

$$J_1 \geq (\leq) J_3 \text{ iff } \delta^2 \leq (\geq) \delta_2 \text{ if } \delta_2 > 0; \text{ and} \quad (A9)$$

$$J_2 \geq (\leq) J_3 \text{ iff } \delta \leq (\geq) \delta_3 \text{ if } \delta_3 > 0. \quad (A10)$$

We then examine whether $\delta_i > 0$ holds and determine the relationships among J_i for $i = 1, 2, 3$. The corresponding results are presented in the following three claims, and proofs are available upon request.

Claim 1. (i) If $2c_1D_2 \in (rc_1, rc_2]$, then $\delta_1 > 0$, and hence $J_1 > J_2$.

(ii) If $2c_1D_2 \in (rc_2, rc_1 + rc_2]$, then $\delta_1 < 0$, and hence $J_1 > J_2$.

(iii) If $2c_1D_2 > rc_1 + rc_2$, then $\delta_1 \in (0, 1)$, and hence the relative sizes of J_1 and J_2 are determined by (A8).

Claim 2. (i) If $2c_1D_2 \leq 2rc_2$, then $\delta_2 < 0$, and hence $J_1 > J_3$.

(ii) If $2c_1D_2 > 2rc_2$, then $\delta_2 \in (0, 1)$, and hence the relative sizes of J_1 and J_3 are determined by (A9).

Claim 3. (i) If $2c_1D_2 \in (rc_1, rc_2]$, then $\delta_3 < 0$, and hence $J_2 > J_3$.

(ii) If $2c_1D_2 \in (rc_2, rc_1 + rc_2]$, then $\delta_3 > 0$, and hence the relative sizes of J_2 and J_3 are given in (A10).

(iii) If $2c_1D_2 \in (rc_1 + rc_2, 2rc_2)$, then $\delta_3 < 0$, and hence $J_2 > J_3$.

(iv) If $2c_1D_2 \geq 2rc_2$, then $\delta_3 \in (0, 1)$, and hence the relative sizes of J_2 and J_3 are given in (A10).

Based on the above, (9)-(10) imply

$$\pi_1^n - \pi_1^c = \frac{D_1[q_1^n + q_1^c][(1 - \delta)(rc_2 - 2c_1D_2)]}{4D_1D_2 - r^2} \geq (\leq) 0 \text{ iff } rc_2 \geq (\leq) 2c_1D_2. \quad (A11)$$

On the other hand, (10)-(11) imply

$$\pi_1^c - \pi_1^m = \frac{q_1^c f(a)}{4JH(D_1D_2 - r^2)} \geq (\leq) 0 \text{ iff } f(a) \geq (\leq) 0, \quad (A12)$$

with a positive denominator since $D_i > 1 > |r|$ and $q_1^c > 0$ by Assumption A1, and where $f(a) \equiv T_1a^2 + T_2a + T_3$.

Based on the above, (A7) gives rise to the following five cases.

Case 1: Suppose $2c_1D_2 \in (rc_1, rc_2]$. Then, by Claims 1(i), 2(i), and 3(i), we have $J_1 > J_2 > J_3$. Thus, there are four sub-cases.

Case 1a: Suppose $J^2 \leq J_3$. By (A1), $T_1 > 0$; by (A2), $T_2 > 0$; and by (A5), $T_3 > 0$. These imply that $f(a) > 0$ for all $a \geq 0$. Hence, by (A11)-(A12), the solution is

$$(S1) \quad \pi_1^n \geq \pi_1^c > \pi_1^m \text{ for all } a \geq 0, \quad (A13)$$

with equality if $2c_1D_2 = rc_2$.

Case 1b: Suppose $J^2 \in (J_3, J_2)$. Again, by (A1), (A2), and (A5), we have $T_i > 0$ for $i = 1, 2, 3$. Thus, the solution is the same as (S1) in (A13).

Case 1c: Suppose $J^2 \in [J_2, J_1]$. Then, (A1) implies $T_1 \geq 0$, (A2) implies $T_2 \leq 0$, and (A5) implies $T_3 > 0$. If $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$, there are three possibilities. First, if $T_2^2 > 4T_1T_3$, then $f(a) \geq 0$ for $a \leq a_1$ or $a > a_2$; and $f(a) < 0$ for $a \in (a_1, a_2)$. Second, if $T_2^2 = 4T_1T_3$, then $f(a) \geq 0$ for all $a \geq 0$. Third, if $T_2^2 < 4T_1T_3$, then $f(a) > 0$ for all $a \geq 0$. These imply that the solution is

If $T_2^2 > 4T_1T_3$, then $\pi_1^n \geq \pi_1^c > \pi_1^m$ for $a \in (0, a_1) \cup (a_2, \infty)$; and

$$(S2) \quad \min\{\pi_1^m, \pi_1^n\} \geq \pi_1^c \text{ for } a \in [a_1, a_2]. \quad (A14)$$

If $T_2^2 \leq 4T_1T_3$, then $\pi_1^n \geq \pi_1^c \geq \pi_1^m$ for all $a \geq 0$,

where $0 < a_1 \equiv \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} < a_2 \equiv \frac{-T_2 + \sqrt{T_2^2 - 4T_1T_3}}{2T_1}$; $\pi_1^n = \pi_1^c$ or $\min\{\pi_1^m, \pi_1^n\} = \pi_1^c$ occurs when $2c_1D_2 = rc_2$; and $\pi_1^c = \pi_1^m$ occurs when $T_2^2 = 4T_1T_3$ and $a = \frac{-T_2}{2T_1}$.

However, if $T_1 = 0$, $T_2 < 0$, and $T_3 > 0$, then $f(a) > (<) 0$ for $a \in (0, \frac{-T_3}{T_2})$ ($a > \frac{-T_3}{T_2}$). Thus, the solution is

$$(S3) \quad \pi_1^n \geq \pi_1^c > \pi_1^m \text{ for } a \in (0, \frac{-T_3}{T_2}); \text{ and } \min\{\pi_1^m, \pi_1^n\} \geq \pi_1^c \text{ for } a \geq \frac{-T_3}{T_2}, \quad (A15)$$

where $\pi_1^n = \pi_1^c$ occurs when $2c_1D_2 = rc_2$, and $\min\{\pi_1^m, \pi_1^n\} = \pi_1^c$ occurs when $a = \frac{-T_3}{T_2}$.

Finally, if $T_1 > 0$, $T_2 = 0$, and $T_3 > 0$, then $f(a) = T_1a^2 + T_3 > 0$ for all $a \geq 0$, and hence the solution is (S1) given in (A13).

Case 1d: Suppose $J^2 > J_1$. Then, by (A1), (A2), and (A5), we have $T_1 < 0$, $T_2 < 0$, and $T_3 > 0$. Thus, $f(a) > 0$ for $a < a_1$; and $f(a) \leq 0$ for $a \geq a_1$, and hence the solution is

$$(S4) \quad \pi_1^n \geq \pi_1^c > \pi_1^m \text{ for } a < a_1; \text{ and } \min\{\pi_1^m, \pi_1^n\} \geq \pi_1^c \text{ for } a \geq a_1, \quad (A16)$$

where $a_1 = \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$; and $\pi_1^n = \pi_1^c$ or $\min\{\pi_1^m, \pi_1^n\} = \pi_1^c$ occurs when $2c_1D_2 = rc_2$.

Case 2: Suppose $2c_1D_2 \in (rc_2, rc_1 + rc_2]$. Then, by (A11), we always have $\pi_1^c > \pi_1^n$ for all $a \geq 0$. Moreover, by Claims 1(ii), 2(i), and 3(ii), we have $J_1 > J_3 > J_2$ with $T_2 < 0$ by (A3) and $T_3 > 0$ by (A5); or $J_1 > J_2 > J_3$ with $T_2 < 0$ by (A3) and $T_3 > 0$ by (A5). However, since both scenarios will lead to the same conclusions, we consider only the former scenario. Since the sign of T_1 depends on the relative sizes of J^2 and J_1 , three sub-cases arise.

Case 2a: Suppose $J^2 < J_1$. Then $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$, so $f(a)$ has the same form as in Case 1c. In this case, the solution is

$$\begin{aligned} & \text{If } T_2^2 > 4T_1T_3, \text{ then } \pi_1^c > \max\{\pi_1^m, \pi_1^n\} \text{ for } a \in (0, a_1) \cup (a_2, \infty); \text{ and} \\ (S5) \quad & \pi_1^m \geq \pi_1^c > \pi_1^n \text{ for } a \in [a_1, a_2]. \end{aligned} \quad (A17)$$

If $T_2^2 \leq 4T_1T_3$, then $\pi_1^c \geq \max\{\pi_1^m, \pi_1^n\}$ for all $a \geq 0$,

where $0 < a_1 = \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} < a_2 = \frac{-T_2 + \sqrt{T_2^2 - 4T_1T_3}}{2T_1}$; $\pi_1^c = \pi_1^m$ holds at $a = a_1$ or $a = a_2$; and $\pi_1^c = \max\{\pi_1^m, \pi_1^n\}$ holds when $T_2^2 = 4T_1T_3$ and $a = \frac{-T_2}{2T_1}$.

Case 2b: Suppose $J^2 = J_1$. Then $T_1 = 0$, $T_2 < 0$, and $T_3 > 0$, so $f(a)$ has the same form as in Case 1c. In this case, the solution is

$$(S6) \quad \pi_1^c > \max\{\pi_1^m, \pi_1^n\} \text{ for } a \in (0, \frac{-T_3}{T_2}); \text{ and } \pi_1^m \geq \pi_1^c > \pi_1^n \text{ for } a \geq \frac{-T_3}{T_2}, \quad (A18)$$

where $\pi_1^c = \pi_1^m$ occurs at $a = \frac{-T_3}{T_2} > 0$.

Case 2c: Suppose $J^2 > J_1$. Then $T_1 < 0$, $T_2 < 0$, and $T_3 > 0$, so $f(a)$ has the same form as in Case 1d. In this case, the solution is

$$(S7) \quad \pi_1^c > \max\{\pi_1^m, \pi_1^n\} \text{ for } a < a_1; \text{ and } \pi_1^m \geq \pi_1^c > \pi_1^n \text{ for } a \geq a_1, \quad (A19)$$

where $a_1 = \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$ and $\pi_1^m = \pi_1^c$ occurs at $a = a_1$.

Case 3: Suppose $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$. Then, by (A11), we have $\pi_1^c > \pi_1^n$ for all $a \geq 0$ with $T_3 > 0$ by (A5), $J_1 > J_3$ by Claim 2(i), and $J_2 > J_3$ by Claim 3(iii). However, the

relative sizes of J_1 and J_2 are uncertain, as indicated by (A8), depending on the values of δ and δ_1 . Therefore, three sub-cases arise.

Case 3a: Suppose $\delta < \delta_1$. Then $J_1 > J_2 > J_3$. Under this scenario, there are four further sub-cases.

Case 3a-1: Suppose $J^2 \leq J_3$. Then $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$. Since the associated function $f(a)$ has the same form as in Case 2a, the solution is (S5) given in (A17).

Case 3a-2: Suppose $J^2 \in (J_3, J_2)$. Then $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$. Again, the solution is (S5) given in (A17).

Case 3a-3: Suppose $J^2 \in [J_2, J_1]$. Then $T_1 \geq 0$, $T_2 \geq 0$, and $T_3 > 0$. The associated function $f(a)$ has the same form as in Case 1a. The solution depends on the specific values of T_1 and T_2 . First, if $T_1 > 0$, $T_2 > 0$, and $T_3 > 0$, then

$$(S8) \quad \pi_1^c > \max\{\pi_1^m, \pi_1^n\} \text{ for all } a \geq 0. \quad (A20)$$

Second, if $T_1 = 0$, $T_2 > 0$, and $T_3 > 0$, then $f(a) = T_2 a + T_3 > 0$ for all $a \geq 0$, and the solution is again (S8). Third, if $T_1 > 0$, $T_2 = 0$, and $T_3 > 0$, then $f(a)$ has the same form as in Case 1c, but with $\pi_1^c > \pi_1^n$. Hence, the solution is still (S8).

Case 3a-4: Suppose $J^2 > J_1$. Then $T_1 < 0$, $T_2 > 0$, and $T_3 > 0$. Under this scenario, $f(a) > 0$ for $a < a_1$; and $f(a) \leq 0$ for $a \geq a_1$. Hence, the solution is (S7) given in (A19).

Case 3b: Suppose $\delta = \delta_1$. Then $J_1 = J_2$ and $J_3 < J_1 = J_2$. There are three sub-cases.

Case 3b-1: Suppose $J^2 \leq J_3$. Then $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$. The results are the same as those in Case 3a-1.

Case 3b-2: Suppose $J^2 \in (J_3, J_2]$. Then $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$. The results are the same as those in Case 3a-2.

Case 3b-3: Suppose $J^2 > J_2$. Then $T_1 < 0$, $T_2 > 0$, and $T_3 > 0$. The results are the same as those in Case 3a-4.

Case 3c: Suppose $\delta > \delta_1$. Then $J_2 > J_1 > J_3$ by (A8) and Claim 2(i). We consider four sub-cases.

Case 3c-1: Suppose $J^2 \leq J_3$. Then $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$. The solution is (S5), as in Case 3a-1.

Case 3c-2: Suppose $J^2 \in (J_3, J_1)$. Then $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$. Again, the solution is (S5), as in Case 3a-2.

Case 3c-3: Suppose $J^2 \in [J_1, J_2]$. Then $T_1 \leq 0$, $T_2 \leq 0$, and $T_3 > 0$. The solution depends on the specific values of T_1 and T_3 . First, if $T_1 < 0$, $T_2 < 0$, and $T_3 > 0$, the solution is (S7) in (A19), as in Case 2c. Second, if $T_1 = 0$, $T_2 < 0$, and $T_3 > 0$, the solution is (S6) in (A18), as in Case 2b. Third, if $T_1 < 0$, $T_2 = 0$, and $T_3 > 0$, then $f(a) > 0$ for $a < \sqrt{-T_3/T_1}$; and $f(a) \leq 0$ for $a \geq \sqrt{-T_3/T_1}$. Hence, the solution is

$$(S9) \quad \pi_1^c > \max\{\pi_1^m, \pi_1^n\} \text{ for } a < \sqrt{-T_3/T_1}; \text{ and } \pi_1^m \geq \pi_1^c > \pi_1^n \text{ for } a \geq \sqrt{-T_3/T_1}, \quad (A21)$$

where $\pi_1^m = \pi_1^c$ occurs at $a = \sqrt{-T_3/T_1}$.

Case 3c-4: Suppose $J^2 > J_2$. Then $T_1 < 0$, $T_2 > 0$, and $T_3 > 0$. Under this scenario, the solution is (S7) given in (A19), as in Case 2c.

Case 4: Suppose $2c_1D_2 > 2rc_2$. Some calculations yield $\delta_2 < \delta_3 < \delta_1$ if $c_1 \neq c_2$, and $\delta_2 < \delta_3 = \delta_1$ if $c_1 = c_2$. Since the latter is a special case of the former, we focus on the first scenario. There are four sub-cases.

Case 4a: Suppose $\delta \leq \delta_2$. Then $J_3 < J_2 < J_1$ by (A8)-(A10). We further consider four sub-cases.

Case 4a-1: Suppose $J^2 < J_3$. Then $T_1 > 0$ by (A1), $T_2 < 0$ by (A4), and $T_3 > 0$ by (A6). The solution is (S5), as given in (A17).

Case 4a-2: Suppose $J^2 \in [J_3, J_2]$. Then $T_1 > 0$ by (A1), $T_2 \leq 0$ by (A4), and $T_3 \leq 0$ by (A6). Consider the following possibilities. First, if $T_1 > 0$, $T_2 < 0$, and $T_3 < 0$, then $f(a)$ is strictly convex, with $f(a) < 0$ for $a < a_2$, $f(a) = 0$ at $a = a_2$, and $f(a) > 0$ for $a > a_2$, where $a_2 = \frac{-T_2 + \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$. Hence, the solution is

$$(S10) \quad \pi_1^m \geq \pi_1^c > \pi_1^n \text{ for } a \leq a_2; \text{ and } \pi_1^c > \max\{\pi_1^m, \pi_1^n\} \text{ for } a > a_2, \quad (A22)$$

where $\pi_1^m = \pi_1^c$ occurs at $a = a_2$.

Second, if $T_1 > 0$, $T_2 < 0$, and $T_3 = 0$, then $f(a) = T_1 a^2 + T_2 a$. Hence, the solution is (S10) in (A22), with a_2 replaced by $\frac{-T_2}{T_1} > 0$. Third, if $T_1 > 0$, $T_2 = 0$, and $T_3 < 0$, then $f(a) = T_1 a^2 + T_3$. Hence, the solution is (S10) in (A22), with a_2 replaced by $\sqrt{-T_3/T_1} > 0$. Fourth, if $T_1 > 0$, $T_2 = T_3 = 0$, then $f(a) = T_1 a^2 > 0$ for all $a > 0$. Thus, the solution is (S8) in (A20), since $\pi_1^c > \pi_1^n$ and $\pi_1^c > \pi_1^m$ for all $a > 0$.

Case 4a-3: Suppose $J^2 \in (J_2, J_1]$. Then $T_1 \geq 0$ by (A1), $T_2 > 0$ by (A4), and $T_3 < 0$ by (A6). First, if $T_1 > 0$, $T_2 > 0$, and $T_3 < 0$, then the solution is (S10) in (A22). Second, if $T_1 = 0$, $T_2 > 0$, and $T_3 < 0$, then $f(a) = T_2 a + T_3$, with $f(a) \leq 0$ for $a \leq -\frac{T_3}{T_2}$, and $f(a) > 0$ for $a > -\frac{T_3}{T_2}$. Hence, the solution is

$$(S11) \quad \pi_1^m \geq \pi_1^c > \pi_1^n \text{ for } a \leq -\frac{T_3}{T_2}; \text{ and } \pi_1^c > \max\{\pi_1^m, \pi_1^n\} \text{ for } a > -\frac{T_3}{T_2}, \quad (A23)$$

where $\pi_1^m = \pi_1^c$ occurs at $a = -\frac{T_3}{T_2}$.

Case 4a-4: Suppose $J^2 > J_1$. Then $T_1 < 0$ by (A1), $T_2 > 0$ by (A4), and $T_3 < 0$ by (A6). If $T_2^2 > 4T_1T_3$, then $f(a) \leq 0$ for $a \in (0, a_2] \cup [a_1, \infty)$; and $f(a) > 0$ for $a \in (a_2, a_1)$. If $T_2^2 \leq 4T_1T_3$, then $f(a) \leq 0$ for all $a \geq 0$; with $f(a) = 0$ at $a = -\frac{T_2}{2T_1} > 0$. Under these circumstances, the solution is

If $T_2^2 > 4T_1T_3$, then $\pi_1^m \geq \pi_1^c > \pi_1^n$ for $a \in (0, a_2] \cup [a_1, \infty)$; and

$$(S12) \quad \pi_1^c > \max\{\pi_1^m, \pi_1^n\} \text{ for } a \in (a_2, a_1). \quad (A24)$$

If $T_2^2 \leq 4T_1T_3$, then $\pi_1^m \geq \pi_1^c > \pi_1^n$ for all $a \geq 0$,

where $0 < a_2 = \frac{-T_2 + \sqrt{T_2^2 - 4T_1T_3}}{2T_1} < a_1 = \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1}$; and $\pi_1^c = \pi_1^m$ occurs at $T_2^2 = 4T_1T_3$ and $a = -\frac{T_2}{2T_1}$.

Case 4b: Suppose $\delta \in (\delta_2, \delta_3]$. Then $J_3 \leq J_2 < J_1$, and the results are the same as those in Case 4a.

Case 4c: Suppose $\delta \in (\delta_3, \delta_1]$. Then $J_1 \geq J_2$ by (A8), $J_1 < J_3$ by (A9), and $J_2 < J_3$ by (A10). There are three possibilities: $J_2 < J_3 < J_1$, $J_2 < J_1 < J_3$, and $J_1 = J_2 < J_3$. These are discussed as follows.

Case 4c-1: Suppose $J_2 < J_3 < J_1$. There are four sub-cases.

Case 4c-1-1: Suppose $J^2 \leq J_2$. Then, $T_1 > 0$ by (A1) and $J^2 < J_1$; $T_2 \leq 0$ by (A4) and $J^2 \leq J_2$; and $T_3 > 0$ by (A6) and $J^2 < J_3$. If $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$, the solution is (S5) in (A17). If $T_1 > 0$, $T_2 = 0$, and $T_3 > 0$, the solution is (S8) in (A20).

Case 4c-1-2: Suppose $J^2 \in (J_2, J_3)$. Then, $T_1 > 0$ by (A1) and $J^2 < J_1$; $T_2 > 0$ by (A4) and $J^2 > J_2$; and $T_3 > 0$ by (A6) and $J^2 < J_3$. The solution is (S8) in (A20).

Case 4c-1-3: Suppose $J^2 \in [J_3, J_1]$. Then, $T_1 \geq 0$ by (A1) and $J^2 \leq J_1$; $T_2 > 0$ by (A4) and $J^2 > J_2$; and $T_3 \leq 0$ by (A6) and $J^2 \geq J_3$. First, if $T_1 > 0$, $T_2 > 0$, and $T_3 < 0$, the solution is (S10) in (A22). Second, if $T_1 = 0$, $T_2 > 0$, and $T_3 < 0$, the solution is (S11) in (A23). Third, if $T_1 > 0$, $T_2 > 0$, and $T_3 = 0$, the solution is (S8) in (A20).

Case 4c-1-4: Suppose $J^2 > J_1$. Then, $T_1 < 0$ by (A1) and $J^2 > J_1$; $T_2 > 0$ by (A4) and $J^2 > J_2$; and $T_3 < 0$ by (A6) and $J^2 > J_3$. The solution is (S12) in (A24).

Case 4c-2: Suppose $J_2 < J_1 < J_3$. There are four sub-cases.

Case 4c-2-1: Suppose $J^2 < J_2$. Then $T_1 > 0$ by (A1), $T_2 < 0$ by (A4), and $T_3 > 0$ by (A6). The solution is (S5) in (A17).

Case 4c-2-2: Suppose $J^2 \in [J_2, J_1]$. Then $T_1 \geq 0$ by (A1), $T_2 \geq 0$ by (A4), and $T_3 > 0$ by (A6). Under these circumstances, $f(a) = T_1 a^2 + T_2 a + T_3 > 0$ for all $a \geq 0$, and the solution is (S8) in (A20).

Case 4c-2-3: Suppose $J^2 \in (J_1, J_3]$. Then $T_1 < 0$ by (A1), $T_2 > 0$ by (A4), and $T_3 \geq 0$ by (A6). If $T_1 < 0$, $T_2 > 0$, and $T_3 > 0$, the solution is (S7) in (A19). If $T_1 < 0$, $T_2 > 0$, and $T_3 = 0$, the solution is

$$(S13) \quad \pi_1^c > \max\{\pi_1^m, \pi_1^n\} \text{ for } a < -\frac{T_2}{T_1}; \text{ and } \pi_1^m \geq \pi_1^c > \pi_1^n \text{ for } a \geq -\frac{T_2}{T_1}, \quad (A25)$$

where $\pi_1^m = \pi_1^c$ occurs at $a = -\frac{T_2}{T_1}$.

Case 4c-2-4: Suppose $J^2 > J_3$. Then $T_1 < 0$ by (A1), $T_2 > 0$ by (A4), and $T_3 < 0$ by (A6). The solution is (S12) in (A24).

Case 4c-3: Suppose $J_1 = J_2 < J_3$. All solutions are the same as those in Case 4c-2, except that Case 4c-2-2 does not occur here.

Case 4d: Suppose $\delta > \delta_1$. By (A8) and (A10), we have $J_1 < J_2 < J_3$. Under this circumstance, four sub-cases arise.

Case 4d-1: Suppose $J^2 \leq J_1$. Then, $T_1 \geq 0$ by (A1) and $J^2 \leq J_1$; $T_2 < 0$ by (A4) and $J^2 < J_2$; and $T_3 > 0$ by (A6) and $J^2 < J_3$. First, if $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$, the solution is (S5) in (A17). Second, if $T_1 = 0$, $T_2 < 0$, and $T_3 > 0$, the solution is (S6) in (A18).

Case 4d-2: Suppose $J^2 \in (J_1, J_2]$. Then, $T_1 < 0$ by (A1) and $J^2 > J_1$; $T_2 \leq 0$ by (A4) and $J^2 \leq J_2$; and $T_3 > 0$ by (A6) and $J^2 < J_3$. First, if $T_1 < 0$, $T_2 < 0$, and $T_3 > 0$, the solution is (S7) in (A19). Second, if $T_1 < 0$, $T_2 = 0$, and $T_3 > 0$, the solution is (S9) in (A21).

Case 4d-3: Suppose $J^2 \in (J_2, J_3]$. Then, $T_1 < 0$ by (A1) and $J^2 > J_1$; $T_2 > 0$ by (A4) and $J^2 > J_2$; and $T_3 \geq 0$ by (A6) and $J^2 \geq J_3$. First, if $T_1 < 0$, $T_2 > 0$, and $T_3 > 0$, the solution is (S7) in (A19). Second, if $T_1 < 0$, $T_2 > 0$, and $T_3 = 0$, the solution is (S13) in (A25).

Case 4d-4: Suppose $J^2 > J_3$. Then, $T_1 < 0$ by (A1) and $J^2 > J_1$; $T_2 > 0$ by (A4) and $J^2 > J_2$; and $T_3 < 0$ by (A6) and $J^2 > J_3$. The solution is therefore (S12) given in (A24).

In summary, for $2c_1D_2 \in (rc_1, rc_2]$, the results of Case 1 imply that either $\pi_1^n \geq \pi_1^c > \pi_1^m$ or $\min\{\pi_1^m, \pi_1^n\} \geq \pi_1^c$, as indicated by solutions (S1)-(S4) in (A13)-(A16), which constitute Proposition 1(i). In contrast, for $2c_1D_2 > rc_2$ in Cases 2-4, the solutions (S5)-(S13) in (A17)-(A25) imply that either $\pi_1^c > \max\{\pi_1^m, \pi_1^n\}$ or $\pi_1^m \geq \pi_1^c > \pi_1^n$, thereby establishing Proposition 1(ii). $\square$

Proof of Proposition 2: Recall that $D_i = 1 + \frac{d_i}{K_i}$ for $i = 1, 2$, and $J = 4D_1D_2 - r^2 > 0$, as defined in Proposition 1. Moreover, we define the following.

$$\begin{aligned}
\hat{H} &\equiv a(2D_1 - r) + r\delta c_1 - 2\delta c_2 D_1 > 0 \text{ by } q_2^c > 0; \\
\hat{T}_1 &\equiv 4D_2(D_1D_2 - r^2)(2D_1 - r)^2 - J^2(D_1 - r); \\
\hat{T}_2 &\equiv 8\delta D_2(2D_1 - r)(D_1D_2 - r^2)(rc_1 - 2c_2D_1) + \delta_m J^2(2c_2D_1 - rc_1 - rc_2); \\
\hat{T}_3 &\equiv 4\delta^2 D_2(D_1D_2 - r^2)[rc_1 - 2c_2D_1]^2 + c_2\delta_m^2 J^2(rc_1 - c_2D_1); \\
\hat{J}_1 &\equiv \frac{4D_2(D_1D_2 - r^2)(2D_1 - r)^2}{D_1 - r} > 0;
\end{aligned}$$

$$\begin{aligned}
\hat{J}_2 &\equiv \frac{8\delta D_2(D_1 D_2 - r^2)(2D_1 - r)(2c_2 D_1 - rc_1)}{\delta_m(2c_2 D_1 - rc_1 - rc_2)} > 0 \text{ by } c_2 \geq c_1 \text{ and } D_1 > 1 > r; \\
\hat{J}_3 &\equiv \frac{4\delta^2 D_2(D_1 D_2 - r^2)[2c_2 D_1 - rc_1]^2}{\delta_m^2 c_2(c_2 D_1 - rc_1)} > 0 \text{ by } c_2 \geq c_1 \text{ and } D_1 > 1 > r; \\
\hat{\delta}_1 &\equiv \frac{\delta_m(2D_1 - r)[2c_2 D_1 - rc_1 - rc_2]}{2(D_1 - r)(2c_2 D_1 - rc_1)}; \\
\hat{\delta}_2 &\equiv \frac{c_2 \delta_m^2(c_2 D_1 - rc_1)(2D_1 - r)^2}{(D_1 - r)[2c_2 D_1 - rc_1]^2}; \text{ and} \\
\hat{\delta}_3 &\equiv \frac{2c_2 \delta_m(2D_1 - r)(c_2 D_1 - rc_1)}{(2c_2 D_1 - rc_1)[2c_2 D_1 - rc_1 - rc_2]}.
\end{aligned}$$

Moreover, some calculations yield

$$\hat{T}_1 \geq (\leq) 0 \text{ iff } J^2 \leq (\geq) \hat{J}_1; \quad (\text{A26})$$

$$\hat{T}_2 \geq (\leq) 0 \text{ iff } J^2 \geq (\leq) \hat{J}_2; \quad (\text{A27})$$

$$\hat{T}_3 \geq (\leq) 0 \text{ iff } J^2 \leq (\geq) \hat{J}_3; \quad (\text{A28})$$

$$\hat{J}_1 \geq (\leq) \hat{J}_2 \text{ iff } \delta \leq (\geq) \hat{\delta}_1; \quad (\text{A29})$$

$$\hat{J}_1 \geq (\leq) \hat{J}_3 \text{ iff } \delta^2 \leq (\geq) \hat{\delta}_2; \text{ and} \quad (\text{A30})$$

$$\hat{J}_2 \geq (\leq) \hat{J}_3 \text{ iff } \delta \leq (\geq) \hat{\delta}_3. \quad (\text{A31})$$

Based on the above, we have the following.

Claim 4. *We have $0 < \hat{\delta}_3 \leq \hat{\delta}_1 \leq \hat{\delta}_2$; and the equalities hold when $c_1 = c_2$.*

Based on the above, (9)-(10) imply

$$\pi_2^c - \pi_2^m = \frac{q_2^c \hat{f}(a)}{4J\hat{H}(D_1 D_2 - r^2)} \geq (\leq) 0 \text{ iff } \hat{f}(a) \geq (\leq) 0, \quad (\text{A32})$$

where $\hat{f}(a) \equiv \hat{T}_1 a^2 + \hat{T}_2 a + \hat{T}_3$. On the other hand, since $q_2^n < q_2^c$ by $r > 0$, $c_1 < c_2$, and $D_1 > r$, we obtain $\pi_2^c > \pi_2^n$ from (9)-(10). Thus, it remains to analyze the sign of $\hat{f}(a)$. According to Claim 4, there are four cases as follows.

Case 1: Suppose $\delta \leq \hat{\delta}_3$. Then (A29)-(A31) imply $\hat{J}_3 < \hat{J}_2 < \hat{J}_1$. Therefore, there are four sub-cases.

Case 1a: Suppose $J^2 \leq \hat{J}_3$. Then, $\hat{T}_1 > 0$ by (A26), $\hat{T}_2 < 0$ by (A27), and $\hat{T}_3 \geq 0$ by (A28). First, consider the case $\hat{T}_1 > 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 > 0$. If $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, then $\hat{f}(a) > 0$

for $a \in (0, \hat{a}_1) \cup (\hat{a}_2, \infty)$; and $\hat{f}(a) \leq 0$ for $a \in [\hat{a}_1, \hat{a}_2]$, where $0 < \hat{a}_1 = \frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} < \hat{a}_2 = \frac{-\hat{T}_2 + \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$. If $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, then $\hat{f}(a) \geq 0$ for $a \geq 0$. By (A32), this implies the following solution.

If $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, then $\pi_2^c > \max\{\pi_2^m, \pi_2^n\}$ for $a \in (0, \hat{a}_1) \cup (\hat{a}_2, \infty)$; and

$$(S\hat{1}) \quad \pi_2^m \geq \pi_2^c > \pi_2^n \text{ for } a \in [\hat{a}_1, \hat{a}_2]. \quad (A33)$$

If $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, then $\pi_2^c \geq \max\{\pi_2^m, \pi_2^n\}$ for all $a \geq 0$,

where $\pi_2^m = \pi_2^c$ holds at $a = \hat{a}_1$ or $\hat{a}_2$; and $\pi_2^c = \max\{\pi_2^m, \pi_2^n\}$ holds at $\hat{T}_2^2 = 4\hat{T}_1\hat{T}_3$ and $a = \frac{-\hat{T}_2}{2\hat{T}_1}$.

Second, if $\hat{T}_1 > 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 = 0$. In this case, $\hat{f}(a) \leq 0$ for $a \leq \hat{a}_1$; and $\hat{f}(a) > 0$ for $a > \hat{a}_1$. Thus, the solution is

$$(S\hat{2}) \quad \pi_2^m \geq \pi_2^c > \pi_2^n \text{ for } a \leq \hat{a}_1, \text{ and } \pi_2^c > \max\{\pi_2^m, \pi_2^n\} \text{ for } a > \hat{a}_1, \quad (A34)$$

where $\hat{a}_1 = \frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} > 0$, and $\pi_2^m = \pi_2^c$ holds at $a = \hat{a}_1$.

Case 1b: Suppose $J^2 \in (\hat{J}_3, \hat{J}_2]$. Then, $\hat{T}_1 > 0$ by (A26), $\hat{T}_2 \leq 0$ by (A27), and $\hat{T}_3 < 0$ by (A28). First, if $\hat{T}_1 > 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 < 0$, then $\hat{f}(a) \leq 0$ for $a \leq \hat{a}_2$; and $\hat{f}(a) > 0$ for $a > \hat{a}_2$. Thus, by (A32), the solution is

$$(S\hat{3}) \quad \pi_2^m \geq \pi_2^c > \pi_2^n \text{ for } a \leq \hat{a}_2; \text{ and } \pi_2^c > \max\{\pi_2^m, \pi_2^n\} \text{ for } a > \hat{a}_2, \quad (A35)$$

where $\hat{a}_2 = \frac{-\hat{T}_2 + \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} > 0$, and $\pi_2^m = \pi_2^c$ holds at $a = \hat{a}_2$. Second, if $\hat{T}_1 > 0$, $\hat{T}_2 = 0$, and $\hat{T}_3 < 0$, then $\hat{f}(a) = \hat{T}_1 a^2 + \hat{T}_3 \leq 0$ for $a \leq \sqrt{-\frac{\hat{T}_3}{\hat{T}_1}}$, and $\hat{f}(a) > 0$ for $a > \sqrt{-\frac{\hat{T}_3}{\hat{T}_1}}$. Thus, the solution is

$$(S\hat{4}) \quad \pi_2^m \geq \pi_2^c > \pi_2^n \text{ for } a \leq \sqrt{-\frac{\hat{T}_3}{\hat{T}_1}}; \text{ and } \pi_2^c > \max\{\pi_2^m, \pi_2^n\} \text{ for } a > \sqrt{-\frac{\hat{T}_3}{\hat{T}_1}}, \quad (A36)$$

where $\pi_2^m = \pi_2^c$ holds at $a = \sqrt{-\frac{\hat{T}_3}{\hat{T}_1}}$.

Case 1c: Suppose $J^2 \in (\hat{J}_2, \hat{J}_1]$. Then, $\hat{T}_1 \geq 0$ by (A26), $\hat{T}_2 > 0$ by (A27), and $\hat{T}_3 < 0$ by (A28). First, if $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$, then $\hat{f}(a) \leq 0$ for $a \leq \hat{a}_2$; and $\hat{f}(a) > 0$ for $a > \hat{a}_2$. Thus, the solution is (S $\hat{3}$) given in (A35). Second, if $\hat{T}_1 = 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$,

then $\hat{f}(a) = \hat{T}_2 a + \hat{T}_3$ with $\hat{f}(a) \leq 0$ for $a \leq -\frac{\hat{T}_3}{\hat{T}_2}$; and $\hat{f}(a) > 0$ for $a > -\frac{\hat{T}_3}{\hat{T}_2}$. Thus, the solution is

$$(S\hat{5}) \quad \pi_2^m \geq \pi_2^c > \pi_2^n \text{ for } a \leq -\frac{\hat{T}_3}{\hat{T}_2}; \text{ and } \pi_2^c > \max\{\pi_2^m, \pi_2^n\} \text{ for } a > -\frac{\hat{T}_3}{\hat{T}_2}, \quad (A37)$$

where $\pi_2^m = \pi_2^c$ holds at $a = -\frac{\hat{T}_3}{\hat{T}_2}$.

Case 1d: Suppose $J^2 > \hat{J}_1$. Then, $\hat{T}_1 < 0$ by (A26), $\hat{T}_2 > 0$ by (A27), and $\hat{T}_3 < 0$ by (A28). If $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, then $\hat{f}(a) < 0$ for $a \in (0, \hat{a}_2] \cup [\hat{a}_1, \infty)$; and $\hat{f}(a) > 0$ for $a \in (\hat{a}_2, \hat{a}_1)$, where $\hat{a}_1 = \frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} > \hat{a}_2 = \frac{-\hat{T}_2 + \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1} > 0$. In contrast, if $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, then $\hat{f}(a) \leq 0$ for $a \geq 0$. These results imply the following.

If $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, then $\pi_2^m \geq \pi_2^c > \pi_2^n$ for $a \in (0, \hat{a}_2] \cup [\hat{a}_1, \infty)$; and

$$(S\hat{6}) \quad \pi_2^c > \max\{\pi_2^m, \pi_2^n\} \text{ for } a \in (\hat{a}_2, \hat{a}_1). \quad (A38)$$

If $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, then $\pi_2^m \geq \pi_2^c > \pi_2^n$ for all $a \geq 0$,

where $\pi_2^m = \pi_2^c$ holds at $a = \hat{a}_1$ or $\hat{a}_2$ in the first part, and at $a = -\frac{\hat{T}_2}{2\hat{T}_1}$ in the second part.

Case 2: Suppose $\delta \in (\hat{\delta}_3, \hat{\delta}_1)$. Then (A29)-(A31) imply $\hat{J}_2 < \hat{J}_3 < \hat{J}_1$. Hence, there are four sub-cases.

Case 2a: Suppose $J^2 \leq \hat{J}_2$. We then have $J^2 < \hat{J}_3$ and $J^2 < \hat{J}_1$. Thus, by (A26)-(A28), either $\hat{T}_1 > 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 > 0$ hold, or $\hat{T}_1 > 0$, $\hat{T}_2 = 0$, and $\hat{T}_3 > 0$ hold. In the former case, port 2's solution is $(S\hat{1})$ given in (A33), and the solution in the latter case is

$$(S\hat{7}), (S\hat{8}), (S\hat{9}) \quad \pi_2^c > \max\{\pi_2^m, \pi_2^n\} \text{ for all } a \geq 0. \quad (A39)$$

Case 2b: Suppose $J^2 \in (\hat{J}_2, \hat{J}_3]$. We then have $J^2 < \hat{J}_1$, $J^2 > \hat{J}_2$, and $J^2 \leq \hat{J}_3$. Consequently, by (A26)-(A28), either $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 > 0$ hold, or $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 = 0$ hold. Port 2's solution is $(S\hat{8})$ in the former case, and is $(S\hat{9})$ in the latter case, which are the same as that in (A39).

Case 2c: Suppose $J^2 \in (\hat{J}_3, \hat{J}_1]$. We then have $J^2 \leq \hat{J}_1$, $J^2 > \hat{J}_2$, and $J^2 > \hat{J}_3$. Accordingly, by (A26)-(A28), either $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$ hold, or $\hat{T}_1 = 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$ hold. In the former case, port 2's solution is $(S\hat{3})$ given in (A35), and the solution in the latter case is $(S\hat{5})$ given in (A37).

Case 2d: Suppose $J^2 > \hat{J}_1$. We then have $J^2 > \hat{J}_1$, $J^2 > \hat{J}_2$, and $J^2 > \hat{J}_3$. Therefore, (A26)-(A28) imply $\hat{T}_1 < 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$, and hence port 2's solution is $(\hat{S}6)$ given in (A38).

Case 3: Suppose $\delta \in [\hat{\delta}_1, \hat{\delta}_2]$. Then (A29)-(A31) imply $\hat{J}_2 < \hat{J}_2$, $\hat{J}_3 < \hat{J}_1$, and $\hat{J}_2 < \hat{J}_3$. These inequalities are mutually contradictory, so this case is impossible.

Case 4: Suppose $\delta > \hat{\delta}_2$. Then (A29)-(A31) imply $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ if $\delta^2 > \hat{\delta}_2$; and there exists no solution as argued in Case 3 if $\delta^2 \leq \hat{\delta}_2$. In the former situation, there are four sub-cases.

Case 4a: Suppose $J^2 \leq \hat{J}_1$. We then have $J^2 < \hat{J}_2$ and $J^2 < \hat{J}_3$. Hence, by (A26)-(A28), either $\hat{T}_1 > 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 > 0$ hold, or $\hat{T}_1 = 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 > 0$ hold. In the former case, port 2's solution is $(\hat{S}1)$ given in (A33), and the solution in the latter case is

$$(S10) \quad \pi_2^c > \max\{\pi_2^m, \pi_2^n\} \text{ for } a < \frac{-\hat{T}_3}{\hat{T}_2}; \text{ and } \pi_2^m \geq \pi_2^c > \pi_2^n \text{ for } a \geq \frac{-\hat{T}_3}{\hat{T}_2}, \quad (A40)$$

and the equality holds at $a = \frac{-\hat{T}_3}{\hat{T}_2}$.

Case 4b: Suppose $J^2 \in (\hat{J}_1, \hat{J}_2]$. We then have $J^2 > \hat{J}_1$, $J^2 \leq \hat{J}_2$, and $J^2 < \hat{J}_3$. Thus, by (A26)-(A28), either $\hat{T}_1 < 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 > 0$ hold, or $\hat{T}_1 < 0$, $\hat{T}_2 = 0$, and $\hat{T}_3 > 0$ hold. Port 2's solution is $(\hat{S}11)$ in the former case, and is $(\hat{S}12)$ in the latter case, where

$$(S11) \text{ or } (S13) \quad \pi_2^c > \max\{\pi_2^m, \pi_2^n\} \text{ for } a < \hat{a}_2; \text{ and } \pi_2^m \geq \pi_2^c > \pi_2^n \text{ for } a \geq \hat{a}_2, \quad (A41)$$

$$(S12) \quad \pi_2^c > \max\{\pi_2^m, \pi_2^n\} \text{ for } a < \frac{-\sqrt{-4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}; \text{ and } \pi_2^m \geq \pi_2^c > \pi_2^n \text{ for } a \geq \frac{-\sqrt{-4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}, \quad (A42),$$

and the equality holds at $\hat{a}_2 = \frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$ or $\frac{-\sqrt{-4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$.

Case 4c: Suppose $J^2 \in (\hat{J}_2, \hat{J}_3]$. We then have $J^2 > \hat{J}_1$, $J^2 > \hat{J}_2$, and $J^2 \leq \hat{J}_3$. Consequently, by (A26)-(A28), either $\hat{T}_1 < 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 > 0$ hold, or $\hat{T}_1 < 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 = 0$ hold. In the former case, port 2's solution is $(\hat{S}13)$, and the solution is $(\hat{S}14)$ in the latter case, where

$$(S14) \quad \pi_2^c > \max\{\pi_2^m, \pi_2^n\} \text{ for } a < \frac{-\hat{T}_2}{\hat{T}_1}; \text{ and } \pi_2^m \geq \pi_2^c > \pi_2^n \text{ for } a \geq \frac{-\hat{T}_2}{\hat{T}_1}, \quad (A43).$$

and the equality holds at $a = \frac{-\hat{T}_2}{T_1}$.

Case 4d: Suppose $J^2 > \hat{J}_3$. We then have $J^2 > \hat{J}_1$, $J^2 > \hat{J}_2$, and $J^2 > \hat{J}_3$. Accordingly, (A26)-(A28) imply $\hat{T}_1 < 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$, and hence port 2's solution is $(S\hat{6})$ given in (A38).

All the solutions $(S\hat{1})$ -($S\hat{14}$) in (A33)-(A43) imply that either $\pi_2^m \geq \pi_2^c > \pi_2^n$ or $\pi_2^c > \max\{\pi_2^m, \pi_2^n\}$, and hence Proposition 2 is proved. $\square$

Proof of Lemma 2: First, if $T_1 < 0$, $T_2 > 0$, $T_3 < 0$, and $T_2^2 > 4T_1T_3$, then the function $f(a)$ is strictly concave, and intersects the horizontal axis (a-axis) at $a = a_1 \equiv \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1}$ and $a = a_2 \equiv \frac{-T_2 + \sqrt{T_2^2 - 4T_1T_3}}{2T_1}$ with $0 < a_2 < a_1$. Under this scenario, $f(a) \leq 0$ for $a \in (0, a_2] \cup [a_1, \infty)$, and $f(a) > 0$ for $a \in (a_2, a_1)$. These then imply (21) due to $\pi_1^c > \pi_1^n$ implied by Proposition 1(ii) under $2c_1D_2 > rc_2$. This proves Lemma 2(i). Second, if $T_1 < 0$, $T_2 < 0$, and $T_3 > 0$, then $f(a)$ remains strictly concave, intersecting the horizontal axis at a_1 and a_2 with $a_2 < 0 < a_1$. This implies that $f(a) > 0$ for $a < a_1$; and $f(a) \leq 0$ for $a \geq a_1$. Therefore, when $2c_1D_2 > rc_2$, we obtain (22). And (22) continues to hold with appropriate adjustments: replacing a_1 with $\sqrt{\frac{-T_3}{T_1}}$ if $T_1 < 0$, $T_2 = 0$, and $T_3 > 0$, or replacing a_1 with $\frac{-T_2}{T_1}$ if $T_1 < 0$, $T_2 > 0$, and $T_3 = 0$. This proves Lemma 2(ii). Third, if $T_1 > 0$, $T_2 < 0$, $T_3 > 0$, and $T_2^2 > 4T_1T_3$, then $f(a)$ is strictly convex, and will intersect the horizontal axis (a-axis) at $a = a_1 \equiv \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1}$ and $a = a_2 \equiv \frac{-T_2 + \sqrt{T_2^2 - 4T_1T_3}}{2T_1}$ with $0 < a_1 < a_2$. Thus, we have $f(a) > 0$ for $a \in (0, a_1) \cup (a_2, \infty)$, and $f(a) < 0$ for $a \in [a_1, a_2]$, and hence (23) holds by $\pi_1^c > \pi_1^n$ implied by Proposition 1(ii) and $2c_1D_2 > rc_2$. This proves Lemma 2(iii). Fourth, if $T_1 > 0$, $T_2 < 0$, and $T_3 \leq 0$, then $f(a)$ remains strictly convex, intersecting the horizontal axis at $a = a_1$ and $a = a_2$ with $a_1 < 0 < a_2$. This implies that $f(a) \leq 0$ for $a \leq a_2$; and $f(a) > 0$ for $a > a_2$. Consequently, when $2c_1D_2 > rc_2$, we can obtain (24) as indicated by Lemma 2(iv). Fifth, if $T_1 = 0$, $T_2 > 0$, and $T_3 < 0$, then $f(a)$ is a positively sloped line, and intersects the horizontal axis at $a = -\frac{T_3}{T_2}$. We then have $f(a) < 0$ for $a \leq -\frac{T_3}{T_2}$; and $f(a) > 0$ for $a > -\frac{T_3}{T_2}$. These results then imply (25) as indicated by Lemma 2(v). Sixth, if $T_1 = 0$, $T_2 < 0$, and $T_3 > 0$, then $f(a)$ is a negatively sloped line, and intersects the horizontal axis at $a = -\frac{T_3}{T_2}$. Under this circumstance, $f(a) > 0$ for $a < -\frac{T_3}{T_2}$; and $f(a) < 0$ for $a \geq -\frac{T_3}{T_2}$. These prove (26) as indicated by Lemma 2(vi). Similar analyses can be applied to function $\hat{f}(a)$ and prove Lemma 2(vii). $\square$

Proof of Theorem 1. The conditions $c_1 = c_2 = c$ and $D_1 = D_2 = D$ imply that $J_i = \hat{J}_i$, $T_i = \hat{T}_i$, $a_1 = \hat{a}_1$, $a_2 = \hat{a}_2$, $rc_1 = rc_2 = rc$, $rc_1 + rc_2 = 2rc$, $2c_1D_2 = 2c_2D_1 = 2cD$, and $2cD > 2rc$. Moreover, some calculations show the following.

Lemma A. Suppose $c_1 \leq c_2$ and $D_i = 1 + \frac{d}{K_i}$ for $i = 1, 2$. Then, we have the following.

- (i) $J^2 > J_1 = \hat{J}_1$.
- (ii) $T_1 = \hat{T}_1 < 0$.
- (iii) $T_2 = \hat{T}_2 \geq (\leq) 0$ iff $\frac{\delta}{\delta_m} \leq (\geq) \frac{(2D+r)^2}{4D(D+r)}$.
- (iv) $T_3 = \hat{T}_3 \geq (\leq) 0$ iff $\frac{\delta}{\delta_m} \leq (\geq) \sqrt{\frac{(2D+r)^2}{4D(D+r)}}$.
- (v) $(T_2^2 - 4T_1T_3) = (\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3) > 0$.

Since $\frac{(2D+r)^2}{4D(D+r)} > 1$ by $D > 1 > r$ and $1 < \sqrt{\frac{(2D+r)^2}{4D(D+r)}} < \frac{(2D+r)^2}{4D(D+r)}$, there are two possible outcomes stated below. First, if $\frac{\delta}{\delta_m} \leq \sqrt{\frac{(2D+r)^2}{4D(D+r)}}$, then $T_1 = \hat{T}_1 < 0$, $T_2 = \hat{T}_2 > 0$, and $T_3 = \hat{T}_3 < 0$ by Lemma A(i)-(iii). Under the circumstance, the equilibrium profit rankings of ports 1 and 2 are given by (S12) and (S6), and hence the equilibrium outcomes are

$$\pi_i^m \geq \pi_i^c > \pi_i^n \text{ for } a \in (0, a_2] \cup [a_1, \infty); \pi_i^c > \max\{\pi_i^m, \pi_i^n\} \text{ for } a \in (a_2, a_1) \quad (A44)$$

for $i = 1, 2$, where a_1 and a_2 are defined in Theorem 1. These imply that (c, c) is always an NE for all $a > 0$, and it is the unique Pareto-optimal NE when $a \in (a_2, a_1)$. However, since under $c_1 = c_2 = c$ and $D_1 = D_2 = D$ we can show that $\pi_i^m = \frac{(a-\delta_m c)^2}{4(D+r)} > \pi_i^n = \frac{D(a-c)^2}{(2D+r)^2}$, (m, m) is also an NE for all $a > 0$. Furthermore, (A44) shows that (m, m) is the unique Pareto-optimal NE when $a \in (0, a_2] \cup [a_1, \infty)$. This completes the proof of Theorem 1(i).

Second, if $\frac{\delta}{\delta_m} > \sqrt{\frac{(2D+r)^2}{4D(D+r)}}$, either $T_1 = \hat{T}_1 < 0$, $T_2 = \hat{T}_2 > 0$, and $T_3 = \hat{T}_3 > 0$ hold or $T_1 = \hat{T}_1 < 0$, $T_2 = \hat{T}_2 < 0$, and $T_3 = \hat{T}_3 > 0$ hold by Lemma A(i)-(iii). In both cases, the equilibrium profit rankings of ports 1 and 2 are given by (S7) in (A19) and (S13) in (A41), and hence the equilibrium outcomes are

$$\pi_i^c > \max\{\pi_i^m, \pi_i^n\} \text{ for } a < a_1; \pi_i^m \geq \pi_i^c > \pi_i^n \text{ for } a \geq a_1 \quad (A45)$$

for $i = 1, 2$. Both (A44) and (A45) imply that (c, c) is always an NE for all $a > 0$, and is Pareto optimal for small $a < a_1$, while (m, m) is always an NE for all $a > 0$ and is Pareto optimal for large $a \geq a_1$. This completes the proof of Theorem 1(ii). $\square$

Proof of Theorem 2: When $c_1 = c_2 = c$, we have $\delta_1 = \hat{\delta}_1 = \delta_3 = \hat{\delta}_3 = \delta_m$ and $\delta_2 = \hat{\delta}_2 = \delta_m^2$. In addition, some straightforward calculations yield the following three claims.

Claim 5. Suppose $c_1 = c_2 = c$, $D_1 \neq D_2$, and $\hat{\delta} \equiv \frac{D_1(D_1-r)(2D_2-r)^2}{D_2(D_2-r)(2D_1-r)^2}$. We then obtain the following results.

- (i) $J_i \geq (\leq) \hat{J}_i$ iff $D_2 \leq (\geq) D_1$ for $i = 1, 2, 3$.
- (ii) $J_1 \geq (\leq) \hat{J}_2$ iff $\frac{\delta}{\delta_m} \leq (\geq) \hat{\delta}$ and $J_1 \geq (\leq) \hat{J}_3$ iff $\frac{\delta^2}{\delta_m^2} \leq (\geq) \hat{\delta}$.
- (iii) $J_2 \geq (\leq) \hat{J}_1$ iff $\frac{\delta}{\delta_m} \geq (\leq) \frac{1}{\hat{\delta}}$ and $J_2 \geq (\leq) \hat{J}_3$ iff $\frac{\delta}{\delta_m} \leq (\geq) \hat{\delta}$.
- (iv) $J_3 \geq (\leq) \hat{J}_1$ iff $\frac{\delta^2}{\delta_m^2} \geq (\leq) \frac{1}{\hat{\delta}}$ and $J_3 \geq (\leq) \hat{J}_2$ iff $\frac{\delta}{\delta_m} \geq (\leq) \frac{1}{\hat{\delta}}$.
- (v) $J_1 \geq (\leq) J_2$ iff $\delta \leq (\geq) \delta_m$; $J_1 \geq (\leq) J_3$ iff $\delta^2 \leq (\geq) \delta_m^2$; and $J_2 \geq (\leq) J_3$ iff $\delta \leq (\geq) \delta_m$.
- (vi) $\hat{J}_1 \geq (\leq) \hat{J}_2$ iff $\delta \leq (\geq) \delta_m$; $\hat{J}_1 \geq (\leq) \hat{J}_3$ iff $\delta^2 \leq (\geq) \delta_m^2$; and $\hat{J}_2 \geq (\leq) \hat{J}_3$ iff $\delta \leq (\geq) \delta_m$.

Claim 6. Suppose $c_1 = c_2 = c$ and $D_1 \neq D_2$. We then obtain the following results.

- (i) $J^2 > J_1$ iff $D_2 \geq D_1$; and $J^2 \geq (\leq) J_1$ iff $D_2 < D_1$ and $4r^2 D_1[(D_2 - r)^2 + D_2(D_2 - D_1)] + r^4(D_2 - r) \geq (\leq) 0$.
- (ii) $J^2 > \hat{J}_1$ iff $D_1 \geq D_2$; and $J^2 \geq (\leq) \hat{J}_1$ iff $D_2 > D_1$ and $4r^2 D_2[(D_1 - r)^2 + D_1(D_1 - D_2)] + r^4(D_1 - r) \geq (\leq) 0$.

Claim 7. Suppose $c_1 = c_2 = c$ and $D_1 \neq D_2$. Then, $T_2^2 \geq 4T_1T_3$ and $\hat{T}_2^2 \geq 4\hat{T}_1\hat{T}_3$. The equalities hold when $\delta = \delta_m$.

Based on Claims 5-7, there are four cases according to the values of δ , δ_m , D_1 , and D_2 .

Case 1: Suppose $\delta_m \geq \delta$ and $D_2 > D_1$. Then $\hat{\delta} < 1$; and $J_1 \geq J_2 \geq J_3$ and $\hat{J}_1 \geq \hat{J}_2 \geq \hat{J}_3 > J_3$ with the equalities held when $\delta = \delta_m$ by Claim 5(v)-5(vi). However, we still need to know the relative values of J_i and $\hat{J}_j$ for $i, j = 1, 2, 3$. According to the relative sizes of $\frac{\delta}{\delta_m}$ and $\hat{\delta}$, four sub-cases arise.

Case 1a: Suppose $\frac{\delta}{\delta_m} > \hat{\delta}$ and $\frac{\delta^2}{\delta_m^2} < \hat{\delta}$. Then $J_3 < J_2 < \hat{J}_3 < J_1 < \hat{J}_2 < \hat{J}_1$ by Claim 5(ii), 5(v), and 5(vi). Since $J^2 > J_1$, three sub-cases arise.

Case 1a-1: Suppose $J^2 \in (J_1, \hat{J}_2]$. Given that $J^2 > J_1$, $2cD_2 > 2rc$ by $D_2 > r > 1$, $J^2 < \hat{J}_2$, $\delta \leq \delta_m$, and $J_3 < J_2 < J_1$, the equilibrium profit rankings of ports 1 and 2 are: (S12)

and (S $\hat{3}$) if $\hat{T}_1 > 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 < 0$; and (S12) and (S $\hat{4}$) if $\hat{T}_1 > 0$, $\hat{T}_2 = 0$, and $\hat{T}_3 < 0$. From (S12) in (A24) and (S $\hat{3}$) in (A35),

$$\begin{cases} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } a \in (\max\{a_2, \hat{a}_2\}, a_1) \text{ if } \hat{a}_2 < a_1; \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \in (0, \min\{a_2, \hat{a}_2\}]. \end{cases} \quad (\text{A46})$$

In contrast, under $\hat{T}_1 > 0$, $\hat{T}_2 = 0$, and $\hat{T}_3 < 0$, (S12) in (A24) and (S $\hat{4}$) in (A36) yield the same equilibrium outcomes as in (A46), except that $\hat{a}_2$ is replaced by $\sqrt{-\frac{\hat{T}_3}{\hat{T}_1}}$.

Case 1a-2: Suppose $J^2 \in (\hat{J}_2, \hat{J}_1]$. The equilibrium profit rankings of ports 1 and 2 are (S12) and (S $\hat{3}$) if $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$; and (S12) and (S $\hat{5}$) if $\hat{T}_1 = 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. In the first situation, the equilibrium outcomes are as given in (A46). In the second situation, the outcomes are the same as that in (A46), but with $\hat{a}_2$ replaced by $-\frac{\hat{T}_3}{\hat{T}_2}$.

Case 1a-3: Suppose $J^2 > \hat{J}_1$. The equilibrium profit rankings of ports 1 and 2 are (S12) and (S $\hat{6}$) if $\hat{T}_1 < 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. By analyzing the relative sizes of a_1 , a_2 , $\hat{a}_1$, and $\hat{a}_2$, (S12) in (A24) and (S $\hat{6}$) in (A38) imply

$$\begin{cases} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } a \in (\max\{a_2, \hat{a}_2\}, \min\{a_1, \hat{a}_1\}) \\ \quad \text{if } \hat{a}_2 < a_2 < \hat{a}_1 < a_1, a_2 < \hat{a}_2 < \hat{a}_1 < a_1, \text{ or } a_2 < \hat{a}_2 < a_1 < \hat{a}_1; \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2] \cup [a_1, \infty) \\ \quad \text{if } \hat{a}_2 < \hat{a}_1 < a_2 < a_1; \text{ and } a \in (0, \min\{a_2, \hat{a}_2\}] \cup [\max\{a_1, \hat{a}_1\}, \infty) \text{ otherwise.} \end{cases} \quad (\text{A47-1})$$

Case 1b: Suppose $\frac{\delta}{\delta_m} = \hat{\delta}$. Then $J_3 < J_2 = \hat{J}_3 < J_1 = \hat{J}_2 < \hat{J}_1$ by Claim 5(i)-5(vi). Since $J^2 > J_1$, two sub-cases arise.

Case 1b-1: Suppose $J^2 \in (\hat{J}_2, \hat{J}_1]$. The equilibrium rankings are (S12) and (S $\hat{3}$) if $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$; and (S12) and (S $\hat{5}$) if $\hat{T}_1 = 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. The results are identical to those of Case 1a-2.

Case 1b-2: Suppose $J^2 > \hat{J}_1$. The equilibrium rankings are (S12) and (S $\hat{6}$) if $\hat{T}_1 < 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. The results are identical to those of Case 1a-3.

Case 1c: Suppose $\frac{\delta}{\delta_m} < \hat{\delta}$. Then $\frac{\delta^2}{\delta_m^2} < \hat{\delta}$, and $J_3 < \hat{J}_3 < J_2 < \hat{J}_2 < J_1 < \hat{J}_1$ by Claim 5(i), 5(iii), 5(v) and 5(vi). Since $J^2 > J_1$, two sub-cases arise.

Case 1c-1: Suppose $J^2 \in (J_1, \hat{J}_1]$. The equilibrium rankings of the two ports are (S12) and (S $\hat{3}$) if $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$; and (S12) and (S $\hat{5}$) if $\hat{T}_1 = 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. The results are identical to those in Case 1a-2.

Case 1c-2: Suppose $J^2 > \hat{J}_1$. The equilibrium rankings of the two ports are (S12) and (S $\hat{6}$). The results are identical to those in Case 1a-3.

Case 1d: Suppose $\frac{\delta}{\delta_m} > \hat{\delta}$ and $\frac{\delta^2}{\delta_m^2} > \hat{\delta}$. Then $J_3 < J_2 < J_1 < \hat{J}_3 < \hat{J}_2 < \hat{J}_1$ by Claim 5(ii), 5(v), and 5(vi). Since $J^2 > J_1$, four sub-cases arise.

Case 1d-1: Suppose $J^2 \in (J_1, \hat{J}_3]$. The equilibrium rankings of the two ports are (S12) and (S $\hat{1}$) if $\hat{T}_1 > 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 > 0$; and (S12) and (S $\hat{2}$) if $\hat{T}_1 > 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 = 0$. Comparing a_1 , a_2 , $\hat{a}_1$, and $\hat{a}_2$ in (S12) from (A24) and (S $\hat{1}$) from (A33) yields the following.

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } a \in (a_2, a_1) \text{ if } \hat{a}_1 < \hat{a}_2 < a_2 < a_1, \\ \quad a_2 < \hat{a}_1 < a_1 < \hat{a}_2, \text{ or } a_2 < a_1 < \hat{a}_1 < \hat{a}_2; a \in (\hat{a}_2, a_1) \text{ if } \hat{a}_1 < a_2 < \hat{a}_2 < a_1; \\ \quad \text{or } a \in (a_2, \hat{a}_1) \cup (\hat{a}_2, a_1) \text{ if } a_2 < \hat{a}_1 < \hat{a}_2 < a_1; \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \in [\hat{a}_1, \min\{a_2, \hat{a}_2\}] \text{ if } \hat{a}_1 < \hat{a}_2 < a_2 < a_1 \\ \quad \text{or } \hat{a}_1 < a_2 < \hat{a}_2 < a_1; \text{ or } a \in [\max\{a_1, \hat{a}_1\}, \hat{a}_2] \text{ if } a_2 < \hat{a}_1 < a_1 < \hat{a}_2 \\ \quad \text{or } a_2 < a_1 < \hat{a}_1 < \hat{a}_2. \end{array} \right. \quad (\text{A48})$$

In contrast, the equilibrium outcomes under (S12) from (A24) and (S $\hat{2}$) from (A34) are those given in (A46), except that $\hat{a}_2$ is replaced by $\hat{a}_1$.

Case 1d-2: Suppose $J^2 \in (\hat{J}_3, \hat{J}_2]$. The equilibrium rankings of the two ports are (S12) and (S $\hat{3}$) if $\hat{T}_1 > 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 < 0$; and (S12) and (S $\hat{4}$) if $\hat{T}_1 > 0$, $\hat{T}_2 = 0$, and $\hat{T}_3 < 0$. Thus, the equilibrium outcomes are identical to those in Case 1a-1.

Case 1d-3: Suppose $J^2 \in (\hat{J}_2, \hat{J}_1]$. The equilibrium rankings of the two ports are (S12) and (S $\hat{3}$) if $\hat{T}_1 > 0$, $\hat{T}_2 < 0$, and $\hat{T}_3 < 0$; and (S12) and (S $\hat{5}$) if $\hat{T}_1 = 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. Thus, the equilibrium outcomes are identical to those in Case 1a-2.

Case 1d-4: Suppose $J^2 > \hat{J}_1$. The equilibrium rankings of the two ports are (S12) and (S $\hat{6}$) if $\hat{T}_1 < 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. Hence, the equilibrium outcomes are identical to those in Case 1a-3.

The equilibrium outcomes given in (A46), (A47-1), and (A48) then imply that (c, c) is always an NE for all $a > 0$, as shown in Theorem 2(ia); (c, c) is the unique Pareto-optimal NE if and only if one of the conditions in set C1 holds, as shown in Theorem 2(ib); and (m, m) is the unique Pareto-optimal NE if and only if one of the conditions in set C2 holds, as shown in Theorem 2(ic).

Case 2: Suppose $\delta_m \geq \delta$ and $D_1 > D_2$. Under the circumstance, $J_i > \hat{J}_i$ for $i = 1, 2, 3$ by Claim 5(i); $J^2 > \hat{J}_1$ by Claim 6(ii); and $T_2^2 > 4T_1T_3$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$ by Claim 7. According to the relative sizes of $\frac{\delta}{\delta_m}$ and $\frac{1}{\delta}$, four sub-cases arise.

Case 2a: Suppose $\frac{\delta}{\delta_m} < \frac{1}{\delta}$. Then, $\frac{\delta^2}{\delta_m^2} < \frac{1}{\delta}$, and hence $\hat{J}_3 < J_3 < \hat{J}_2 < J_2 < \hat{J}_1 < J_1$ by Claim 5(i), 5 (iii), and 5(iv). Since $J^2 > \hat{J}_1$, two sub-cases arise.

Case 2a-1: Suppose $J^2 \in (\hat{J}_1, J_1]$. Since $J^2 > \hat{J}_1$, $2cD_1 > 2cD_2 > 2rc$, $\delta < \delta_2 = 1$, and $J_3 < J_2 < J_1$, the equilibrium rankings of both ports are (S10) and (S6) if $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$; and (S11) and (S6) if $\hat{T}_1 = 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. From (S10) in (A22) and (S6) in (A38), we have that

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } a \in (\max\{a_2, \hat{a}_2\}, \hat{a}_1); \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \in (0, \hat{a}_2] \cup [\hat{a}_1, a_2] \text{ if } \hat{a}_2 < \hat{a}_1 < a_2; \\ \text{or } a \in (0, \min\{a_2, \hat{a}_2\}] \text{ otherwise.} \end{array} \right. \quad (\text{A49})$$

In the latter case, (S11) in (A23) and (S6) in (A38) imply that the equilibrium outcomes are identical to those in (A49), except that a_2 is replaced by $-\frac{T_3}{T_2}$.

Case 2a-2: Suppose $J^2 > J_1$. The equilibrium rankings of the two ports are (S12) and (S6) if $\hat{T}_1 < 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. Thus, the equilibrium outcomes are identical to those in (A47-1).

Case 2b: Suppose $\frac{\delta}{\delta_m} = \frac{1}{\delta}$. By Claim 5(i), 5(iii), 5(vi), and 5(vii), we have $\hat{J}_3 < J_3 = \hat{J}_2 < \hat{J}_1 = J_2 < J_1$. Since $J^2 > \hat{J}_1$, two possibilities remain: $J^2 \in (\hat{J}_1, J_1]$ or $J^2 > J_1$. In either case, the equilibrium outcomes coincide with those of Case 2a.

Case 2c: Suppose $\frac{\delta}{\delta_m} > \frac{1}{\delta}$ and $\frac{\delta^2}{\delta_m^2} \leq \frac{1}{\delta}$. By Claim 5(i)-5(vi), we have $\hat{J}_3 < \hat{J}_2 < J_3 < \hat{J}_1 < J_2 < J_1$. Since $J^2 > \hat{J}_1$, three sub-cases arise.

Case 2c-1: Suppose $J^2 \in (\hat{J}_1, J_2]$. The equilibrium rankings of the two ports are (S10) and (S $\hat{6}$) if $T_1 > 0$, $T_2 \leq 0$, and $T_3 \leq 0$; and (S8) and (S $\hat{6}$) if $T_1 > 0$, $T_2 = 0$, and $T_3 = 0$. In the former case, the equilibrium outcomes are those given in (A49). In the latter case, (S8) in (A20) and (S $\hat{6}$) in (A38) imply that

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } a \in (\hat{a}_2, \hat{a}_1); \\ (m, m) \text{ cannot be Pareto-optimal but may or may not be an NE.} \end{array} \right. \quad (\text{A50})$$

Case 2c-2: Suppose $J^2 \in (J_2, J_1]$. The equilibrium rankings of the two ports are (S10) and (S $\hat{6}$) if $T_1 > 0$, $T_2 > 0$, and $T_3 < 0$; and (S11) and (S $\hat{6}$) if $T_1 = 0$, $T_2 > 0$, and $T_3 < 0$. In the former case, the equilibrium outcomes are those given in (A49). In the latter case, the equilibrium outcomes are those given in (A49), with a_2 replaced by $\frac{-T_3}{T_2}$.

Case 2c-3: Suppose $J^2 > J_1$. The equilibrium rankings of the two ports are (S12) and (S $\hat{6}$), with the equilibrium outcomes given in (A47-1).

Case 2d: Suppose $\frac{\delta}{\delta_m} \geq \frac{1}{\delta}$ and $\frac{\delta^2}{\delta_m^2} > \frac{1}{\delta}$. Then we have $\hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_3 < J_2 < J_1$ by Claim 5(iv)-5(vi). Since $J^2 > \hat{J}_1$, four sub-cases arise.

Case 2d-1: Suppose $J^2 \in (\hat{J}_1, J_3]$. The equilibrium rankings of the two ports are (S5) and (S $\hat{6}$) if $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$. From (S5) in (A17) and (S $\hat{6}$) in (A38), we obtain that

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } a \in (\hat{a}_2, \hat{a}_1) \text{ if } \hat{a}_2 < \hat{a}_1 < a_1 < a_2 \\ \quad \text{or } a_1 < a_2 < \hat{a}_2 < \hat{a}_1; a \in (\hat{a}_2, a_1) \text{ if } \hat{a}_2 < a_1 < \hat{a}_1 < a_2; \text{ or } a \in (a_2, \hat{a}_1) \\ \quad \text{if } a_1 < \hat{a}_2 < a_2 < \hat{a}_1; \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \in [a_1, a_2] \text{ if } \hat{a}_2 < \hat{a}_1 < a_1 < a_2; \\ \quad a \in [\hat{a}_1, a_2] \text{ if } \hat{a}_2 < a_1 < \hat{a}_1 < a_2; \\ \quad a \in [a_1, \hat{a}_2] \cup [\hat{a}_1, a_2] \text{ if } a_1 < \hat{a}_2 < \hat{a}_1 < a_2; \\ \quad a \in [a_1, \hat{a}_2] \text{ if } a_1 < \hat{a}_2 < a_2 < \hat{a}_1; \text{ or} \\ \quad a \in [a_1, a_2] \text{ if } a_1 < a_2 < \hat{a}_2 < \hat{a}_1. \end{array} \right. \quad (\text{A51})$$

Case 2d-2: Suppose $J^2 \in (J_3, J_2]$. The equilibrium rankings of the two ports are (S8) and (S $\hat{6}$) if $T_1 > 0$, $T_2 = 0$, and $T_3 = 0$; and (S10) and (S $\hat{6}$) if $T_1 > 0$, $T_2 \leq 0$, and $T_3 \leq 0$. In

the former case, the equilibrium outcomes are those given in (A50), whereas in the latter case, the equilibrium outcomes are those given in (A49).

Case 2d-3: Suppose $J^2 \in (J_2, J_1]$. The equilibrium rankings of the two ports are (S10) and (S $\hat{6}$) if $T_1 > 0$, $T_2 > 0$, and $T_3 > 0$; and (S11) and (S $\hat{6}$) if $T_1 = 0$, $T_2 = 0$, and $T_3 < 0$. In the former case, the equilibrium outcomes are those given in (A49), while in the latter case, the equilibrium outcomes are those given in (A49), except that a_2 is replaced by $\frac{-T_3}{T_2}$.

Case 2d-4: Suppose $J^2 > J_1$. The equilibrium rankings of the two ports are (S12) and (S $\hat{6}$), with the equilibrium outcomes given in (A47-1).

The equilibrium outcomes given in (A47-1) and (A49)-(A51) jointly imply that (c, c) is always an NE for all $a > 0$; (c, c) is the unique Pareto-optimal NE if and only if one of the conditions in set C3 holds; and (m, m) is the unique Pareto-optimal NE if and only if one of the conditions in set C4 holds. This completes the proof of Theorem 2(ii).

Case 3: Suppose $\delta_m < \delta$ and $D_2 > D_1$. Then $\frac{\delta}{\delta_m} > 1 > \hat{\delta}$, implying $\frac{\delta^2}{\delta_m^2} > \hat{\delta}$. Moreover, by Claim 5(v)-5(vi), we have $J_1 < J_2 < J_3$ and $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$. Depending on the relative sizes of $\frac{\delta}{\delta_m}$ and $\frac{1}{\hat{\delta}}$, two sub-cases arise.

Case 3a: Suppose $\frac{\delta}{\delta_m} \geq \frac{1}{\hat{\delta}}$. By Claims 5(i)-5(iii), $J_1 < \hat{J}_1 < J_2 < \hat{J}_2 < J_3 < \hat{J}_3$. Since $D_2 > D_1$, Claim 6(i) implies that $J^2 > J_1$. Therefore, six sub-cases need to be considered.

Case 3a-1: Suppose $J^2 \in (J_1, \hat{J}_1]$. Because $J_1 < J_2 < J_3$ and $J^2 > J_1$, port 1's equilibrium profit ranking follows either (S7) or (S9). Moreover, since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 \leq \hat{J}_1$, port 2's equilibrium profit ranking follows either (S $\hat{1}$) or (S $\hat{10}$). Combining (S7) in (A19) with (S $\hat{1}$) in (A33), the equilibrium outcomes are

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } a \in (0, \min\{a_1, \hat{a}_1\}) \text{ when } a_1 < \hat{a}_1 < \hat{a}_2 \text{ or} \\ \quad \hat{a}_1 < a_1 < \hat{a}_2; a \in (0, \hat{a}_1) \cup (\hat{a}_2, a_1) \text{ when } \hat{a}_1 < \hat{a}_2 < a_1; \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \in [\max\{a_1, \hat{a}_1\}, \hat{a}_2] \text{ when } a_1 < \hat{a}_1 < \hat{a}_2 \text{ or} \\ \quad \hat{a}_1 < a_1 < \hat{a}_2. \end{array} \right. \quad (\text{A52})$$

In contrast, combining (S9) in (A21) with (S $\hat{1}$) in (A33) yields the same equilibrium outcomes as in (A52), except that a_1 is replaced by $\sqrt{-\frac{T_3}{T_1}}$. Combining (S7) in (A19) with

(S10) in (A40) yields the following equilibrium outcomes:

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } a \in (0, \min\{a_1, \frac{-\hat{T}_3}{\hat{T}_2}\}) \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \geq \max\{a_1, \frac{-\hat{T}_3}{\hat{T}_2}\}. \end{array} \right. \quad (\text{A53})$$

Finally, combining (S9) in (A21) with (S10) in (A40) yields the same equilibrium outcomes as in (A53), except that a_1 is replaced by $\sqrt{\frac{-T_3}{T_1}}$.

Case 3a-2: Suppose $J^2 \in (\hat{J}_1, J_2]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_1$, port 1's equilibrium profit ranking is either (S7) or (S9). Moreover, because $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_1$, port 2's equilibrium profit ranking is either (S11) or (S12). Combining (S7) in (A19) with (S11) in (A41) yields the equilibrium outcomes in (A53), except that $\frac{-\hat{T}_3}{\hat{T}_2}$ is replaced by $\frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$. Combining (S7) in (A19) with (S12) in (A42) yields the equilibrium outcomes in (A53), except that $\frac{-\hat{T}_3}{\hat{T}_2}$ is replaced by $\frac{-\sqrt{-4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$. Combining (S9) in (A21) with (S11) in (A41) yields the equilibrium outcomes in (A53), except that a_1 is replaced by $\sqrt{\frac{-T_3}{T_1}}$ and $\frac{-\hat{T}_3}{\hat{T}_2}$ is replaced by $\frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$. Finally, combining (S9) in (A21) with (S12) in (A42) yields the equilibrium outcomes in (A53), except that a_1 is replaced by $\sqrt{\frac{-T_3}{T_1}}$ and $\frac{-\hat{T}_3}{\hat{T}_2}$ is replaced by $\frac{-\sqrt{-4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$.

Case 3a-3: Suppose $J^2 \in (J_2, \hat{J}_2]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_2$, port 1's equilibrium profit ranking is either (S7) or (S13). Moreover, because $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_1$, port 2's equilibrium profit ranking is either (S11) or (S12). The equilibrium outcomes corresponding to (S7) with (S11) and (S7) with (S12) are presented in Case 3a-2. Combining (S13) in (A25) with (S11) in (A41) yields the same equilibrium outcomes as in (A53), except that a_1 is replaced by $\frac{-T_2}{T_1}$ and $\frac{-\hat{T}_3}{\hat{T}_2}$ is replaced by $\frac{-\hat{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$. Similarly, combining (S13) in (A25) with (S12) in (A42) yields the equilibrium outcomes in (A53), except that a_1 is replaced by $\frac{-T_2}{T_1}$ and $\frac{-\hat{T}_3}{\hat{T}_2}$ is replaced by $\frac{-\sqrt{-4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$.

Case 3a-4: Suppose $J^2 \in (\hat{J}_2, J_3]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_2$, port 1's equilibrium profit ranking is either (S7) or (S13). Moreover, because $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_2$, port 2's equilibrium profit ranking is either (S13) or (S14). Combining (S7) in (A19) with (S13) in (A43) gives the same outcomes as those obtained from (S7) and (S11). Combining (S7) with (S14) in (A44) yields the equilibrium outcomes in (A53), except that $\frac{-\hat{T}_3}{\hat{T}_2}$ is replaced

by $\frac{-\hat{T}_2}{\hat{T}_1}$. Combining (S13) in (A25) with (S13) in (A43) yields the outcomes in (A53), except that a_1 is replaced by $\frac{-T_2}{T_1}$ and $\frac{-\hat{T}_3}{\hat{T}_2}$ is replaced by $\frac{-\bar{T}_2 - \sqrt{\hat{T}_2^2 - 4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$. Finally, combining (S13) in (A25) with (S14) in (A44) yields the outcomes in (A53), except that a_1 is replaced by $\frac{-T_2}{T_1}$ and $\frac{-\hat{T}_3}{\hat{T}_2}$ is replaced by $\frac{-\hat{T}_2}{\hat{T}_1}$.

Case 3a-5: Suppose $J^2 \in (J_3, \hat{J}_3]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_3$, port 1's equilibrium profit ranking is (S12). Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_2$, port 2's equilibrium profit ranking is either (S13) or (S14). Combining (S12) in (A24) with (S13) in (A43) gives

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } a \in (a_2, \hat{a}_2) \text{ when } a_2 < \hat{a}_2 < a_1; \text{ or } a \in (a_2, a_1) \\ \quad \text{if } a_2 < a_1 < \hat{a}_2; \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \in [\hat{a}_2, a_2] \cup [a_1, \infty) \text{ when } \hat{a}_2 < a_2 < a_1; \\ \quad \text{or } a \geq \max\{a_2, \hat{a}_2\} \text{ otherwise.} \end{array} \right. \quad (\text{A54})$$

The equilibrium outcomes corresponding to (S12) in (A24) and (S14) in (A44) are identical to those in (A54), except that $\hat{a}_2$ is replaced by $-\frac{\hat{T}_2}{\hat{T}_1}$.

Case 3a-6: Suppose $J^2 > \hat{J}_3$. Since $J_1 < J_2 < J_3$ and $J^2 > J_3$, port 1's equilibrium profit ranking is (S12). Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_3$, port 2's equilibrium profit ranking is (S6). The equilibrium outcomes corresponding to (S12) in (A24) and (S6) in (A38) are those reported in (A47-1).

Case 3b: Suppose $\frac{\delta}{\delta_m} < \frac{1}{\delta}$. By Claim 5(ii), 5(v), and 5(vi), $J_1 < J_2 < J_3 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3$. Hence, six sub-cases arise.

Case 3b-1: Suppose $J^2 \in (J_1, J_2]$. This case is identical to Case 3a-1.

Case 3b-2: Suppose $J^2 \in (J_1, J_3]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_2$, port 1's equilibrium profit ranking is either (S7) or (S13). Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 < \hat{J}_1$, port 2's equilibrium profit ranking is either (S1) or (S10). The combinations involving (S7) coincide with Case 3a-1. Combining (S13) with (S1) yields the outcomes in (A52), except that a_1 is replaced by $-\frac{T_2}{T_1}$. Combining (S13) with (S10) yields the outcomes in (A53), except that a_1 is replaced by $-\frac{T_2}{T_1}$.

Case 3b-3: Suppose $J^2 \in (J_3, \hat{J}_1]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_3$, port 1's equilibrium profit ranking is (S12). Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 < \hat{J}_1$, port 2's equilibrium profit

ranking is either $(S\hat{1})$ or $(S\hat{1}0)$. The equilibrium outcomes corresponding to $(S12)$ and $(S\hat{1})$ are those presented in (A48). The outcomes corresponding to $(S12)$ and $(S\hat{1}0)$ are those reported in (A54), except that $\hat{a}_2$ is replaced by $\frac{-\hat{T}_3}{\hat{T}_2}$.

Case 3b-4: Suppose $J^2 \in (\hat{J}_1, \hat{J}_2]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_3$, port 1's equilibrium profit ranking is $(S12)$. Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_1$, port 2's equilibrium profit ranking is either $(S\hat{1}1)$ or $(S\hat{1}2)$. The equilibrium outcomes corresponding to $(S12)$ and $(S\hat{1}1)$ are those presented in (A54). The outcomes corresponding to $(S12)$ and $(S\hat{1}2)$ are those reported in (A54), except that $\hat{a}_2$ is replaced by $-\frac{\sqrt{-4\hat{T}_1\hat{T}_3}}{2\hat{T}_1}$.

Case 3b-5: Suppose $J^2 \in (\hat{J}_2, \hat{J}_3]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_3$, port 1's equilibrium profit ranking is $(S12)$. Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_2$, port 2's equilibrium profit ranking is either $(S\hat{1}3)$ or $(S\hat{1}4)$. The equilibrium outcomes corresponding to $(S12)$ and $(S\hat{1}3)$ are those presented in (A54). The outcomes corresponding to $(S12)$ and $(S\hat{1}4)$ are those reported in (A54), except that $\hat{a}_2$ is replaced by $-\frac{\hat{T}_2}{\hat{T}_1}$.

Case 3b-6: Suppose $J^2 > J_3$. The case is identical to Case 3a-6.

The equilibrium outcomes reported in (A47-1) and (A52)-(A54) jointly imply that (c, c) is always an NE for all $a > 0$; (c, c) is the unique Pareto-optimal NE if and only if one of the conditions in set $C\hat{1}$ holds; and (m, m) is the unique Pareto-optimal NE if and only if one of the conditions in set $C\hat{2}$ holds. These results establish Theorem 2(iii).

Case 4: Suppose $\delta_m < \delta$ and $D_2 < D_1$. Then $\frac{\delta}{\delta_m} > 1$, $\hat{\delta} > 1$, and $J^2 > \hat{J}_1$. Moreover, by Claim 5(v)-5(vi), we have $J_1 < J_2 < J_3$ and $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$. Depending on the relative magnitude of $\frac{\delta}{\delta_m}$ and $\hat{\theta}$, two sub-cases arise.

Case 4a: Suppose $\frac{\delta}{\delta_m} \leq \hat{\delta}$. By Claim 5(ii), 5(v), and 5(vi), either $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1 < J_2 < J_3$ or $\hat{J}_1 < \hat{J}_2 < J_1 < \hat{J}_3 < J_2 < J_3$. Since these two cases yield similar equilibrium outcomes, we consider only the former. Thus, six sub-cases arise.

Case 4a-1: Suppose $J^2 \in (\hat{J}_1, \hat{J}_2]$. Since $J_1 < J_2 < J_3$ and $J^2 < J_2$, port 1's equilibrium profit ranking is either $(S5)$ or $(S6)$. Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_1$, port 2's equilibrium

profit ranking is either (S11) or (S12). Combining (S5) with (S11) gives

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } a \in (0, \min\{a_1, \hat{a}_2\}) \text{ when } \hat{a}_2 < a_1 < a_2 \text{ or} \\ \quad a_1 < \hat{a}_2 < a_2; \text{ or } a \in (0, a_1) \cup (a_2, \hat{a}_2) \text{ otherwise;} \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \in [\max\{a_1, \hat{a}_2\}, \hat{a}_2] \text{ when } \hat{a}_2 < a_1 < a_2 \text{ or} \\ \quad a_1 < \hat{a}_2 < a_2. \end{array} \right. \quad (\text{A55})$$

The equilibrium outcomes corresponding to (S5) and (S12) coincide with those in (A55), except that $\hat{a}_2 = -\sqrt{\frac{-4\hat{T}_1\hat{T}_3}{2\hat{T}_1}}$.

Case 4a-2: Suppose $J^2 \in (\hat{J}_2, \hat{J}_3]$. Since $J_1 < J_2 < J_3$ and $J^2 < J_2$, port 1's equilibrium profit ranking is either (S5) or (S6). Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_2$, port 2's equilibrium profit ranking is either (S13) or (S14). Combining (S5) with (S13) yields the equilibrium outcomes in (A55). Combining (S5) with (S14) also yields the outcomes in (A55), except that $\hat{a}_2 = -\frac{\hat{T}_2}{\hat{T}_1}$. Combining (S6) with (S13) yields the equilibrium outcomes in (A53), except that a_1 is replaced by $-\frac{T_3}{T_2}$ and $-\frac{\hat{T}_3}{\hat{T}_2}$ is replaced by $\hat{a}_2$. Combining (S6) and (S14) yields the equilibrium outcomes in (A53), except that a_1 is replaced by $-\frac{T_3}{T_2}$ and $\hat{a}_2$ is replaced by $-\frac{\hat{T}_2}{\hat{T}_1}$.

Case 4a-3: Suppose $J^2 \in (\hat{J}_3, J_1]$. Since $J_1 < J_2 < J_3$ and $J^2 < J_2$, port 1's equilibrium profit ranking is either (S5) or (S6). Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_3$, port 2's equilibrium profit ranking is (S6). Combining (S5) and (S6) yields the equilibrium outcomes in (A51). Combining (S6) and (S6) yields

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } a \in (\hat{a}_2, -\frac{T_3}{T_2}) \text{ when } \hat{a}_2 < -\frac{T_3}{T_2} < \hat{a}_1; \text{ or } , \\ \quad a \in (\hat{a}_2, a_1) \text{ when } \hat{a}_2 < \hat{a}_1 < -\frac{T_3}{T_2}; \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \in [-\frac{T_3}{T_2}, \hat{a}_2] \cup [\hat{a}_1, \infty) \text{ when } -\frac{T_3}{T_2} < \hat{a}_2 < \hat{a}_1; \\ \quad \text{or } a \geq \max\{\hat{a}_1, -\frac{T_3}{T_2}\} \text{ otherwise.} \end{array} \right. \quad (\text{A56})$$

Case 4a-4: Suppose $J^2 \in (J_1, J_2]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_1$, port 1's equilibrium profit ranking is either (S7) or (S9). Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_3$, port 2's equilibrium profit ranking is (S6). Combining (S5) with (S6) yields the outcomes in (A51). Combining (S9) and (S6) yields the outcomes in (A56), except that $-\frac{T_3}{T_2}$ is replaced by $\sqrt{-\frac{T_3}{T_1}}$.

Case 4a-5: Suppose $J^2 \in (J_2, J_3]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_2$, port 1's equilibrium profit ranking is either (S7) or (S13). Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_3$, port 2's equilibrium profit ranking is (S6). Combining (S7) with (S6) yields the outcomes in (A56). Combining (S13) with (S6) yields the outcomes in (A56), except that $-\frac{T_3}{T_2}$ is replaced by $-\frac{T_2}{T_1}$.

Case 4a-6: Suppose $J^2 > J_3$. This case is identical to Case 3a-6.

Case 4b: Suppose $\frac{\delta}{\delta_m} > \hat{\delta}$. Then, by Claim 5(i)-5(vi), $\hat{J}_1 < J_1 < \hat{J}_2 < J_2 < \hat{J}_3 < J_3$. Consequently, six sub-cases arise.

Case 4b-1: Suppose $J^2 \in (\hat{J}_1, J_1]$. This case is identical to Case 4a-1.

Case 4b-2: Suppose $J^2 \in (J_1, \hat{J}_2]$. This case is identical to Case 3a-2.

Case 4b-3: Suppose $J^2 \in (\hat{J}_2, J_2]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_1$, port 1's equilibrium profit ranking is either (S7) or (S9). Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_3$, port 2's equilibrium profit ranking is either (S13) or (S14). Combining (S7) with (S13) yields the equilibrium outcomes in (A53), except that $-\frac{\hat{T}_3}{\hat{T}_2}$ is replaced by $\hat{a}_2$.

Case 4b-4: Suppose $J^2 \in (J_2, \hat{J}_3]$. Since $J_1 < J_2 < J_3$ and $J^2 > J_2$, port 1's equilibrium profit ranking is either (S7) or (S13). Since $\hat{J}_1 < \hat{J}_2 < \hat{J}_3$ and $J^2 > \hat{J}_2$, port 2's equilibrium profit ranking is either (S13) or (S14). Combining (S7) with (S13) yields the equilibrium outcomes in (A53), except that $-\frac{\hat{T}_2}{\hat{T}_1}$ is replaced by $\hat{a}_2$. Combining (S7) with (S14) yields the outcomes in (A53), except that $-\frac{\hat{T}_3}{\hat{T}_2}$ is replaced by $-\frac{\hat{T}_2}{\hat{T}_1}$.

Case 4b-5: Suppose $J^2 \in (\hat{J}_3, J_3]$. This case is identical to Case 4a-5.

Case 4b-6: Suppose $J^2 > J_3$. This case is identical to Case 4a-6.

Collecting the equilibrium outcomes in (A51), (A53), (A55), and (A56), we conclude that (c, c) is always an NE for all $a > 0$; (c, c) is the unique Pareto-optimal NE if and only if one of the conditions in set $C\hat{3}$ holds; and (m, m) is the unique Pareto-optimal NE if and only if one of the conditions in set $C\hat{4}$ holds. This completes the proof of Theorem 2(iv). $\square$

Proof of Theorem 3. Under $c_2 > c_1$ and $D_1 = D_2 = D$, the following claims are straightforward.

Claim 8. Suppose $D_1 = D_2 = D$, $c_2 > c_1$, $\delta^\circ = \frac{2c_2(2D-r)(c_2D-rc_1)}{(2c_2D-rc_1)(2c_2D-rc_1-rc_2)}$, and $\bar{\delta} = \frac{(2D-r)(2c_2D-rc_1-rc_2)}{2(D-r)(2c_2D-rc_1)}$. Then, we have the following.

- (i) $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ for $\frac{\delta}{\delta_m} \leq \delta^\circ$; $\hat{J}_3 > \hat{J}_1 > \hat{J}_2$ or $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ for $\frac{\delta}{\delta_m} \in (\delta^\circ, \bar{\delta})$; and $\hat{J}_3 > \hat{J}_2 > \hat{J}_1$ for $\frac{\delta}{\delta_m} \geq \bar{\delta}$.
- (ii) $J_1 = \hat{J}_1$, $J_2 > \hat{J}_2$, and $J_3 > \hat{J}_3$.
- (iii) $J^2 > J_1 = \hat{J}_1$.

According to the values of $\frac{\delta}{\delta_m}$, three case arise.

Case 1: Suppose $\frac{\delta}{\delta_m} \leq \delta^\circ$. We then have $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$. Since $D > 1$ and $c_2 > c_1$, we have $2c_2D > 2rc_2 > rc_1 + rc_2 > rc_2 > rc_1$, which give rise to the following four cases.

Case 1a: Suppose $2c_1D \in (rc_1, rc_2]$. Then $J_1 > J_2$ by Claim 1(i); $J_1 > J_3$ by Claim 2(i); and $J_2 > J_3$ by Claim 3(i). Together with Claim 8(i)-8(ii), we obtain $\hat{J}_3 < J_3 < J_2 < J_1 = \hat{J}_1$ and $\hat{J}_2 < J_2$. Since $J^2 > J_1$ by Claim 8(iii), the equilibrium profit rankings of the two ports are (S4) from (A16) and (S $\hat{6}$) from (A38), which imply the following equilibrium outcomes.

$$\begin{cases} (c, c) \text{ cannot be an NE for all } a > 0; \\ (m, m) \text{ cannot be an NE unless } \min\{\pi_1^m, \pi_1^n\} = \pi_1^n. \end{cases} \quad (\text{A57})$$

Case 1b: Suppose $2c_1D \in (rc_2, rc_1 + rc_2]$. Then $J_1 > J_2$ by Claim 1(ii); $J_1 > J_3$ by Claim 2(i); and $J_2 < J_3$ by Claim 3(ii). Combining these with Claim 8(i)-8(ii), we obtain $\hat{J}_3 < \hat{J}_2 < J_2 < J_3 < J_1 = \hat{J}_1$. Since $J^2 > J_1$ by Claim 8(iii), the equilibrium profit rankings of the two ports are (S7) in (A19) and (S $\hat{6}$) in (A38), with the equilibrium outcomes given in (A56), except that $-\frac{T_3}{T_2}$ is replaced by $\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$.

Case 1c: Suppose $2c_1D \in (rc_1 + rc_2, 2rc_2]$. Then, we have an uncertain ordering between J_1 and J_2 by Claim 1(iii), $J_1 > J_3$ by Claim 2(i), and $J_2 > J_3$ by Claim 3(iii). Since the relative ordering of J_1 and J_2 depends on the relative sizes of δ and δ_1 , three sub-cases arise.

Case 1c-1: Suppose $\delta < \delta_1$. Then $J_1 > J_2$, and hence $\hat{J}_3 < J_3 < J_2 < J_1 = \hat{J}_1$, and $\hat{J}_2 < J_2$. Since $J^2 > J_1$ by Claim 8(iii), the equilibrium profit rankings of the two ports are (S7) and (S $\hat{6}$). These equilibrium outcomes are given in (A56), except that $-\frac{T_3}{T_2}$ is replaced by $a_1 = \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$.

Case 1c-2: Suppose $\delta = \delta_1$. Then $J_1 = J_2$, and hence $\hat{J}_3 < J_3 < J_1 = J_2 = \hat{J}_1$, and $\hat{J}_2 < J_2$. Since $J^2 > J_1$ by Claim 8(iii), the equilibrium profit rankings of the two ports are (S7) and (S6). These equilibrium outcomes are given in (A56), except that $-\frac{T_3}{T_2}$ is replaced by $\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$.

Case 1c-3: Suppose $\delta > \delta_1$. Then $J_2 > J_1$, which implies $\hat{J}_3 < J_3 < J_1 < J_2$ and $\hat{J}_2 < J_1$. Two sub-cases arise.

Case 1c-3-1: Suppose $J^2 \in (J_1, J_2]$. The equilibrium profit rankings of the two ports are (S7) and (S6) if $T_1 < 0$, $T_2 < 0$, and $T_3 > 0$; (S6) and (S6) if $T_1 = 0$, $T_2 < 0$, and $T_3 > 0$; and (S9) and (S6) if $T_1 < 0$, $T_2 = 0$, and $T_3 > 0$. In the first case, the equilibrium outcomes are those given in (A56) but replacing $-\frac{T_3}{T_2}$ with $a_1 = \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$. In the second case, (S6) in (A18) and (S6) in (A38) imply that the equilibrium outcomes coincide with those in (A56). In the third case, (S9) in (A21) and (S6) in (A38) imply that the equilibrium outcomes are the same as those in (A56), except that $-\frac{T_3}{T_2}$ is replaced with $\sqrt{\frac{-T_3}{T_1}}$.

Case 1c-3-2: Suppose $J^2 > J_2$. The equilibrium profit rankings of both profits are (S7) and (S6), and hence the equilibrium outcomes are those given in (A56) but replacing $-\frac{T_3}{T_2}$ with $a_1 = \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$.

Case 1d: Suppose $2c_1D > 2rc_2$. Claims 1(iii), 2(ii), and 3(iv) imply that the relative sizes of J_1 , J_2 , and J_3 depend on the relative sizes of δ_1 , δ_2 , and δ_3 . Thus, five sub-cases arise.

Case 1d-1: Suppose $\delta_2 < \delta_3 < \delta_1$ with $\delta \leq \delta_2$. Then $J_3 < J_2 < J_1$, which implies $\hat{J}_3 < J_3 < J_2 < J_1 = \hat{J}_1$ and $\hat{J}_2 < J_2$. Since $J^2 > J_1$ by Claim 8(iii), the equilibrium profit rankings of the two ports are (S12) and (S6). Unlike (A47-1), which presents the equilibrium outcomes of (S12) and (S6) derived under $T_2^2 > 4T_1T_3$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, these two conditions may not always hold here. Depending on whether they do, there are four possibilities. First, if $T_2^2 > 4T_1T_3$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, the equilibrium outcomes are those given in (A47-1). Second, if $T_2^2 > 4T_1T_3$ and $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, the equilibrium outcomes are

$$\begin{cases} (c, c) \text{ is an NE for all } a > 0, \text{ but never Pareto-optimal;} \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \in (0, a_2] \cup [a_1, \infty). \end{cases} \quad (\text{A47-2})$$

Third, if $T_2^2 \leq 4T_1T_3$ and $\hat{T}_2^2 > 4\hat{T}_1\hat{T}_3$, the equilibrium outcomes are

$$\begin{cases} (c, c) \text{ is an NE for all } a > 0, \text{ but never Pareto-optimal;} \\ (m, m) \text{ is the unique Pareto-optimal NE iff } a \in (0, \hat{a}_2] \cup [\hat{a}_1, \infty). \end{cases} \quad (\text{A47-3})$$

Fourth, if $T_2^2 \leq 4T_1T_3$ and $\hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3$, the equilibrium outcomes are

$$\begin{cases} (c, c) \text{ is an NE for all } a > 0, \text{ but never Pareto-optimal;} \\ (m, m) \text{ is the unique Pareto-optimal NE for all } a > 0. \end{cases} \quad (\text{A47-4})$$

Case 1d-2: Suppose $\delta_2 < \delta_3 < \delta_1$ with $\delta \in (\delta_2, \delta_3]$. Then $J_3 \leq J_2 < J_1$, which implies $\hat{J}_3 < J_3 < J_2 < J_1 = \hat{J}_1$ and $\hat{J}_2 < J_2$. Whether $J_3 < J_2$ or $J_3 = J_2$, the equilibrium profit rankings of the two ports remain (S12) and (S $\hat{6}$) by $J^2 > J_1$. Thus, the equilibrium outcomes are identical to those in (A47-1)-(A47-4).

Case 1d-3: Suppose $\delta_2 < \delta_3 < \delta_1$, $\delta \in (\delta_3, \delta_1]$, and $J_2 < J_3 < J_1$. Then $J_3 < \hat{J}_2 < J_2 < J_3 < J_1 = \hat{J}_1$. Again, since $J^2 > J_1$, the equilibrium profit rankings of the two ports are (S12) and (S $\hat{6}$). Hence, the equilibrium outcomes are also identical to those in (A47-1)-(A47-4).

Case 1d-4: Suppose $\delta_2 < \delta_3 < \delta_1$, $\delta \in (\delta_3, \delta_1]$, and $J_1 = J_2 < J_3$. Then $\hat{J}_3 < \hat{J}_2 < J_1 = J_2 < J_3$. Since $J^2 > J_1$, two sub-cases arise.

Case 1d-4-1: Suppose $J^2 \in (J_2, J_3]$. The equilibrium profit rankings of the two ports are (S7) and (S $\hat{6}$) if $T_1 < 0$, $T_2 > 0$, and $T_3 > 0$; and (S13) and (S $\hat{6}$) if $T_1 < 0$, $T_2 > 0$, and $T_3 = 0$. In the former situation, the equilibrium outcomes are those given in (A56) but replacing $-\frac{T_3}{T_2}$ with $a_1 = \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$. In the latter situation, (S13) in (A25) and (S $\hat{6}$) in (A38) imply that the equilibrium outcomes are the same as those given in (A56), except that $-\frac{T_3}{T_2}$ is replaced with $\frac{-T_2}{T_1}$.

Case 1d-4-2: Suppose $J^2 > J_3$. The equilibrium profit rankings of the two ports are (S12) and (S $\hat{6}$) if $T_1 < 0$, $T_2 > 0$, and $T_3 < 0$. Accordingly, the equilibrium outcomes are those given in (A47-1)-(A47-4).

Case 1e: Suppose $\delta_2 < \delta_3 < \delta_1$ and $\delta > \delta_1$. Then $J_1 < J_2 < J_3$, and hence $\hat{J}_3 < \hat{J}_2 < J_1 = \hat{J}_1 < J_2 < J_3$. Since $J^2 > J_1$, there are three sub-cases.

Case 1e-1: Suppose $J^2 \in (J_1, J_2]$. The equilibrium profit rankings of the two ports are (S7) and (S $\hat{6}$) if $T_1 < 0$, $T_2 < 0$, and $T_3 > 0$; and (S9) and (S $\hat{6}$) if $T_1 < 0$, $T_2 = 0$, and $T_3 > 0$. In the former situation, the equilibrium outcomes are those given in (A56) but replacing $-\frac{T_3}{T_2}$ with $\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$. In the latter situation, (S9) in (A21) and (S $\hat{6}$) in (A38) imply that the equilibrium outcomes are the same as those given in (A56), except that $-\frac{T_3}{T_2}$ is replaced with $\sqrt{\frac{-T_3}{T_1}}$.

Case 1e-2: Suppose $J^2 \in (J_2, J_3]$. The equilibrium profit rankings of the two ports are (S7) and (S $\hat{6}$) if $T_1 < 0$, $T_2 > 0$, and $T_3 > 0$; and (S13) and (S $\hat{6}$) if $T_1 < 0$, $T_2 > 0$, and $T_3 = 0$. In the former situation, the equilibrium outcomes are those given in (A56) but replacing $-\frac{T_3}{T_2}$ with $a_1 = \frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$. In the latter situation, the equilibrium outcomes are the same as those given in (A56), except that $-\frac{T_3}{T_2}$ is replaced with $\frac{-T_2}{T_1}$.

Case 1e-3: Suppose $J^2 > J_3$. The equilibrium profit rankings of the two ports are (S12) and (S $\hat{6}$) if $T_1 < 0$, $T_2 > 0$, and $T_3 < 0$. Hence, the equilibrium outcomes are those given in (A47-1)-(A47-4).

Finally, the equilibrium outcomes in (A47-1)-(A47-4), (A56), and (A57) together imply that for $2c_1D \in (rc_1, rc_2]$, (c, c) cannot be an NE and neither is (m, m) unless $\min\{\pi_1^m, \pi_1^n\} = \pi_1^n$. In contrast, for $2c_1D > rc_2$, (c, c) is always an NE; (c, c) is the unique Pareto-optimal NE iff one of the conditions given in set C5 holds; and (m, m) is the unique Pareto-optimal NE iff one of the conditions given in set C6 holds. The conditions in sets C5 and C6 are provided in Appendix A. These results establish Theorem 3(i).

Case 2: Suppose $\frac{\delta}{\delta_m} \in (\hat{\delta}, \bar{\delta})$. Then either $\hat{J}_3 > \hat{J}_1 = J_1 > \hat{J}_2$ or $\hat{J}_1 > \hat{J}_3 > \hat{J}_2$. Since the former ordering admits no solution, we consider only the latter. Therefore, seven sub-cases arise.

Case 2a: Suppose $\hat{J}_2 < J_2 < \hat{J}_3 < J_3 < J_1 = \hat{J}_1$. Since $J^2 > J_1$ port 1's equilibrium profit ranking is (S7). Moreover, because $\hat{J}_2 < \hat{J}_3 < \hat{J}_1$ and $J^2 > \hat{J}_1$, port 2's equilibrium profit ranking is (S $\hat{6}$). Combining (S7) with (S $\hat{6}$) yields the equilibrium outcomes in (A56).

Case 2b: Suppose $\hat{J}_2 < J_2 < \hat{J}_3 < J_1 = \hat{J}_1 < J_3$. Since $J^2 > J_1$, two sub-cases arise.

Case 2b-1: Suppose $J^2 \in (J_1, J_3]$. Port 1's equilibrium profit ranking is either (S7) or (S13), whereas port 2's equilibrium profit ranking is (S6) because $J^2 > \hat{J}_1$. Combining

(S7) with (S $\hat{6}$) yields the equilibrium outcomes in (A56). The same outcomes are obtained by combining (S13) with (S $\hat{6}$).

Case 2b-2: Suppose $J^2 > J_3$. Port 1's equilibrium profit ranking is (S12), whereas port 2's equilibrium profit ranking is (S $\hat{6}$) because $J^2 > \hat{J}_1$. Combining (S12) with (S1 $\hat{2}$) yields the equilibrium outcomes in (A47-1)-(A47-4).

Case 2c: Suppose $\hat{J}_2 < \hat{J}_3 < J_2 < J_3 < J_1 = \hat{J}_1$. This case is identical to Case 2a.

Case 2d: Suppose $\hat{J}_2 < \hat{J}_3 < J_2 < J_1 = \hat{J}_1 < J_3$. This case is identical to Case 2b.

Case 2e: Suppose $\hat{J}_2 < \hat{J}_3 < J_1 = \hat{J}_1 < J_2 < J_3$. Two sub-cases arise.

Case 2e-1: Suppose $J^2 \in (J_1, J_2]$. Port 1's equilibrium profit ranking is either (S7) or (S13), whereas port 2's equilibrium profit ranking is (S $\hat{6}$) because $J^2 > \hat{J}_1$. Combining (S7) with (S $\hat{6}$) yields the equilibrium outcomes in (A56), except that $-\frac{T_3}{T_2}$ is replaced by $\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$. Combining (S9) with (S $\hat{6}$) yields the equilibrium outcomes in (A56), except that $-\frac{T_3}{T_2}$ is replaced by $\sqrt{\frac{-T_3}{T_2}}$.

Case 2e-2: Suppose $J^2 > J_2$. This case is identical to Case 2b.

Case 2f: Suppose $\hat{J}_2 < \hat{J}_3 < J_1 = \hat{J}_1 < J_3 < J_2$. This ordering admits no solutions.

Case 2g: Suppose $\hat{J}_2 < \hat{J}_3 < J_3 < J_2 < \hat{J}_1 = J_2$. Port 1's equilibrium profit ranking is (S4), (S7) or (S12), whereas port 2's equilibrium profit ranking is (S $\hat{6}$) because $J^2 > \hat{J}_1$. Combining (S4) with (S $\hat{6}$) yields the equilibrium outcomes in (A57). Combining (S7) with (S $\hat{6}$) yields the equilibrium outcomes in (A56), except that $-\frac{T_3}{T_2}$ is replaced by $\frac{-T_2 - \sqrt{T_2^2 - 4T_1T_3}}{2T_1} > 0$. Combining (S12) with (S $\hat{6}$) yields the equilibrium outcomes in (A47-1)-(A47-4).

Collecting the equilibrium outcomes in (A47-1)-(A47-4) and (A56)-(A57), we conclude that for $2c_1D \in (rc_1, rc_2]$, (c, c) cannot be an NE and neither is (m, m) unless $\min\{\pi_1^m, \pi_1^n\} = \pi_1^n$. In contrast, for $2c_1D > rc_2$, (c, c) is always an NE; (c, c) is the unique Pareto-optimal NE if and only if one of the conditions in set C $\hat{5}$ holds; and (m, m) is the unique Pareto-optimal NE if and only if one of the conditions in set C $\hat{6}$ holds. The conditions defining C $\hat{5}$ and C $\hat{6}$ are provided in Appendix A. This establishes Theorem

3(ii).

Case 3: Suppose $\frac{\delta}{\delta_m} \geq \bar{\delta}$. Then $\hat{J}_3 > \hat{J}_2 > \hat{J}_1 = J_1$ by Claims 8(i) and 8(ii). Moreover, for $2c_1D \in (rc_1, rc_2]$, we have $J_1 > J_2 > J_3$ and $J^2 > J_1 = \hat{J}_1$ by Claim 1(i), Claim 2(i), Claim 3(i), and Claim 8(iii) as in Case 1a. Hence, both ports' equilibrium profit rankings are S4 and S6, and hence the equilibrium outcomes are in (A57). However, for $2c_1D > rc_2]$, the relative ordering between J_2 and $\hat{J}_3$, as well as that between J_2 and J_3 , cannot be determined, three sub-cases arise.

Case 3a: Suppose $J_1 = \hat{J}_1 < \hat{J}_2 < J_2 < \hat{J}_3 < J_3$. Since $J^2 > J_1 = \hat{J}_1$, five sub-cases arise.

Case 3a-1: Suppose $J^2 \in (J_1, \hat{J}_2]$. This case is identical to Case 3a-2 of Theorem 2.

Case 3a-2: Suppose $J^2 \in (\hat{J}_2, J_2]$. This case is identical to Case 3a-4 of Theorem 2.

Case 3a-3: Suppose $J^2 \in (J_2, \hat{J}_3]$. This case is identical to Case 4a-5 of Theorem 2.

Case 3a-4: Suppose $J^2 \in (\hat{J}_3, J_3]$. This case is identical to Case 3a-4 of Theorem 2.

Case 3a-5: Suppose $J^2 > J_3]$. This case is identical to Case 3a-6 of Theorem 2.

Case 3b: Suppose $J_1 = \hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_2 < J_3$. Since $J^2 > J_1 = \hat{J}_1$, five sub-cases arise.

Case 3b-1: Suppose $J^2 \in (J_1, \hat{J}_2]$. This case is identical to Case 3a-2 of Theorem 2.

Case 3b-2: Suppose $J^2 \in (\hat{J}_2, \hat{J}_3]$. This case is identical to Case 3a-4 of Theorem 2.

Case 3b-3: Suppose $J^2 \in (\hat{J}_3, J_2]$. This case is identical to Case 4a-4 of Theorem 2.

Case 3b-4: Suppose $J^2 \in (J_2, J_3]$. This case is identical to Case 4a-5 of Theorem 2.

Case 3b-5: Suppose $J^2 > J_3$. This case is identical to Case 3a-6 of Theorem 2.

Case 3c: Suppose $J_1 = \hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_3 < J_2$. This ordering admits no solution.

Collecting the equilibrium outcomes in (A47-1)-(47-4), (A51), (A53), and (A56), we conclude that for $2c_1D \in (rc_1, rc_2]$, (c, c) cannot be an NE and neither is (m, m) unless $\min\{\pi_1^m, \pi_1^n\} = \pi_1^n$. In contrast, for $2c_1D > rc_2$, (c, c) is always an NE; (c, c) is the unique Pareto-optimal NE if and only if one of the conditions in set C5 holds; and (m, m)

is the unique Pareto-optimal NE if and only if one of the conditions in set $\hat{C}\hat{6}$ holds. The conditions defining $\hat{C}\hat{5}$ and $\hat{C}\hat{6}$ are provided in Appendix A. This completes the proof of Theorem 3(iii). $\square$

Proof of Theorem 4. Under $c_2 > c_1$ and $D_1 > D_2$, we can establish the following results.

Claim 9. Suppose $c_2 > c_1$, $D_1 > D_2$, $\delta|_{D=D_1} = \frac{2c_2(2D_1-r)(c_2D_1-rc_1)}{(2c_2D_1-rc_1)(2c_2D_1-rc_1-rc_2)}$, $\bar{\delta}|_{D=D_1} = \frac{(2D_1-r)(2c_2D_1-rc_1-rc_2)}{2(D_1-r)(2c_2D_1-rc_1)}$, $\check{\delta}_1 = \frac{D_1(2D_2-r)^2(2c_2D_1-rc_1-rc_2)}{2D_2(2D_1-r)(D_2-r)(2c_2D_1-rc_1)} > 0$, $\check{\delta}_2 = \frac{2c_1D_2(2D_1-r)(2c_2D_1-rc_1)(c_1D_2-rc_2)}{D_1(2c_1D_2-rc_2)^2(2c_2D_1-rc_1-rc_2)}$, and $\check{\delta}_3 = \frac{2c_2D_1(2D_2-r)(2c_1D_2-rc_2)(c_2D_1-rc_1)}{D_2(2c_2D_1-rc_1)^2(2c_1D_2-rc_1-rc_2)}$ with $\delta|_{D=D_1} < \bar{\delta}|_{D=D_1}$. Then, we have the following.

- (i) $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ for $\frac{\delta}{\delta_m} \leq \delta|_{D=D_1}$; $\hat{J}_3 > \hat{J}_1 > \hat{J}_2$ or $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ for $\frac{\delta}{\delta_m} \in (\delta|_{D=D_1}, \bar{\delta}|_{D=D_1})$; and $\hat{J}_3 > \hat{J}_2 > \hat{J}_1$ for $\frac{\delta}{\delta_m} \geq \bar{\delta}|_{D=D_1}$.
- (ii) $J_1 > \hat{J}_1$.
- (iii) $J_1 \geq (\leq) \hat{J}_2$ iff $\frac{\delta}{\delta_m} \leq (\geq) \check{\delta}_1$.
- (iv) $J_2 < \hat{J}_2$ if $2c_1D_2 \in (rc_2, rc_1+rc_2)$; otherwise, the relative sizes of J_2 and $\hat{J}_2$ are indeterminate.
- (v) $\hat{J}_3 > J_3$ if $2c_1D_2 \in (rc_1, 2rc_2]$; otherwise, the relative sizes of $\hat{J}_3$ and J_3 are indeterminate.
- (vi) $\hat{J}_2 > J_3$ for $2c_1D_2 \in (rc_1, 2rc_2]$, and $\hat{J}_2 \geq (\leq) J_3$ iff $\frac{\delta}{\delta_m} \leq (\geq) \check{\delta}_2$.
- (vii) $\hat{J}_3 > J_2$ for $2c_1D_2 \in (rc_2, rc_1+rc_2)$; and $\hat{J}_3 \geq (\leq) J_2$ iff $\frac{\delta}{\delta_m} \geq (\leq) \check{\delta}_3$.
- (viii) $J^2 > \hat{J}_1$.

Since there exists no solution for $\frac{\delta}{\delta_m} \in (\delta|_{D=D_1}, \bar{\delta}|_{D=D_1})$ with $\hat{J}_3 > \hat{J}_1 > \hat{J}_2$, there are three cases as follows.

Case 1: Suppose $\frac{\delta}{\delta_m} \leq \delta|_{D=D_1}$. Then $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 9(i). Since $rc_1 < rc_2 < rc_1 + rc_2 < 2rc_2 < 2c_2D_1$ and $2c_1D_2 < 2c_2D_1$ by $c_2 > c_1$ and $D_1 > D_2 > 1 > r > 0$, four possible cases arise.

Case 1a: Suppose $2c_1D_2 \in (rc_1, rc_2]$. Then $J_1 > J_2 > J_3$ by Claim 1(i) and Claim 3(i); $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 9(i); $J_3 < \hat{J}_3$ by Claim 9(vi); $\hat{J}_2 > J_3$ by Claim 9(v); and $J_1 > \hat{J}_1$ by Claim 9(ii). These imply $J_3 < \hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_1$, while the relative ordering between $\hat{J}_1$ and J_2 remains uncertain. Thus, two sub-cases arise.

Case 1a-1: Suppose $J_3 < \hat{J}_3 < \hat{J}_2 < J_2 < \hat{J}_1 < J_1$. Since $J^2 > \hat{J}_1$ by Claim 9(viii), two sub-cases arise.

Case 1a-1-1: Suppose $J^2 \in (\hat{J}_1, J_1]$. The equilibrium profit rankings of the two ports are

(S2) and (S $\hat{6}$) if $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$; (S3) and (S $\hat{6}$) if $T_1 = 0$, $T_2 < 0$, and $T_3 > 0$; and (S1) and (S $\hat{6}$) if $T_1 > 0$, $T_2 = 0$, and $T_3 > 0$. In the first case, (S2) in (A14) and (S $\hat{6}$) in (A38) imply the following equilibrium outcomes.

$$\begin{cases} (c, c) \text{ cannot be an NE for all } a > 0; \\ (m, m) \text{ cannot be a (Pareto-optimal) NE unless } \max\{\pi_1^m, \pi_1^n\} = \pi_1^m. \end{cases} \quad (\text{A58})$$

In the second case, (S3) in (A15) and (S $\hat{6}$) in (A38) yield the same equilibrium outcomes as those in (A58).¹ In the third case, (S1) in (A13) and (S $\hat{6}$) in (A38) imply the following equilibrium outcomes.

$$\begin{cases} (c, c) \text{ cannot be an NE for all } a > 0; \\ (m, m) \text{ cannot be an NE for all } a > 0. \end{cases} \quad (\text{A59})$$

Case 1a-1-2: Suppose $J^2 > J_1$. The equilibrium profit rankings of the two ports are (S4) and (S $\hat{6}$), leading to the equilibrium outcomes given in (A57).

Case 1a-2: Suppose $J_3 < \hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_2 < J_1$. Since $J^2 > \hat{J}_1$ by Claim 9(viii), three sub-cases arise.

Case 1a-2-1: Suppose $J^2 \in (\hat{J}_1, J_2]$. The equilibrium profit rankings of the two ports are (S1) and (S $\hat{6}$), yielding the equilibrium outcomes in (A59).

Case 1a-2-2: Suppose $J^2 \in (J_2, J_1]$. The results coincide with those in Case 1a-1-1.

Case 1a-2-3: Suppose $J^2 > J_1$. The results coincide with those in Case 1a-1-2.

Case 1b: Suppose $2c_1D_2 \in (rc_2, rc_1 + rc_2]$. Then $J_1 > J_2$ by Claim 1(ii); $J_1 > J_3$ by Claim 2(ii); $J_2 < J_3$ by Claim 3(ii); $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 9(i); $J_1 > \hat{J}_1$ by Claim 9(ii); $J_2 < \hat{J}_2$ by Claim 9(iv); $\hat{J}_2 > J_3$ by Claim 9(v); $\hat{J}_3 > J_3$ by Claim 9(vi); and $\hat{J}_3 > J_2$ by Claim 9(vii). Together these imply $J_2 < J_3 < \hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_1$. Since $J^2 > \hat{J}_1$ by Claim 9(viii), two sub-cases arise.

Case 1b-1: Suppose $J^2 \in (\hat{J}_1, J_1]$. The equilibrium profit rankings of the two ports are (S5) and (S $\hat{6}$). Then (S5) in (A17) and (S $\hat{6}$) in (A38) imply the following equilibrium

¹Once (m, m) is an NE, the conditions for it to be Pareto-optimal implied by (S2) and (S $\hat{6}$) differ from those implied by (S3) and (S $\hat{6}$).

outcomes.

$$\left\{ \begin{array}{l}
 (c, c) \text{ is an NE for all } a > 0. \\
 (c, c) \text{ is the unique Pareto-optimal NE iff one of the following holds:} \\
 \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, \hat{a}_2 < \hat{a}_1 < a_1 < a_2, a \in (\hat{a}_2, \hat{a}_1); \\
 \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, \hat{a}_2 < a_1 < \hat{a}_1 < a_2, a \in (\hat{a}_2, a_1); \\
 \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a_1 < \hat{a}_2 < a_2 < \hat{a}_1, a \in (a_2, \hat{a}_1); \\
 \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a_1 < a_2 < \hat{a}_2 < \hat{a}_1, a \in (\hat{a}_2, \hat{a}_1); \\
 \quad T_2^2 \leq 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a \in (\hat{a}_2, \hat{a}_1). \\
 (m, m) \text{ is the unique Pareto-optimal NE iff one of the following holds:} \\
 \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, \hat{a}_2 < \hat{a}_1 < a_1 < a_2, a \in [a_1, a_2]; \\
 \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, \hat{a}_2 < a_1 < \hat{a}_1 < a_2, a \in [\hat{a}_1, a_2]; \\
 \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a_1 < \hat{a}_2 < \hat{a}_1 < a_2, a \in [a_1, \hat{a}_2] \cup [\hat{a}_1, a_2]; \\
 \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a_1 < \hat{a}_2 < a_2 < \hat{a}_1, a \in [a_1, \hat{a}_2]; \\
 \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a_1 < a_2 < \hat{a}_2 < \hat{a}_1, a \in [a_1, a_2]; \\
 \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3, a \in [a_1, a_2].
 \end{array} \right. \quad (\text{A60})$$

Case 1b-2: Suppose $J^2 > J_1$. The equilibrium profit rankings of the two ports are (S7) and (S6), and the resulting equilibrium outcomes are those given in (A56).

Case 1c: Suppose $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$. Then, three sub-cases arise.

Case 1c-1: Suppose $\delta < \delta_1$. Then $J_1 > J_2$ by Claim 1(iii) and $\delta < \delta_1$; $J_1 > J_3$ by Claim 2(i); $J_2 > J_3$ by Claim 3(iii); $J_1 > \hat{J}_1$ by Claim 9(ii); $\hat{J}_3 > J_3$ by Claim 9(vi); and $\hat{J}_2 > J_3$ by Claim 9(v). According to the relative sizes of J_2 and $\hat{J}_1$, there are two possibilities.

Case 1c-1-1: Suppose $J_3 < \hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_1$ and $J_2 < \hat{J}_1$. Since $J^2 > \hat{J}_1$ by Claim 9(viii), two sub-cases arise.

Case 1c-1-1-1: Suppose $J^2 \in (\hat{J}_1, J_1]$. The equilibrium profit rankings of the two ports are (S8) and (S6). Then (S8) in (A20) and (S6) in (A38) imply the following equilibrium outcomes.

$$\left\{ \begin{array}{l}
 (c, c) \text{ is an NE for all } a > 0; \\
 (c, c) \text{ is the unique Pareto-optimal NE iff } \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3 \text{ and } a \in (\hat{a}_2, \hat{a}_1); \\
 (m, m) \text{ cannot be a (Pareto-optimal) NE.}
 \end{array} \right. \quad (\text{A61})$$

Case 1c-1-1-2: Suppose $J^2 > J_1$. The equilibrium profit rankings of the two ports are (S7) and (S $\hat{6}$), and the corresponding equilibrium outcomes are those given in (A56).

Case 1c-1-2: Suppose $J_3 < \hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_2 < J_1$. Four sub-cases arise.

Case 1c-1-2-1: Suppose $J^2 \in (\hat{J}_1, J_2]$. The equilibrium profit rankings of the two ports are (S5) and (S $\hat{6}$), with equilibrium outcomes given in (A60).

Case 1c-1-2-2: Suppose $J^2 \in (J_2, J_1]$. The equilibrium profit rankings of the two ports are (S8) and (S $\hat{6}$), with equilibrium outcomes given in (A61).

Case 1c-1-2-3: Suppose $J^2 > J_1$. The equilibrium profit rankings of the two ports are (S7) and (S $\hat{6}$), with equilibrium outcomes given in (A56).

Case 1c-2: Suppose $\delta = \delta_1$. Then, by (A8), we have $J_1 = J_2$, and hence $J_3 < \hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_2 = J_1$, as argued in Case 1c-1. There are two sub-cases.

Case 1c-2-1: Suppose $J^2 \in (\hat{J}_1, J_2]$. The equilibrium profit rankings of the two ports are (S5) and (S $\hat{6}$), with equilibrium outcomes given in (A60).

Case 1c-2-2: Suppose $J^2 > J_1$. The equilibrium profit rankings of the two ports are (S7) and (S $\hat{6}$), with equilibrium outcomes given in (A56).

Case 1c-3: Suppose $\delta > \delta_1$. Then, by (A8), we have $J_1 < J_2$, and hence $J_3 < \hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_1 < J_2$. There are three sub-cases.

Case 1c-3-1: Suppose $J^2 \in (\hat{J}_1, J_1]$. The equilibrium profit rankings of the two ports are (S5) and (S $\hat{6}$), with equilibrium outcomes given in (A60).

Case 1c-3-2: Suppose $J^2 \in (J_1, J_2]$. The equilibrium profit rankings of the two ports are (S7) and (S $\hat{6}$) if $T_1 < 0$, $T_2 < 0$, and $T_3 > 0$; (S6) and (S $\hat{6}$) if $T_1 = 0$, $T_2 < 0$, and $T_3 > 0$; and (S9) and (S $\hat{6}$) if $T_1 < 0$, $T_2 = 0$, and $T_3 > 0$. In the first case, the equilibrium outcomes are those in (A56); in the second case, the outcomes are those in (A56) but with a_1 replaced by $\frac{-T_3}{T_2}$; and in the third case, the outcomes are those in (A56) but with a_1 replaced by $\sqrt{\frac{-T_3}{T_1}}$.

Case 1c-3-3: Suppose $J^2 > J_2$. The equilibrium profit rankings of the two ports are (S7) and (S $\hat{6}$), with equilibrium outcomes given in (A56).

Case 1d: Suppose $2c_1D_2 \in (2rc_2, 2c_2D_1]$. We have $\delta_2 < \delta_3 < \delta_1$ since $c_1 \neq c_2$, by some calculations. Thus, four sub-cases arise.

Case 1d-1: Suppose $\delta \leq \delta_3$. Then $J_3 < J_2 < J_1$ by (A8)-(A10); $J_1 > \hat{J}_1$ by Claim 9(ii); and $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 9(i). However, the relative sizes of the other parameters are indeterminate. Hence, six possibilities arise.

Case 1d-1-1: Suppose $J_3 < \hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_1$ and $J_2 < \hat{J}_1$. Two sub-cases follow.

Case 1d-1-1-1: Suppose $J^2 \in (\hat{J}_1, J_1]$. The equilibrium profit rankings of the two ports are (S10) and (S6) if $T_1 > 0$, $T_2 > 0$, and $T_3 < 0$; and (S11) and (S6) if $T_1 = 0$, $T_2 > 0$, and $T_3 < 0$. In the former case, (S10) in (A22) and (S6) in (A38) imply that the following equilibrium outcomes:

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3 \text{ and } a \in (\max\{a_2, \hat{a}_2\}, \hat{a}_1); \\ (m, m) \text{ is the unique Pareto-optimal NE iff one of the following holds:} \\ \quad \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3 \text{ and } a \in (0, \max\{a_2, \hat{a}_2\}], \\ \quad \hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3 \text{ and } a \in (0, a_2]. \end{array} \right. \quad (\text{A62})$$

In the latter case, the equilibrium outcomes of (S11) and (S6) are the same as those in (A62), except that a_2 is replaced by $\frac{-T_3}{T_2}$.

Case 1d-1-1-2: Suppose $J^2 > J_1$. The equilibrium profit rankings of the two ports are (S12) and (S6), and the corresponding equilibrium outcomes are those given in (A47-1)-(A47-4).

Case 1d-1-2: Suppose $J_3 < \hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_2 < J_1$. Three sub-cases arise.

Case 1d-1-2-1: Suppose $J^2 \in (\hat{J}_1, J_2]$. The equilibrium profit rankings of the two ports are (S10) and (S6) if $T_1 > 0$, $T_2 \leq 0$, and $T_3 \leq 0$; and (S8) and (S6) if $T_1 > 0$, $T_2 = 0$, and $T_3 = 0$. The equilibrium outcomes in the former case are those given in (A62), while the equilibrium outcomes in the latter case are those given in (A61).

Case 1d-1-2-2: Suppose $J^2 \in (J_2, J_1]$. The equilibrium profit rankings of the two ports are (S10) and (S6) if $T_1 > 0$, $T_2 > 0$, and $T_3 < 0$; and (S11) and (S6) if $T_1 = 0$, $T_2 > 0$, and $T_3 < 0$. The equilibrium outcomes in the former case are those given in (A62), whereas

the equilibrium outcomes in the latter case are those given in (A62) with a_2 replaced by $\frac{-T_3}{T_2}$.

Case 1d-1-2-3: Suppose $J^2 > J_1$. The equilibrium profit rankings of the two ports are (S12) and (S6), with equilibrium outcomes given in (A47-1)-(A47-4).

Case 1d-1-3: Suppose $\hat{J}_3 < J_3 < \hat{J}_2 < \hat{J}_1 < J_1$ and $J_2 < \hat{J}_1$. The results are identical to those in Case 1d-1-1.

Case 1d-1-4: Suppose $\hat{J}_3 < J_3 < \hat{J}_2 < \hat{J}_1 < J_2 < J_1$. The results are identical to those in Case 1d-1-2.

Case 1d-1-5: Suppose $\hat{J}_3 < \hat{J}_2 < J_3 < J_2 < \hat{J}_1 < J_1$. The results are identical to those in Case 1d-1-1.

Case 1d-1-6: Suppose $\hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_3 < J_2 < J_1$. Four sub-cases arise.

Case 1d-1-6-1: Suppose $J^2 \in (\hat{J}_1, J_3]$. The equilibrium profit rankings of the two ports are (S5) and (S6), with equilibrium outcomes given in (A60).

Case 1d-1-6-2: Suppose $J^2 \in (J_3, J_2]$. The results are identical to those in Case 1d-1-2-1.

Case 1d-1-6-3: Suppose $J^2 \in (J_2, J_1]$. The results are identical to those in Case 1d-1-1-1.

Case 1d-1-6-4: Suppose $J^2 > J_1$. The results are identical to those in Case 1d-1-1-2.

Case 1d-2: Suppose $\delta \in (\delta_3, \delta_1]$ and $J_2 < J_3 < J_1$. Then $J_1 > \hat{J}_1$ by (A8), and $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 9(i). The relative sizes of the remaining parameters are uncertain. Thus, three sub-cases arise.

Case 1d-2-1: Suppose $\hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_1$ and $J_2 < J_3 < \hat{J}_1$. Since $J^2 > \hat{J}_1$ by Claim 9(viii), two sub-cases arise.

Case 1d-2-1-1: Suppose $J^2 \in (\hat{J}_1, J_1]$. The equilibrium profit rankings of the two ports are (S10) and (S6) if $T_1 > 0$, $T_2 > 0$, and $T_3 < 0$; (S11) and (S6) if $T_1 = 0$, $T_2 > 0$, and $T_3 < 0$; and (S8) and (S6) if $T_1 > 0$, $T_2 > 0$, and $T_3 = 0$. The equilibrium outcomes for the first case are given in (A62); for the second case, they are as in (A62) with a_2 replaced by $\frac{-T_3}{T_2}$; and for the third case, they are those in (A61).

Case 1d-2-1-2: Suppose $J^2 > J_1$. The equilibrium profit rankings of the two ports are (S12) and (S6), with equilibrium outcomes given in (A47-1)-(A47-4).

Case 1d-2-2: Suppose $\hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_2 < J_3$. Since $J^2 > \hat{J}_1$, four sub-cases arise.

Case 1d-2-2-1: Suppose $J^2 \in (\hat{J}_1, J_2]$. The equilibrium profit rankings of the two ports are (S5) and (S6) if $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$; and (S8) and (S6) if $T_1 > 0$, $T_2 = 0$, and $T_3 > 0$. The equilibrium outcomes for the first case are given in (A60), while those for the second case are given in (A61).

Case 1d-2-2-2: Suppose $J^2 \in (J_2, J_3]$. The equilibrium profit rankings of the two ports are (S8) and (S6), with equilibrium outcomes given in (A61).

Case 1d-2-2-3: Suppose $J^2 \in (J_3, J_1]$. The results are identical to those in Case 1d-2-1-1.

Case 1d-2-2-4: Suppose $J^2 > J_1$. The results are identical to those in Case 1d-2-1-2.

Case 1d-2-3: Suppose $\hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_3 < J_1$ and $J_2 < \hat{J}_1$. Since $J^2 > \hat{J}_1$, three sub-cases arise.

Case 1d-2-3-1: Suppose $J^2 \in (\hat{J}_1, J_3]$. The results are identical to those in Case 1d-2-2-2.

Case 1d-2-3-2: Suppose $J^2 \in (J_3, J_1]$. The results are identical to those in Case 1d-2-2-3.

Case 1d-2-3-3: Suppose $J^2 > J_1$. The results are identical to those in Case 1d-2-2-4.

Case 1d-3: Suppose $\delta \in (\delta_3, \delta_1]$ and $J_1 = J_2 < J_3$. Then, $J_1 > \hat{J}_1$ by (A8) and $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 9(i), which together imply $\hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_1 = J_2 < J_3$. Since $J^2 > \hat{J}_1$, there are three sub-cases.

Case 1d-3-1: Suppose $J^2 \in (\hat{J}_1, J_1]$. The equilibrium profit rankings of the two ports are (S5) and (S6) if $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$; (S8) and (S6) if $T_1 > 0$, $T_2 = 0$, and $T_3 > 0$; and (S6) and (S6) if $T_1 = 0$, $T_2 < 0$, and $T_3 > 0$. The corresponding equilibrium outcomes are those in (A60) in the first situation, those in (A61) in the second situation, and those in (A56) with a_1 replaced by $\frac{-T_3}{T_2}$ in the third situation.

Case 1d-3-2: Suppose $J^2 \in (J_1, J_3]$. The equilibrium profit rankings of the two ports are (S7) and (S6) if $T_1 < 0$, $T_2 > 0$, and $T_3 > 0$; and (S13) and (S6) if $T_1 < 0$, $T_2 > 0$, and

$T_3 = 0$. The corresponding equilibrium outcomes are those in (A56) in the first situation, and those in (A56) with a_1 replaced by $\frac{-T_3}{T_1}$ in the second situation.

Case 1d-3-3: Suppose $J^2 > J_3$. The equilibrium profit rankings of the two ports are (S12) and (S6), with equilibrium outcomes given in (A47-1)-(A47-4).

Case 1d-4: Suppose $\delta > \delta_1$. Then $J_1 < J_2 < J_3$ by (A8)-(A10); $J_1 > \hat{J}_1$ by Claim 9(ii); and $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 9(i). Hence, $\hat{J}_3 < \hat{J}_2 < \hat{J}_1 < J_1 < J_2 < J_3$. Since $J^2 > \hat{J}_1$, there are four sub-cases.

Case 1d-4-1: Suppose $J^2 \in (\hat{J}_1, J_1]$. The equilibrium profit rankings of the two ports are (S5) and (S6) if $T_1 > 0$, $T_2 < 0$, and $T_3 > 0$; and (S6) and (S6) if $T_1 = 0$, $T_2 < 0$, and $T_3 > 0$. The equilibrium outcomes are those in (A60) in the first situation, and those in (A56) with a_1 replaced by $\frac{-T_3}{T_2}$ in the second situation.

Case 1d-4-2: Suppose $J^2 \in (J_1, J_2]$. The equilibrium profit rankings of the two ports are (S7) and (S6) if $T_1 < 0$, $T_2 < 0$, and $T_3 > 0$; and (S9) and (S6) if $T_1 < 0$, $T_2 = 0$, and $T_3 > 0$. The equilibrium outcomes are those in (A56) in the first situation, and those in (A56) with a_1 replaced by $\sqrt{\frac{-T_3}{T_1}}$ in the second situation.

Case 1d-4-3: Suppose $J^2 \in (J_2, J_3]$. All results are identical to those in Case 1d-3-2.

Case 1d-4-4: Suppose $J^2 > J_3$. All results are identical to those in Case 1d-3-3.

The equilibrium outcomes are those given in (A47-1)-(A47-4), (A56), and (A58)-(A62). These outcomes imply that (c, c) is not an NE whenever $2c_1D_2 \in (rc_1, rc_2]$, whereas (m, m) can be an NE only if $\max\{\pi_1^m, \pi_1^n\} = \pi_1^m$, as shown by Theorem 4(ia). Moreover, for $2c_1D_2 > rc_2$, (c, c) is always an NE; (c, c) is the unique Pareto-optimal NE if and only if one of the conditions in set C7 holds, as indicated by Theorem 4(ib); and (m, m) is the unique Pareto-optimal NE if and only if one of the conditions in set C8 holds, as indicated by Theorem 4(ic). Condition sets C7 and C8 are provided in Appendix A.

Since the analyses of Cases 2 and 3 are considerably more involved than that of Case 1, we present only the equilibrium outcomes together with the corresponding supporting conditions below. The detailed derivations are available upon request.

Case 2: Suppose $\frac{\delta}{\delta_m} \in (\bar{\delta}|_{D=D_1}, \bar{\delta}|_{D=D_1})$. By Claim 9(i), we have $\hat{J}_1 > \hat{J}_3 > \hat{J}_2$. The possible equilibrium outcomes are summarized as follows.

(i) The equilibrium profit rankings of the two ports can be (S1) and ($\hat{S}6$), with the equilibrium outcomes given in (A59), under one of the following conditions:

- (ia) $J_2, J_3, \hat{J}_2, \hat{J}_3 < \hat{J}_1 < J_1$, with $J^2 \in (\hat{J}_1, J_1]$;
- (ib) $\min\{J_3, \hat{J}_1\}, \hat{J}_2, \hat{J}_3 < \max\{J_3, \hat{J}_1\} < J_2 < J_1$, with $J^2 \in (\hat{J}_1, J_2]$.

(ii) The equilibrium profit rankings of the two ports can be (S2) and ($\hat{S}6$), with the equilibrium outcomes given in (A58), under either of the conditions stated in part (i).

(iii) The equilibrium profit rankings of the two ports can be (S3) and ($\hat{S}6$), with the equilibrium outcomes given in (A58), under either of the conditions stated in part (i).

(iv) The equilibrium profit rankings of the two ports can be (S4) and ($\hat{S}6$), with the equilibrium outcomes given in (A57), under one of the following conditions:

- (iva) $J_2, J_3, \hat{J}_2 < \hat{J}_3 < \hat{J}_1 < J_1$, with $J^2 > J_1$;
- (ivb) $J_3, \hat{J}_2, \hat{J}_3 < J_2 < \hat{J}_1 < J_1$, with $J^2 > J_1$;
- (ivc) $J_3, \hat{J}_1, \hat{J}_2, \hat{J}_3 < J_2 < J_1$, with $J^2 > J_1$.

(v) The equilibrium profit rankings of the two ports can be (S5) and ($\hat{S}6$), with the equilibrium outcomes given in (A60), under one of the following conditions:

- (va) $J_2, J_3, \hat{J}_1, \hat{J}_2, \hat{J}_3 < J_1$, with $J^2 > \hat{J}_1$;
- (vb) $\hat{J}_2 < \hat{J}_3 < \hat{J}_1 < J_2 < J_1 < J_3$, with $J^2 \in (\hat{J}_1, J_2]$;
- (vc) $J_3, \hat{J}_1, \hat{J}_2, \hat{J}_3 < J_1 < J_2$, with $J^2 \in (\hat{J}_1, J_1]$;
- (vd) $\hat{J}_2 < \hat{J}_3 < \hat{J}_1 < J_1 < J_2 < J_3$, with $J^2 \in (\hat{J}_1, J_1]$.

(vi) The equilibrium profit rankings of the two ports can be either (S6) and ($\hat{S}6$) or (S7) and ($\hat{S}6$), with the equilibrium outcomes given in (A56), under one of the following conditions:

- (via) $J_2, J_3, \hat{J}_1, \hat{J}_2, \hat{J}_3 < J_1$, with $J^2 > \hat{J}_1$;
- (vib) $\hat{J}_2 < J_2, \hat{J}_1, \hat{J}_3 < J_1 < J_3$, with $J^2 \in (J_1, J_3]$;
- (vic) $\hat{J}_2 < \hat{J}_3 < \hat{J}_1 < J_2 < J_3 < J_1$, with $J^2 > J_1$;
- (vid) $J_3, \hat{J}_1, \hat{J}_2, \hat{J}_3 < J_1 < J_2$, with $J^2 > J_1$.

(vii) The equilibrium profit rankings of the two ports can be (S9) and ($\hat{S}6$), with the

equilibrium outcomes given in (A56), under one of the following conditions:

- (viiia) $J_3, \hat{J}_1, \hat{J}_2, \hat{J}_3 < J_1 < J_2$, with $J^2 \in (J_1, J_2]$;
- (viiib) $\hat{J}_2 < \hat{J}_3 < \hat{J}_1 < J_1 < J_2 < J_3$, with $J^2 \in (J_1, J_2]$.

(viii) The equilibrium profit rankings of the two ports can be (S12) and (S6), with the equilibrium outcomes given in (A47-1)-(A47-4), if $J_3 > \max\{J_1, J_2, \hat{J}_1, \hat{J}_2, \hat{J}_3\}$, with $J^2 > J_3$.

(ix) The equilibrium profit rankings of the two ports can be (S13) and (S6), with the equilibrium outcomes given in (A56), if $\hat{J}_2 < \hat{J}_3 < J_1, J_2, \hat{J}_1 < J_3$, with $J^2 \in (\max\{J_1, J_2\}, J_3]$.

The corresponding equilibrium outcomes are given in (A47-1)-(A47-4) and (A56)-(A60). These outcomes imply that (c, c) is not an NE when $2c_1D_2 \in (rc_1, rc_2]$, whereas (m, m) can be an NE only if $\max\{\pi_1^m, \pi_1^n\} = \pi_1^m$. Moreover, for $2c_1D_2 > rc_2$, (c, c) is always an NE; and we identify the conditions under which either (c, c) or (m, m) is the unique Pareto-optimal NE. This completes the proof of Theorem 4(ii).

Case 3: Suppose $\frac{\delta}{\delta_m} \geq \bar{\delta}|_{D=D_1}$. By Claim 9(i), we have $\hat{J}_3 > \hat{J}_2 > \hat{J}_1$. The possible equilibrium outcomes are summarized as follows.

(i) The equilibrium profit rankings of the two ports can be (S1) and (S11), (S1) and (S12), (S1) and (S13), (S1) and (S14), (S2) and (S11), (S2) and (S12), (S2) and (S13), (S2) and (S14), (S3) and (S11), (S3) and (S12), (S3) and (S13), or (S3) and (S14), with the equilibrium outcomes given in (A59), under one of the following conditions:

- (ia) $J_2, J_3, \hat{J}_1 < J_1, \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (ib) $J_3, \hat{J}_1, \hat{J}_2 < J_2 < J_1 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (ic) $J_2, J_3, \hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1$, with $J^2 \in (\hat{J}_1, \hat{J}_3]$;
- (id) $J_3, \hat{J}_1, \hat{J}_2 < J_2, \hat{J}_3 < \hat{J}_1$, with $J^2 \in (\hat{J}_1, \hat{J}_3]$;
- (ie) $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_3 < J_2 < J_1$, with $J^2 \in (\hat{J}_1, \hat{J}_3]$.

(ii) The equilibrium profit rankings of the two ports can be (S4) and (S11), (S4) and (S12), (S4) and (S13), and (S4) and (S14), with the equilibrium outcomes given in (A59), under one of the following conditions:

- (iia) $J_3 < J_2 < \hat{J}_1 < J_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_2, \hat{J}_3]$;
- (iib) $J_2, J_3, \hat{J}_1 < J_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (J_1, \hat{J}_3]$;

(iic) $J_2, J_3, \hat{J}_1, \hat{J}_2 < J_1 < \hat{J}_3$, with $J^2 \in (J_1, \hat{J}_3]$.

(iii) The equilibrium profit rankings of the two ports can be (S5) and (S11), (S5) and (S12), (S5) and (S13), or (S5) and (S14), with the equilibrium outcomes given in (A55), under one of the following conditions:

- (iiia) $J_1, J_2, J_3, \hat{J}_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiib) $\hat{J}_1 < J_2 < J_1 < \hat{J}_2$, $\min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 \in (\hat{J}_1, J_2]$;
- (iiic) $J_1, J_2, J_3, \hat{J}_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiid) $\hat{J}_1 < J_1 < J_2 < \hat{J}_2 < J_3, \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiie) $J_1, J_3, \hat{J}_1 < \hat{J}_2 < J_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiif) $\hat{J}_1 < J_1 < \hat{J}_2 < J_2 < J_3, \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiig) $J_1, J_3, \hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_2$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiih) $\hat{J}_1 < J_1 < \hat{J}_2 < \hat{J}_3 < J_2 < J_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiij) $J_2 < J_3, \hat{J}_1, \hat{J}_2 < J_1 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiik) $\hat{J}_1 < J_2 < J_3, \hat{J}_2 < J_1 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiil) $\hat{J}_1 < J_2 < \hat{J}_2 < J_1 < J_3, \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_2]$;
- (iiim) $\hat{J}_1 < \hat{J}_2 < J_2 < J_3 < J_1 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiin) $J_3, \hat{J}_1, \hat{J}_2 < J_1 < J_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiio) $\hat{J}_1 < \hat{J}_2 < J_1 < J_2 < J_3, \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiip) $J_3, \hat{J}_1, \hat{J}_2 < J_1 < \hat{J}_3 < J_2$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiiq) $\hat{J}_1 < \hat{J}_2 < J_1 < \hat{J}_3 < J_2 < J_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiir) $J_2 < J_3, \hat{J}_1, \hat{J}_2 < \hat{J}_3 < J_1$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiis) $J_2 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_3 < J_1$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiit) $\hat{J}_1 < J_2 < J_3, \hat{J}_2 < \hat{J}_3 < J_1$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiiu) $\hat{J}_1 < J_2 < \hat{J}_2 < \hat{J}_3 < J_1, J_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiiv) $\hat{J}_1 < \hat{J}_2 < J_2 < J_3, \hat{J}_3 < J_1$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiiw) $\hat{J}_1 < \hat{J}_2 < J_2 < \hat{J}_3 < J_1 < J_3$, with $J^2 \in (\hat{J}_1, J_2]$;
- (iiix) $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_2 < J_3 < J_1$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiiy) $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_2 < J_1 < J_3$, with $J^2 \in (\hat{J}_1, J_2]$;
- (iiiz) $J_3, \hat{J}_1, \hat{J}_2, \hat{J}_3 < J_1 < J_2$, with $J^2 \in (\hat{J}_1, J_1]$;
- (iiiz-1) $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1 < J_2 < J_3$, with $J^2 \in (\hat{J}_1, \hat{J}_2]$.

(iv) The equilibrium profit rankings of the two ports can be (S6) and (S11), (S6) and (S12), (S6) and (S13), (S6) and (S14), (S7) and (S11), (S7) and (S12), (S7) and (S13), (S7) and (S14), (S9) and (S11), (S9) and (S12), (S9) and (S13), (S9) and (S14), (S13) and (S11), (S13) and (S12), (S13) and (S13), or (S13) and (S14), with the equilibrium outcomes given in (A53), under one of the following conditions:

- (iva) $J_2 < J_3, \hat{J}_1 < J_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivb) $\hat{J}_1 < J_2 < J_3 < J_1 < \hat{J}_2 < J_3$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivc) $\hat{J}_1 < J_1 < J_2 < \hat{J}_2 < J_3, \hat{J}_3$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivd) $\hat{J}_1 < J_1 < J_2 < \hat{J}_2, J_3 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_3]$;
- (ive) $\hat{J}_1 < J_1 < J_2 < \hat{J}_2 < \hat{J}_3 < J_3$, with $J^2 \in (\hat{J}_1, J_3]$;
- (ivf) $J_1, J_3, \hat{J}_1 < \hat{J}_2 < J_2 < \hat{J}_3$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivg) $\hat{J}_1 < J_1 < \hat{J}_2 < J_2 < J_3, \hat{J}_3$, with $J^2 \in (\hat{J}_1, \min\{J_3, \hat{J}_3\}]$;
- (ivh) $J_1, J_3, \hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_2, \hat{J}_3$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivi) $\hat{J}_1 < J_1 < \hat{J}_2 < \hat{J}_3 < J_2 < J_3$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivk) $J_2 < J_3, \hat{J}_1 < \hat{J}_2 < J_1 < \hat{J}_3$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivl) $J_2 < \hat{J}_1 < \hat{J}_2 < J_1, \min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 \in (J_1, \min\{J_3, \hat{J}_3\}]$;
- (ivm) $\hat{J}_1 < J_2 < J_1, \hat{J}_2, \min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivn) $\hat{J}_1 < \hat{J}_2 < J_2 < J_3 < J_1 < \hat{J}_3$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivo) $\hat{J}_1 < \hat{J}_2 < J_2 < J_1 < J_3, \hat{J}_3$, with $J^2 \in (J_1, \min\{J_3, \hat{J}_3\}]$;
- (ivp) $J_3 < \hat{J}_1 < \hat{J}_2 < J_1 < J_2 < \hat{J}_3$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivq) $\hat{J}_1 < J_1, J_3, \hat{J}_2 < J_2 < \hat{J}_3$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivr) $\hat{J}_1 < \hat{J}_2 < J_1 < J_2 < J_3, \hat{J}_3$, with $J^2 \in (\hat{J}_1, \min\{J_3, \hat{J}_3\}]$;
- (ivs) $J_3, \hat{J}_1, \hat{J}_2 < J_1 < \hat{J}_3 < J_2$, with $J^2 \in (J_1, \hat{J}_3]$;
- (ivt) $\hat{J}_1 < \hat{J}_2 < J_1 < \hat{J}_3 < J_2 < J_3$, with $J^2 \in (\hat{J}_2, J_3]$;
- (ivu) $J_2 < \hat{J}_1 < J_1 < \hat{J}_2 < J_3, \hat{J}_3$, with $J^2 \in (J_1, \min\{J_3, \hat{J}_3\}]$;
- (ivv) $\hat{J}_1 < J_2 < J_1, \hat{J}_2, \min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 \in (J_1, \min\{J_3, \hat{J}_3\}]$.

(v) The equilibrium profit rankings of the two ports can be (S8) and (S11), (S8) and (S12), (S8) and (S13), or (S8) and (S14), with the following equilibrium outcomes:

$$\begin{cases} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE for } a < \hat{a}_2; \\ (m, m) \text{ cannot be an NE for any } a > 0. \end{cases} \quad (\text{A63})$$

The equilibrium outcomes in (A63) arise under one of the following conditions:

- (va) $J_2 < \hat{J}_1 < J_1, \hat{J}_2 < J_3, \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (vb) $\hat{J}_1 < J_2 < J_1 < J_3, \hat{J}_2, \hat{J}_3$, with $J^2 \in (J_2, J_1]$;
- (vc) $\hat{J}_1 < J_2, \hat{J}_2 < J_1 < J_3, \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (vd) $J_2, \hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1 < J_3$, with $J^2 \in (\hat{J}_1, J_1]$;
- (ve) $\hat{J}_1 < \hat{J}_2 < J_2, \hat{J}_3 < J_1 < J_3$, with $J^2 \in (J_2, J_1]$.

(vi) The equilibrium profit rankings of the two ports can be (S12) and (S11), (S12) and (S12), (S12) and (S13), or (S12) and (S14), with the equilibrium outcomes given in (A54), under one of the following conditions:

- (via) $J_2 < \hat{J}_1 < J_1 < \hat{J}_2 < J_3 < \hat{J}_3$, with $J^2 \in (J_3, \hat{J}_3]$;
- (vib) $\hat{J}_2 < J_2 < J_1 < J_3 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (J_3, \hat{J}_3]$;
- (vic) $\hat{J}_1 < J_1 < J_2, J_3, \hat{J}_2 < \hat{J}_3$, with $J^2 \in (J_3, \hat{J}_3]$;
- (vid) $J_2 < \hat{J}_1 < \hat{J}_2 < J_1 < J_3 < \hat{J}_3$, with $J^2 \in (J_3, \hat{J}_3]$;
- (vie) $\hat{J}_1 < J_2, \hat{J}_2 < J_1 < J_3 < \hat{J}_3$, with $J^2 \in (J_3, \hat{J}_3]$;
- (vif) $\hat{J}_1 < \hat{J}_2 < J_1 < J_2 < J_3 < \hat{J}_3$, with $J^2 \in (J_3, \hat{J}_3]$.

(vii) The equilibrium profit rankings of the two ports can be (S1) and (S6), with the equilibrium outcomes given in (A59), under one of the following conditions:

- (viia) $J_3, \hat{J}_1, \hat{J}_2 < \hat{J}_3 < J_2 < J_1$, with $J^2 \in (\hat{J}_3, J_2]$;
- (viib) $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_3 < J_2 < J_1$, with $J^2 \in (\hat{J}_3, J_2]$.

(viii) The equilibrium profit rankings of the two ports can be (S2) and (S6), with the equilibrium outcomes given in (A58), under one of the following conditions:

- (viiiia) $\hat{J}_1 < J_3, \hat{J}_2 < J_2 < \hat{J}_3 < J_1$, with $J^2 \in (\hat{J}_3, J_1]$;
- (viiiib) $J_3 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_2 < J_1$, with $J^2 \in (J_2, J_1]$;
- (viiiic) $\hat{J}_1 < J_3, \hat{J}_2 < \hat{J}_3 < J_2 < J_1$, with $J^2 \in (J_2, J_1]$;
- (viiiid) $J_3 < J_2 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1$, with $J^2 \in (\hat{J}_3, J_1]$.

The same conclusions apply to (S3) and (S6).

(ix) The equilibrium profit rankings of the two ports can be (S4) and (S6), with the equilibrium outcomes given in (A57), under one of the following conditions:

- (ixa) $J_3, \hat{J}_1 < J_2 < J_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 > \hat{J}_3$;
- (ixb) $J_2, J_3, \hat{J}_1, \hat{J}_2 < J_1 < \hat{J}_3$, with $J^2 > \hat{J}_3$;
- (ixc) $J_2, J_3, \hat{J}_1, \hat{J}_2 < \hat{J}_3 < J_1$, with $J^2 > J_1$;

(ixd) $J_3, \hat{J}_1, \hat{J}_2 < \hat{J}_3 < J_2 < J_1$, with $J^2 > J_1$.

(x) The equilibrium profit rankings of the two ports can be (S5) and ($\hat{S}6$), with the equilibrium outcomes given in (A60), under one of the following conditions:

(xa) $J_2, \hat{J}_1, \hat{J}_2, \hat{J}_3 < J_3 < J_1$, with $J^2 \in (\hat{J}_3, J_1]$;

(xb) $\hat{J}_1 < J_2, J_3, \hat{J}_2 < \hat{J}_3 < J_1$, with $J^2 \in (\hat{J}_3, J_1]$;

(xc) $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_2 < J_1 < J_3$, with $J^2 \in (\hat{J}_3, J_2]$;

(xd) $J_3, \hat{J}_1, \hat{J}_2 < \hat{J}_3 < J_1 < J_2$, with $J^2 \in (\hat{J}_3, J_1]$;

(xe) $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1, J_2, J_3$, with $J^2 \in (\hat{J}_3, J_1]$.

(xi) The equilibrium profit rankings of the two ports can be (S6) and ($\hat{S}6$), with the equilibrium outcomes given in (A56), under one of the following conditions:

(xia) $J_2, J_3, \hat{J}_1, \hat{J}_2, \hat{J}_3 < J_1$, with $J^2 = J_1$;

(xib) $J_3 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1 < J_2$, with $J^2 \in (J_1, J_2]$;

(xic) $\hat{J}_1 < J_3, \hat{J}_2 < \hat{J}_3 < J_1 < J_2$, with $J^2 \in (J_1, J_2]$;

(xid) $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_1 < J_2 < J_3$, with $J^2 \in (\hat{J}_3, J_1]$.

The same conclusions apply to (S7) and ($\hat{S}6$), as well as (S9) and ($\hat{S}6$).

(xii) The equilibrium profit rankings of the two ports can be (S8) and ($\hat{S}6$), with the equilibrium outcomes given in (A61), under one of the following conditions:

(xiia) $J_2, \hat{J}_1, \hat{J}_2 < \hat{J}_3 < J_1 < J_3$, with $J^2 \in (\hat{J}_3, J_1]$;

(xiib) $\hat{J}_1 < \hat{J}_2 < \hat{J}_3 < J_2 < J_1 < J_3$, with $J^2 \in (J_2, J_1]$.

(xiii) The equilibrium profit rankings of the two ports can be (S12) and ($\hat{S}6$), with the equilibrium outcomes given in (A47-1)-(A47-4), under one of the following conditions:

(xiia) $J_2 < \hat{J}_1 < J_1 < \hat{J}_2$, $\min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 > \max\{J_3, \hat{J}_3\}$;

(xiib) $\hat{J}_1 < J_2 < J_1 < \hat{J}_2$, $\min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 > \max\{J_3, \hat{J}_3\}$;

(xiic) $\hat{J}_1 < J_1 < J_2 < \hat{J}_2$, $\min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 > \max\{J_3, \hat{J}_3\}$;

(xiid) $\hat{J}_1 < J_1 < \hat{J}_2 < J_2$, $\min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 > \max\{J_3, \hat{J}_3\}$;

(xiie) $J_2 < \hat{J}_1 < \hat{J}_2 < J_1 < \min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 > \max\{J_3, \hat{J}_3\}$.

(xiv) The equilibrium profit rankings of the two ports can be (S13) and ($\hat{S}6$), with the equilibrium outcomes given in (A56), under one of the following conditions:

(xiva) $J_1, J_2, \hat{J}_1, \hat{J}_2 < \hat{J}_3 < J_3$, with $J^2 \in (\hat{J}_3, J_3]$;

- (xivb) $J_1, \hat{J}_1, \hat{J}_2, \hat{J}_3 < J_2 < J_3$, with $J^2 \in (J_2, J_3]$;
(xivc) $J_2, \hat{J}_1, \hat{J}_2, \hat{J}_3 < J_1 < J_3$, with $J^2 \in (J_1, J_3]$.

The corresponding equilibrium outcomes are given in (A47-1)-(A47-4), (A53)-(A61), and (A63). These results imply that (c, c) is not an NE whenever $2c_1D_2 \in (rc_1, rc_2]$, whereas (m, m) can be an NE only if $\max\{\pi_1^m, \pi_1^n\} = \pi_1^m$. Furthermore, for $2c_1D_2 > rc_2$, (c, c) is always an NE; and we identify the conditions under which either (c, c) or (m, m) is the unique Pareto-optimal NE. This completes the proof of Theorem 4(iii). $\square$

Proof of Theorem 5. Under $c_2 > c_1$ and $D_2 > D_1$, the following results hold.

Claim 10. *Suppose $c_2 > c_1$ and $D_2 > D_1$. Then, we have the following.*

- (i) *The relationship among $\hat{J}_1, \hat{J}_2$, and $\hat{J}_3$ is the same as that specified in Claim 9(i).*
- (ii) $J_1 < \hat{J}_1$.
- (iii) $J_2 < \hat{J}_2$ if $2c_1D_2 \in (rc_2, rc_1+rc_2)$; otherwise, the relative order of J_2 and $\hat{J}_2$ is indeterminate.
- (iv) $\hat{J}_2 > J_3$ if $2c_1D_2 \in (rc_1, 2rc_2]$; otherwise, the relative order of $\hat{J}_2$ and J_3 is indeterminate.
- (v) $\hat{J}_3 > J_3$ for $2c_1D_2 \in (rc_1, 2rc_2]$; otherwise, the relative order of $\hat{J}_3$ and J_3 is indeterminate.
- (vi) $\hat{J}_3 > J_2$ for $2c_1D_2 \in (rc_2, rc_1+rc_2)$; otherwise, the relative order of $\hat{J}_3$ and J_2 is indeterminate.
- (vii) $J^2 > \hat{J}_1$.

According to the values of $\frac{\delta}{\delta_m}$, three cases arise.

Case 1: Suppose $\frac{\delta}{\delta_m} \leq \hat{\delta}|_{D=D_1}$. WE then have $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 10(i). Since $c_2 > c_1$ and $D_1 > 1 > r > 0$, we obtain $rc_1 < rc_2 < rc_1 + rc_2 < 2rc_2 < 2c_2D_1$. Given $c_2 > c_1$ and $D_2 > D_1$, the relative order between c_1D_2 and c_2D_1 is indeterminate. Thus, five sub-cases should be considered. However, since the results for $2c_1D_2 \in (2rc_2, 2c_2D_1]$ coincide with those for $2c_1D_2 > 2c_2D_1$, both are combined into the fourth sub-case, i.e., $2c_1D_2 > 2rc_2$, as shown below.

Case 1a: Suppose $2c_1D_2 \in (rc_1, rc_2]$. Then $J_1 > J_2 > J_3$ by Claim 1(i) and Claim 3(i); $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 10(i); $\hat{J}_2 > J_3$ by Claim 10(iv); $J_3 < \hat{J}_3$ by Claim 10(v); and $J^2 > J_1$ by Claim 10(vii). Thus, it suffices to consider the configuration $J_3 < \hat{J}_3 < J_2 < J_1 < \hat{J}_1$, with $\hat{J}_2 < J_1$.² Since $J^2 > J_1$, there are two sub-cases.

²The other possibility is $J_3 < \hat{J}_3 < J_1 < \hat{J}_2 < \hat{J}_1$ with $J_2 < J_1$. Since all possible equilibrium outcomes in this case are identical to those under $J_3 < \hat{J}_3 < J_2 < J_1 < \hat{J}_1$ with $\hat{J}_2 < J_1$, they are omitted.

Case 1a-1: Suppose $J^2 \in (J_1, \hat{J}_1]$. The equilibrium profit rankings of the two ports are (S4) and (S $\hat{3}$) if $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$; and (S4) and (S $\hat{5}$) if $\hat{T}_1 = 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. In the former situation, (S4) in (A16) and (S $\hat{3}$) in (A35) together imply the equilibrium outcomes are those in (A58). In the latter situation, (S4) in (A16) and (S $\hat{5}$) in (A37) together imply the equilibrium outcomes are also those in (A58).

Case 1a-2: Suppose $J^2 > \hat{J}_1$. The equilibrium profit rankings of the two ports are (S4) and (S $\hat{6}$) if $\hat{T}_1 < 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. The corresponding equilibrium outcomes are the same as those in (A58).

Case 1b: Suppose $2c_1D_2 \in (rc_2, rc_1 + rc_2]$. Then, $J_1 > J_3 > J_2$ by Claim 1(ii), Claim 2(i), and Claim 3(ii). Moreover, $J_1 < \hat{J}_1$ by Claim 10(ii); $J_2 < \hat{J}_2$ by Claim 10(iii); $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 10(i); $\hat{J}_2 > J_3$ by Claim 10(iv); $J_3 < \hat{J}_3$ by Claim 10(v); $\hat{J}_3 > J_2$ by Claim 10(vi); and $J^2 > J_1$ by Claim 10(vii). Together, these imply $J_2 < J_3 < \hat{J}_3 < \hat{J}_2 < J_1 < \hat{J}_1$. Consequently, there are two sub-cases.

Case 1b-1: Suppose $J^2 \in (J_1, \hat{J}_1]$. The equilibrium profit rankings of the two ports are (S7) and (S $\hat{3}$) if $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$; and (S7) and (S $\hat{5}$) if $\hat{T}_1 = 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. In the former situation, (S7) in (A19) and (S $\hat{3}$) in (A35) imply that

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE for } a \in (\hat{a}_2, a_1) \text{ if } \hat{a}_2 < a_1; \\ (m, m) \text{ is the unique Pareto-optimal NE for } a \in [\frac{-T_3}{T_2}, \hat{a}_2] \text{ if } \hat{a}_2 > \frac{-T_3}{T_2}. \end{array} \right. \quad (\text{A64})$$

In the latter situation, (S7) in (A19) and (S $\hat{5}$) in (A37) imply that the equilibrium outcomes are the same as those in (A64), but with $\hat{a}_2$ replaced by $\frac{-\hat{T}_3}{\hat{T}_2}$.

Case 1b-2: Suppose $J^2 > \hat{J}_1$. The equilibrium profit rankings of the two ports are (S7)

and (S6). Thus, (S7) in (A19) and (S6) in (A38) yield the following equilibrium outcomes.

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3 \text{ with one of the following holds:} \\ \quad \hat{a}_2 < a_1 < \hat{a}_1 \text{ and } a \in (\hat{a}_2, a_1), \\ \quad \hat{a}_2 < \hat{a}_1 < a_1 \text{ and } a \in (\hat{a}_2, \hat{a}_1); \\ (m, m) \text{ is the unique Pareto-optimal NE iff one of the following holds:} \\ \quad \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a_1 \leq \hat{a}_2, \text{ and } a \in [a_1, \hat{a}_2] \cup [\hat{a}_1, \infty), \\ \quad \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, \hat{a}_2 < a_1 < \hat{a}_1, \text{ and } a \in [\hat{a}_1, \infty), \\ \quad \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a > \hat{a}_1, \text{ and } a \in [a_1, \infty), \\ \quad \hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3 \text{ and } a \in [a_1, \infty). \end{array} \right. \quad (\text{A65})$$

Case 1c: Suppose $2c_1D_2 \in (rc_1 + rc_2, 2rc_2]$. There are two sub-cases.

Case 1c-1: Suppose $\delta \leq \delta_1$. Then $J_1 > J_2 > J_3$ by Claim 2(i) and Claim 3(iii); $J_1 < \hat{J}_1$ by Claim 10(ii); $J_1 < \hat{J}_1$ by Claim 10(ii); $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 10(i); $J^2 > J_1$ by Claim 10(vii); and $J_3 < \hat{J}_3$ by Claim 10(v). These jointly imply $J_3 < \hat{J}_3 < \hat{J}_2 < J_1 < \hat{J}_1$ and $J_2 < J_1$. Since $J^2 > J_1$, the results coincide with those of Case 2.

Case 1c-2: Suppose $\delta > \delta_1$. Then $J_2 > J_1 > J_3$ by (A8), Claim 2(i), and Claim 3(iii). Moreover, we have $J_1 < \hat{J}_1$ by Claim 10(ii); $J^2 > J_1$ by Claim 10(vii); $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 10(i); $\hat{J}_2 > J_3$ by Claim 10(iv); and $J_3 < \hat{J}_3$ by Claim 10(v). Thus, the rankings are either $J_3 < \hat{J}_3 < \hat{J}_2 < J_1 < J_2 < \hat{J}_1$ or $J_3 < \hat{J}_3 < \hat{J}_2 < J_1 < \hat{J}_1 < J_2$.

Case 1c-2-1: Suppose $J_3 < \hat{J}_3 < \hat{J}_2 < J_1 < J_2 < \hat{J}_1$. Since $J^2 > J_1$, three sub-cases arise.

Case 1c-2-1-1: Suppose $J^2 \in (J_1, J_2]$. Then, port 1's equilibrium profit ranking is (S6) if $T_1 = 0$, $T_2 < 0$, and $\hat{T}_3 > 0$; (S7) if $T_1 < 0$, $T_2 < 0$, and $\hat{T}_3 > 0$; or (S9) if $T_1 < 0$, $T_2 = 0$, and $\hat{T}_3 > 0$. Port 2's equilibrium profit ranking is (S3) if $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$; or (S5) if $\hat{T}_1 = 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$. Hence, given (S6) and (S3), the equilibrium outcomes are those in (A64), with a_1 replaced by $\frac{-T_3}{T_2}$. Given (S6) and (S5), the equilibrium outcomes are those in (A64), with a_1 replaced by $\frac{-T_3}{T_2}$. Given (S7) and (S3), the equilibrium outcomes are those in (A64). Given (S7) and (S5), the equilibrium outcomes are those in (A64), with $\hat{a}_2$ replaced by $\frac{-\hat{T}_3}{\hat{T}_2}$. Given (S9) and (S3), the equilibrium outcomes are those in (A64), with a_1 replaced by $\sqrt{\frac{-T_3}{T_1}}$. Given (S9) and (S5), the equilibrium outcomes are

those in (A64), with $\hat{a}_2$ replaced by $\frac{-\hat{T}_3}{\hat{T}_2}$.

Case 1c-2-1-2: Suppose $J^2 \in (J_2, \hat{J}_1]$. The equilibrium profit rankings of both ports are (S7) and (S $\hat{3}$) or (S7) and (S $\hat{5}$). In the former case, the equilibrium outcomes are those in (A64); in the latter case, the equilibrium outcomes are those in (A64) with $\hat{a}_2$ replaced by $\frac{-\hat{T}_3}{\hat{T}_2}$.

Case 1c-2-1-3: Suppose $J^2 > \hat{J}_1$. The equilibrium profit rankings of both ports are (S7) and (S $\hat{3}$), with equilibrium outcomes given in (A64).

Case 1c-2-2: Suppose $J_3 < \hat{J}_3 < \hat{J}_2 < J_1 < \hat{J}_1 < J_2$. Since $J^2 > J_1$. Since $J^2 > J_1$, three sub-cases arise.

Case 1c-2-2-1: Suppose $J^2 \in (J_1, \hat{J}_1]$. All results coincide with those of Case 1c-2-1-1.

Case 1c-2-2-2: Suppose $J^2 \in (\hat{J}_1, J_2]$. The equilibrium profit rankings of both ports are (S7) and (S $\hat{6}$), (S6) and (S $\hat{6}$), or (S9) and (S $\hat{6}$). In the first case, the equilibrium outcomes are those in (A65); in the second case, they are those in (A56) with a_1 replaced by $\frac{-T_3}{T_2}$; and in the third case, they are those in (A56) with a_1 replaced by $\sqrt{\frac{-T_3}{T_2}}$.

Case 1c-2-2-3: Suppose $J^2 > J_2$. The equilibrium profit rankings of both ports are (S7) and (S $\hat{6}$), with the equilibrium outcomes given in (A65).

Case 1d: Suppose $2c_1D_2 > 2rc_2$. There are four cases as follows.

Case 1d-1: Suppose $\delta \leq \delta_3$. Then $J_1 > J_2 > J_3$ by (A10); $J_1 < \hat{J}_1$ by Claim 10(ii); $\hat{J}_1 > \hat{J}_2 > \hat{J}_3$ by Claim 10(i); and $J^2 > J_1$ by Claim 10(vii). Hence, $J_3 < \hat{J}_3 < \hat{J}_2 < J_1 < \hat{J}_1$ and $J_2 < J_1$. Since $J^2 > J_1$, two sub-cases arise.

Case 1d-1-1: Suppose $J^2 \in (J_1, \hat{J}_1]$. The equilibrium profit rankings of both ports are (S12) and (S $\hat{3}$) if $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, and $\hat{T}_3 < 0$; and (S12) and (S $\hat{5}$) if $\hat{T}_1 = 0$, $\hat{T}_2 > 0$, and

$\hat{T}_3 < 0$. In the former situation, (S12) in (A24) and (S $\hat{3}$) in (A35) imply that

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0; \\ (c, c) \text{ is the unique Pareto-optimal NE iff } T_2^2 > 4T_1T_3, \hat{a}_2 \leq a_1, \\ \quad \text{and } a \in (\max\{\hat{a}_2, a_2\}, a_1); \\ (m, m) \text{ is the unique Pareto-optimal NE iff one of the following holds:} \\ \quad T_2^2 > 4T_1T_3 \text{ and } a \in (0, \min\{\hat{a}_2, a_2\}], \\ \quad T_2^2 \leq 4T_1T_3 \text{ and } a \in (0, a_2] \cup [a_1, \infty). \end{array} \right. \quad (\text{A66})$$

In the latter situation, the equilibrium outcomes of (S12) in (A24) and (S $\hat{5}$) in (A36) are those in (A66), with $\hat{a}_2$ replaced by $\frac{-\hat{T}_3}{\hat{T}_2}$.

Case 1d-1-2: Suppose $J^2 > \hat{J}_1$. The equilibrium profit rankings of both ports are (S12) and (S $\hat{6}$). Thus, (S12) in (A24) and (S $\hat{6}$) in (A38) imply the following equilibrium outcomes.

$$\left\{ \begin{array}{l} (c, c) \text{ is an NE for all } a > 0. \\ (c, c) \text{ is the unique Pareto-optimal NE iff one of the following holds:} \\ \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, \hat{a}_2 < a_2 < \hat{a}_1 < a_1, a \in (a_2, \hat{a}_1); \\ \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a_2 < \hat{a}_2 < \hat{a}_1 < a_1, a \in (\hat{a}_2, \hat{a}_1); \\ \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a_2 < \hat{a}_2 < a_1 < \hat{a}_1, a \in (\hat{a}_2, a_1). \\ (m, m) \text{ is the unique Pareto-optimal NE iff one of the following holds:} \\ \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, \hat{a}_2 < \hat{a}_1 < a_2 < a_1, a \in (0, \hat{a}_2) \cup [\hat{a}_1, a_2] \cup [a_1, \infty); \\ \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, \hat{a}_2 < a_2 < \hat{a}_1 < a_1, a \in (0, \hat{a}_2] \cup [a_1, \infty); \\ \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a_2 < \hat{a}_2 < \hat{a}_1 < a_1, a \in (0, a_2] \cup [a_1, \infty); \\ \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a_2 < \hat{a}_2 < a_1 < \hat{a}_1, a \in (0, a_2] \cup [\hat{a}_1, \infty); \\ \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 > 4\hat{T}_1\hat{T}_3, a_2 < a_1 < \hat{a}_2 < \hat{a}_1, a \in (0, a_2] \cup [a_1, \hat{a}_2] \cup [\hat{a}_1, \infty); \\ \quad T_2^2 > 4T_1T_3, \hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3, a \in (0, a_2] \cup [a_1, \infty); \\ \quad T_2^2 \leq 4T_1T_3, \hat{T}_2^2 \geq 4\hat{T}_1\hat{T}_3, a \in (0, \hat{a}_2] \cup [\hat{a}_1, \infty); \\ \quad T_2^2 \leq 4T_1T_3, \hat{T}_2^2 \leq 4\hat{T}_1\hat{T}_3, a > 0. \end{array} \right. \quad (\text{A67})$$

Case 1d-2: Suppose $\delta \in (\delta_3, \delta_1]$ with $J_2 < J_3 < J_1$. Since $J^2 > J_1$, all results coincide with those of Case 1d-1.

Case 1d-3: Suppose $\delta \in (\delta_3, \delta_1]$ with $J_1 = J_2 < J_3$. Then, we have either $\hat{J}_3 < \hat{J}_2 < J_1 = J_2 < J_3 < \hat{J}_1$ or $\hat{J}_3 < \hat{J}_2 < J_1 = J_2 < \hat{J}_1 < J_3$. Accordingly, we consider the following sub-cases.

Case 1d-3-1: Suppose $\hat{J}_3 < \hat{J}_2 < J_1 = J_2 < J_3 < \hat{J}_1$. Since $J^2 > J_1$, there are three further sub-cases.

Case 1d-3-1-1: Suppose $J^2 \in (J_1, J_3]$. Port 1's equilibrium profit ranking is either (S7) or (S13), while port 2's is either (S $\hat{3}$) or (S $\hat{5}$). Given (S7) and (S $\hat{3}$), the equilibrium outcomes are those in (A64). Given (S7) and (S $\hat{5}$), the equilibrium outcomes are those in (A64) with $\hat{a}_2$ replaced by $\frac{-\hat{T}_3}{\hat{T}_2}$. Given (S13) and (S $\hat{3}$), the equilibrium outcomes are those in (A64) with a_1 replaced by $\frac{-T_2}{T_1}$. Given (S13) and (S $\hat{5}$), the equilibrium outcomes are those in (A64) with a_1 replaced by $\frac{-T_2}{T_1}$ and $\hat{a}_2$ with $\frac{-\hat{T}_3}{\hat{T}_2}$.

Case 1d-3-1-2: Suppose $J^2 \in (J_3, \hat{J}_1]$. The equilibrium profit rankings of the two ports are (S12) and (S $\hat{3}$), or (S12) and (S $\hat{5}$). In the former situation, the equilibrium outcomes are those in (A66). In the latter situation, the equilibrium outcomes remain those in (A66), but with $\hat{a}_2$ replaced by $\frac{-\hat{T}_3}{\hat{T}_2}$.

Case 1d-3-1-3: Suppose $J^2 > \hat{J}_1$. The equilibrium profit rankings of the two ports are (S12) and (S $\hat{6}$), with equilibrium outcomes given in (A67).

Case 1d-3-2: Suppose $\hat{J}_3 < \hat{J}_2 < J_1 = J_2 < \hat{J}_1 < J_3$. Since $J^2 > J_1$, three sub-cases arise.

Case 1d-3-2-1: Suppose $J^2 \in (J_1, \hat{J}_1]$. All results coincide with those of Case 1d-3-1-1.

Case 1d-3-2-2: Suppose $J^2 \in (\hat{J}_1, J_3]$. The equilibrium profit rankings of the two ports are (S7) and (S $\hat{6}$), or (S13) and (S $\hat{6}$). In the former situation, the equilibrium outcomes are those in (A65). In the latter situation, the equilibrium outcomes remain those in (A65), but with a_1 replaced by $\frac{-T_2}{T_1}$.

Case 1d-3-2-3: Suppose $J^2 > J_3$. The equilibrium profit rankings of the two ports are (S12) and (S $\hat{6}$), with equilibrium outcomes given in (A67).

Case 1d-4: Suppose $\delta > \delta_1$. Then $J_1 < J_2 < J_3$ by (A8) and (A10); $J_1 < \hat{J}_1$ by Claim 10(ii); $\hat{J}_3 < \hat{J}_2 < \hat{J}_1$ by Claim 10(i); and $J^2 > J_1$ by Claim 10(vii). Depending on the relative sizes among J_1 , J_2 , J_3 , and $\hat{J}_1$, three possibilities arise.

Case 1d-4-1: Suppose $\hat{J}_3 < \hat{J}_2 < J_1 < J_2 < J_3 < \hat{J}_1$. Four sub-cases arise.

Case 1d-4-1-1: Suppose $J^2 \in (J_1, J_2]$. Port 1's equilibrium profit ranking is either (S7) or

(S9), while port 2's is either (S $\hat{3}$) or (S $\hat{5}$). Given (S7) and (S $\hat{3}$), the equilibrium outcomes are those in (A64). Given (S7) and (S $\hat{5}$), the equilibrium outcomes are those in (A64) but with $\hat{a}_2$ replaced by $\frac{-\hat{T}_3}{\hat{T}_2}$. Given (S9) and (S $\hat{3}$), the equilibrium outcomes are those in (A64) but with a_1 replaced by $\sqrt{\frac{-T_3}{T_1}}$. Given (S9) and (S $\hat{5}$), the equilibrium outcomes are those in (A64) but with a_1 replaced by $\frac{-T_3}{T_2}$ and $\hat{a}_2$ replaced by $\frac{-\hat{T}_3}{\hat{T}_2}$.

Case 1d-4-1-2: Suppose $J^2 \in (J_2, J_3]$. All results coincide with those of Case 1d-3-1-1.

Case 1d-4-1-3: Suppose $J^2 \in (J_3, \hat{J}_1]$. All results coincide with those of Case 1d-3-1-2.

Case 1d-4-1-4: Suppose $J^2 > \hat{J}_1$. All results coincide with those of Case 1d-3-1-3.

Case 1d-4-2: Suppose $\hat{J}_3 < \hat{J}_2 < J_1 < \hat{J}_2 < \hat{J}_1 < J_3$. There are four sub-cases.

Case 1d-4-2-1: Suppose $J^2 \in (J_1, J_2]$. All results coincide with those of Case 1d-4-1-1.

Case 1d-4-2-2: Suppose $J^2 \in (J_2, \hat{J}_1]$. All results coincide with those of Case 1d-3-1-1.

Case 1d-4-2-3: Suppose $J^2 \in (\hat{J}_1, J_3]$. All results coincide with those of Case 1d-3-2-2.

Case 1d-4-2-4: Suppose $J^2 > J_3$. All results coincide with those of Case 1d-3-1-1.

Case 1d-4-3: Suppose $\hat{J}_3 < \hat{J}_2 < J_1 < \hat{J}_1 < J_2 < J_3$. Since $J^2 > J_1$, four sub-cases arise.

Case 1d-4-3-1: Suppose $J^2 \in (J_1, \hat{J}_1]$. All results coincide with those of Case 1d-4-1-1.

Case 1d-4-3-2: Suppose $J^2 \in (\hat{J}_1, J_2]$. The equilibrium profit rankings of the two ports are either (S7) and (S $\hat{6}$) or (S9) and (S $\hat{6}$). In the former case, the equilibrium outcomes are those given in (A65). In the latter case, the equilibrium outcomes are also given in (A65) but with a_1 replaced by $\sqrt{\frac{-T_3}{T_1}}$.

Case 1d-4-3-3: Suppose $J^2 \in (J_2, J_3]$. All results coincide with those of Case 1d-3-2-2.

Case 1d-4-3-4: Suppose $J^2 > J_3$. All results coincide with those of Case 1d-3-1-1.

The equilibrium outcomes given in (A58), (A64)-(A67) imply that (c, c) is not an NE whenever $2c_1D_2 \in (rc_1, rc_2]$, whereas (m, m) can be an NE only if $\max\{\pi_1^m, \pi_1^n\} = \pi_1^m$. Furthermore, for $2c_1D_2 > rc_2$, (c, c) is always an NE; (c, c) is the unique Pareto optimal NE if and only if one of the conditions in set C9 holds; and (m, m) is the unique Pareto-

optimal NE if and only if one of the conditions in set C10 holds. Conditions sets C9 and C10 are provided in Appendix A. This completes the proof of Theorem 5(i).

Case 2: Suppose $\frac{\delta}{\delta_m} \in (\hat{\delta}|_{D=D_1}, \bar{\delta}|_{D=D_1})$. By Claim 10(i), we have either $\hat{J}_3 > \hat{J}_1 > \hat{J}_2$ or $\hat{J}_1 > \hat{J}_3 > \hat{J}_2$. Since the former case yields no solution, we focus on the latter case. Moreover, because $J^2 > \hat{J}_1$ by Claim 10(vii), three sub-cases arise.

Case 2a: Suppose $J_1 = \max\{J_1, J_2, J_3\}$. There are three possible pairs of equilibrium profit rankings for the two ports: (S4) and (S $\hat{6}$), (S7) and (S $\hat{6}$), and (S12) and (S $\hat{6}$). The corresponding equilibrium outcomes are given in (A57), (A65), and (A67), respectively.

Case 2b: Suppose $J_2 < J_1 < J_3$ with $2c_1D_2 > 2rc_2$ and $\delta \in (\delta_3, \delta_1]$. According to Claim 10(ii), either $J_2 < J_1 < J_3 < \hat{J}_1$ or $J_2 < J_1 < \hat{J}_1 < J_3$. Since $J^2 > \hat{J}_1$ by Claim 10(vii), the first ordering cannot satisfy the required condition. Therefore, when $J_2 < J_1 < J_3 < \hat{J}_1$, the equilibrium profit rankings are (S12) and (S $\hat{6}$), with the corresponding equilibrium outcomes given in (A67). For the second ordering, $J_2 < J_1 < \hat{J}_1 < J_3$, the equilibrium profit rankings are (S7) and (S $\hat{6}$) or (S13) and (S $\hat{6}$) if $J^2 \in (\hat{J}_1, J_3]$, whereas they are (S12) and (S $\hat{6}$) if $J^2 > J_3$. The corresponding equilibrium outcomes for (S7) and (S $\hat{6}$) and for (S13) and (S $\hat{6}$) are given in (A65) and (A56), respectively. For (S12) and (S $\hat{6}$), the equilibrium outcomes are given in (A67).

Case 2c: Suppose $J_1 < J_2 < J_3$ with $2c_1D_2 > 2rc_2$ and $\delta > \delta_1$. Three sub-cases arise.

Case 2c-1: Suppose $J_1 < J_2 < J_3 < \hat{J}_1$. The equilibrium profit rankings of the two ports are (S12) and (S $\hat{6}$), with the corresponding equilibrium outcomes given in (A67).

Case 2c-2: Suppose $J_1 < J_2 < \hat{J}_1 < J_3$. The results are identical to those in Case 2b with $J_2 < J_1 < \hat{J}_1 < J_3$.

Case 2c-3: Suppose $J_1 < \hat{J}_1 < J_2 < J_3$. If $J^2 \in (\hat{J}_1, J_2]$, The equilibrium profit rankings of the two ports are (S7) and (S $\hat{6}$), with the corresponding equilibrium outcomes given in (A65). If $J^2 \in (J_2, \hat{J}_3]$, the results are identical to those in Case 2c-2. If $J^2 > \hat{J}_3$, the results are identical to those in Case 2c-1.

The equilibrium outcomes given in (A56), (A57), (A65), and (A67) imply that (c, c) is not an NE whenever $2c_1D_2 \in (rc_1, rc_2]$, whereas (m, m) can be an NE only if

$\max\{\pi_1^m, \pi_1^n\} = \pi_1^m$. Furthermore, for $2c_1D_2 > rc_2$, (c, c) is always an NE; and we identify the conditions under which either (c, c) or (m, m) is the unique Pareto-optimal NE. This completes the proof of Theorem 5(ii).

Case 3: Suppose $\frac{\delta}{\delta_m} \geq \bar{\delta}|_{D=D_1}$. By Claim 10(i), we have $\hat{J}_3 > \hat{J}_2 > \hat{J}_1$. As in the proof of Theorem 4, this case involves numerous sub-cases. Therefore, we present only the equilibrium outcomes together with the corresponding supporting conditions.

(i) The equilibrium profit rankings of the two ports can be (S4) and (S11), (S4) and (S12), (S4) and (S13), or (S4) and (S14), with the equilibrium outcomes given in (A59), if $J_3 < J_2 < J_1 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, \hat{J}_3]$.

(ii) The equilibrium profit rankings of the two ports can be (S4) and (S6), with the equilibrium outcomes given in (A57), if $J_3 < J_2 < J_1 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 > \hat{J}_3$.

(iii) The equilibrium profit rankings of the two ports can be (S7) and (S11), (S7) and (S12), (S7) and (S13), or (S7) and (S14), with the equilibrium outcomes given in (A53), under one of the following conditions:

- (iiia) $J_2 < J_3 < J_1 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, \hat{J}_3]$;
- (iiib) $J_2 < J_1 < \hat{J}_1 < J_3, \hat{J}_2, \hat{J}_3$, with $J^2 \in (\hat{J}_1, \min\{J_3, \hat{J}_3\}]$;
- (iiic) $J_3 < J_1 < J_2 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, \hat{J}_3]$;
- (iiid) $J_1 < J_2 < \hat{J}_1 < J_3, \hat{J}_2, \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_3]$;
- (iiie) $J_3 < J_1 < \hat{J}_1 < J_2 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, \hat{J}_3]$;
- (iiif) $J_1 < \hat{J}_1 < J_2 < J_3, \hat{J}_2, \hat{J}_3$, with $J^2 \in (\hat{J}_1, \min\{J_3, \hat{J}_3\}]$;
- (iiig) $J_3 < J_1 < \hat{J}_1 < \hat{J}_2 < J_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, \hat{J}_3]$;
- (iiih) $J_1 < \hat{J}_1 < \hat{J}_2 < J_2, J_3, \hat{J}_3$, with $J^2 \in (\hat{J}_1, \min\{J_3, \hat{J}_3\}]$.

(iv) The equilibrium profit rankings of the two ports can be (S6) and (S11) or (S6) and (S12), with the equilibrium outcomes given in (A53), if $J_3 < J_1 < \hat{J}_1 < J_2, \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, \min\{J_2, \hat{J}_2\}]$.

(v) The equilibrium profit rankings of the two ports can be (S9) and (S11), (S9) and (S12), (S9) and (S13), or (S9) and (S14), with the equilibrium outcomes given in (A53), under one of the following conditions:

- (va) $J_3 < J_1 < \hat{J}_1 < J_2 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_2]$;

- (vb) $J_1 < \hat{J}_1 < J_2 < J_3, \hat{J}_2, \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_2]$;
- (vc) $J_3 < J_1 < \hat{J}_1 < \hat{J}_2 < J_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_2]$;
- (vd) $J_1 < \hat{J}_1 < \hat{J}_2 < J_2, J_3, \hat{J}_3$, with $J^2 \in (\hat{J}_1, J_2]$.

(vi) The equilibrium profit rankings of the two ports can be (S12) and (S1 $\hat{1}$), (S12) and (S1 $\hat{2}$), (S12) and (S1 $\hat{3}$), or (S12) and (S1 $\hat{4}$), with the equilibrium outcomes given in (A54), under one of the following conditions:

- (via) $J_2 < J_1 < J_3 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, \hat{J}_2]$;
- (vib) $J_2 < J_1 < \hat{J}_1 < J_3, \hat{J}_2 < \hat{J}_3$, with $J^2 \in (J_3, \hat{J}_3]$;
- (vic) $J_1 < J_2 < J_3 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 \in (\hat{J}_1, \hat{J}_3]$;
- (vid) $J_1 < J_2 < \hat{J}_1 < J_3, \hat{J}_2 < \hat{J}_3$, with $J^2 \in (J_3, \hat{J}_3]$;
- (vie) $J_1 < \hat{J}_1 < J_2, J_3, \hat{J}_2 < \hat{J}_3$, with $J^2 \in (J_3, \hat{J}_3]$.

(vii) The equilibrium profit rankings of the two ports can be (S13) and (S1 $\hat{1}$), (S13) and (S1 $\hat{2}$), (S13) and (S1 $\hat{3}$), or (S13) and (S1 $\hat{4}$), with the equilibrium outcomes given in (A53), under one of the following conditions:

- (viia) $J_2 < J_1 < \hat{J}_1 < \hat{J}_2, \min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 \in (\hat{J}_1, \min\{J_3, \hat{J}_3\}]$;
- (viib) $J_1 < J_2 < \hat{J}_1 < \hat{J}_2, \min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 \in (\hat{J}_1, \min\{J_3, \hat{J}_3\}]$;
- (viic) $J_1 < \hat{J}_1 < J_2 < \hat{J}_2, \min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 \in (J_2, \min\{J_3, \hat{J}_3\}]$;
- (viid) $J_1 < \hat{J}_1 < \hat{J}_2 < J_2 < J_3, \hat{J}_3$, with $J^2 \in (J_2, \min\{J_3, \hat{J}_3\}]$.

(viii) The equilibrium profit rankings of the two ports can be (S4) and (S $\hat{6}$), with the equilibrium outcomes given in (A57), if $J_3 < J_2 < J_1 < \hat{J}_1 < \hat{J}_2 < \hat{J}_3$, with $J^2 > \hat{J}_3$.

(ix) The equilibrium profit rankings of the two ports can be (S7) and (S $\hat{6}$), with the equilibrium outcomes given in (A65), if $\min\{J_2, J_3\} < \max\{J_2, J_3\}$, $J_1, \hat{J}_1, \hat{J}_2 < \hat{J}_3$, with $J^2 > \hat{J}_3$.

(x) The equilibrium profit rankings of the two ports can be (S12) and (S $\hat{6}$), with the equilibrium outcomes given in (A67), under one of the following conditions:

- (xa) $J_2 < J_1 < \hat{J}_1, \hat{J}_2, \min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 > \max\{J_3, \hat{J}_3\}$;
- (xb) $J_1 < J_2 < \hat{J}_1, \hat{J}_2, \min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 > \max\{J_3, \hat{J}_3\}$;
- (xc) $J_1 < \hat{J}_1 < J_2 < \hat{J}_2, \min\{J_3, \hat{J}_3\} < \max\{J_3, \hat{J}_3\}$, with $J^2 > \max\{J_3, \hat{J}_3\}$;
- (xd) $J_1 < \hat{J}_1 < \hat{J}_2 < J_2 < J_3, \hat{J}_3$, with $J^2 > \max\{J_3, \hat{J}_3\}$.

(xi) The equilibrium profit rankings of the two ports can be (S13) and (S6 $\hat{}$), with the equilibrium outcomes given in (A56), if $\min\{J_1, J_2\} < \max\{J_1, J_2\}$, $\hat{J}_1, \hat{J}_2 < \hat{J}_3 < J_3$, with $J^2 \in (\hat{J}_3, J_3]$.

The equilibrium outcomes given in (A53), (A54), (A56), (A57), (A59), (A65), and (A67) imply that (c, c) is not an NE whenever $2c_1D_2 \in (rc_1, rc_2]$, whereas (m, m) can be an NE only if $\max\{\pi_1^m, \pi_1^n\} = \pi_1^m$. Furthermore, for $2c_1D_2 > rc_2$, (c, c) is always an NE; and we identify the conditions under which either (c, c) or (m, m) is the unique Pareto-optimal NE. This completes the proof of Theorem 5(iii). $\square$