SECTION 7.2
Partial Derivatives (偏微分)

- Situation
  - In many problems involving functions of two variables, the goal is to find the rate of change of the function with respect to one of its variables when the other is held constant.
  - That is, the goal is to differentiate the function with respect to the particular variable in question when keeping the other variable fixed.
  - This process is known as partial differentiation.
- Examples: $Q(x,y)=5x^2+7xy$

Partial Derivatives

- Suppose $z = f(x,y)$.
- The partial derivative of f with respect to x is denoted by

$$\frac{\partial z}{\partial x} \quad \text{or} \quad f_x(x, y)$$

- The partial derivative of f with respect to y is denote by

$$\frac{\partial z}{\partial y} \quad \text{or} \quad f_y(x, y)$$

- The basic definitions of partial derivative, see the notes on p574 (similar to the case of f(x)).

Examples 7.2.1-3

- Find the partial derivatives *fx* and *fy* if

$$f(x,y) = x^2 + 2xy^2 + \frac{2y}{3x}$$

- Find the partial derivatives

$$\frac{\partial z}{\partial x} \quad \text{and } \frac{\partial z}{\partial y} \text{ if } \; z = (x^2 + xy + y)^5$$

- Find the partial derivatives *fx* and *fy* if *f(x,y)* = *xe-2xy*.

Geometric interpretation of partial derivatives (p576)

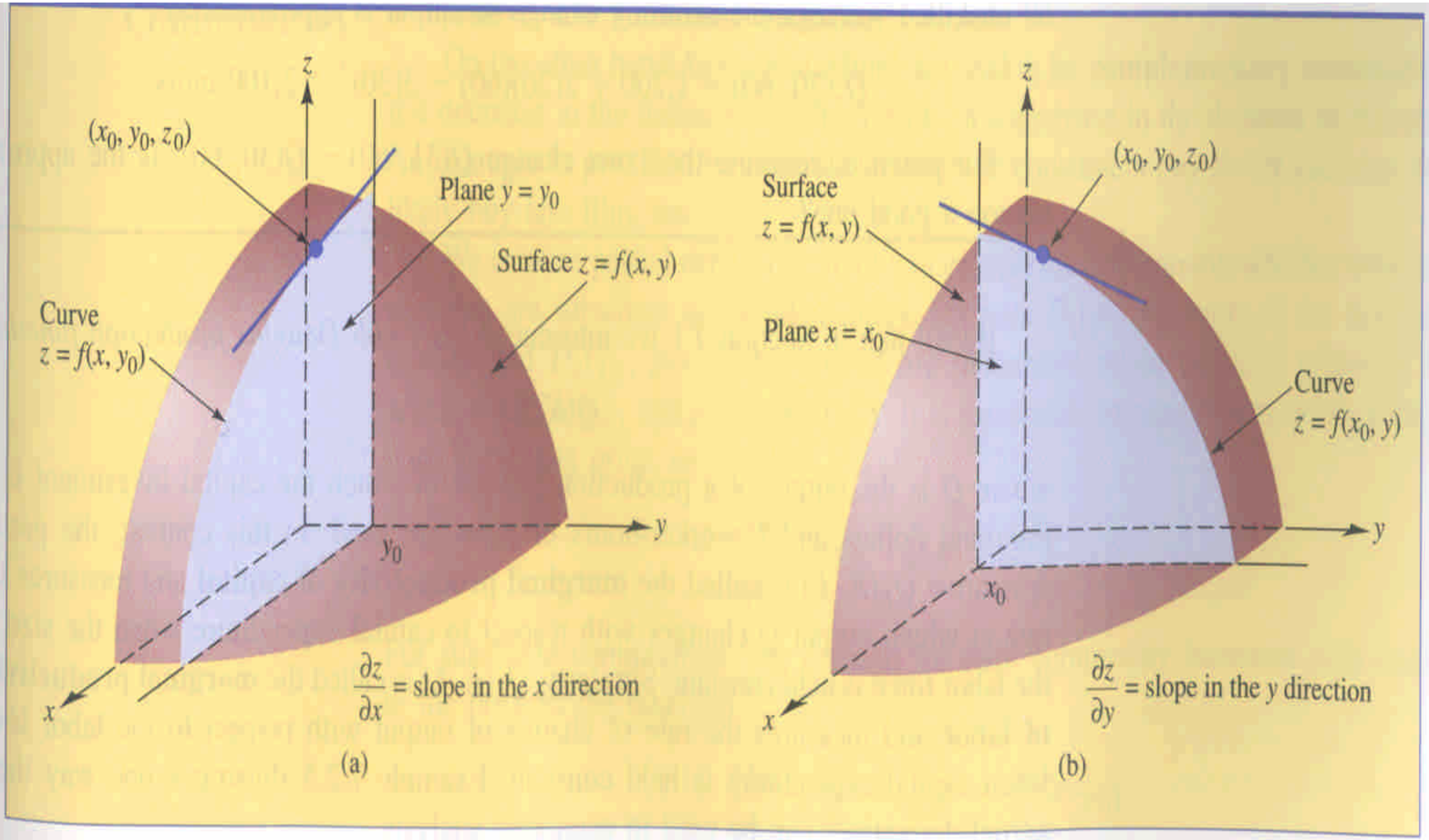

FIGURE 7.10 Geometric interpretation of partial derivatives.

Example 7.2.4

- It is estimated that the weekly output of a certain plant is given by the function $Q(x,y)=1{,}200x + 500y + x^2y - x^3 - y^2$ units, where
  - x is the number of skilled workers and
  - y the number of unskilled workers .
- Currently the workforce consists of 30 skilled workers and 60 unskilled workers.

- **Q**: Use marginal analysis to estimate the change in the weekly output that will result from the addition of 1 more skilled worker if the number of unskilled workers is not changed..

Example 7.2.5

- A manufacturer estimates that the monthly output at a certain factory is given by the Cobb-Douglas function

$$Q(K,L) = 50K^{0.4}L^{0.6}$$

where K is the capital expenditure in units of \$1,000 and L is the size of the labor force, measured in worker-hours.

Q1: Find the marginal productivity of capital Q_K and the marginal productivity of labor Q_L when the capital expenditure is \$750,000, and the level of labor is 991 worker-hours.

Q2: Should the manufacturer consider adding capital or increasing the labor level in order to increase output?

Example 7.2.6

Suppose the demand function for flour in a certain community is given by $D_1(p_1, p_2) = 500 + \frac{10}{p_1 + 2} - 5p_2$

While the corresponding demand for bread is given by

$$D2(p_1, p_2) = 400 - 2p_1 + \frac{7}{p_2 + 3}$$

where p_1 is the dollar price of a pound of flour and p_2 is the price of a loaf of bread.

Q: Determine whether flour and bread are substitute or complementary commodities or neither.

Note: The definitions of substitute and complementary commodities are seen on p578.

Second-order partial derivatives (p579)

Example 7.2.7

Compute the four second-order partial derivatives of the function.

$$f(x, y) = xy^3 + 5xy^2 + 2x + 1$$

Example 7.2.8

- Suppose the output Q at a factory depends on the amount K of capital invested in the plant and equipment and also on the size L of the labor force, measured in worker-hours.
- Give an economic interpretation of the sign of the second-order partial derivative

Chain Rule for Partial Derivatives (p581)

- Suppose z is a function of x and y, each of which is a function of t. Then z can be regarded as a function of t and

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

Example 7.2.9

A health store carries two kinds of multiple vitamins, brand *A* and brand *B*. Sales figures indicate that if brand *A* is sold for *x* dollars per bottle and brand B for y dollars per bottle, the demand for brand *A* will be

$$Q(x, y) = 300 - 20x^2 + 30y \quad \text{bottles per month}$$

It is estimated that *t* months from now the price of brand *A* will be

$$x = 2 + 0.05t \quad \text{dollars per bottle}$$

And the price of brand *B* will be

$$y = 2 + 0.1\sqrt{t} \quad \text{dollars per bottle}$$

Q: At what rate will the demand for brand *A* be changing with respect to time 4 months from now?

Example 7.2.10

- At a certain factory, the daily output is $Q=60K^{1/2}L^{1/3}$ units, where K denotes the capital investment measured in units of \$1,000 and L the size of the labor force measured in worker-hours.
- The current capital investment is \$900,000, and 1,000 worker-hours of labor are used each day.
- **Q**: Estimate the change in output that will result if capital investment is increased by \$1,000 and labor is increased by 2 worker-hours.

Note: Incremental approximation formula for functions of two variables, see p582.